23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

MOTORCYCLE TIRE-WHEEL ANALYTICAL MODELING INCLUDING NON-UNIFORMITIES FOR RIDE COMFORT ANALYSIS

Matheus de Barros Vallim
Faculty of Mechanical Engineering, University of Campinas, 200 Rua Mendeleyev, Campinas, São Paulo 13083-860 CP 6122, Brazil
matheusvallim@gmail.com

Argemiro Luis de Aragão Costa
Pirelli Pneus Ltda, 871 Avenida Giovanni Battista Pirelli, Santo André, 09111-340 São Paulo
Argemiro.Costa@pirelli.com

José Maria Campos Dos Santos
Faculty of Mechanical Engineering, University of Campinas, 200 Rua Mendeleyev, Campinas, São Paulo 13083-860 CP 6122, Brazil
zema@fem.unicamp.br

Abstract. *The transmission of vibrations in motorcycles and their perception by the passengers are fundamental in comfort analysis. Tire non-uniformities can generate self-excitations and contribute to the ride vibration environment. In this work a multi-body tire-wheel model is built for evaluate the ride comfort with respect to tyre non-uniformities. The aim is to obtain a tire-wheel model that allows the tire non-uniformities to predict the vertical and longitudinal force variations. The tire-wheel assembly is a three degrees of freedom model. It consists of external rigid ring and internal disc connected by radial and torsional springs and damping elements. The model includes three types of non-uniformities: lumped mass imbalance, radial run-out and radial stiffness variation. At first, the non-uniform tire-wheel model is evaluated separately for each type of uniformity. Then, the model is tested with two types of non-uniformity together. Simulated results are compared with ones from literature.*

Keywords: *motorcycle tire non-uniformity; radial run-out; lumped mass imbalance; stiffness variation*

1. INTRODUCTION

The study of dynamic behavior of pneumatic tires is essential in automobile industry. Vehicle manufactures have tried to reduce noise and vibration transmitted to occupants, generated by steering systems, engine, suspension, exhaust system and tires. Two-wheeled vehicles are increasingly present on the roads and deserve more attention with respect to vibrations, which can cause discomfort and even the failure of mechanical components.

Tire non-uniformities are material or manufacturing anomalies that can induce varying forces that act on the hub (Marshall, 2006). These forces are generated when the tire-wheel assembly rolls, being directly related to the velocity of the vehicle. The presence of imperfections such as tire run-out, rim run-out, tire-bead area placement variations and other bead-seating errors (Pottinger, 2009) can be categorized as mass imbalance, geometric variations or stiffness variations. Actually, Dorfi (2005) mentions that these three types of non-uniformities can be combined with their respective phases, creating force variation at the spindle.

In this work, a motorcycle tire-wheel assembly is modeled to study how the tire non-uniformities influence the vibrations transmitted to the hub of the wheel and affect ride comfort. An analytical model is built based on the study of Dillinger (2005). A three degrees of freedom tire-wheel model is built considering a disc and a rigid ring connected by radial and torsional springs and dampers. Since it is an easy and fast model to be developed, it consumes less time in engineering analysis and can serve as a beginning for further studies. To check the validity of the model, it is compared with one from literature. The values of model parameters are obtained experimentally from a real motorcycle tire.

In fact, the model proposed here is widely used for cars and trucks. However, particular parameters as stiffness, mass and moment of inertia can characterize the system like a motorcycle tire-wheel assembly at least in-plane analysis.

Tire non-uniformities are included in the model as lumped mass imbalance, run-out and stiffness variation. The relationship between tire rolling frequency and each non-uniformity are analyzed and discussed.

2. TIRE-WHEEL MODEL

The model proposed is shown in Figure 1. It consists of a rigid ring connected to a disc by two radial springs (horizontal and vertical) and one torsional spring. The springs represent the tire sidewall stiffness and the stiffness caused by the tire internal pressure in radial and angular directions. A vertical spring is included between the rigid ring and the road to represent the tread stiffness. Damper elements are also placed in parallel with springs to constitute the damping behavior of the system.

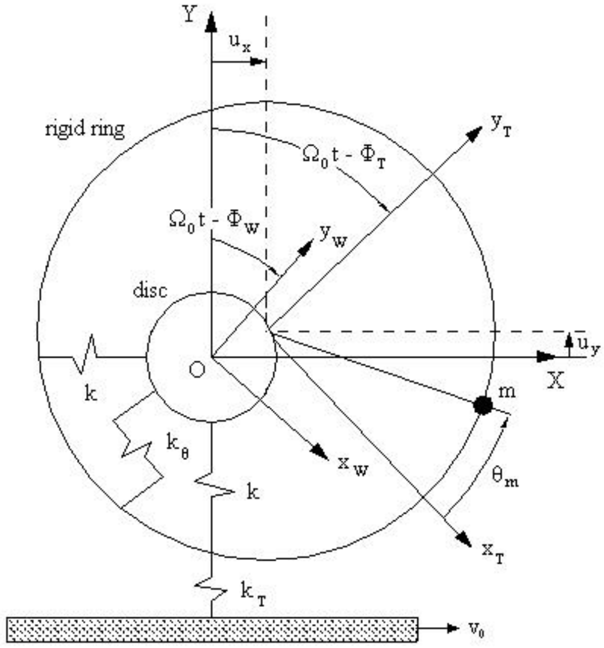

Figure 1. Tire-wheel model

This kind of representation is able to exhibit one radial and two torsional modes shapes, i.e., the main first rigid modes of the tire. The vibratory behavior modeling is critical when non-uniformity force excitations are studied, because the transmission of forces and vibrations affect the system response accordingly to its structural characteristics.

Some assumptions and simplifications need to be take into account. According to Dillinger (2005), the mass of sidewall is negligible, the mass imbalance is represented as a single point on the tread, the wheel is fixed, the damping is considered viscous, the springs are linear and there is no slippage between the tire and the road.

The model has three coordinate systems. Coordinate system XYZ is located on the center of the wheel and is stationary. Coordinate system $x_w y_w z_w$ rotates with the disc and $x_T y_T z_T$ is attached to the rigid ring with an angular velocity $\vec{\Omega} = (\Omega_0 - \dot{u}_x / R)\hat{K}$, where Ω_0 is the average rotation of the rigid ring, defined as $\Omega_0 = V_0 / R$, where V_0 is the velocity of the road. The term $\dot{u}_x / R$ is superimposed on the rotation of the rigid ring, where $\dot{u}_x$ is the velocity in the direction x and R is the radius of the rigid ring. Since there is no slippage, $u_x = R\Phi_T$, so $\dot{\Phi}_T = \dot{u}_x / R$. Therefore, the angular velocity of the ring is expresses as $(\Omega_0 - \dot{\Phi}_T)\hat{K}$ and for the disc, the rotational velocity is $(\Omega_0 - \dot{\Phi}_W)\hat{K}$. $\dot{\Phi}_T$ and $\dot{\Phi}_W$ are, respectively, the angular velocity imposed on the ring and the angular velocity imposed on the disc.

Including mass imbalance effect, energy equations can be derived as follow:

$$T = \frac{1}{2} I_T (\Omega_0 - \dot{\Phi}_T)^2 + \frac{1}{2} M_T \dot{u}_y{}^2 + \frac{1}{2} M_T \dot{u}_x{}^2 + \frac{1}{2} I_W (\Omega_0 - \dot{\Phi}_W)^2 + \frac{1}{2} m \vec{v}_m \cdot \vec{v}_m \tag{1}$$

$$U = \frac{1}{2} k_\theta \left[\frac{u_x}{R} - \Phi_W \right]^2 + \frac{1}{2} k u_x{}^2 + \frac{1}{2} (k + k_T) u_y{}^2 \tag{2}$$

$$D = \frac{1}{2} c_\theta \left[\frac{\dot{u}_x}{R} - \dot{\Phi}_W \right]^2 + \frac{1}{2} c \dot{u}_x{}^2 + \frac{1}{2} (c + c_T) \dot{u}_y{}^2 \tag{3}$$

where

$$\vec{v}_m = \left[\dot{u}_x + R\Omega\cos(\Omega_0 t + \theta_m)\right]\hat{I} + \left[\dot{u}_y + R\Omega\sin(\Omega_0 t + \theta_m)\right]\hat{J} \tag{4}$$

T is the kinetic energy of the system, U is the potential energy and D is the dissipated power.
Substituting $u_x = R\Phi_T$ and using Lagrange equations, the mathematical representation becomes:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{Q\} \tag{5}$$

where

$$[M] = \begin{bmatrix} I_W & 0 & 0 \\ 0 & 2I_T + mR^2 & 0 \\ 0 & 0 & M_T + m \end{bmatrix} \tag{6}$$

$$[C] = \begin{bmatrix} c_\theta & -c_\theta & 0 \\ -c_\theta & c_\theta + cR^2 & 0 \\ 0 & 0 & c + c_T \end{bmatrix} \tag{7}$$

$$[K] = \begin{bmatrix} k_\theta & -k_\theta & 0 \\ -k_\theta & k_\theta + kR^2 & 0 \\ 0 & 0 & k + k_T \end{bmatrix} \tag{8}$$

$$\{Q\} = \left\{0 \quad \left[mR^2\Omega_0{}^2 \sin(\Omega_0 t + \theta_m)\right] \quad \left[-mR\Omega_0{}^2 \cos(\Omega_0 t + \theta_m)\right]\right\}^T \tag{9}$$

and the generalized coordinates of the system are

$$q = \left\{\Phi_W \quad \Phi_T \quad u_y\right\} \tag{10}$$

I_W is the mass moment of inertia of disc, I_T is the mass moment of inertia of rigid ring, m is the single point mass, M_T is the rigid ring mass, R is the rigid ring radius, c_θ, c and c_T are the torsional, radial and tread damping, respectively, k_θ, k and k_T are the torsional, radial and tread stiffness, respectively, and θ_m is the angular position of single point mass.

Radial run-out can be represented by a force inserted in vector $\{Q\}$, based in Stutts *et al.* (1991):

$$\{Q\} = \left\{0 \quad 0 \quad \left[k_T u_{run-out} + c_T \dot{u}_{run-out}\right]\right\}^T \tag{11}$$

$$u_{run-out} = U_y \cos(\Omega_0 t + \theta_{run-out}) \tag{12}$$

$$\dot{u}_{run-out} = -U_y \dot{\Omega}_0 \sin(\Omega_0 t + \theta_{run-out}) \tag{13}$$

where U_y is the amplitude of tire run-out and $\theta_{run-out}$ is the angular position of maximum run-out.

Finally, stiffness variation is modeled as an excitation term dependent of rigidity, based at Dorfi (2005):

$$\{Q\} = \{0 \quad 0 \quad [(Rk\varepsilon)\cos(\Omega_0 t + \theta_{stiff})]\}^T \tag{14}$$

Equation (14) represents the self-excitation created by stiffness non-uniformity present in the tire, where ε is the percentage of stiffness variation in the system and θ_{stiff} is the angular position of maximum tire radial stiffness.

This work aims to analyze the varying forces in the center of the wheel. The dynamic force vector acting on the hub of tire-wheel assembly depends of the system response. It is defined by the effect of the horizontal and vertical displacement of the rigid ring in the stiffness in horizontal and vertical directions, as showed below:

$$\vec{F} = (ku_x + c\dot{u}_x)\hat{I} + (ku_y + c\dot{u}_y)\hat{J} \tag{15}$$

$$F_x = kR\Phi_T + cR\dot{\Phi}_T \tag{16}$$

$$F_y = ku_y + c\dot{u}_y \tag{17}$$

The dynamic model was implemented in a Matlab computational code and simulated results were obtained. In order to verify the computational code, Fig. 3 were obtained to compare with Dillinger's (2005) model, showed in Fig. 2. It was inserted an imbalance mass of 5 g and a rolling velocity of 10 km/h.

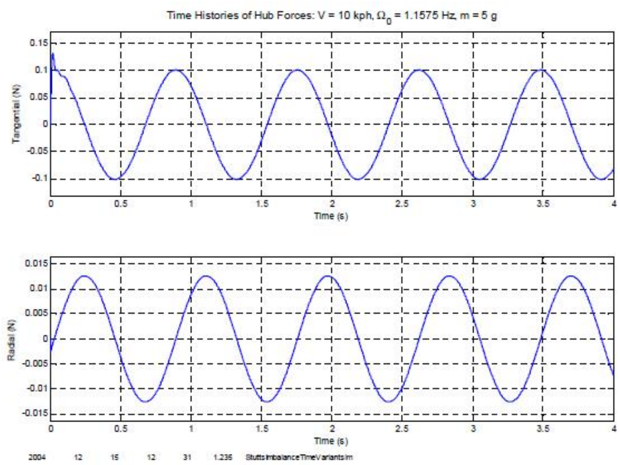

Figure 2. Reference model (Dillinger, 2005)

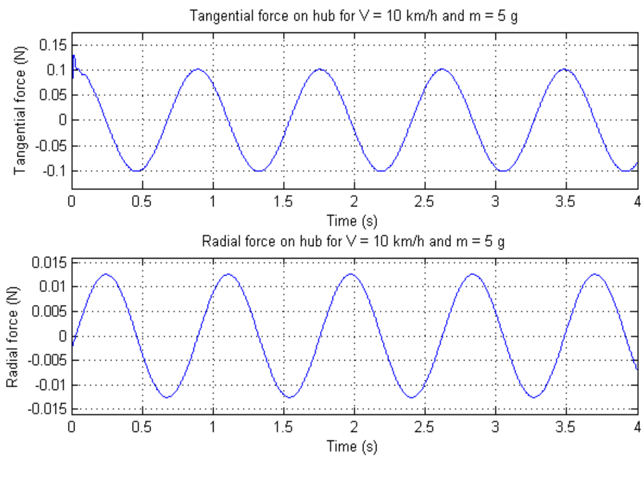

Figure 3. Tire-wheel assembly model simulation

3. MOTORCYCLE TIRE PROPERTIES AND NON-UNIFORMITIES EVALUATION

The motorcycle tire used in this project is a 90/90 - 21M/C 54V TL MT90 Front A/T. It is a bias tire, attached to a rim 1.85 x 21".

Experimental tests and simulations were performed by Pirelli Pneus Ltda. to define the values of physical parameters necessary to analyze the model.

Table 1 shows the parameter values obtained by Pirelli Pneus Ltda. No damping parameters are considered because they demand complex tests. Mass moment of inertia of the rigid ring is the mass moment of inertia of the tire, acquired in Abaqus software. The mass moment of inertia of the disc is the mass moment of inertia of the wheel (established in a CAD software). Torsional stiffness was obtained by measuring the angular displacement of the sidewall of the tire after having applied a moment, showed in Fig 4. A radial equivalent stiffness k_{eq} was measured by pressing the tire to the ground, also showed in Fig. 4. Due to the difficulty to determine the tread stiffness, it was approached to 2 000 000 N/m, based on the values utilized by Dillinger (2005). Radial stiffness k was determined considering that k_{eq} is the equivalent stiffness of k and k_T disposed in series, as Fig. 1 shows.

Table 1. Tire-wheel model parameters

Parameter	Value
Mass moment of inertia of disc I_W	1.08 kg.m²
Mass moment of inertia of rigid ring I_T	0.491 kg.m²
Rigid ring radius R	0.35127 m
Rigid ring mass M_T	5.35 kg
Torsional stiffness k_θ	12 000 N.m/rad
Radial equivalent stiffness k_{eq}	122 000 N/m
Tread stiffness k_T	2 000 000 N/m
Radial stiffness k	129 930 N/m

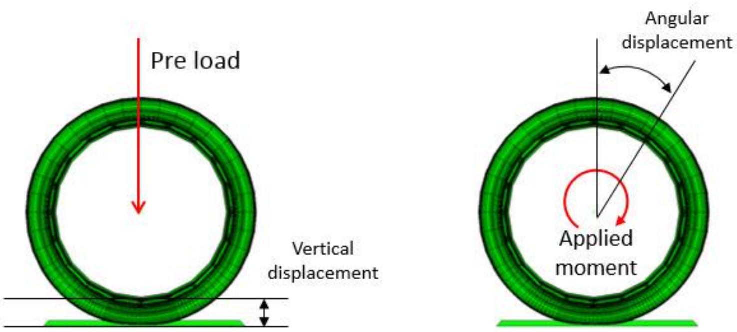

Figure 4. Radial equivalent and torsional stiffness determination

Modal analysis was performance in Matlab. The model has three natural frequencies: f_1 = 11.56 Hz, f_2 = 29.50 Hz and f_3 = 100.37 Hz, where f_1 is the first torsional mode, f_2 is the second torsional mode and f_3 is the first radial/vertical mode.

3.1 Mass imbalance

As previously commented, tire non-uniformities are related to anomalies generated during the manufacturing process. Mass imbalance phenomenon appears when there is a mass distribution concentrated on a single point. Stiffness variation are due to geometric variations such as ply overlap, splices and other dimensional aspects (Dorfi, 2005). Run-out and out-of-roundness also belong to geometric variations class. However, in the model, stiffness and geometry are considered uncoupled to simplify the mathematical analysis and to provide an easy conception between stiffness and geometric variations. The analysis will be done in frequency domain and for 1st harmonic excitations, regarding the peaks in horizontal and vertical force amplitude on hub, described in Eq. (16) and Eq. (17).

Mass imbalance was analyzed for m = 0.005 kg and velocity of V = 40, 91.85 and 120 km/h (f = 5.06, 11.56 and 15.10 Hz), as shown in Fig. 5.

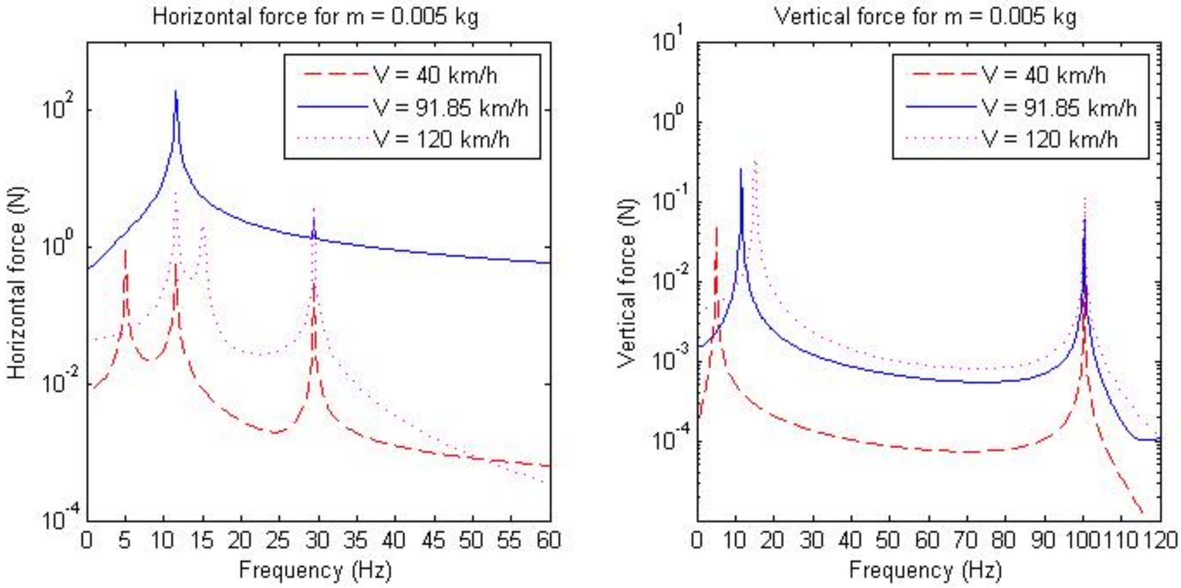

Figure 5. Mass imbalance analysis

It can be noted that mass imbalance is more perceptible in horizontal direction. For 120 km/h, horizontal peak values are 6.17, 2.10 and 3.94 N, while in vertical, peaks values are 0.34 and 0.11 N. It is important to realize that for 91.85 km/h it occurs a resonance phenomenon in horizontal direction, because this velocity corresponds to the first natural frequency of the system (f = 11.56 Hz). Once the peak value of force is amplified, ride comfort can be affected.

Another important point is the speed dependence of the peaks for mass imbalance phenomenon. Analyzing both directions, it is clear that the peaks increase with the velocity. In fact, Eq. 9 shows us that the mass imbalance effect is represented by an extern excitation with an angular velocity squared (Ω_0^2).

3.2 Geometric variation

For geometric and stiffness variation analysis, it is important to highlight that the tire acts like a spring in radial direction. According to Dorfi (2005), this "spring" can vary both in stiffness (stiffness variation) and in length (out-of-roundness, which generates radial run-out).

Geometric variation was take into account by inserting a radial run-out of U_y = 0.0005 and 0.0020 m in the tire-wheel assembly for V = 40, 80 and 120 km/h, shown in Fig. 6, and represented by Eqs. (11), (12) and (13).

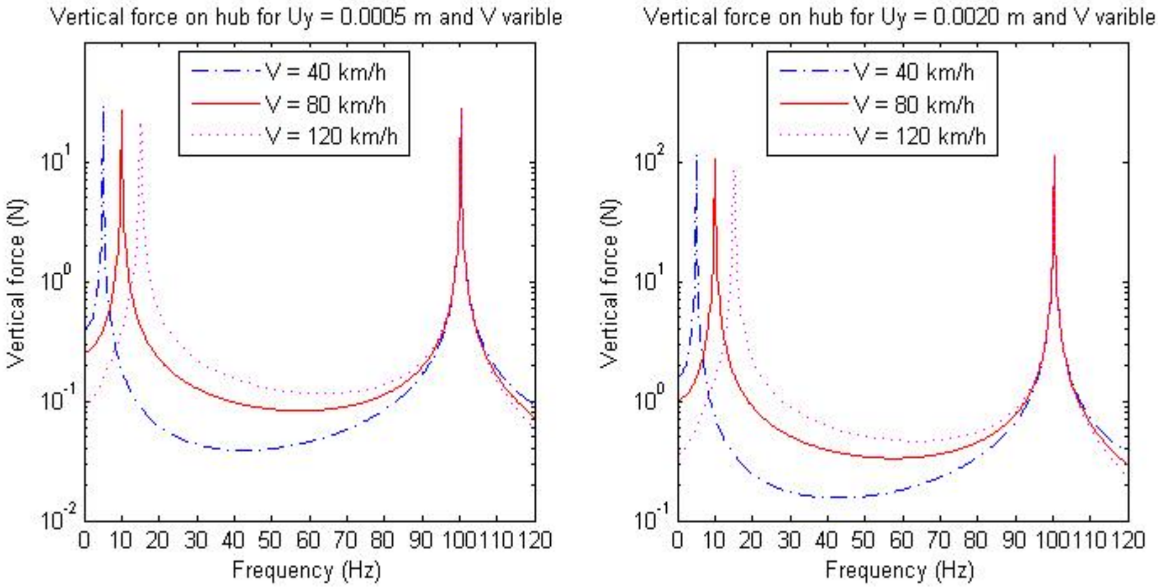

Figure 6. Geometric variation represented as a run-out

Figure 5 shows that the vertical force for *Uy* = 0.0020 m is bigger than for *Uy* = 0.0002 m, i. e, vertical force increases with the run-out magnitude. Moreover, the amplitudes decrease with the velocities (from *f* = 5.03 Hz (*V* = 40 km/h) to *f* = 15.02 Hz (*V* = 120 km/h), passing by *f* = 10.07 Hz (*V* = 80 km/h)) in both figures.

3.3 Stiffness variation

The third non-uniformity analyzed, stiffness variation was inserted in the model as a variation of k parameter, as showed in Eq. (14).

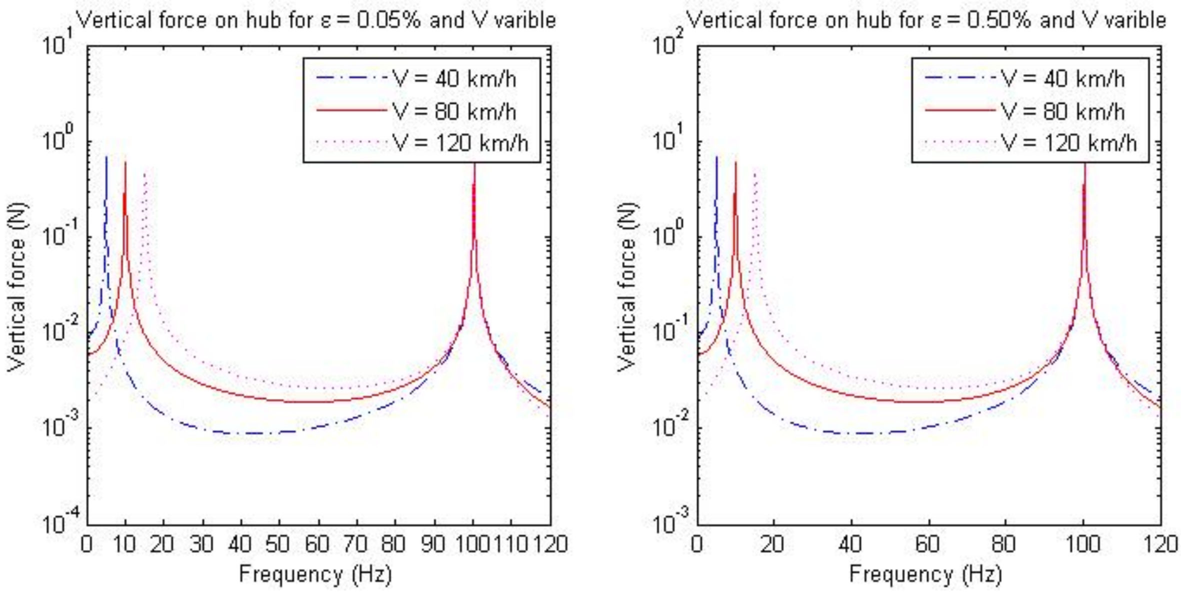

Figure 7. Stiffness variation analysis

Stiffness variation shows the same behavior observed in geometric analysis. Increasing the percentage of stiffness variation, it is clear that the force amplitude also increases. Actually, the stiffness modeling proposed by Dorfi (2005) is very simple and should be used only with 1st harmonic analysis. Since its representation proposed has a cosine multiplied by an amplitude, its comportment is similar to the ones introduced by Sttuts *et al.* (1991) for run-out evaluation.

4. CONCLUSIONS

Rigid ring modeling is capable to represent the main structures of a tire-wheel assembly, including the rigid body modes. It can be accommodated to easily receive geometric and mass imbalance modeling, since springs, damper elements, mass single points and run-out positions can be placed accordingly to the polar coordinate system of the tire. Furthermore, a three-degrees-of-freedom system can be easily modeled if compared with other methods. However, more researches need to be done for using this model for motorcycle tire modeling, accompanied with experimental tests for validation. Furthermore, the tread stiffness value was obtained by a rough estimate, instead of a stress-compression test of the tread material, for example.

A relevant point of this work was the decoupling of stiffness and geometric non-uniformities. Geometry anomalies due to bad adjustments of tire components can also cause out-of-roundness or run-out and variations in the structure. Once the tire is considered an assembly of springs, it can be generated variations in the rigidity of the system. That was why the similarity between the response of geometric and stiffness variation behaviors.

For future works, some actions can be taken to improve the motorcycle tire-wheel analysis. Modal analysis is an important tool for determining the vibratory behavior of the structure and can be used for validating the model. Improving the contact region can help to understand how road excitations and non-uniformities self-excitations are transmitted between the ground and the tire. In addition, a flexibly ring model is capable to represent other ring modes rather than only rigid modes because high tire modes can be excited. Finally, adding a sprung mass and more degrees of freedom to represent the motorcycle vehicle is an essential point to understand how the vibrations affect the pilot.

5. ACKNOWLEDGEMENTS

This paper is a part of an initial motorcycle tire study related to tire non-uniformities. I would like to thank Pirelli Pneus Ltda. for technical support and Faculty of Mechanical Engineering (*Faculdade de Engenharia Mecânica*) from Unicamp for academic support, especially to Dr. Argemiro Luis de Aragão Costa and Dr. José Maria Campos Dos Santos.

6. REFERENCES

Dillinger, B.L., 2005. *The development of analytical tire models with stiffness non-uniformity, mass imbalance, and radial run-out*. M.Sc thesis, Clemson University, Clemson.

Dorfi, H.R., 2005. "Tire non-uniformities and steering wheel vibrations". *Tire Science and Technology*, Vol. 33, no. 2, p. 64-102.

Marshall, K.D., 2006. "An overview of tire technology". The pneumatic tire, p. 231-285.

Pottinger, M., 2009. "Uniformity: a crucial attribute of tire/wheel assemblies". *Tire Technology International 2009* <http://www.tiretechnologyinternational.com>

Stutts, D.S., Soedel W. and Jha, S. K., 1991. "Fore-aft forces in tire-wheel assemblies generated by unbalances and the influence of balancing". *Tire Science and Technology*, Vol. 19, no. 3, p. 142-162.

7. RESPONSIBILITY NOTICE