

A QUARTER-CAR MODEL COMPRISING GEOMETRICAL NONLINEARITIES AND ASYMMETRICAL DAMPERS

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Abstract. *The objective of this work is to study the dynamical behaviour of vehicle suspension systems employing asymmetrical viscous damping, with a focus on improving passenger comfort, considering geometrical nonlinearities which are present in the system. Previous studies have shown that the use of asymmetrical dampers in these types of systems can be advantageous with regard to the comfort of the passengers. Two configurations, one with symmetrical viscous damping and another with asymmetrical viscous damping are presented and compared. The simulations are performed using a 2-DOF quarter-car model under harmonic excitation. The geometrical configuration of a suspension system is taken into account in modeling the damping force and spring force, to include geometrical nonlinearities in the model. Mathematical models of the described suspension systems are analysed by numerical simulation. For this analysis, the values of the damping coefficient for the compression and extension of the damper were varied, establishing an asymmetry ratio. A proper choice of the asymmetry ratio can diminish the effects that the uneven pavement causes on the displacement and acceleration of the sprung mass, improving the comfort for the passengers.*

Keywords: vehicle dynamics, asymmetrical damper, harmonic excitation, geometrical nonlinearities

1. INTRODUCTION

All ground vehicles are subjected to vibrations induced by the road. These vibrations affect directly the handling and comfort of the passengers, and are considered disturbances. Suspension systems are designed to minimize these unwanted effects to improve comfort, while also providing the driver with a good ride control over the vehicle (Els *et al.*, 2007). Besides these aspects, the suspension system must also provide a good level of comfort, minimizing the motions and acceleration imposed or perceived by them. However, in ground vehicles, improving one of these objectives generally can compromise the other (Silveira *et al.*, 2014). Generally, comfort is associated to a softer suspension while that, to keep a good handling, the suspension system should be stiffer (Nguyen *et al.*, 2012).

The vast majority of vehicles uses passive suspension systems. However, because they utilize non-adjustable passive elements, passive suspension systems present some limitations (Bouazara *et al.*, 2006). Motivated to develop suspension systems capable of self adjusting to any type of pavement, automotive engineers began to develop controlled semi-active and active suspension systems (Guglielmino *et al.*, 2008). The evolution of computers and electronic devices as, actuators and sensors, aided the development of these types of suspension systems.

According to Gysen *et al.* (2010), active suspension systems can provide up to 60% more comfort than passive suspension system. The first practical applications of active suspension systems were in the decade of 1980 (Hrovat, 1997; Nguyen *et al.*, 2012). Active suspensions were first developed for Formula 1 cars, with Lotus being the first to equip a car with an active system (Milliken and Milliken, 1995). Much effort has been put to improve such systems (Yoshimura *et al.*, 1999; Fischer and Isermann, 2004; Gysen *et al.*, 2010; Deshpande *et al.*, 2014). Although active suspension systems provide good comfort compared to passive suspension system, its use implies an increase in the global cost of the vehicle. Thus, it has been used only in high performance vehicles (Bouazara *et al.*, 2006).

With the aim of reducing the overall cost of the vehicle without compromising the level of passenger comfort, alternatives to this question have been sought. A proposed solution is to explore the use of passive dampers whose damping forces act asymmetrically. Some studies involving the dynamic analysis of asymmetrical damping shows satisfactory results when applying this type of damping in suspension systems (Ahmed and Rakheja, 1992; Rajalingham and Rakheja, 2003; Silveira *et al.*, 2014).

Asymmetrical dampers have different damping coefficient during the extension phase and the compression phase. This asymmetrical characteristic result in larger damping force during the extension. As a consequence, the wheel velocity is larger in the upward direction. (Rajalingham and Rakheja, 2003). Suspension systems which employs asymmetrical damping can change the operating equilibrium position of the vehicle sprung mass, and this phenomenon is attributed to

the asymmetrical nature of the dampers. The potential energy stored in the spring during compression is dissipated during extension under larger damping, diminishing the stroke during the extension of damper. This effect is called jacking down or packing (Rajalingham and Rakheja, 2003). Numerical simulations and analytic solutions have been used to analyse the behaviour of the system employing asymmetrical damping (Silveira *et al.*, 2015).

The geometry of the suspension system is an important factor to be considered in the design of a vehicle. In passenger vehicles, there are two most common types of suspension systems. The McPherson type is the most used in such vehicles, because of advantages such as its simplicity of construction, reduced weight and volume (Hurel *et al.*, 2012). The other common type of suspension is the double wishbone. The geometry of this suspension type has significant effect on handling and passenger comfort as compared with other types such as McPherson and other types of suspension (Balike, 2010).

According to ISO 2631, the human body has distributed inertia and stiffness resulting in different natural frequencies for each of its parts. In occupational vibration evaluation, several factors influence the risk characterisation, among which are: vibration amplitude, frequency, its direction and the exposure time (ISO-2631, 1997). Therefore, in ground vehicles, the suspension system has an important role in reducing the vertical displacement and acceleration of the sprung mass, maintaining the comfort levels as high as possible.

Therefore, the analyses presented in this paper are focused in decreasing the discomfort caused by the uneven roads, combining the asymmetrical damping and analyzing the role of suspension system geometry on the response of the system.

Two aspects are investigated in this paper. The first one is the influence of asymmetrical damping in the response of the system. In the second one, the effects of the combination of the asymmetrical damping and the suspension geometry are analysed with respect to the amplitude of displacement and maximum acceleration of the sprung mass. A two-degree-of-freedom quarter-car model with harmonic base excitation is presented in section 2; results of maximum displacements, accelerations and amplitude of displacement are presented in section 3; and analysis of suspension geometrical nonlinearities as inclination of spring-damper assembly and variation between unsprung and sprung mass are presented in section 4. Finally conclusions are drawn in section 5.

2. QUARTER-CAR MODEL WITH GEOMETRICAL NONLINEARITIES

In this study, two aspects are analysed. The first one is to investigate the influence of asymmetrical damping in the response of the system. A quarter-car model for analyzing vertical vibrations having two degrees of freedom is used, as shown in Fig. (1).

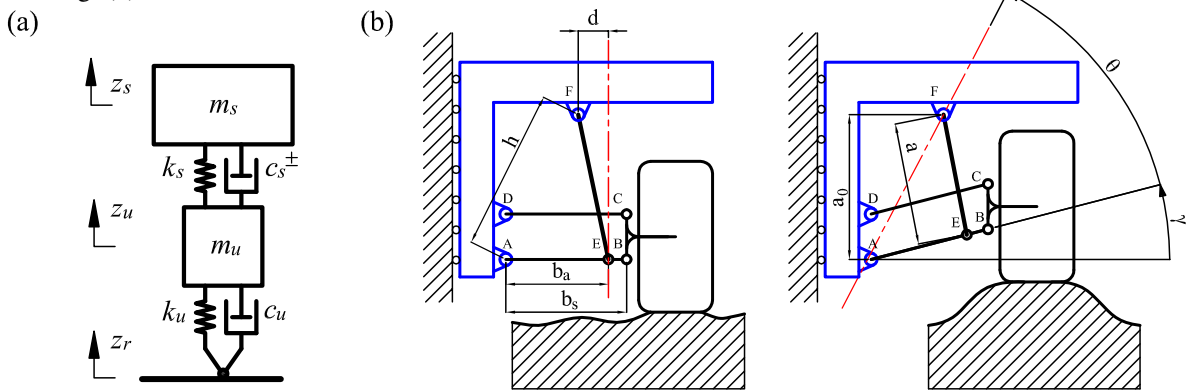


Figure 1. Representation of quarter-car model with its parameters (a) and representation of the model considering the geometrical nonlinearities with respective parameters (b).

This system consists of two subsystems. The lower subsystem consists of a block of mass m_u , a spring with stiffness k_u and symmetrical damping coefficient c_u . The mass m_u is the unsprung mass, which includes the lower control arm, the wheel and tire. The vertical displacement of this mass is given by z_u . The stiffness and damping characteristics of this subsystem are derived from characteristics of the tire and wheel. The upper subsystem consists of a block of mass m_s , a spring stiffness k_s . This subsystem is the sprung mass of the vehicle, including the chassis, the bodywork, the passengers, the engine and transmission. Its vertical displacement is represented by z_s . The viscous damper between the block of mass m_s and m_u is considered asymmetrical. Thus, the damping coefficient for extension is given by c_s^+ , and the damping coefficient during compression is given by c_s^- . In the two subsystems, the stiffness is considered linear. In modeling this quarter-car system, the excitation of the base represents the irregularities from the road, and it is given by the harmonic function:

$$z_r = z_{max} \sin(\omega t) \quad (1)$$

in which z_{max} is the amplitude of excitation, ω is the frequency of excitation and t is time.

A Lagrangian approach is used to obtain the dynamical equations, so that the two equations of motion for a system with two degrees of freedom with asymmetrical damping can be written as:

$$\begin{aligned} m_u \ddot{z}_u - k_s(z_s - z_u) - c_s^\pm(\dot{z}_s - \dot{z}_u) + c_u(\dot{z}_u - \dot{z}_r) + k_u(z_u - z_r) &= 0 \\ m_s \ddot{z}_s + k_s(z_s - z_u) + c_s^\pm(\dot{z}_s - \dot{z}_u) &= 0 \end{aligned} \quad (2)$$

in which a dot denotes time derivative. The damping force is calculated by the product of the viscous damping coefficient and the relative velocity, and is given by:

$$F_{cs}^\pm = c_s^\pm (\dot{z}_s - \dot{z}_u) \quad (3)$$

wherein the asymmetrical damping coefficient for extension and compression is defined as:

$$c_s^\pm \Rightarrow \begin{cases} c_s^+ & \text{if } \dot{z}_s - \dot{z}_u \geq 0 \\ c_s^- & \text{if } \dot{z}_s - \dot{z}_u < 0 \end{cases} \quad (4)$$

The second aspect is to analyze the role of the geometrical parameters in the amplitude of displacement and maximum acceleration of the sprung mass. To perform out a study of the effects of geometrical nonlinearities resulting from the geometry of a double wishbone type suspension, the parameters relating to the geometry of this suspension are defined as shown in Fig. (1-b). Due to imperfections of the road and consequent relative displacement between the subsystems, there is variation of the suspension geometry.

The relative displacement between m_s and m_u is represented by a , and is given by:

$$a = \sqrt{b_a^2 + h^2 - 2b_a h \cos \theta} \quad (5)$$

The spring force is calculated by the product of spring stiffness and the relative displacement, and is given by:

$$F_{ks} = k_s(a_i - a) \quad (6)$$

in which a_i is the relative displacement at rest. The relative velocity between the m_s and m_u is represented by $\dot{a}$, and is given by:

$$\dot{a} = \frac{b_a h \sin(\theta_f) (\dot{z}_s - \dot{z}_u)}{\sqrt{b_a^2 + h^2 - 2b_a h \cos(\theta_f) (z_u - z_s + b_s)(-z_u + z_s + b_s)}} \quad (7)$$

For a viscous damper, the damping force is proportional to the velocity on the ends of the spring-damper assembly (Rao, 2009). When considering geometrical nonlinearities, the relative velocity is given by Eq. (7). Consequently, the damping force is given by:

$$F_{cs}^\pm = c_s^\pm \dot{a} \quad (8)$$

The two dynamical equations for a system with two degrees of freedom with asymmetrical damping considering the geometrical nonlinearities can be written as follows:

$$\begin{aligned} m_u \ddot{z}_u + k_u(z_u - z_r) + c_u(\dot{z}_u - \dot{z}_r) - k_s(a_i - a) - c_s^\pm \dot{a} &= 0 \\ m_s \ddot{z}_s + k_s(a_i - a) + c_s^\pm \dot{a} &= 0 \end{aligned} \quad (9)$$

The parameters used here are based on values for a medium size passenger vehicle (Silveira *et al.*, 2014). The unsprung mass is $m_u = 200$ kg, the stiffness of the unsprung system is $k_u = 400000$ N/m and the damping of the unsprung system is $c_u = 40$ Ns/m. The sprung mass is $m_s = 1500$ kg, the stiffness of the sprung system is $k_s = 44000$ N/m. For the simulations with symmetrical damping, the coefficients for extension and compression are $c_s^+ = 4000$ and $c_s^- = 4000$ Ns/m, and for the asymmetrical system the damping coefficient receive different values for compression and extension according to an asymmetry ratio β , defined as Eq. (10).

$$\beta = \frac{c_s^+}{c_s^-} \quad (10)$$

The amplitude of road profile utilized on the simulations is $z_{max.} = 0.02$ m. The values of c_s^+ and c_s^- were chosen such that the dissipated energy in one cycle can be similar to the symmetrical case (Silveira *et al.*, 2014). The two resonance peaks for this system are at $\omega_2 = 5.13$ rad/s and $\omega_1 = 47.14$ rad/s. Therefore, the simulations are focused in the range of frequencies comprising these values.

The parameters related to the geometry of the system are: $b_s = 400$ mm, $b_a = 340$ mm, a_0 varies between 280 mm and 480 mm, and d varies between 0 mm and 300 mm. The responses were obtained by numerical integration using a fixed step 4th order Runge-Kutta method.

3. SUSPENSION SYSTEM RESPONSE USING ASYMMETRICAL DAMPING

This section presents the response of suspension system with asymmetrical damping compared with symmetrical damping. Figure (2) shows the influence of asymmetrical damper on maximum displacement ($z_{s,max}$) of the sprung mass. Depending on β , the maximum displacement presents positive and negative values. This phenomenon is attributed to the asymmetrical characteristic of the damping, and the mean position depends on the value of β .

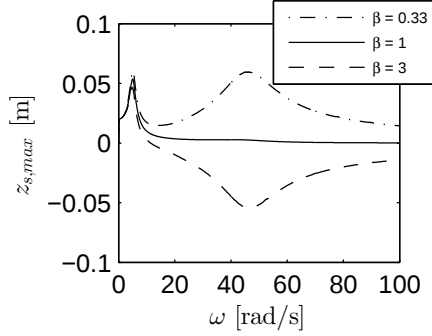


Figure 2. Maximum displacement ($z_{s,max}$) of the sprung mass for three values of asymmetry ratio (β).

This effect happens when the asymmetrical damper is used under harmonic force. The potential energy stored in the spring during compression is dissipated during extension at higher damping, thereby reducing the stroke of the damper during extension, cause a negative shift on the mean position of the sprung mass of the vehicle (Rajalingham and Rakheja, 2003). The mean position of the suspended mass varies with the excitation frequency, and the shift is larger close to ω_2 . Figure (3) shows the time history of the sprung mass for $\omega = 20$ rad/s. The mean position presents positive values when $\beta = 0.33$, negative values with $\beta = 3$ and is null for $\beta = 1$.

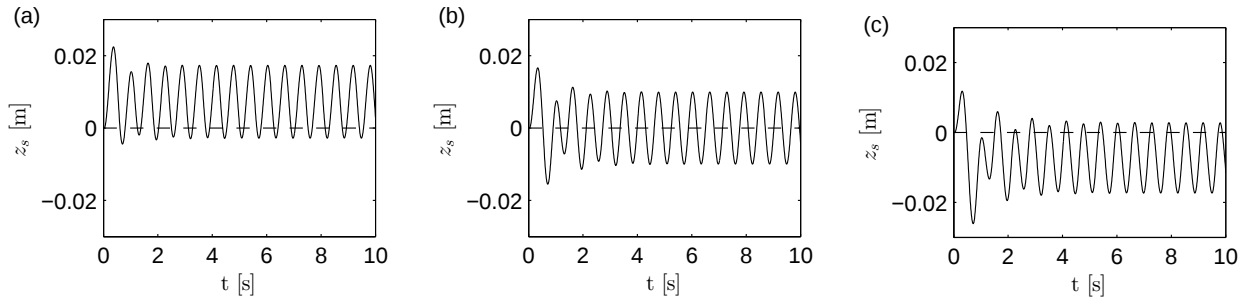


Figure 3. Time history of the sprung mass for $\beta = 0.33$ (a), $\beta = 1$ (b) and $\beta = 3$ (c).

Figure (4) shows the maximum acceleration of the sprung mass as function of the excitation frequency. It is seen that $\beta > 1$ is more advantageous at higher frequencies. Close to ω_1 , using $\beta = 3$ results in a reduction of 11% on the maximum acceleration, while $\beta = 0.33$ results in an increase of 13% when compared to the symmetrical case ($\beta = 1$).

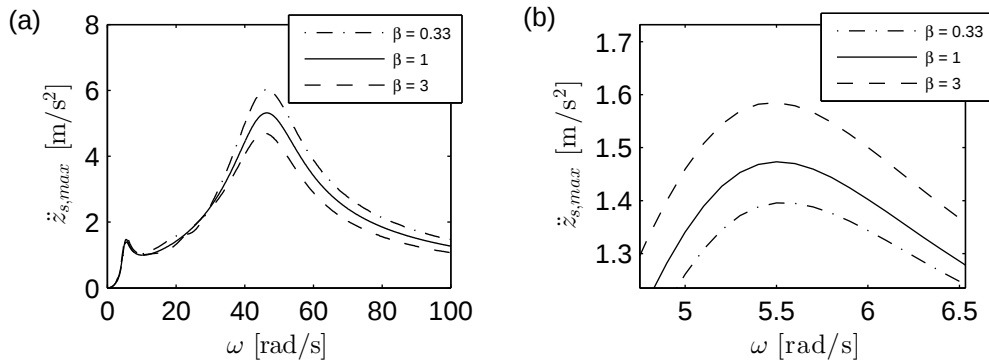


Figure 4. Maximum acceleration ($\ddot{z}_{s,max}$) relative to ω for $\beta = 0.33$, $\beta = 1$ and $\beta = 3$ (a) and detail of ($\ddot{z}_{s,max}$) close to the first resonance peak (b).

However, at low frequencies close to the first resonance peak this tendency inverts, and β values lower than 1 showed

best results. In this case there was a reduction of 5.2% using $\beta = 0.33$ and the suspension system with $\beta = 3$ provides an increase of 7.6% in the maximum acceleration of the sprung mass. Figure (4-b) shows detail of maximum acceleration of the sprung mass close to the first resonance peak.

Although the asymmetrical damping may provide good results on decreasing the maximum acceleration, the amplitude of displacement of the unsprung and sprung masses are not influenced. The use of asymmetrical damping only affects the maximum displacement of the sprung mass. The amplitude of displacement of the sprung mass is more sensitive at lower frequencies. The amplitude of displacement of the unsprung mass shows higher values in high frequencies. Figure (5) shows a comparison of three cases of suspension systems adopting $\beta = 0.33$, $\beta = 1$ and $\beta = 3$.

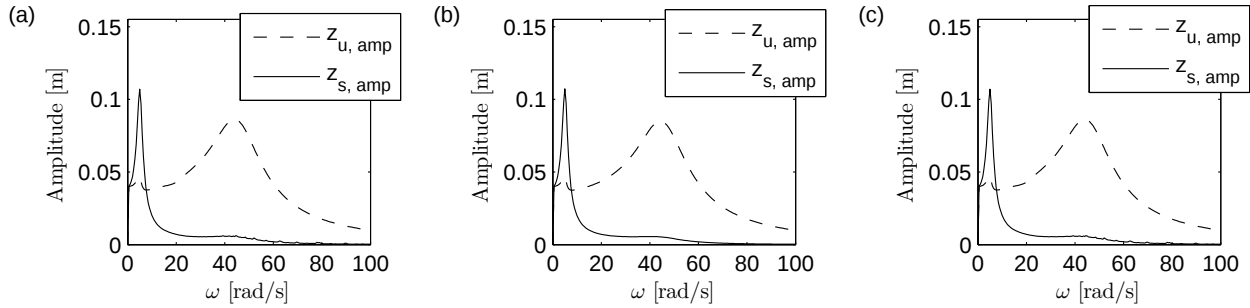


Figure 5. Amplitude of displacement of the unsprung and sprung masses relative to ω with $\beta = 0.33$ (a), $\beta = 1$ (b) and $\beta = 3$ (c).

4. INFLUENCE OF SUSPENSION GEOMETRICAL NONLINEARITIES

Previous studies have shown satisfactory results in diminishing the amplitude of displacement and acceleration of the sprung mass, when the parameters of suspension geometry are combined with the asymmetrical damping (Fernandes *et al.*, 2015).

Figure (6) shows the maximum acceleration of the sprung mass as function of the asymmetry ratio β with $\omega \approx \omega_1$ and $\omega \approx \omega_2$. The inclination of spring-damper assembly has more influence in the maximum acceleration at lower frequencies than at higher frequencies. For example, considering a system with $\beta = 3$ and $\omega \approx \omega_2$, the maximum acceleration when $d = 0$ mm is 1.416 m/s² and 1.142 m/s² with $d = 300$ mm, a reduction of 19%.

For the system with $\beta = 3$ and $\omega \approx \omega_1$, the inclination of spring-damper assembly causes only 3% of reduction in the maximum acceleration of the sprung mass.

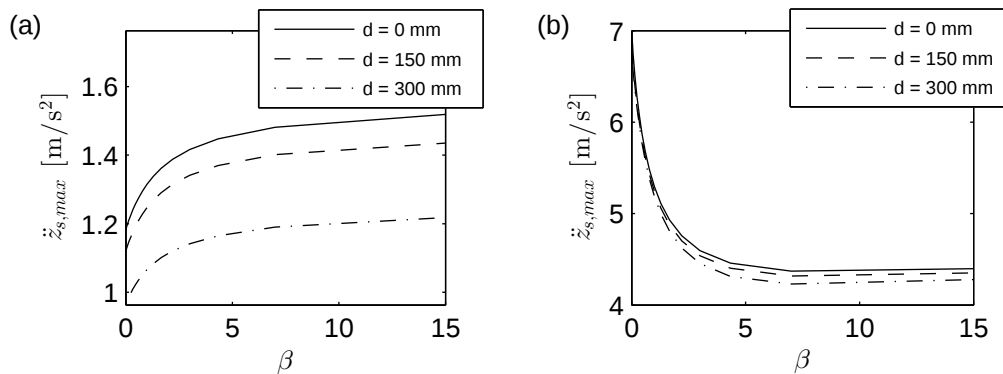


Figure 6. Influence of spring-damper inclination on the maximum acceleration of the sprung mass ($\ddot{z}_{s,max}$) relative to β with $d = 0$ mm, $d = 150$ mm and $d = 300$ mm for $\omega = 5.13$ rad/s (a), ($\ddot{z}_{s,max}$) relative to β for $\omega = 47.14$ rad/s (b).

Figure (7) shows the influence of the distance a_0 in the amplitude of displacement ($z_{s,amp}$) of the sprung mass. In Fig. (7-c) one notes a more significant reduction in amplitude of displacement. For $a_0 = 480$ mm and $\beta = 3$, the amplitude is 0.0849 m and for $a_0 = 280$ mm the amplitude is 0.0597 m. The reduction in the amplitude of displacement in this case is approximately 30%. Comparing the configuration using $d = 300$ mm and $d = 150$ mm, it is possible to note that the worst case using $d = 300$ mm has smaller amplitude of displacement than the best case for $d = 150$ mm. In the first resonance peak a_0 shows more influence on the amplitude of displacement than β .

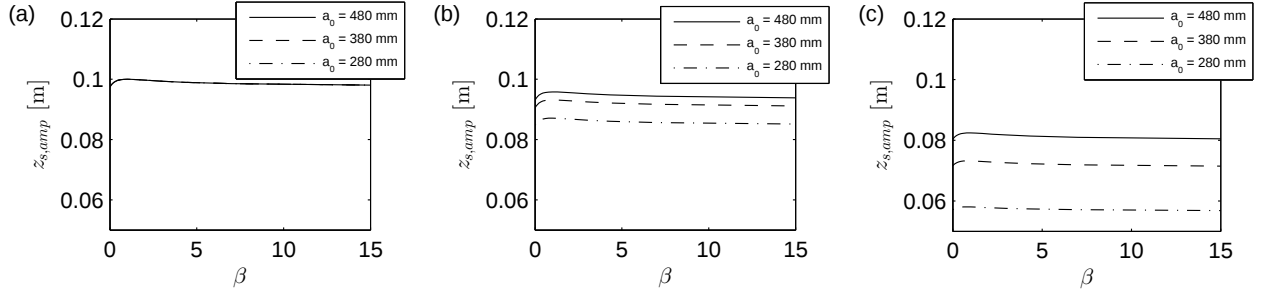


Figure 7. Influence of the distance a_0 in the amplitude of displacement ($z_{s,amp}$) of the sprung mass, for three different values of inclination of the spring-damper assembly, on the first resonance peak, being $d = 0$ mm (a), $d = 150$ mm (b) and $d = 300$ mm.

Figure (8) shows the influence of the spring-damper inclination and variation of a_0 . This comparison shows some particularities over the changes on the geometry. The system with $\beta = 0.33$ without inclination and changes in a_0 is the worst case among all analyzed, as shown in Fig. (8-a), where the maximum acceleration is greater for all the frequency values. The system with $\beta = 3$ showed better results when compared to $\beta = 0.33$ and $\beta = 1$. However, it can be improved combining the asymmetrical damping with the changes on the geometry. Figure (8-c) shows that the gain in the second resonance peak is low, only 3.9%. However, away from the second peak, the reduction on the maximum acceleration is greater. For example, for $\omega = 60$ rad/s the reduction on maximum acceleration is 29.3%.

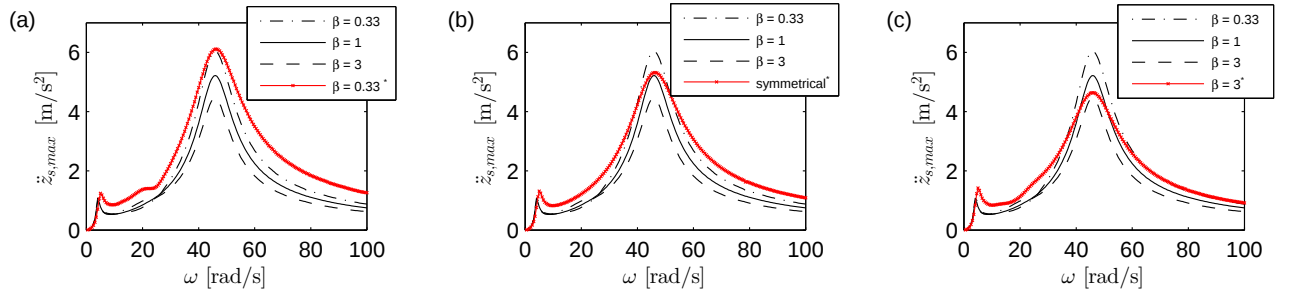


Figure 8. Maximum acceleration of the sprung mass ($\ddot{z}_{s,max}$), comparing three suspension systems with $d = 300$ mm and $a_0 = 280$ mm for $\beta = 0.33$, $\beta = 1$ and $\beta = 3$ with suspension systems with $d = 0$ mm and $a_0 = 480$ mm and $\beta = 0.33^*$ (a), $\beta = 1^*$ (b) and $\beta = 3^*$ (c).

5. CONCLUSIONS

In this study it was possible to note that the ratio of asymmetry influences the maximum displacement and maximum acceleration. However, the amplitude of displacement is less sensitive. The use of this type of asymmetrical damper implies the variation of the mean position of the sprung mass. These different positions depend on the asymmetry ratio, and the maximum displacement depends on the frequency of excitation. However, this effect only occurs at higher frequencies, which are close to the second eigenvalue. It is noteworthy that in the work related to suspension systems with asymmetrical damping, the asymmetry ratio used is always greater than 1. However, it was found in this work that for some excitation frequency bands, it is more advantageous to have this asymmetry ratio smaller than 1.

The dynamics of a suspension system with geometrical nonlinearities using asymmetrical damping is also of great importance for system optimization. Through parametric analysis it was possible to verify the influence of the spring-damper assembly inclination in the system dynamics. It was also noted that the choice of the asymmetry ratio combined with variations on the inclination of spring-damper assembly may be advantageous. It was observed that the variation in the inclination of spring-damper assembly can reduce the maximum acceleration and amplitude of displacement of the sprung mass. The variation in a_0 can also improve the comfort. In this case, while considering the variation of a_0 , the inclination of the spring-damper assembly and the asymmetry ratio, a 29.3% reduction was obtained in the maximum acceleration of the sprung mass.

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