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NUMERICAL ANALYSIS FOR THE PROBLEM OF STEADY NATURAL CONVECTION HEAT TRANSFER BY DOUBLE DIFFUSION FROM A HEATED CYLINDER BURIED IN A SATURATED POROUS MEDIA

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Abstract. *In this paper, numerical solutions are presented for the steady natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium where both, the cylinder and the medium surfaces, are kept at constant uniform temperature and concentration. This situation occurs, for instance, in control of dissemination of pollutant contaminants from repository sites of industrial chemical waste and radioactive waste that provide solutions for problems in subject of radioactive scattering on soils, the contamination of water sheets, where knowledge of these mechanisms means have conditions for a better estimation of scattering and disseminations. Governing equations are expressed in bi-polar coordinates in the stream function formulation and handled numerically by a control volume method. Heat and mass transfer are studied as a function of Rayleigh, Lewis and the buoyancy ratio numbers.*

Keywords: *Heat and mass transfer, natural convection, steady solution, cylinder buried*

1. INTRODUCTION

Many transport processes presented in the environment occur due to the flow off with a simultaneous occurrence of temperature and concentration gradients. Some oceanographic phenomena like the salt sources are explained by the coupled presence of thermal gradients and saline. There is also an explanation for the dissemination control of pollutants contaminators proceeding from chemical industrial refuse replaceable places and radioactive waste that solves the problem in the area of radioactive spreading out in the ground, in water contamination and in other correlates that still ask for solution such as Chaves and Trevisan (1992).

In literature is found some works to solve heat transfer problems in steady state, such as Bau (1984) that presented analytical solutions to natural convection in cases of Rayleigh numbers smaller than 1, which the convection is induced by a cylinder heated in a saturated and permeable porous medium, where both, cylinder and ground, are kept in a constant temperature.

A study about steady natural convection promoted by double diffusion in saturated porous medium was done by Chaves (1990). This study proposed a numerical solution to variations of Rayleigh number (0 to 1,000), of Lewis number (0 to 100) and of the buoyancy ratio number (-3 to +3), using the control volume method idealized by Patankar (1980). This method has been widely used and its implementation to a bi-polar coordinates system was realized.

Pantakar's method was applied to the case of transient natural convection heat transfer by double diffusion from heated cylinders buried in saturated porous medium by Chaves (1990). The transient study was motivated by some simplifications made by Freitas and Prata (1992) in the problem of heat and mass transfer around electrical cables buried in porous medium, where a transient analyses was done, however effectuated a numerical simplification taking that the cable had square shape transversal section, solving the problem of buried cylinders.

Natural convection applied to heated cylinder buried can be found in Chaves *et al.* (2005).

Numerical solutions for the transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium where both, the cylinder and the medium surfaces, were kept at constant uniform temperature and concentration was presented by Chaves *et al.* (2005), where heat and mass transfer are studied as a function of Rayleigh, Lewis and the buoyancy ratio numbers.

The steady natural convection in a square cavity filled with a porous medium with the non-Darcy model (Darcy-Forchheimer model) was numerically studied (Saeid and Pop, 2005) and it was found that increasing the inertial effects parameter leads to a slowdown in the natural convection currents in the cavity and reduces the average Nusselt number for constant values of Rayleigh number.

The problem of the steady natural convection boundary layer flow over a downward-pointing vertical cone in porous media saturated with non-Newtonian power-law fluids under mixed thermal boundary conditions was studied by Cheng (2009) and the generalized governing equations derived can be applied to the cases of prescribed surface temperature and prescribed heat flux.

The steady natural convection in a partially opened enclosure filled with porous media using the Brinkman-Forchheimer model was examined by Oztop *et al.* (2011) and the results of this study can be used in the design of an effective cooling system for electronic components to help ensure effective and safe operational conditions.

This paper is a continuation of the study made by Chaves *et al.* (2005) and Chaves *et al.* (2015), applied to the case of the problem of steady natural convection heat transfer by double diffusion from heated cylinders buried in saturated porous medium presented by Bau (1984).

This paper aims to present numerical solutions for the problem of steady natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium, exposed to constant uniform temperature and concentration in the cylinder and in the medium surface.

2. PROBLEM ANALYSIS

Consider an infinite cylinder buried in a saturated porous medium. The cylinder has a radius r_1 and is buried in d depth from the porous medium superior surface. The cylinder outside cover is kept at T_W temperature and at C_W concentration, constant while the porous medium superior surface is kept at T_S temperature and at C_S concentration, according to Figure 1. The hypothesis of transient regime and impermeable wall are considered as well.

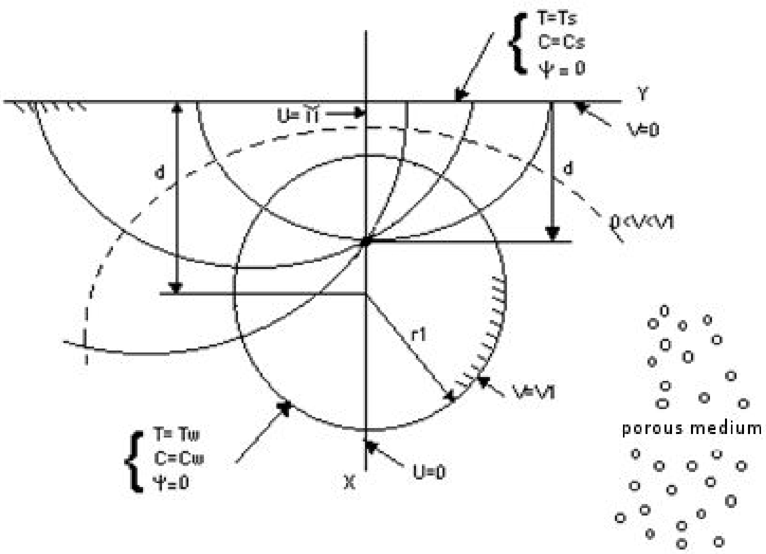


Figure 1. Bi-polar coordinates system

To obtain the equations that describe the problem, it is assumed that (Chaves, 1990):

a) the porous medium and the fluid that saturates it are isotropic and homogeneous; the Boussinesq approximation is valid to intensities variation due to changes in both temperature and concentration. It is possible, this way, to express the specific mass by Bird *et al.* (1976), Eq. (1):

$$\rho = \rho_0 \cdot \left[1 - \beta \cdot \left(T - T_0\right) - \beta_c \cdot \left(C - C_0\right)\right], \tag{1}$$

where β and β_c are the coefficients of thermal and chemical expansion, respectively defined by:

$$\beta = \frac{-1}{\rho_0} \cdot \left(\frac{\partial \rho}{\partial T}\right)_p \text{ and } \beta_c = \frac{-1}{\rho_0} \cdot \left(\frac{\partial \rho}{\partial C}\right)_p$$

b) Darcy law is assumed to describe the fluid flow in porous medium, this way, by Bejan (1984) is possible to reach Eq. (4);

$$\left(\frac{\mu}{K}\right) \nabla \times \bar{V} = -(\nabla \rho) \times \bar{g} \tag{2}$$

c) the porous medium is rigid and the thermodynamics proprieties (except the density in the buoyancy ratio term) is considered constant;

d) there are no chemical reactions and the viscous dissipation are negligible;

e) the porous medium and the fluid presents thermodynamic equilibrium.

Based on considered hypothesis for the Eqs. (1) and (2), the problem governing equations of transient natural convection heat transfer by double diffusion from a heated cylinder buried in a saturated porous medium can be written for transient regimes and incompressible fluids, in bi-polar coordinates (u, v) presented on Figure 1 in dimensionless terms in the formulation of the stream function Chaves *et al.* (2005) by, Eqs. (3) to (5).

$$\nabla^2 \Psi^* = a^* \left[\left(H \frac{\partial T^*}{\partial u^*} + G \frac{\partial T^*}{\partial v^*}\right) + N\left(H \frac{\partial C^*}{\partial u^*} + G \frac{\partial C^*}{\partial v^*}\right)\right] \tag{3}$$

where:

$$H = \frac{1 - \cos u^* \cosh v^*}{(\cosh v^* - \cos u^*)^2}, \quad G = \frac{\sinh v^* \sin u^*}{(\cosh v^* - \cos u^*)^2}, \quad V_u^* = \frac{1}{h_v} \cdot \frac{\partial \Psi^*}{\partial v^*}, \quad V_v^* = -\frac{1}{h_u} \cdot \frac{\partial \Psi^*}{\partial u^*} \quad \text{and}$$

$$h_u = h_v = \frac{a^*}{(\cosh v^* - \cos u^*)}$$

$$\nabla^2 T^* = Ra\left(\frac{\partial \Psi^*}{\partial v^*} \cdot \frac{\partial T^*}{\partial u^*} - \frac{\partial \Psi^*}{\partial u^*} \cdot \frac{\partial T^*}{\partial v^*}\right) \tag{4}$$

$$\nabla^2 C^* = RaLe\left(\frac{\partial \Psi^*}{\partial v^*} \cdot \frac{\partial C^*}{\partial u^*} - \frac{\partial \Psi^*}{\partial u^*} \cdot \frac{\partial C^*}{\partial v^*}\right) \tag{5}$$

where:

$$a^* = \frac{a}{r_1} = \sinh v_1, \quad d^* = \frac{d}{r_1} = \cosh v_1, \quad T^* = \frac{T - T_s}{T_w - T_s}, \quad C^* = \frac{C - C_s}{C_w - C_s}, \quad \Psi^* = \frac{\Psi}{(\alpha Ra)}, \quad N = \frac{\beta_c \cdot \Delta C}{\beta \cdot \Delta T},$$

$$Le = \frac{\alpha}{D}, \quad Ra = \frac{K \cdot g \cdot \beta \cdot \Delta T \cdot r_1}{(\nu \cdot \alpha)}, \quad \Delta T = T_w - T_s \quad \text{and} \quad \Delta C = C_w - C_s$$

3. METHODOLOGY

To solve the problem numerically, it integrates the Eqs. (3) to (5) related to u and v variables, already described in dimensionless bi-polar coordinates on a generic volume control. Such dominion is described on Figure 2 and the integration is done following the volume control method formulation developed by Patankar (1980) where potential law schematic is taken, to calculate the flux term through the limits of each internal control volume.

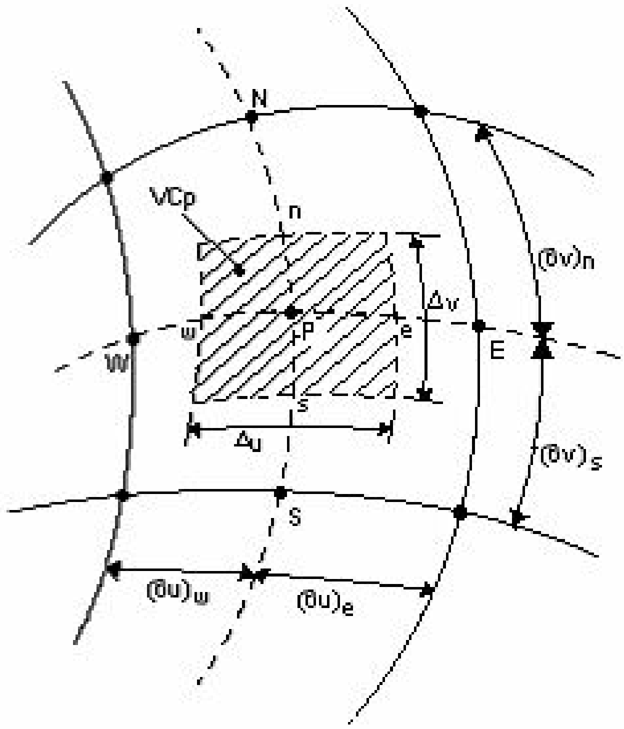


Figure 2. Typical cell of the volume control method

For dimensionless terms, in the new coordinate system (u, v) is established the following boundary as the dominion shows in Figure 1:

$u = 0$ and $v_1 < v \le 0 \Rightarrow \Psi^* = 0$ (6)

$u = \pi$ and $v_1 < v < 0 \Rightarrow \Psi^* = 0, \; \dfrac{\partial T^*}{\partial v} = \dfrac{\partial C^*}{\partial v} = 0$ (7)

$v = v_1$ and $0 \le u \le \pi \Rightarrow \Psi^* = 0, T^* = C^* = 1$ (over the buried cylinder) (8)

$v = 0$ and $0 \le u \le \pi \Rightarrow \Psi^* = 0, T^* = C^* =0$ (over floor surface) (9)

Boundary conditions, presented at Eqs. (6)-(9) refers to the flow off domain covering the heated cylinder. The condition $\Psi^* = 0$ refers to the stagnated fluid. The condition $\dfrac{\partial T^*}{\partial v} = \dfrac{\partial C^*}{\partial v} = 0$ refers to the mass and fluid absence.

Distinguished equations together with boundary and initial conditions make a coupled system involving stream function, temperature and concentration variables. The numerical solution in this system is treated using simple schematic purposed by Patankar (1980). To solve this simultaneous mathematical equations that come from distinguish process, is used line-to-line iterative method.

In each process, interaction there was a need for updating ψ^*, T^* and C^* values and ψ^* equation was solved three times for each interaction. To reach T* and C* values it was solved only once by iteration. Such process was widely useful in cases where Ra and Le are high, that causes stronger convection streams.

The acceptance standard of a solution as converged is based on the maximum mistake possible inside the whole calculation range. The obtained results convergence was accepted when relative changes in the dependent variables were below 1 x 10^{-5}.

4. RESULTS AND DISCUSSION

A flow class basically dominated by thrust due the cylinder heating was considered in this section. In this case, N << 1 and the distribution of chemical constituent have a small influence on fluid movement. The value of N was fixed on zero for simulation purposes. In this boundary, temperatures field is linked to flow field, but is independent of concentration distribution. Both fields are dependent of Rayleigh number, but are not dependent of Lewis number. However, chemical constituents' distribution depends of both fields. So, concentration field is influenced by both numbers, Rayleigh and Lewis, and the difference between temperature and concentration fields is dictated by Lewis number. How it can be shown from Eqs. (3) and (5), both fields are identical for Lewis equal to 1 and only one of these equations is sufficient to describe the problem of this singular case.

In Figure 3 can observe the influence of Rayleigh on the Nusselt numbers in the cylinder and the wall. Heat exchange with the cylinder increases monotonically with the Rayleigh number. This behaviour also occurs for Nusselt in the wall.

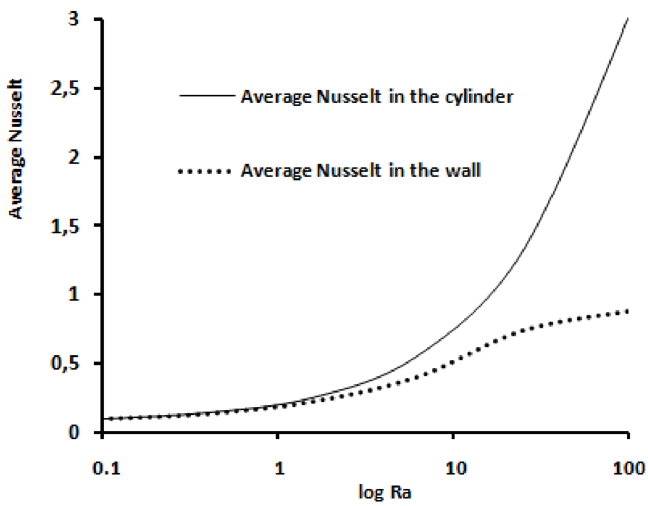


Figure 3. Influence of Ra on Nu in the cylinder and in the wall for N = 0 and Le = 1

Figure 4 shows the influence of Rayleigh and Lewis parameters on the average Sherwood number of the flow for N = 0, where it can see the same Sherwood behaviour as a function of Rayleigh and Lewis.

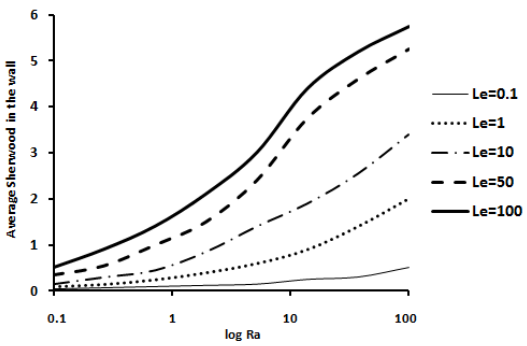


Figure 4a. Influence of Le and Ra on Sherwood in the cylinder (N = 0)

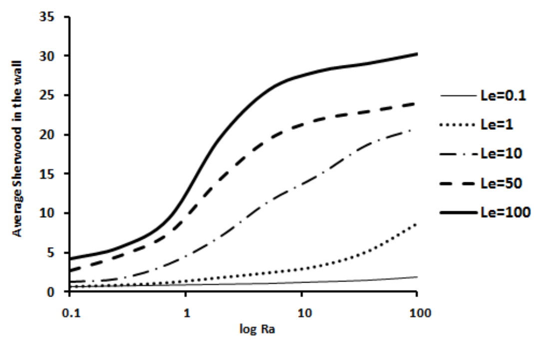


Figure 4b. Influence of Le and Ra on Sherwood in the wall (N = 0)

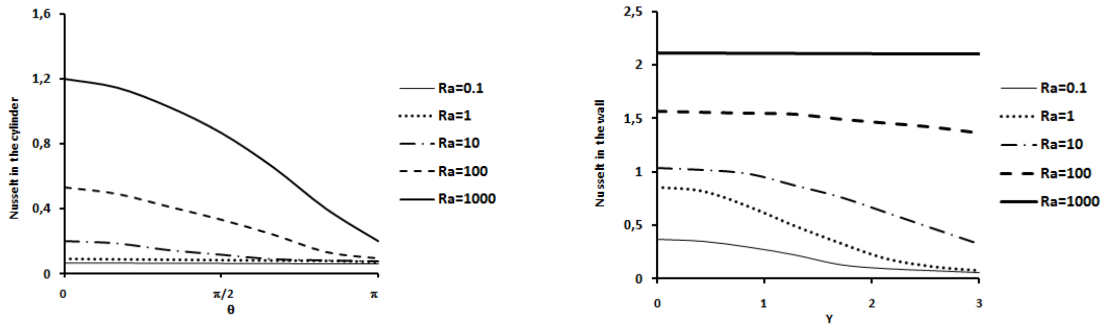
Figure 4a shows that for low Rayleigh, the conductive effect is predominant, for every value of Lewis. Similar effect can be observed in Figure 4b where the small influence of Rayleigh in the range of low values of the Lewis number. Figure 4a shows the monotonically behaviour of the average Sherwood curves. The results obtained with

Le = 0.1 already approximate closely to the pure conduction effect, that is Sh = 1, where it can also be noted that curves for Le = 50 and Le = 100 are quite close and parallel indicating a fully convective flow.

Figure 4b confirms previous observations. It will be seen also from this figure that for Ra = 1,000 the predominant effect on the convection flow is indicated by a linear curve with variation of Lewis number.

Figure 4b shows the monotonically behaviour of Sherwood curves in the wall due to Rayleigh. Such behaviour is very similar to that shown in Figure 9a for Sherwood in the cylinder due to Rayleigh.

Figure 5 shows the distribution of the local Nusselt around the cylinder (Figure 5a) and along the wall (Figure 5b) for a broad range of Rayleigh numbers.



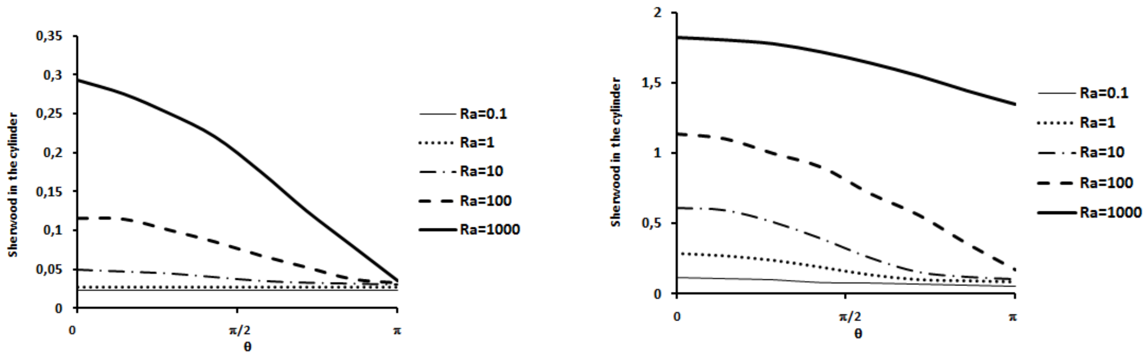
(a) (b)

Figure 5. Influence of Ra on the distribution of local Nu (N = 0): a) in the cylinder; b) in the wall

Figure 5a showed that the local Nusselt numbers are higher in the cylinder region opposite to the wall (θ = 0). Although the fluid movement in this region is smaller, the fluid is completely cold to come into contact with it causing a high gradient. For all values of Rayleigh considered the local Nusselt number decays continuously as the fluid moves toward the plume region. Figure 5a also shows that the value for Ra to 10, the transfer of cylinder heat is predominantly conductive.

Figure 5b showed that for large values of Rayleigh there is a tendency for a constant heat exchange in the wall, which in this case, it is important to note that this is a wall with constant temperature imposed.

Figure 6 shows the distribution of the local Sherwood along the cylinder for Le = 0.1 and 10.



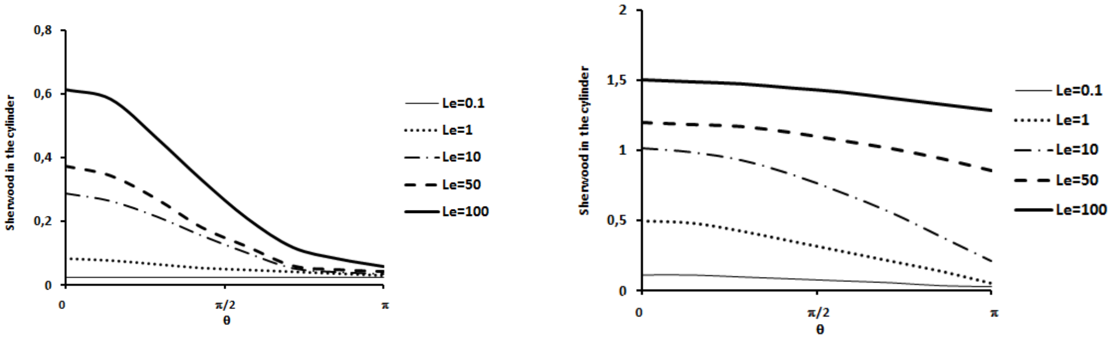
(a) (b)

Figure 6. Influence of Ra on local Sherwood distribution in the cylinder for: a) Le = 0.1 and N = 0; b) Le = 10 and N = 0

It can observe that for Ra approximately of 10 with Le = 0.1, the diffusive characteristic is modified as shown in Figure 6a. With increasing of Lewis, this limit drops for Ra = 0.1 with Le = 10, seen in Figure 6b.

Figure 6a showed that for low Lewis even with high Rayleigh it is difficult to achieve purely convective effects across the surface of the cylinder. The change is noted in Figure 6b for Le = 10.

Figure 7 shows the influence of the Rayleigh distribution on the local Sherwood distribution in the cylinder which is similar to that observed in Figure 6 for varying of the Lewis number. For the case N = 0 is noted the great similarity of Rayleigh and Lewis influences on the distribution of local Sherwood in the cylinder.



(a) (b)

Figure 7. Influence of Le on local Sherwood distribution in the cylinder for: a) Ra = 1 and N = 0; b) Ra = 100 and N = 0

5. CONCLUSIONS

The study of doubly diffusive natural convection for a cylinder buried in a homogeneous porous and saturated medium was investigated numerically.The equations describing the problem were expressed in bipolar coordinates according to stream function formulation and numerically solved by the method of control volume. The program implemented allowed to achieve satisfactory results and enabled a better understanding of the influence of Rayleigh and Lewis numbers on flows driven by heat. The Nusselt and Sherwood numbers as a function of Rayleigh, Lewis and the ratio of thrust N were widely analysed and their effects relatively explained within the range of values specified in this work. The results for the heat driven flow directed allowed basically demonstrating purely conductive, conductive and convective, and strongly convective regions for different values of Rayleigh and Lewis considered.

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