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NONLINEAR DYNAMICS OF LATTICED BAMBOO STRUCTURES

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Abstract. *We perform a nonlinear time domain analysis using vector approach for a latticed bamboo roof under wind action. We divide fluid-structure interaction forces in two parts, one concerning the average wind velocity, considered permanent, and another related to the fluctuation of the velocity around the average, of random nature. The computer program developed for this paper solves the equations of motion via the Central Difference Method. We compute (large) displacements and restoring forces at each time step, using an appropriate nonlinear stress-strain relationship for the bamboo. We simulate dynamic wind loadings, of random nature, by applying a superposition of several harmonic loads on the structure with randomly set phase angles and amplitudes corresponding to spectral density function bands. The computer processing will result series of displacements and member tractions for determining the characteristic design values using a probability density function.*

Palavras-chave: *synthetic Wind 1, nonlinear dynamics 2, bamboo 3, roofs 4*

1. INTRODUCTION

We perform a nonlinear time domain analysis using vector approach for a 3D model of a tensioned hyperbolic paraboloid latticed bamboo roof under wind action. We divide fluid-structure interaction forces in two parts, one concerning the average wind velocity, considered permanent, and another related to the fluctuation of the velocity around the average, of random nature. The computer program developed for this paper solves the equations of motion via the Central Difference Method. We compute (large) displacements and restoring forces at each time step, using an appropriate stress-strain relationship for the bamboo. We simulate dynamic wind loadings, of random nature, by applying a superposition of several harmonic loads on the structure with randomly set phase angles and amplitudes corresponding to spectral density function bands. The computer processing will result series of displacements and member tractions for determining the characteristic design values using a probability density function.

2. NONLINEAR DYNAMIC ANALYSIS OF TENSION LATTICED

2.1 Central difference time step integration of EDO motion equations

The three displacements of the NN nodes of our structural model are the n = 3NN generalized coordinates x_i (i = 1,…,n). Our system of second order nonlinear differential equations of motion are:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{r} = \mathbf{p} \tag{1}$$

where **M** and **C** are the mass and damping matrices (diagonal in our model), **x** the displacements vector, superposed dots indicating successive time differentiation, **r** is the restoring elastic forces vector and **p** the loading vector.

The time domain is divided into (small) time steps h. A Central Difference approach is used to determine second order approximations to velocity and acceleration at each time step, depending on the known present and previous displacements and on future unknown displacements.

$$\dot{\mathbf{x}}(t_i) = \frac{\mathbf{x}(t_{i+1}) - \mathbf{x}(t_{i-1})}{2h} \tag{2}$$

$$\ddot{\mathbf{x}}(t_i) = \frac{\mathbf{x}(t_{i+1}) - 2\mathbf{x}(t_i) + \mathbf{x}(t_{i-1})}{h^2} \tag{3}$$

Approximations (2) and (3) plugged into the dynamic equilibrium equation (1) at the present time step renders an explicit recurrence formula to compute the sought for unknown displacements in the next step:

$$\hat{\mathbf{K}}\mathbf{x}(t_{i+1}) = \hat{\mathbf{p}}(t_i) \tag{4}$$

where

$$\hat{\mathbf{K}} = \frac{1}{h^2}\mathbf{M} + \frac{1}{2h}\mathbf{C} \tag{5}$$

and

$$\hat{\mathbf{p}}(t_i) = \mathbf{p}(t_i) - \mathbf{r}(\mathbf{x}(t_i)) + \frac{2}{h^2}\mathbf{M}\,\mathbf{x}(t_i) - \left(\frac{1}{h^2}\mathbf{M} - \frac{1}{2h}\mathbf{C}\right)\mathbf{x}(t_{i-1}) \tag{6}$$

As the procedure is dependent in the knowledge of both present and previous displacements, it is not self-starting. Thus, we compute the initial acceleration at initial time by plugging the given initial displacement and velocity into the dynamic equilibrium equation (1):

$$\ddot{\mathbf{x}}_0 = \mathbf{M}^{-1}\left(\mathbf{p}_0 - \mathbf{r}(\mathbf{x}_0) - \mathbf{C}\dot{\mathbf{x}}_0\right) \tag{7}$$

Next, we assume that this initial acceleration keeps constant along the first time step, a uniformly accelerated motion, thus allowing for the initiation of the recurrence process:

$$\ddot{\mathbf{x}}(t_1) = \ddot{\mathbf{x}}_0 \qquad \dot{\mathbf{x}}(t_1) = \ddot{\mathbf{x}}_0 h + \dot{\mathbf{x}}_0 \qquad \mathbf{x}(t_1) = \frac{1}{2}\ddot{\mathbf{x}}_0 h^2 + \dot{\mathbf{x}}_0 h + \mathbf{x}_0 \tag{8}$$

As in our algorithm we consider nodal lumped masses and damping, and we compute the restoring forces components at each time step, no mass, damping and stiffness matrices are necessary and the procedure becomes a vector one, were no sets of algebraic equations are solved or matrix inversions are needed. This allows for a very efficient computational procedure.

2.2 Computation of nonlinear restoring forces components

At each integration time step, the 3D structural nodal restoring forces components of **r** are computed as follows. First, we determine the new coordinates of each node of the structure by adding the displacements of the previous step. This allows for the knowledge of the present length of the lattices that connect the nodes and thus the Engineering strain level at that lattice. From this strain level, we calculate the acting stress by considering an appropriate stress-strain relationship for the selected bamboo material. From this stress value, we obtain the axial forces at each lattice member and then the structural restoring forces components at the nodes, in each of the 3 space directions.

With the increments of displacements obtained in each time step, the new coordinates $Y_{ik,t+h}$ for the nodes are calculated in a given time and therefore the lengths of the new elements.

$$\Delta L_{j,t+h} = L_{j,t+h} - L_{j,t} \tag{9}$$

Also new direction cosines for the axes of the elements $cos \propto_{jk,t+h}$ are calculated.

The following algorithm gives an estimate of the normal force at time $t + h$:

If for *j*-th element $\Delta L_{j,t+\Delta t}$ is less than or equal to the element unstressed length L_{jN0}, then the normal force in the element sections, $N_{j,t+h}$, is considered to be zero (no compression accepted), otherwise, the tensile normal force is computed assuming this force is within the linear elastic range.

Properties adopted for the bamboo used in our example are given in Fig. 1, due to Amaral et al. (2003). The assumed experimentally measured elasticity modulus is set to 10 GPa.

In this paper, we consider linear elastic properties for our material. In our ongoing research a nonlinear material model is being developed and will be reported in future publications.

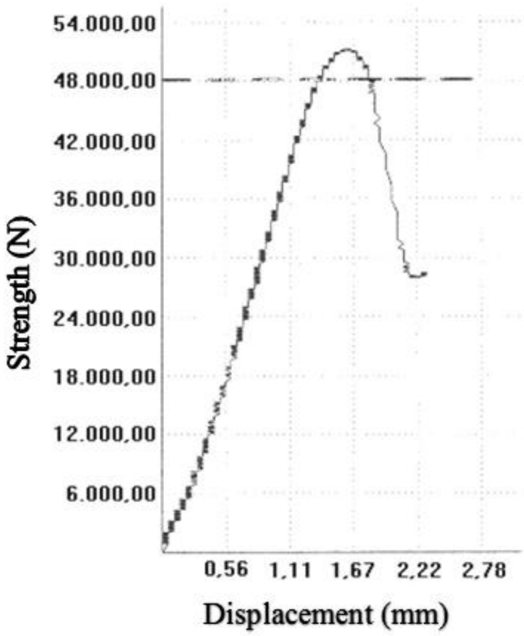

Figure 1: Experimental load x displacements for a bamboo specimen

Projecting the normal forces $N_{j,t+h}$ applied to node n_i in the direction k through its direction cosines $cos \propto_{jk,t+h}$, we obtain $r_{ik,t+h}$ and proceed to a new iteration, writing the equilibrium equation for time $t+h$.

2.3 Nonlinear static analysis by dynamic relaxation

As self-weight forces, as well as the steady state wind forces due to average wind velocity, are static loads, we need to perform a previous nonlinear static analysis. To this end we use the same already described nonlinear dynamic analysis algorithm to carry on a so called *dynamic relaxation* procedure. The easiest way is to set structural damping of the numerical model close to the critical damping value, so that we nearly eliminate unwanted vibrations.

2.4 Damping estimation

For the nonlinear dynamic analysis, we estimate structural damping by assuming a given usual practical damping ratio and determining the structure natural frequencies from previously performed undamped response time histories.

3. SYNTHETIC WIND LOADING

3.1 Franco's synthetic wind method and Davenport's PSDF function

We present the so called synthetic wind procedure due to Franco (1993). In this approach, related to Monte Carlo methods, a large set of artificially generated wind loading time series are applied to a structure. The resulting large set of response time series is statistically treated in order to define appropriate process average characteristics allowing for sound Engineering probabilistic assessment of structural reliability.

The used load time series are generated as follows. We adopt an adequate wind velocity Power Spectrum Density Function, in this case one adapted from Davenport, apud Franco (1993), shown in Fig. 4. The PSDF is divided in a large number of narrow bands of frequencies. The average PSD value in each band is transformed into a wind pressure amplitude for that frequency and used as the amplitude of a harmonic load time series. We set a pseudo-random phase angle to each of these harmonic load function and apply the superposition these functions to the structure to obtain a response time history. The procedure is repeated a large number of times, in each one new pseudo-random phase angles are set for each harmonic function to simulate the actual random process.

Finally, we compute process averages and we adopt a Probability Density Function, either Gaussian or Gumble, and we are able to predict the probability of safe operation of the structure and its live expectancy, see Newland (1993), for example.

3.2 Adaptation to Brazilian Code NBR 6123:1988

Operationally, we partially follow the guidelines of Brazilian Wind Forces upon Structures Code NBR 6123:1988, ABNT (1988). Thus, we use Brazilian Code isoplethe map and drag coefficients to compute the basic dynamic pressure value. Further, we consider the wind velocity to be divided into two portions. The first, related to average wind velocity is supposed to be applied as static loading to be consider along with dead weight. The other is the fluctuating portion of wind velocity around average values responsible for the dynamic effects upon the structure. This is the portion to which we apply Francos's synthetic wind method, Franco (1993).

The Design Codes only provide average wind velocities in time intervals. Thus, it was decided to adopt as a peak velocity the average for the shortest time interval of NBR6123:1988, ABNT (1988) standard, which is 3 seconds, V_3. This peak speed V_3 includes the mentioned average and the floating parcels (gusts).

It will be assumed that the peak average velocity is the standard average for a much longer time interval, adopted equal to 10 minutes or 600s, which will be denoted V_{600}.

Having the ratio between the average and the peak velocities, one has also the ratio between the floating and the peak parcels. The NBR6123 standard provides, for flat terrain, group II construction, category IV terrain, as would be the case of the urban area of São Paulo, Brazil, a ratio between the velocities V_{600} and V_3 equals to:

$$V_{600}/V_3 = 0.57 \tag{10}$$

from where:

$$P_{600}/P_3 = (V_{600}/V_3)^2 = 0.325 \tag{11}$$

In the same conditions, V_3, the peak velocity, on a point at z meters height above the ground is, according to NBR6123:1988 (1988):

$$V_3 = 33.25\ (z/10)^{0,12}\, m/s \tag{12}$$

Then, the peak pressure is:

$$p = 0.613.\, C_p.V_3^2 = 637.56\ C_p.(z/10)^{0,24} N/m^2 \tag{13}$$

and, therefore, the average parcel of the peak pressure is:

$$\bar{p} = 0.325\ p = 207.21\ C_p\ (z/10)^{0,24} N/m^2 \tag{14}$$

and the floating parcel is:

$$p' = 0.675p\ = 434.4\ C_p\ (z/10)^{0,24} N/m^2 \tag{15}$$

3.3 Spatial correlation

Another interesting feature of Franco's synthetic wind method, Franco (1993) is that to consider the spatial random nature of wind gusts, we use correlation functions to estimate the variation of pressure values in the other nodes of the structure when a pick level of pressure is applied to only one of them. Of course, one does not know where this peak value will actually be applied. Thus we use some Engineering feeling and always suppose that this gust center is the one that will generate the largest possible internal forces in the structure. In our example, the central node of the roof.

4. SAMPLE LATTICED ROOF

4.1 Geometry

Here, we apply the proposed method to the analysis of a tensioned hyperbolic paraboloid shaped latticed bamboo network which constitutes the structure of a hypothetical roof, subjected to wind forces, following Buchholdt (1999), for example. We adapted the model of Figs. 2, where the Z coordinate is plotted in a different scale of X and Y, for a better understanding, from a cable network structure proposed by Paik [5]. The equation of the surface is

$$z = [(x^2 - y^2)/780] + 6.096\ m \tag{16}$$

The two diagonally opposed upper fixed points in X direction have 9.144 m Z coordinate. The two diagonally opposed lower fixed points in Y direction have 3.048 m Z coordinate. The central node has 6.096 m Z coordinate.

The area of the transversal section of all lattices is set to 0.0012 m².

The adopted mash has a regular distance of 3.048 m between lattice lines. Thus, we have 545 nodes and 1024 lattices. The full structures is a square with 97.54 m diagonals, a total covered area of about 4,757 m².

4.2 Mass; static loading; pretension; damping

We adopted the density of bamboo to be 1.000 kg/m³. A lumped mass of 242 kg is considered in each node, comprising the mass of the structure and covering material.

We adopted constant static loading of 2136 N per node. This load comprises both self-weight and the constant part of the wind pressure. We adopted pretension of 66.7 KN for each bamboo lattice member. The vertical static displacement of the central node of the roof due to self-weight was found to be 2.16 m. Including the constant part of wind, it becomes 1.74m.

We considered that damping is also lumped, 38.5 Ns/m in each node, corresponding to a 3 % damping ratio. The critical damping was estimated by performing a dynamic relaxation procedure with the already mentioned constant static loading. Starting with zero damping and slowly increasing damping proportional to mass, several such procedures were carried out until no vibrations were detected, indicating critical damping conditions. We found that numerical values of lumped dampers for the critical condition corresponds roughly to 5.3 times the numerical value of the lumped masses.

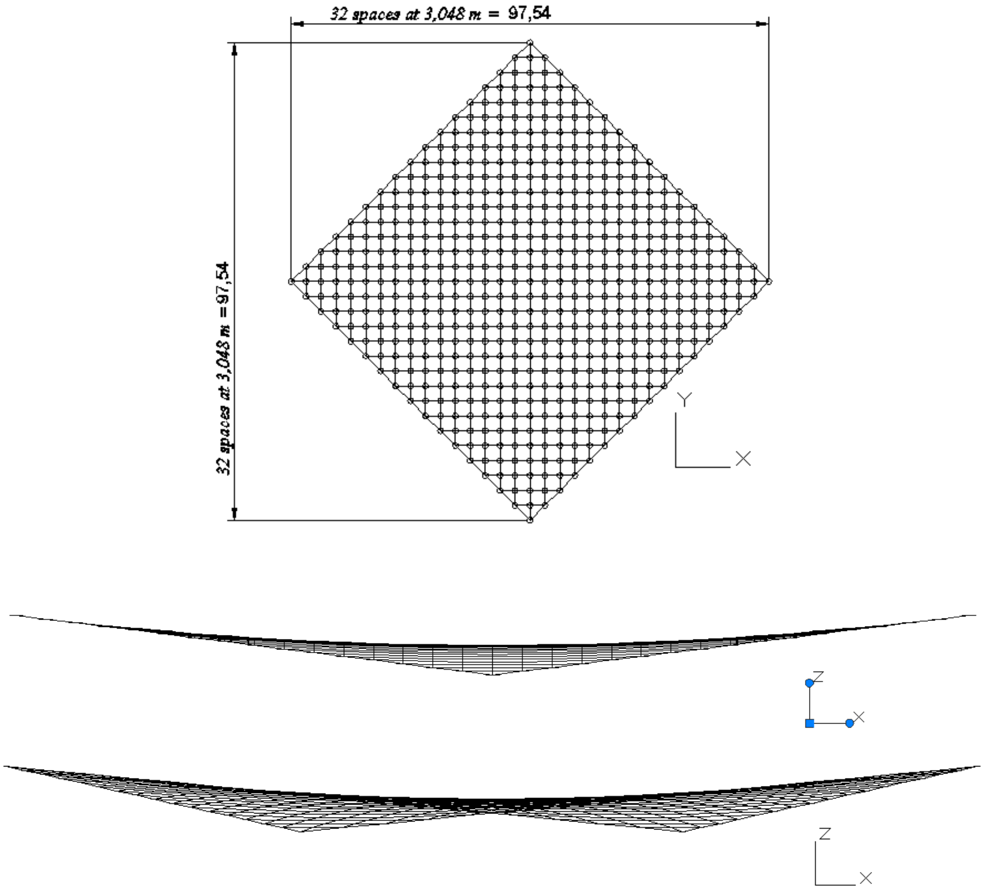

Figure 2: Sample bamboo roof structure

4.3 Wind

Wind characteristics were based on the Brazilian code for wind forces on structures NBR 6123:1988, ABNT (1988). We took wind velocity to be 37.5 m/s, as if the structure would be built in urban environment in São Paulo city, Brazil.

The drag coefficients C_p for this structure, displayed in Table 1, were obtained for different wind directions from wind tunnel tests, are given by Esquilland and Saillard, 1963, as used by Paik (1975). In a real structural design, it will be necessary to consider all these coefficients in the structural analysis. It will be considered in this paper, for simplicity, the coefficients in just one of these directions. Location of the 25 nodes where these coefficients are available are displayed in Fig. 3.

Table 1 - Drag coefficients according Esquilland and Saillard, 1963, apud Paik [5]

Node	1	2	3	4	5	6	7	8	9	0	11	12
C_p	-.50	-.33	-.45	-.60	.20	-.10	-.36	-.68	-.83	.10	.43	.44

Node	13	14	15	16	17	18	19	20	21	22	23	24	25	
C_p	.10	-.33	-.63	-.80	-.83	-.68	-.36	-.10	.20	-.60	-.45	-.33	-.50	

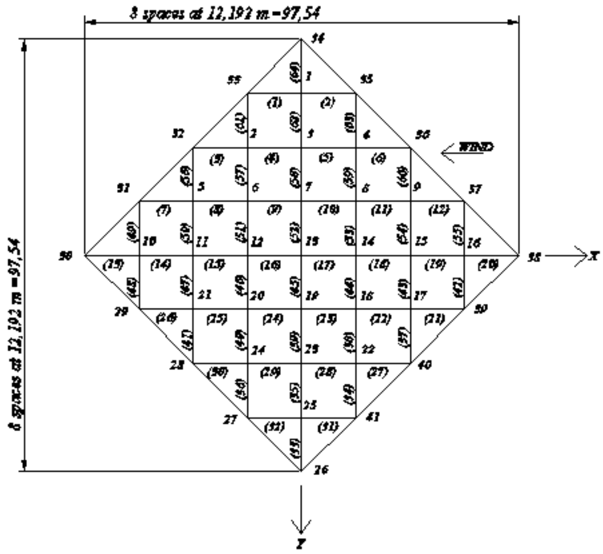

Figure 3: Location of the nodes where drag coefficients are available

We adopted Davenport's PSDF, Fig. 4, as modified by Franco (1993), to generate 11 harmonic loading functions, whose amplitudes are proportional to 11 frequency bands of the chosen wind velocity spectrum. The proportions used to compute the amplitude of each harmonic function and the corresponding frequencies are given in Table 2.

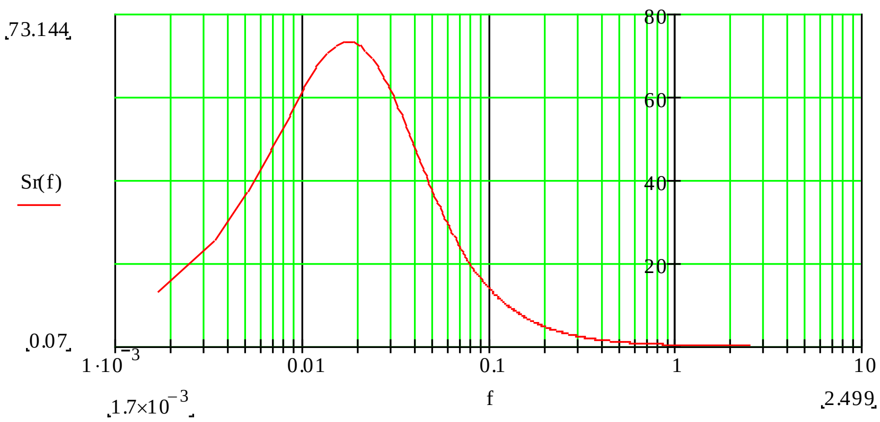

Figure 4: Davenport's reduced wind velocity PSD

Table 2 – Proportional amplitude and frequency of each harmonic function

r	c_k	f_R(Hz)
1	4,0667E-02	3,3755E+00
2	5,1233E-02	1,6878E+00
3	6,4531E-02	8,4388E-01
4	8,1208E-02	4,2194E-01
5	1,0184E-01	2,1097E-01
6	1,2598E-01	1,0549E-01
7	1,4841E-01	5,2743E-02
8	1,5201E-01	2,6371E-02
9	1,2054E-01	1,3186E-02
10	7,4031E-02	6,5930E-03
11	3,9567E-02	3,2960E-03

Next, 20 load time histories were computed by superposing these functions, each one affected by a pseudo randomly set phase angle, different for each of the 20 time histories. These pseudo-random loading time histories were applied to the roof structure, one at a time. Thus, we obtained 20 response time histories.

Each analysis is a fully dynamic nonlinear procedure, allowing for geometrical nonlinearities due to large displacements and material nonlinearities for bamboo, if available, not the case in the present paper.

Results are presented in Table 3.

Table 3 – Maximum displacements of central node and tension forces for each load history

Comb.	1	2	3	4	5	6	7	8	9	10
Displ. (m)	-2,012	-2,819	-2,039	-2,139	-2,278	-2,278	-2,198	-2,148	-2,077	-1,868
Tension (N)	114366	136886	115891	118134	122794	122355	119748	118929	117255	114837

Comb.	11	12	13	14	15	16	17	18	19	20
Displ. (m)	-2,105	-2,057	-2,124	-1,995	-2,079	-2,415	-2,187	-2,173	-1,994	-2,169
Tension (N)	119038	116476	117679	114262	116863	125213	119604	120719	115661	120072

We computed the average μ and standard deviations σ of maximum displacement of center node and tension force of the structure. We used these statistical properties of the process to obtain parameters α e β of a maximums probability distribution function, the Gumbel distribution. Statistical results and characteristic values presented in Table 4.

Table 4 – Computation of characteristic values of maximum displacements and tension force

Statistical parameters	μ	σ	β	α
Displ. m)	-2,158	0,196715	0,153378	-2,24618
Tension (N)	119.339	5054,695	3941,13	117064,5

Characteristic values	Displ.(m) = -2,414
	Tension (N) = 128770

Finally, we can use this adopted probability function to evaluate characteristic values of traction forces in the bamboo lattices in order to verify their resistance in a LRFD, *Load and Resistance Factor Design,* kind of procedure.

Of course, today's computer memory and speed capacities allow for the use of far larger number of basic harmonic functions and loading histories. Thus a real Monte Carlo type analysis may be performed, and more reliable Engineering assessment of structural safety may be obtained.

5. CONCLUSIONS AND COMMENTS

The analysis of a tensioned hyperbolic paraboloid latticed bamboo roof structure of geometrically non-linear behavior subjected to wind loads of random nature has been presented. To perform this analysis, a program using the Central Difference Method to solve the equations of motion was developed. A method for considering the random nature of wind is proposed, based on Franco's Synthetic Wind Method, Franco (1993). Time displacements series and time forces series were generated for a sample structure. Characteristic values to be used in the design of the structure were obtained.

In this paper, we consider linear elastic properties for our material, bamboo. In our ongoing research a nonlinear material model is being developed and will be reported in future publications.

We are also researching about feasible details of the nodes of the latticed bamboo members and how to apply pretension tractions forces to them.

6. ACKNOWLEDGEMENTS

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