23rd ABCM International Congress of Mechanical Engineering
December 6-11, 2015, Rio de Janeiro, RJ, Brazil

A SIMPLE ANALYSIS OF THE EXERGETIC PERFORMANCE OF THE HOT WATER STORAGE AND THE TANKLESS PRODUCTION SYSTEMS

Carlos Pinho
CEFT- FEUP – Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
ctp@fe.up.pt

Abstract. *An exergy analysis of a domestic hot water production system compares the irreversibility of the water storage and the tankless production systems. To carry out such analysis some basic assumptions were made: a constant temperature heat source and no heat losses from the system towards the surroundings. Under these circumstances it is shown that if hot water is only required for limited periods of time, the hot water storage system leads to lower exergy losses, being thus the preferred option. It is also demonstrated that the use of the joule effect electricity as the system heat source, be it a water storage one or a tankless one, maximizes the system irreversibility and that the tankless system, with the joule effect electricity heat source, is the worst situation.*

Keywords: *Thermodynamics, energy, exergy*

1. INTRODUCTION

Most recent analysis of domestic hot water systems are either concerned with the static (Fernández-Seara, 2007a) or dynamic (Fernández-Seara, 2007b) performance of electric heated storage cylinders, the use of phase change materials for thermal energy accumulation (Solé *et al*., 2008), while deeper analysis are concerned with the exergetic performance of water heat storage tanks (Dammel *et al*., 2012). The book of Dincer and Rosen (2011) also covers many energy storage situations, decomposing the operating cycle into three phases, charging, storage and discharging, discussing thoroughly the merits of the system efficiency quantification, either in energetic or exergetic terms. Other studies are more concerned with sanitary issues (Armstrong *et al*., 2014a) or with the conflicting issues between the thermal and the sanitary performance (Armstrong *et al*., 2014b).

In sanitary hot water systems two layouts are usually used, water storage tanks or cylinders equipped with heating equipment or tankless heating systems. In the first situation the installation can supply hot water mass flow rates well above those that can be available through the direct use of the water heating device alone, as in the tankless situation. A thermodynamic analysis is here applied to such kind of systems to evaluate their operating conditions and dimensions. Both situations are analyzed starting from a common layout, the relative exergetic merits are compared and simple design equations are derived, for an operating cycle composed only of two phases, heating with hot water consumption and heating without hot water consumption.

2. ENERGY ANALYSIS

Steady state operating conditions are assumed where a constant mass flow $\dot{m}_u$ of hot water is required at a temperature of T_u, during a certain utilization time interval t_u. This time interval can be variable, what is assumed constant in the present analysis is the system operating cycle which is the summation of the hot water utilization time t_u with the time without hot water consumption required to reposition the system in its initial conditions, t_a. The duration of this operation cycle will then be,

$$t = t_u + t_a \tag{1}$$

Figure 1 shows schematically the installation under analysis. In the analysis it will be assumed no heat losses from the heating system towards the environment. Initially it will be analyzed the first part of the operating cycle, with a duration of t_u. For an elemental time interval Δt, hot water is being consumed $\dot{m}_u \neq 0$, and the energy balance to the control volume depicted in Figure 1 leads to

$$\dot{m}\, c_L T_o \Delta t + \dot{m}_o c_L T_o \Delta t + \dot{Q}\, \Delta t = \rho_L V\, c_L \Delta T + \dot{m}_u c_L T_u \Delta t \tag{2}$$

where:

$\dot{m}_o$ - mixing cold water mass flow rate;

$\dot{m}$ - tank supply cold water mass flow rate;
$\dot{m}_u$ - consumed hot water mass flow rate;
$\dot{Q}$ - thermal power supplied to the storage tank;
V - tank volume;
ΔT - average temperature reduction of water in the tank, for the time interval Δt .

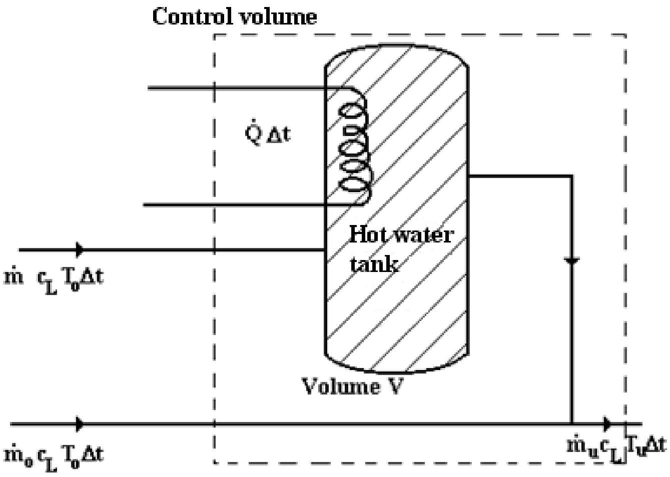

Figure 1. Schematic representation of a water heating and accumulation system.

Diving both members of Eq. (2) Δt, applying the continuity equation to the control volume of Fig. 1, $\dot{m} + \dot{m}_o = \dot{m}_u$ and subsequently determining the limit when $\Delta t \rightarrow 0$, the result is,

$$\rho_L V \ c_L \frac{dT}{dt} = \dot{m}_u c_L (T_o - T_u) + \dot{Q} \tag{3}$$

In the previous equation only the temperature T of the water contained in the tank is a time variable, so its integration is rather simple. This is true if steady state conditions can be assumed for the other parameters and time average values can be used in the analysis,

$$\frac{\rho_L V \ c_L}{\dot{m}_u (T_o - T_u) c_L + \dot{Q}} \int_{T_i}^{T_f} dT = \int_0^{t_u} dt \tag{4}$$

leading to,

$$t_u = \frac{\rho_L V \ c_L (T_f - T_i)}{\dot{m}_u c_L (T_o - T_u) + \dot{Q}} \tag{5}$$

where:
T_i - initial higher hot water temperature inside the tank;
T_f - final lower hot water temperature inside the tank. Usually $T_f = T_u$.

To know the evolution of the mass flow rate of hot water supplied by the tank $\dot{m}$, as well as the cold water flow rate $\dot{m}_o$ that must be mixed with the mass flow rate output of the tank, to achieve the desired water mass flow rate consumed, an energy balance is now applied to the water mixing process, Fig. 2, and the result is.

$$\dot{m} = \dot{m}_u \frac{(T_u - T_o)}{(T - T_o)} \tag{6}$$

During the second part of the operating cycle, of duration t_a, there is only heating of the water contained in the tank and no hot water consumption. So, making another energy balance to the control volume of Fig. 1, it gives

$$t_a = \frac{\rho_L V\ c_L\left(T_i - T_f\right)}{\dot{Q}} \tag{7}$$

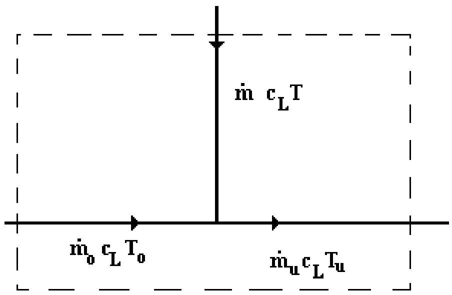

Figure 2. Water mixing process.

This operating cycle works for $T_i > T_f \geq T_u$. Considering now Eqs. (5) and (7) for the time duration of both parts of the operating cycle, reworking them

$$t_u \dot{m}_u c_L\left(T_u - T_o\right) - \dot{Q}\ t_u = \rho_L V\ c_L\left(T_i - T_f\right) \tag{8}$$

$$\dot{Q}\ t_a = \rho_L V\ c_L\left(T_i - T_f\right) \tag{9}$$

and equalizing both first members

$$\dot{Q}\ = \frac{t_u}{t_u + t_a}\dot{m}_u c_L\left(T_u - T_o\right) \tag{10}$$

But from Eq. (7) that defines t_a,

$$V\ = \frac{\dot{Q}\ t_a}{\rho_L c_L\left(T_i - T_f\right)} \tag{11}$$

and then,

$$V\ = \frac{t_u t_a}{t_u + t_a}\frac{\dot{m}_u}{\rho_L}\frac{T_u - T_o}{T_i - T_f} \tag{12}$$

Writing it in a different form,

$$\frac{V\ \rho_L}{\dot{m}_u} = \frac{t_u t_a}{t_u + t_a}\frac{T_u - T_o}{T_i - T_f} \tag{13}$$

this ratio is a kind of compaction factor as it gives the volume of the water tank per volume flow rate of the consumed hot water and if it is assumed that it must be as small as possible, the minimum of this ratio can be determined through the calculation of,

$$\frac{\partial\left(\frac{V\ \rho_L}{\dot{m}_u}\right)}{\partial t_u} = 0 \tag{14}$$

The result of this calculation is,

$$\frac{1}{t_u+t_a}=\frac{t_u}{\left(t_u+t_a\right)^2}\Rightarrow t_a=0 \tag{15}$$

The meaning of this is simply that to minimize the volume of the hot water accumulator tank the system will be only composed by a hot water heater with a mass flow rate capacity equal to the required hot water consumption rate, i. e. the tankless system. With $t_a=0$ Eq. (10) gives,

$$\dot{Q}=\dot{m}_u c_L\left(T_u-T_o\right) \tag{16}$$

while Eq. (12) gives,

$$V=0 \tag{17}$$

Defining the hot water time utilization factor as,

$$f_u=\frac{t_u}{t_u+t_a} \tag{18}$$

equations (10) and (12) will become,

$$\dot{Q}=f_u\dot{m}_u c_L\left(T_u-T_o\right) \tag{19}$$

$$V=f_u t_a\frac{\dot{m}_u}{\rho_L}\frac{T_u-T_o}{T_i-T_f} \tag{20}$$

and the value of this utilization factor for the condition stated by equation (15) is $f_u=1$. Through Eqs. (19) and (20) the sizing of the water heating system can be carried out and plots of them, in terms of f_u, are shown in Figs. 3 and 4, for $T_o=15$ ºC , $T_u=45$ ºC , $T_i=65$ ºC , an operating cycle of one hour and five different values of $\dot{m}_u$.

From Eq. (20) and considering that $t_a=\left(1-f_u\right)t$, remembering that t is the duration of the operating cycle,

$$V=f_u\left(1-f_u\right)t\ \frac{\dot{m}_u}{\rho_L}\frac{T_u-T_o}{T_i-T_f} \tag{21}$$

and determining $\left.\frac{\partial V}{\partial f_u}\right|_{t=const}=0$, the result is,

$$f_u=\frac{1}{2} \tag{22}$$

which explains the shape of tank volume curves shown in Fig. 4.

So according to the energy analysis there are two extreme situations, one corresponding to the largest tank size, $f_u={}^1\!/_2$, leading to,

$$V_{\max}=\frac{1}{4}t\ \frac{\dot{m}_u}{\rho_L}\frac{T_u-T_o}{T_i-T_f} \tag{23}$$

where in practical terms, $T_f=T_u$, and consequently the corresponding power input will be,

$$\dot{Q}_{V\max} = \frac{1}{2}\dot{m}_u c_L \left(T_u - T_o\right) \tag{24}$$

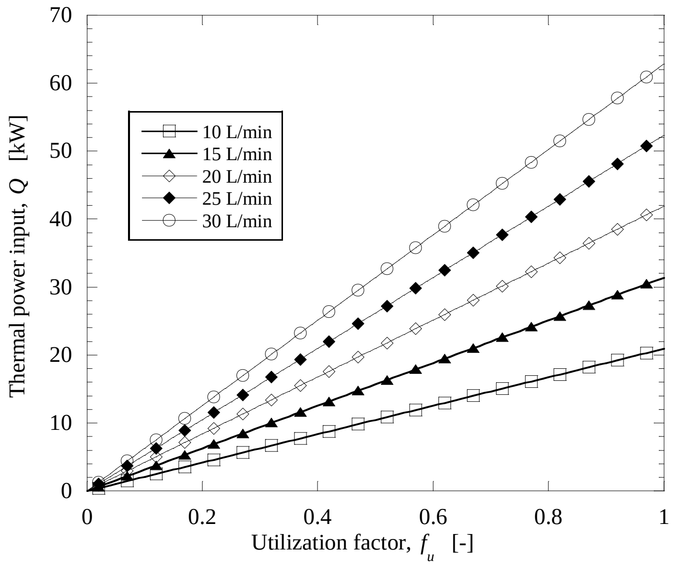

Figure 3. Thermal power input as a function of the hot water utilization factor. Ambient temperature 15 ºC and hot water temperature of 45 ºC.

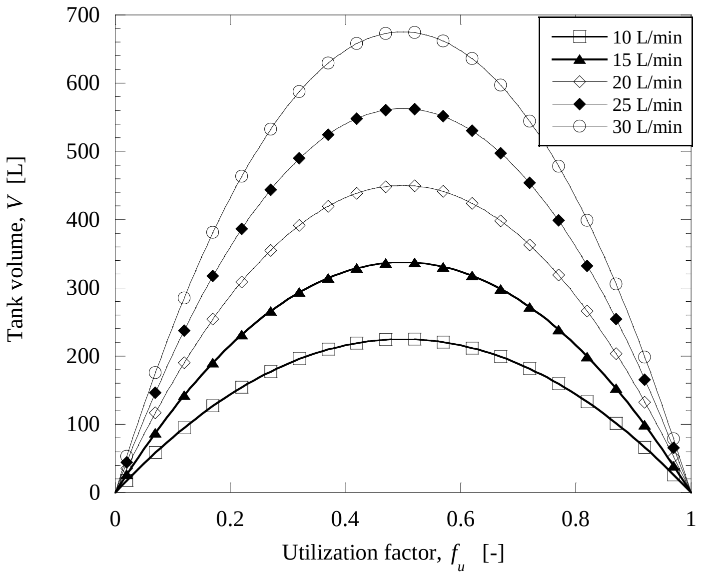

Figure 4. Tank volume as a function of the hot water utilization factor. Ambient temperature 15 ºC and hot water temperature of 45 ºC. Duration of the operating cycle, 1 hour.

and the situation corresponding to the absence of hot water tank, $f_u = 1 \Rightarrow V = 0$, leading to the tankless situation when the power input will be,

$$\dot{Q}_{\max} = \dot{m}_u c_L \left(T_u - T_o\right) \tag{25}$$

3. EXERGY ANALYSIS

An exergy analysis will now be carried out at the control volume under analysis. For an evolution between two time instants *t* and *t'*, so that $\Delta t = t' - t$, it can be written for a uniform flow that,

$$ {}_t\left(E_{xq}\right)_{t'} = J\left(t'\right) - J\left(t\right) + \sum_{out} m_i k_i - \sum_{in} m_i k_i + {}_t I_{t'} \tag{26}$$

where,

$$ {}_t\left(E_{xq}\right)_{t'} = \dot{Q}\left(1 - \frac{T_o}{T_H}\right)\Delta t \tag{27}$$

$$ J\left(t'\right) = m\left(t'\right) j\left(t'\right) = \rho_L V\ j\left(t'\right) \text{ and } J\left(t\right) = m\left(t\right) j\left(t\right) = \rho_L V\ j\left(t\right) \tag{28}$$

The choice of Eq. (27) for the definition of the thermal power exergy, assumes that the present analysis is only for a constant temperature heat source. The coenergy (Borel, 1986) is defined for an incompressible liquid as,

$$ j = \left(u - u_o\right) - T_o\left(s - s_o\right) = c_L\left(T - T_o\right) - T_o c_L \ln\left(\frac{T}{T_o}\right) \tag{30}$$

while the coentalpy (Borel, 1986) for an incompressible liquid is defined as,

$$ k = \left(h - h_o\right) - T_o\left(s - s_o\right) = c_L\left(T - T_o\right) - c_L T_o \ln\frac{T}{T_o} \tag{31}$$

So applying Eqs. (27) to (31), into equation (26) and considering that $\Delta T = T' - T$ and that $\Delta t = t' - t$

$$ \begin{aligned} &\dot{Q}\left(1 - \frac{T_o}{T_H}\right)\Delta t = \rho_L V\ c_L\left[\Delta T - T_o \ln\left(\frac{T + \Delta T}{T}\right)\right] + \\ &\dot{m}_u\left[c_L\left(T_u - T_o\right) - T_o c_L \ln\left(\frac{T_u}{T_o}\right)\right]\Delta t + {}_t I_{t'} \end{aligned} \tag{32}$$

and defining an average irreversibility power as,

$$ {}_t I_{t'} = \dot{I}\ \Delta t \tag{33}$$

$$ \begin{aligned} &\dot{Q}\left(1 - \frac{T_o}{T_H}\right) - \dot{I} + \dot{m}_u c_L\left[T_o \ln\left(\frac{T_u}{T_o}\right) - \left(T_u - T_o\right)\right] = \\ &\rho_L V\ c_L \frac{\Delta T - T_o \ln\left(\frac{T + \Delta T}{T}\right)}{\Delta t} \end{aligned} \tag{34}$$

Using the approximation that $\ln\left(1 + \frac{\Delta T}{T}\right) \approx \frac{\Delta T}{T}$ and considering that $\Delta t \rightarrow 0$,

$$ \begin{aligned} &\dot{Q}\left(1 - \frac{T_o}{T_H}\right) - \dot{I} + \dot{m}_u c_L\left[T_o \ln\left(\frac{T_u}{T_o}\right) - \left(T_u - T_o\right)\right] = \\ &\rho_L V\ c_L\left(1 - \frac{T_o}{T}\right)\frac{\mathrm{d}T}{\mathrm{d}t} \end{aligned} \tag{35}$$

Integrating this equation for the hot water consumption period, $\begin{matrix} t = t_u & T = T_f \\ t = 0 & T = T_i \end{matrix}$, and solving the resulting equation in order to the irreversibility, and using t_u from Eq. (5), $\dot{I}_u$ is obtained

$$\dot{I}_u = \dot{Q}\left(1 - \frac{T_o}{T_H}\right) + \dot{Q}\left[\frac{T_o}{T_f - T_i}\ln\left(\frac{T_f}{T_i}\right) - 1\right] + \dot{m}_u c_L T_o\left[\ln\left(\frac{T_u}{T_o}\right) - \left(\frac{T_u - T_o}{T_f - T_i}\right)\ln\left(\frac{T_f}{T_i}\right)\right] \tag{36}$$

On the other end during the t_a period there is only water heating, with its temperature raising inside the tank from T_f to T_i, and no hot water consumption, the exergy balance is simpler, and the result is,

$$\dot{I}_a = \dot{Q}\left(1 - \frac{T_o}{T_H}\right) - \dot{Q}\left[1 - \frac{T_o}{(T_i - T_f)}\ln\left(\frac{T_i}{T_f}\right)\right] \tag{37}$$

For the operating cycle $t = t_u + t_a$, it can be written that, $\dot{I} = f_u \dot{I}_u + (1 - f_u)\dot{I}_a$ and consequently combining Eqs. (36) and (37),

$$\dot{I} = \dot{Q}\left(1 - \frac{T_o}{T_H}\right) + \dot{Q}\left[\frac{T_o}{(T_f - T_i)}\ln\left(\frac{T_f}{T_i}\right) - 1\right] + f_u \dot{m}_u c_L T_o\left[\ln\left(\frac{T_u}{T_o}\right) - \frac{(T_u - T_o)}{(T_f - T_i)}\ln\left(\frac{T_f}{T_i}\right)\right] \tag{39}$$

Now considering once again that $T_f = T_u$, and using Eq. (19), the average irreversibility power is given by,

$$\dot{I} = f_u \dot{m}_u c_L T_o\left[\ln\left(\frac{T_u}{T_o}\right) - \frac{T_u - T_o}{T_H}\right] \tag{40}$$

T_i does not affect the overall irreversibility because in the last instance the heat will “drop down” until reaching T_u. Whatever is T_i, the water at this temperature will mix with the cooled water at T_o to reach T_u. The total irreversibility is dependent upon T_H, $T_u = T_f$ and T_o, while T_i is an intermediate step on the thermal energy transfer.

For a set of operating conditions (T_H, T_u, T_o, $\dot{m}_u$) the irreversibility increases with f_u. For $f_u = 0 \Rightarrow \dot{I} = 0$, see Fig. 5, because there is no hot water consumption, thus no water heating process, while for $f_u = 1$ the irreversibility is the higher for the above mentioned set of operating conditions,

$$\dot{I}_1 = \dot{m}_u c_L T_o\left[\ln\left(\frac{T_u}{T_o}\right) - \frac{T_u - T_o}{T_H}\right] \tag{41}$$

The above equation means that if no heat accumulation is being used a maximum of thermal power is required for water heating, Eq. (25), and consequently the exergy losses increase.

When the heat source is the joule effect electricity $T_H \to \infty$, then Eq. (40) becomes,

$$\dot{I}_{upp} = f_u \dot{m}_u c_L T_o \ln\left(\frac{T_u}{T_o}\right) \tag{42}$$

This Eq. (42) gives the upper limit of the expected values for the irreversibility of the process and it is the upper line in Fig. 5. The worst situation is for the combination of the use of electricity as heat source and $f_u = 1$,

$$\dot{I}_{\max} = \dot{m}_u c_L T_o \ln\left(\frac{T_u}{T_o}\right) \tag{43}$$

The minimum theoretical value for the heat source temperature is $T_H = T_i$, leading to a lower boundary for the irreversibility,

$$\dot{I}_{low} = f_u \dot{m}_u c_L T_o \left[\ln \left(\frac{T_u}{T_o} \right) - \left(\frac{T_u - T_o}{T_i} \right) \right] \tag{44}$$

This limit $T_H = T_i$, for $f_u \neq 0$, gives a condition of minimum irreversibility but for an infinite time as when the water is being heated in the tank through a high temperature source at this same temperature, the time required to achieve the temperature equilibrium in the water tank will be infinite.

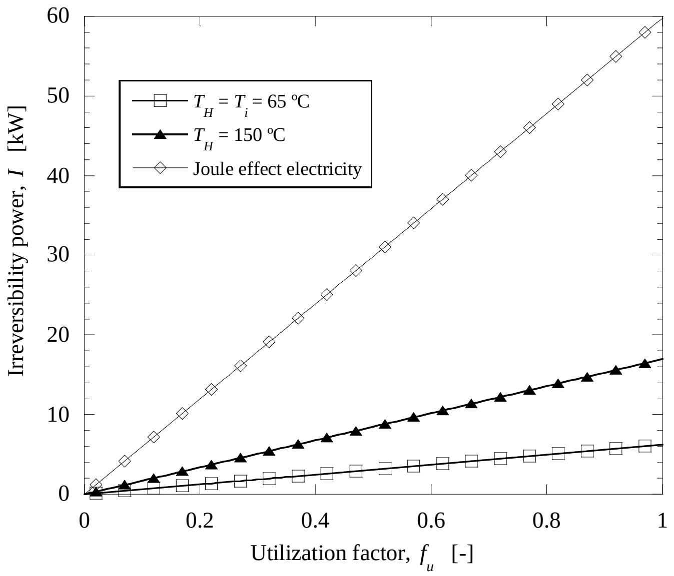

Figure 5. Irreversibility power values for $\dot{m}_u$ = 30 L/min, according to the heat source temperature.

4. CONCLUSIONS

Energy and exergy analysis of a hot water heating and storage system were carried out and it was demonstrated that for an adiabatic situation the use of a hot water storage tank minimizes the required thermal power input with clear advantages in term of the minimization of exergy destruction. It is also shown that the use of joule effect electricity augments the exergy losses and that the worst situation is the tankless joule effect electricity system. Simple equations for the sizing of the tank volume and thermal power input were also proposed.

5. REFERENCES

Armstrong, P., Ager, D., Thompson, I. and McCulloch, M., 2014a. “Domestic hot water storage: Balancing thermal and sanitary performance”. *Energy Policy*, Vol. 68, p. 334-339.

Armstrong, P.M., Uapipatanakul, M., Thompson, I., Ager, D. and McCulloch, M., 2014b, “Thermal and sanitary performance of domestic hot ater cylinders: Conflicting requirements”. *Applied Energy*, Vol. 131, p. 171-179.

Borel; L., 1986, *Thermodynamique et Énergétique*. Presses Polytechniques Romandes, Lausanne, Suisse, 1ére edition.

Dammel, F., Winterling, J., Langeheinecke, K.-J. and Stephan, P., 2012, “Exergy Analysis of a Water Heat Storaage Tank”. *Proceedings of the 2012 COMSOL Conference*, Milan, Italy.

Dincer, I. and Rosen, M., 2011, *Thermal Energy Storage Systems and Applications*. John Wiley and Sons, Chichester, UK.

Fernández-Seara, J., Uhía, F.J. and Sieres, J., 2007a, “Experimental analysis of a domestic electric hot water storage tank. Part I: Static mode of operation”. Applied Thermal Engineering, Vol. 27, p. 129-136.

Fernández-Seara, J., Uhía, F.J. and Sieres, J., 2007b, “Experimental analysis of a domestic electric hot water storage tank. Part II: Dynamic mode of operation”. Applied Thermal Engineering, Vol. 27, p. 137-144.

Solé, C., Medrano, M., Castell, A., Nogués, M., Mehling, H. and Cabeza, L.F., 2008, “Energetic and exergetic analysis of a domestic water tank with phase change material”. International Journal of Energy Research, Vol. 32, p. 204-214.

6. RESPONSIBILITY NOTICE

The author is the only responsible for the printed material included in this paper.