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MODELLING AND OPTIMAL TOPOLOGY DESIGN OF THE AIRPLANE FUSELAGE FOR SAE AERODESIGN COMPETITION

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Abstract. *The support structure of the airplane fuselage for the SAE Aerodesign Competition is usually a truss. This structure supports several loads from different aircraft subsystems, such as, the undercarriage, engine thrust, etc. In this work, it is purposed a methodology for generating the best layout or material distribution of the airplane fuselage support structure with maxima stiffness using topology optimization. By using the Matlab® software, it was created a function for the computation of the fuselage structure optimal topology based on the magnitude and direction of the several forced applied on the truss. In this way, this function in Matlab® language can generate automatically the optimal material distribution by considering the final volume of the airplane fuselage design space.*

Keywords: *airplane fuselage, modelling, stiffness, topology optimization*

1. INTRODUCTION

Topology optimization will be used in this work for the optimal structural design of the airplane fuselage with maximum stiffness for SAE Aerodesign competition. Topology optimization can be defined as an optimal structural design technique that seeks the best layout or connectivity of the members or elements of a structure. Furthermore, others optimization techniques may also be applied in the optimal design of the mechanical structures, such as, the shape and sizing (Bendsoe and Sigmund, 2008). In the shape optimization, only the boundaries or the shape of the structure are optimized and the geometry of the holes is preserved in the design space. For the sizing optimization, the structure parameters, that is, the length, height and width are optimized during this procedure. Among these techniques, topological optimization can be considered as the more general structural optimization procedure since as the geometry of the holes, as the boundaries are optimized in the structure design space (Christensen and Klarbring, 2009). In the literature, there are several applications of this technique in the structure optimal design, as for example: in the generation of the topologies with maxima flexibility and stiffness (Sigmund, 2001; Nishiwaki et al., 2001); in forced and free vibration problems of the structures (Jog, 2002); in the computation of the optimal topologies of the structures subjected to the plastic strain (Maute et al., 1998; Guimarães et al., 2008); etc.

In the case of the airplane fuselage structure problem, the design space is subjected to the several loads, such as, the propulsion force of the engine, the force applied by the undercarriage on the fuselage and the aerodynamic forces applied by the wings on the fuselage. Usually, the airplane fuselage support structure for SAE Aerodesign competition is made of aluminum alloy or carbon fiber bars which are distributed in order to maximize the stiffness of the design space (Megson, 1999; Rodrigues, 2011). For this purpose, the designers use their intuition, experience and knowledge for the creation of the best layout of the fuselage structure truss. However, since the geometry of the design space is irregular and there are several forces applied in different regions of the fuselage, it is very difficult to analytically predict an optimal fuselage truss able to support these loads with maximum stiffness.

In this sense, the objective of this work is to use the topology optimization technique as a useful design methodology to be used in the layout definition of the airplane fuselage truss. Indeed, the topology optimization with maximum stiffness or minimum compliance is a well-established problem in the literature which has been studied by several researchers in this field (Sigmund, 2001; Bendsoe and Sigmund, 2008). Nevertheless, by using the topological optimization it is possible to generate automatically the best airplane fuselage support structure by depending on the loads magnitude applied to the design space and their locations. After the computation of the fuselage optimal topology, a re-analysis procedure was also applied in order to validate the fuselage structure stiffness, that is, if the generated optimal truss is able to support the loads applied on structural design space.

2. LOADS APPLIED ON THE AIRPLANE FUSELAGE STRUCTURE FOR SAE AERODESIGN COMPETITION

The figure 1 illustrates the truss of the fuselage of an airplane for SAE Aerodesign competition (Rodrigues, 2011). In the practice, when the airplane is operating, it is subjected to the static and dynamic loads which are applied on the fuselage, wings and tail. Actually, all loads applied on the airplane structure are dynamic since when it is flying these forces are varying with the time. However, for the computation of the fuselage optimal topology, it will be considered that all loads are static, that is, they do not depend on the time. Therefore, the inertia loads by acting on the airplane will be neglected in this work and the analysis of the loads will be done by considering the airplane in static equilibrium.

Figure 1. Fuselage support truss of an airplane for SAE Aerodesign competition (Rodrigues, 2011).

In a general way, the main loads applied on the airplane structure can be seen in the Fig. (2). According to this figure, the vector L represents the lift load acting on the aerodynamic center of the wings of the airplane, T is the engine thrust, W is the weight of the airplane and the P is the tail horizontal load. Furthermore, there are also the undercarriage load which is significant when the airplane lands. In fact, when the airplane lands, the reaction load acting on the undercarriage becomes an impact force which can be cause damages in the fuselage support truss. All these loads abovementioned will be considered in the modelling and the optimal topology design of the airplane fuselage truss for SAE Aerodesign competition.

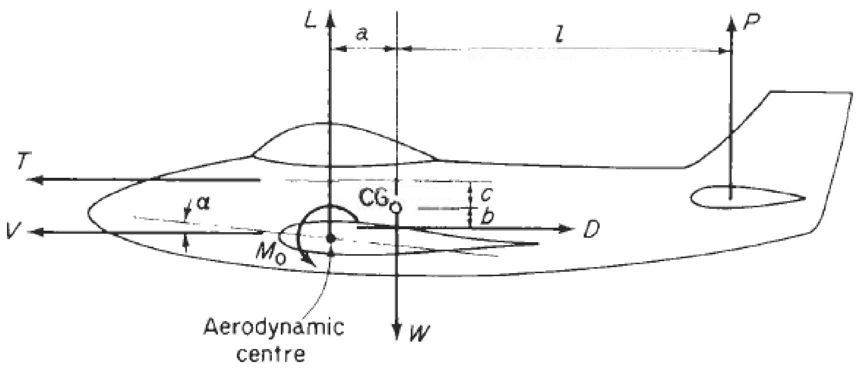

Figure 2. Loads acting on the airplane structure (Megson, 1999).

3. TOPOLOGY OPTIMIZATION AS A TOOL DESIGN OF THE AIRPLANE FUSELAGE

Topology optimization can be defined as a structural design tool which generates automatically the best layout of the elements or components of a structure (Sigmund, 2001). This procedure must locate the structure elements that will be either solid or void. Solid elements have relative density equal to 1 (one) and void elements have relative density equal to 0 (zero). For example, Fig. (3) shows a rectangular cantilever beam discretized by finite elements. The optimal topology shown at the same figure is the layout of the beam with minimal strain energy. In this case, the volume of the optimal topology is 40% of original design space volume (rectangular beam).

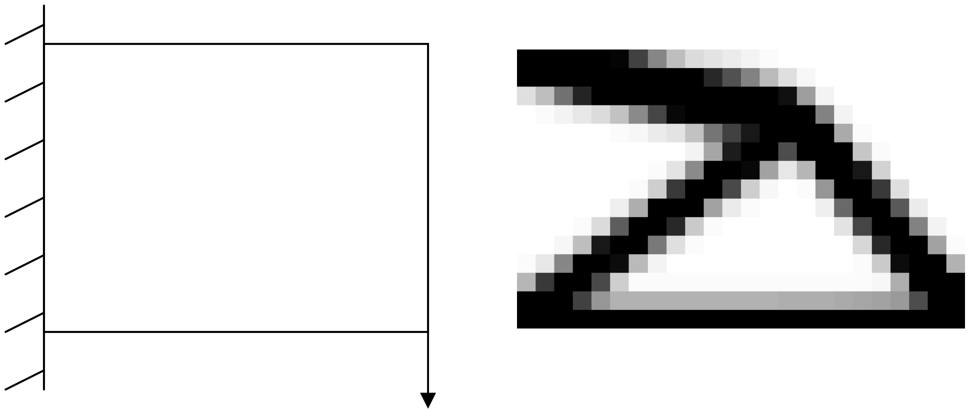

Figure 3. Optimal topology of a cantilever beam.

It has been mentioned that the airplane support structure should be stiff when operating. Physically, the topology of an ideally stiff airplane fuselage has minimal strain energy. It means that the fuselage support truss should not deform when the airplane is flying. Mathematically, this topology optimization problem can be written as:

Minimize: $C(x) = \sum U^T K U$ (1)

Subject to: $\dfrac{V(x)}{V} = f$ (2)

$KU = F$ (3)

$0 < x_{min} \leq x \leq 1$ (4)

where $C(x)$ represents compliance or strain energy of the structure. Because the airplane fuselage is subjected to several loads, the compliance will be the sum of the strain energy for each force acting on the truss. The variable K denotes the global stiffness matrix of the finite element model, U is the displacement vector and F the load vector applied to the structure. During the optimization, the user selects the optimal topology volume $V(x)$. This parameter divided by the design space volume, V, defines the optimal topology volume fraction, f. The relative density of the structure elements, x, is the design variable of this optimization problem.

The objective function (1) has been extensively used in the topology optimization field (Sigmund, 2001). The reason of this is due to the highly convex nature of strain energy. The optimal topology volume fraction, f, is an equality constraint which must be satisfied during the whole optimization process. From the equation (3), it is extracted the nodal displacement vector of the structure, U, to be inserted in the objective function (1). Equation (4) represents a side constraint applied to the relative density of each element of the structure. The parameter x_{min}, usually equals to 0.01, is the minimal value that can be assumed by the design variables. The importance of this constraint is to avoid a singularity in the global stiffness matrix K, during the calculation of the displacement vector, U.

For calculation of the sensitivity of the topology optimization problem, it is necessary to define a material model for the finite elements relative density from design space. In the literature, the material models more used are the homogenization method and the power law (Bendsoe and Sigmund, 2008). By using this model, the relationship between the original and modified element stiffness matrix is given by:

$$k = x^p k_o \quad (5)$$

where k_o and k represents the original and modified element stiffness matrix original and modified respectively and p is the penalization power. In the practice, the material of the fuselage structure (carbon fiber) is orthotropic, however, for the computation of the optimal topology, it will be considered that the truss material is isotropic. In this case, the penalization power is equal to 3 in order to Poisson Ration be 1/3 (Bendsoe and Kikuchi, 2008). By using this material model and applying the Optimality Criteria to the Lagrangian of the optimization problem, it can be demonstrated that the strain energy density of the structure is constant for the whole design space (Bendsoe and Kikuchi, 2008):

$$B = \sum \frac{p x^{p-1} u^T k_o u}{\lambda \frac{\partial V}{\partial x}} \tag{6}$$

where B is the strain energy density, u is nodal displacement vector of the finite elements from design space and λ is the Lagrangian multiplier has calculated from the active volume constraint (2). Once again, since the design space of the airplane fuselage is subjected to several loads, it necessary to sum the sensitivities caused by each force acting on the truss. The optimal topology of the airplane fuselage structure will be updated by using the strain energy density has defined in the Eq. (6).

4. NUMERICAL AND STRUCTURAL MODELLING OF THE AIRPLANE FUSELAGE

4.1 Numerical implementation of the procedure

The topology optimization problem (6) has been solved in the literature by using either Sequential Linear Programming (Sigmund, 1997; Nishiwaki et all; 2001) or the optimality criteria method (Yin and Ananthasuresh, 2001). In the Sequential Linear Programming method, the objective function and constraints are estimated using a first order approximation of a Taylor series. In this procedure, the user should choose moving limits for decreasing the values of side constraints applied to relative density of each finite element. However, it is very difficult to predict suitable values of these moving limits which produce a feasible optimal solution (Vanderplaats, 1984).

The optimization problem defined in the Eq. (6) has only one active constraint (volume constraint). Because of this, the implementation of the optimality criteria method becomes more efficient and simpler than Sequential Linear Programming procedure (Sigmund, 2001; Yin and Ananthasuresh, 2001). For this reason, the optimality criteria method will be used in this work as an optimizer for updating the airplane fuselage topology.

The Lagrangian multiplier, λ, to be used in the topology updating scheme is obtained from the active volume constraint of the optimization problem:

$$V(x) - fV = 0 \tag{7}$$

Hence, the value of the Lagrangian multiplier λ has obtained from the solution of the Eq. (7) provides the minimum of the optimization problem stated in the Eq. (7). This concept will be used in this work in order to derivate an updating scheme of the airplane fuselage topology. Equation (6) multiplied by –1 is the gradient of this optimization problem which represents the steepest descent direction. In this context, an updating scheme of the relative density can be defined by (Bendsoe and Kikuchi, 2008):

$$x^{j+1} = x^j B^{\eta} \tag{8}$$

which is an usual representation of most numerical optimization algorithms (Vanderplaats, 1999). Although the Eq. (8) be the simplest way of updating the design variables, it has proven be enough efficient for this optimization problem (Yin and Ananthasuresh, 2001). The index j denotes the number of iteration used in the updating of relative density and the parameter η is the numerical damping coefficient has used to decrease the convergence rate of the procedure. Unfortunately, the equation (16) does not guarantee stable convergence during the optimization since abrupt changes in the topology can occur. It will be used in this work the procedure have proposed by Bendsϕe and Sigmund (2008) in order to stabilize the formation of the optimal topology:

$$\max(0.001, x^j - m) \le x^{j+1} \le \min(1, x^j + m) \tag{9}$$

where m represents a limit value of the change of relative density. The role of this parameter is to avoid the formation of discontinuities in the topology during the optimization (Bendsϕe, 1995). In the Eq. (7), 0.001 and 1 are respectively the minimum and maximum values which the relative density can assume (side constraints).

The figure 4 shows the steps of the topology optimization algorithm of the stent cell to be used in this work. Initially, all finite elements from structure have relative density equal to volume fraction chosen by the user. After, the nodal displacement field are obtained by solving the equilibrium equation (3). These displacements are used in the calculation of the objective function (1) and its sensibility. Subsequently, the objective function sensibility is filtered in order to avoid the checkerboards patterns in the stent optimal topology (Sigmund, 2001). The Lagrangian multiplier λ is determined by using a bi-sectioning algorithm for the search of the volume constraint root and the topology of structure is updated. All subroutines of the flowchart were implemented in Matlab® R2009.

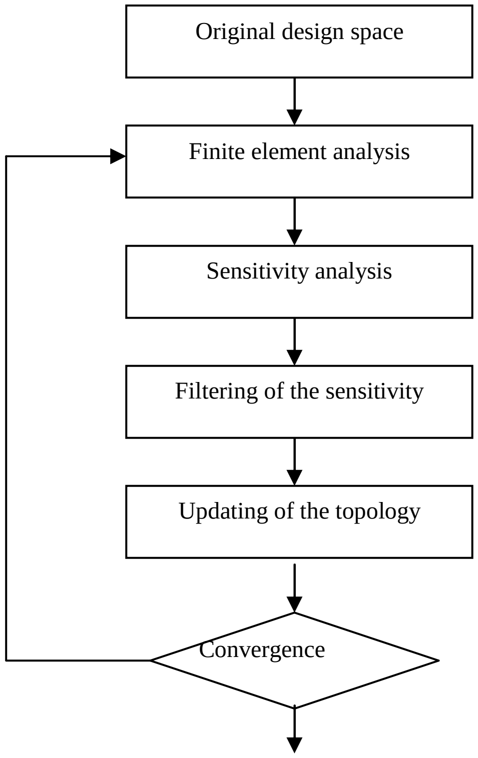

Figure 4. Flowchart of airplane fuselage topology optimization algorithm with maxima stiffness.

4.2 Structural Modeling of the airplane fuselage

The figure 5 shows the geometry, loads and boundary conditions from design space of the airplane fuselage to be considered in this work. The dimensions of the design space are in millimeters. It can be seen that the geometry of the design space is irregular, that is, near from the location of the engine thrust the area is not rectangular. This geometry was selected in order to reduce the final volume of the fuselage optimal topology to be generated. In this figure, the vectors T, L, P and C represent the engine thrust, lift, tail and undercarriage loads acting on the fuselage respectively.

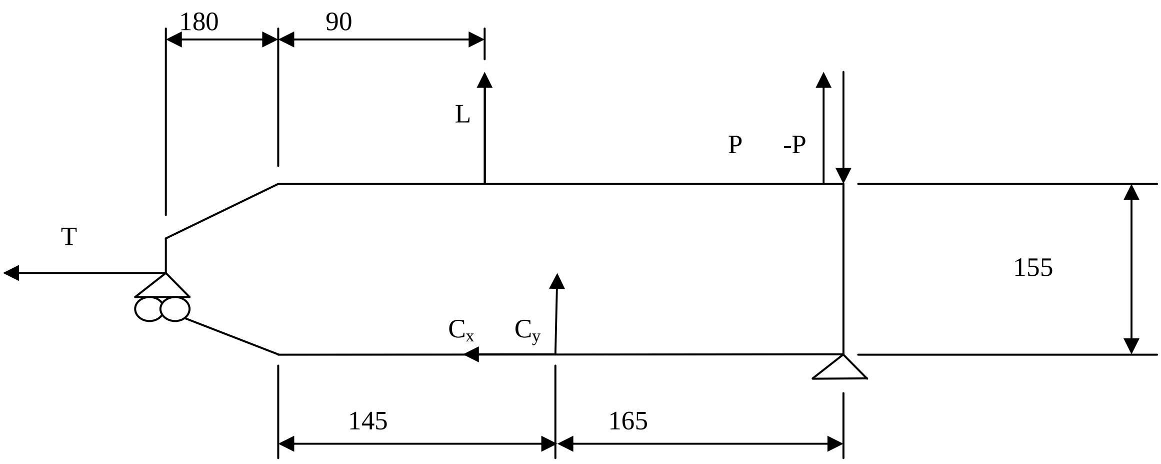

Figure 5. Design space of the topology optimization problem of the airplane fuselage for SAE Aerodesign competition.

There are two boundary conditions have applied on the design space of the topology optimization problem of the airplane fuselage structure. The middle left edge is free to move in the horizontal direction and the lower right corner is supported in both directions, vertical and horizontal. The first constraint is the location of the engine thrust in the horizontal direction and the second one was imposed in order to guarantee stability for the fuselage structure. The load magnitude have applied on the fuselage are described in the Tab. 1. These values were calculated based on the structural criteria of the fuselage, wings and tail (Campofredo , 2015). There is a force binary has applied in the left bottom corner from design space. This loading simulates the moment caused by the distance between the tail and the right edge of the design space.

Table 1. Material properties and load magnitude acting on the design space of the airplane fuselage.

Parameters	Values
Elasticity Modulus, E	1.0 Pa
Poisson's Ratio, ν	0.3
Engine thrust, T	32.0 N
Lift load, L	266.0 N
Horizontal component of undercarriage load, C_x	108.0 N
Vertical component of undercarriage load, C_y	353.2 N
Tail load, P	17.8 N

The material properties of the fuselage structure to be used in the topology optimization are described in the Tab. 1. For the computation of the nodal displacements of the design space, it was discretized using quadrilateral finite elements with 4 nodes in plane state of stress. In this type of finite element, each node has two degree of freedom, that is, it can move in horizontal and vertical directions. During the topology updating of the airplane fuselage for SAE Aerodesign competition, the Elasticity modulus of the fuselage structure material was normalized to 1 (one).

5. ANALYSIS OF THE RESULTS

The figure 5 illustrates the optimal topology of the airplane fuselage with maxima stiffness has obtained from the procedure above described. It is interesting to note that the topological optimization technique distributed automatically material in the regions where the loads acting on the design space produce large strain in the structure. For example, close to the points of application of the engine thrust and the lift load the bars of the optimal topology have small thickness. On the other hand, in the right edge of the topology the material distribution is more concentrated, that is, the truss bars have large thickness because the moment caused by the tail load is high. In the upper edge from optimal topology where the undercarriage load is applied, there are two bars connected in order to guarantee the stability of the fuselage truss when the airplane lands.

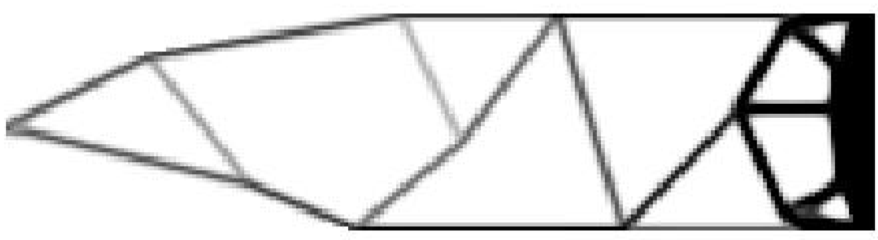

Figure 5. Optimal topology of the airplane fuselage for SAE Aerodesign competition.

After the computation of the optimal topology of the fuselage, a re-analysis procedure was applied in order to validate if the generated structure can support the loads acting on the airplane fuselage. First of all, the contours of the optimal topology matrix was generated using the software Matlab® R2009. Hence, the isolines with the same material distribution generated by the contours represent the boundaries of the fuselage structure. In the next step, the coordinates of the several lines of the fuselage structure boundaries was obtained by using the functions available in language Matlab® R2009. The vector with the coordinates of the boundaries lines was exported for the software ANSYS® 2014 in order to evaluate the finite elements analysis of the airplane fuselage optimal topology.

Finally, it was created the truss model of the airplane fuselage based on the optimal topology isolines. The generated truss structure is illustrated in the Fig. 6. Furthermore, this figure also show the boundaries conditions and the loads have applied on the nodes of the fuselage truss. These loads and constraints are similar to the design space used in the topology optimization procedure. It can be seen that the fuselage truss structure used in the re-analysis procedure is also similar to the optimal topology of the plane model of the airplane fuselage discretized with quadrilateral finite elements. The fuselage truss was created by using beam finite elements with circular section. The bars diameters were dimensioned using the thickness of the optimal topology links have shown in the Fig. 5. The material of bars is carbon fiber with Elasticity Modulus equals to 70 MPa. After the finite elements analysis of this fuselage truss model, it was observed that that strains of the links were small (Fig. 7), that is, the equivalent truss model to the optimal topology can be used in the building of the airplane fuselage for SAE Aerodesign competition.

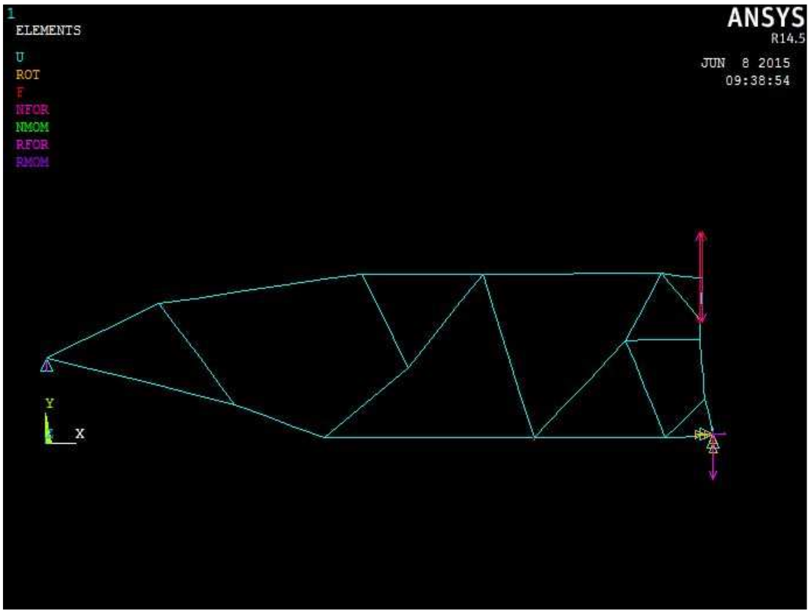

Figure 6. Optimal airplane fuselage truss for the SAE Aerodesign competition.

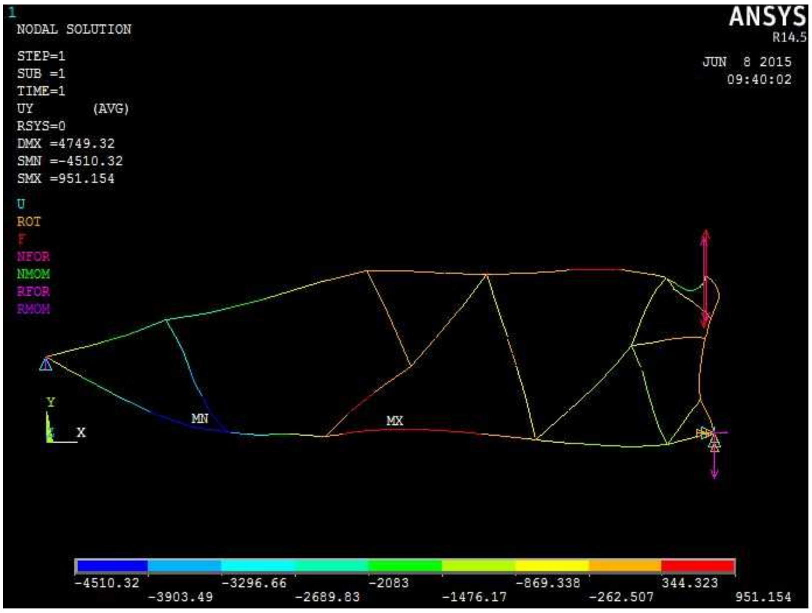

Figure 7. Strain field of the fuselage truss subjected to the boundary conditions and loads used to generate the optimal topology.

6. CONCLUSIONS

In this work, it was purposed a methodology for the optimal topology design of the airplane fuselage structure for SAE Aerodesign competition. The loads to be considered in the computation of the fuselage optimal topology are the engine thrust, lift, tail load and undercarriage load. The boundaries conditions used in modelling of the design space of the optimal topology are similar to the fuselage truss structure ones. After the generation of the optimal topology of the airplane fuselage using quadrilateral finite elements with 4 four nodes, it was created an equivalent fuselage truss by using beam finite elements with circular section. Hence, it is possible to build the airplane fuselage support structure for SAE Aerodesign competition using carbon fiber pipes.

The results prove that the purposed methodology can aid the designer to define the best geometry or layout of truss links. By simulating the finite elements model of the truss, it was verified that the strain and stress of the structure was very small, that is, the airplane fuselage is sufficiently stiff in order to support all loads. The same methodology can also be applied to others airplane fuselage models, change the loads or boundary conditions of the design space.

7. ACKNOWLEDGEMENTS

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