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EXPERIMENTAL RESEARCH ABOUT THE FIRST NATURAL FREQUENCY OF LARGELY DEFORMED BEAMS

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Abstract. *This paper presents a study about the first vibration mode of largely deformed cantilevered beams. This study is important because the vibration of beams with this level of deformation occurs from a position that differs to a great extent from the undistorted configuration. Thus, a component of the generalized force starts to appear that changes the stiffness of the system as a function of the deflection assumed by the piece. As the deformation of the rod increases, the component of the lateral force that acts tangentially to the curved shape that the beam assumes is also increased, changing its stiffness and influencing the natural frequency of the system. This experimental study shows that as the deflection of the beam increases, the difference between the results of the frequency considering the straight shaft and those computed from the deformed position increases with the length of the piece. However, the downward trend is interrupted and the frequency rises slightly, precisely indicating the moment in which the said component starts to influence the stiffness of the bending piece. A computer model to obtain the natural frequency of the beam was then prepared using the Finite Element Method in order to follow the experimental results in two ways. The first considered the beam in its horizontal configuration. The second was based on nonlinear static analysis processing that was previously performed with the large displacement theory; the second method results show a good approximation with the experimental values.*

Keywords: *Experimental Research, Beams, Geometric Stiffness, Fundamental Frequency, Large Deformed.*

1. INTRODUCTION

The vibration analysis response of framed structures that are modeled as beams and columns has been studied by many researchers; the response continues to be studied extensively. The dynamic bending of beams, also known as the flexural vibration of beams, was first investigated by Daniel Bernoulli in the late 18^{th} century. Bernoulli's equation of motion of a vibrating beam tended to overestimate the natural frequencies of beams and was marginally improved by Rayleigh (1877) with the addition of a mid-plane rotation. Timoshenko (1921-1922) improved the theory further by incorporating the effect of shear on the dynamic response of bending beams. A study about Timoshenko`s contribution has been presented by T. Kaneko (1975) and H.E. Rosinger-Ritchie (1977). This allowed the theory to be used for problems involving high frequency vibrations where the dynamic Euler-Bernoulli theory is inadequate. The Euler-Bernoulli and Timoshenko theories for the dynamic bending of beams continue to be widely used by engineers.

However, is important to consider that in the function of structural system characteristics, the applied loads can have a significant effect on the response of the structure when it is subjected to dynamic loading. For this reason, it is important to evaluate the influence of stiffness changes in the dynamic response of the structure because they may be caused by the forces applied to it, including the calculation of the frequency by means of known geometric stiffness. In general, the compression force tends to reduce the stiffness of the structural system, while the tension forces produce the opposite effect, i.e., tension forces increase the stiffness.

In the case of a cantilevered beam, the geometric stiffness is characterized by the action of a normal force over the deformed element, which is responsible for the stiffness changes in the beam. A transversely loaded beam is deformed by the action of the active forces. For this reason, at the same time that the rod extends, the difference between the deformed and undistorted configuration also increases. If the loaded beam is excited, it vibrates, and the oscillatory movement occurs from the deformed configuration, even at small amplitudes, as illustrated in Figure 1.

The situation described previously indicates that one should consider that oscillatory motion occurs, taking into account the curved shape about which the bending beam deforms. In this case, the vibration system is established around the deformed configuration of the beam. Thus, the efforts that were perpendicular to the straight beam shaft at the start are altered and now show a normal component that changes the system stiffness, influencing the determination of the natural frequency of the vibration.

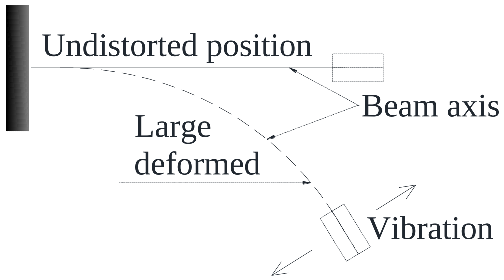


Figure 1. Large deformation of the beam from which the oscillatory movement occurs

Beams and columns constitute a continuous system with infinite degrees of freedom; for that reason, the development of equations to understand it are generally complex and are difficult to practically apply. An alternative to study the behavior of these systems is the use discretization techniques, in which the structure is transformed into subsystems that are defined by points called joints. This technique is employed in numerical solutions, such as the Finite Element Method (FEM); this technique can be found today in several commercial and modern software packages.

The purpose of this article is to present an experimental study of the first natural vibration of largely deformed cantilevered beams that stay under the influence of their geometric stiffness. The evaluation of the experimental results is made by a comparison of the Finite Element Method with modal analysis for eigenvalues that are obtained on the modified stiffness of the beam after large static displacement processing that is realized as indicated by SAP 2000 (2015).

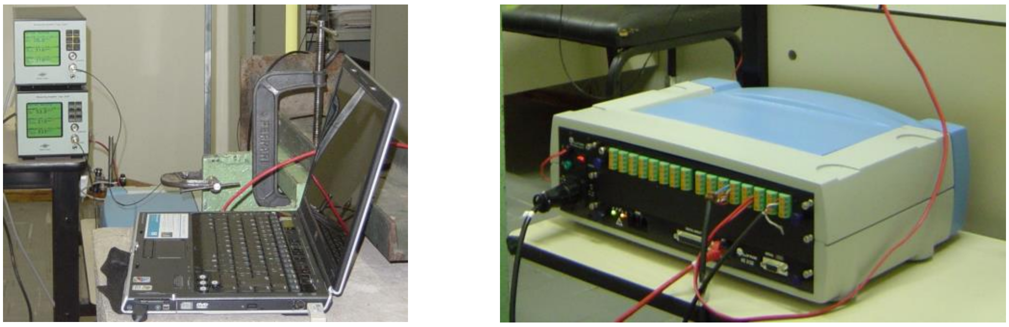
2. DESCRIPTION OF THE EXPERIMENTAL ACTIVITY

Electrical strain gages and piezoelectric accelerometers were used. The former were manufactured by Excel Sensory (2015), and the latter were manufactured by Brüel & Kjaer (2015). The arrangement that was adopted for connecting the extensometers to the data acquisition system and the characteristics of the equipment were as follows in Table 1. The accelerometers were calibrated using a Brüel & Kjaer type 4294 manual caliper driver; they were connected to the acquisition system through a differential tension configuration with a gain of 1. The connection of the accelerometers to the data acquisition system was preceded by connecting the accelerometer to the Brüel & Kjaer type 2525 amplifier. The ADS-2000 automatic data acquisition system AqDados (2013) was used with AI-2161 conversing plates, an AC-2122VA controlling plate (LYNX Informatics) and 16-bit resolution. Fig. 2 shows the set of data acquisition.

Table 1. Characteristics of the sensors.

Dispositive							
Strain gage	Resistance	Factor	Arrangement	Gain		Excitation tension	
	120 Ω	2.1	¼ 3-wire bridge	2000		5 V	
Accelerometer	Model	Sensitivity	Frequency interval	Resonance frequency	Residual noise level	Maximum operational level	Mass
	4393	3.1 pC/g	0.1 at 16500 Hz	55 kHz	0.52 g	5 g	2.4 grams
	4371	10 pC/g	0.1 at 12600 Hz	42 kHz	0.24 g	6 g	11 grams

Interfacing with the microcomputer was achieved through Ethernet networking. The connection of the sensors to the data acquisition system was achieved through input connectors located at the rear of the equipment. The test sample consisted of a nominally 1/2" (12.70 mm) by 1/8” (3.17 mm) flat metal bar that had two metallic masses fixed to its free extremity by lateral pressure. Its mass and the masses of the accelerometers and their magnetic bases resulted in a total of 1,595 grams on the top of the rod. Because the model of longitudinal elasticity was a steel piece, it was assumed to be 205 GPa. The density of the rod material was experimentally determined in a materials laboratory using the helium picnometry technique. The relative density obtained was 8.19 (8190 kg/m^3). The other masses involved were measured using an electronic scale. The sample test was instrumented with three extensometers and two accelerometers, according to the layout shown in Fig. 3. The extensometers were glued to the extension of the beam, and the accelerometers were attached to the magnetic bases.



(a) Amplifiers and portable computer (b) Data acquisition system (ADS)

Figure 2. Set up of the acquisition system

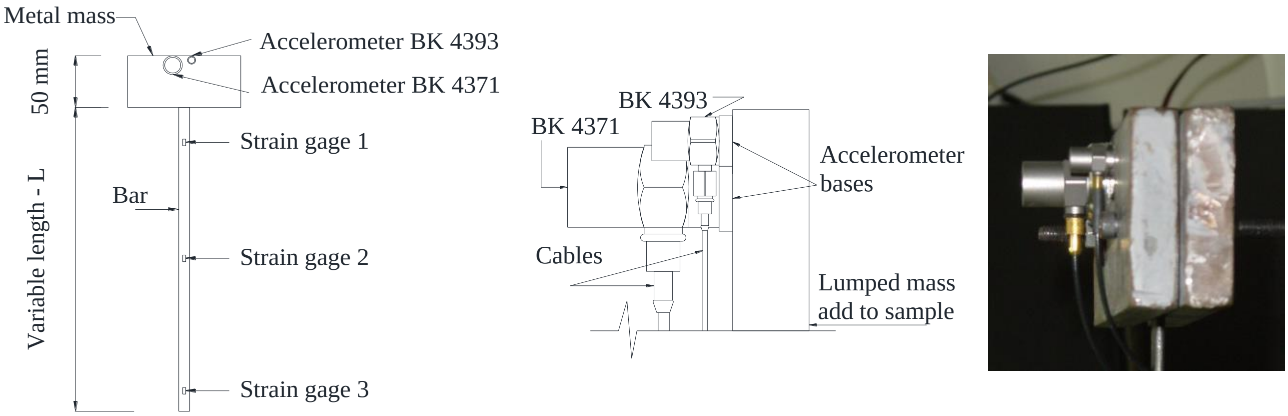


Figure 3. Instrumentation of the test sample and details of the accelerometer settings

With the metallic masses added to the rod, a beam position was adopted and the set was installed in the horizontal position as a cantilever beam. Figure 4 illustrates how the experimental activity was executed.

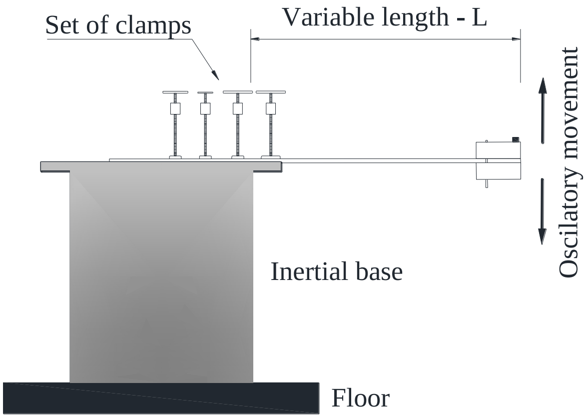


Figure 4. Position adopted in the test

The test sample was fixed to the supporting device by metal clamps. The same fixation pattern was used for all of the models. The contact surface of the inertial base was carefully prepared to reduce imperfections and roughness. The accelerometer cables were fixed to prevent signal interference. The support ensemble provided safe inertial conditions for carrying out the tests. The reference experimental length (L) was visually controlled and measured using a metallic tapeline to compensate for the uncertainty in the real fixation point of the models on the base and in the real position where the vertical lumped force was applied, as shown in Fig. 5 and Fig. 6. The same references were maintained for the different positions. The length varied by 5 cm up to the physical limit of the possible fixation. In Fig. 7, the initial configuration of the beam, for some positions, is presented before start of the vibration.

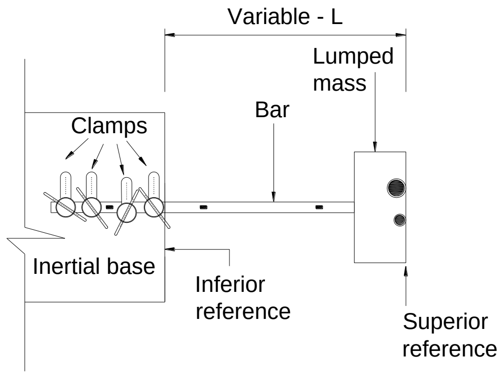


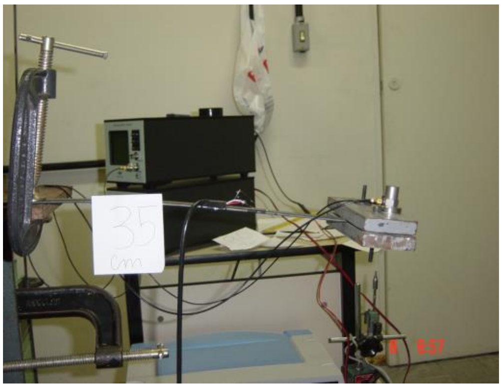
Figure 5. Scheme to define the length of the models – view from the top

(a) Details of the fixation set

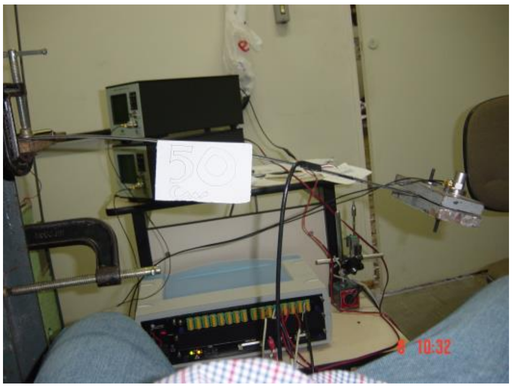
(b) Plate base of fixation set

(c) Zoom - Inferior reference

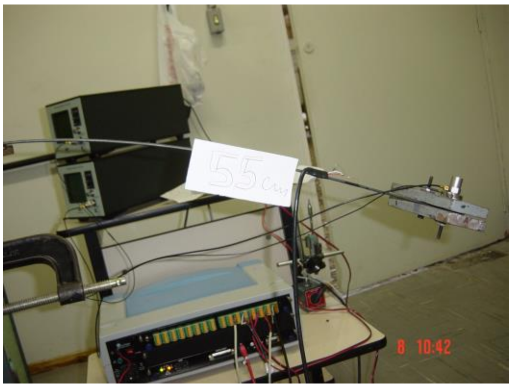
Figure 6. Establishing the reference samples



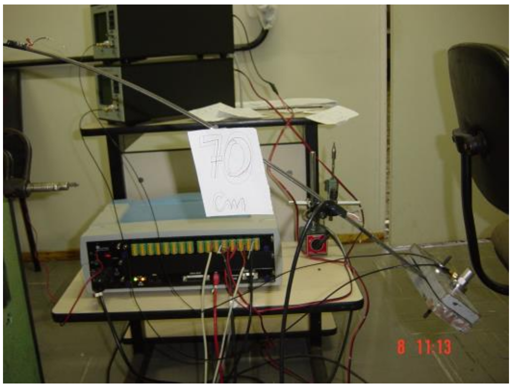
(a) L = 35 cm



(b) L = 50 cm



(c) L = 55 cm



(d) L = 70 cm

(e) L = 85 cm

(f) L = 90 cm

Figure 7. Deformed samples

In both tests, models with different lengths were excited by a random force of sufficient magnitude to set the system into oscillatory motion. After the excitation, the systems oscillated around the initially deformed position. The time-response signals were recorded and subsequently analyzed. The fundamental frequency of the models was obtained using the *Fourier* transform in the AqDAnalysis 7 (2013) program. The auto-spectrum of the program was configured as follows: a Hanning-type compensation window; a data window for calculating the average spectrum; a FFT (Fast-Fourier Transform) zoom equal to 1; and the maximum resolution possible for the number of samples. It is important to note the conclusions of Carneiro (1996), who states that for small amplitudes, both experience and theoretical solutions show that the influence of the initial displacement in relation to body length is negligible and that the influence of damping on the vibration period can generally be ignored.

3. DISCUSSION AND RESULTS

The objective of this work is to experimentally research the first vibration frequency of isolated cantilever beams, emphasizing that activity, and to process a mathematical procedure using a Finite Element Method to calculate the corresponding frequency. For this purpose, a set of dynamic tests was conducted in the laboratory using the elastic and geometric parameters of flat metallic bar samples as beams, varying their length from 0.20 m to 0.9 m at short intervals; the deformation is shown in Fig. 8 and Table 2. Specifically, this work addresses the identification of the first natural frequency in cantilevered beams with large deformations that are caused by geometric effects or when the stiffness of the beam is altered by a transversal force component acting tangentially to the beam axis, including its own weight, which generates a tension for each position in the beam.

Some researchers that have studied the vibration problem in large displacements, including Yaman (2007), who used the Adomian decomposition method to determine the vibrations of beams/columns with a variable rotation relative to the initial straight axis, obtaining results that are compatible with the finite elements method. Additionally, Jurjo *et al.* (2010) performed tests to verify the efficacy of a computational vision system in studying the dynamic behavior of a clamped-free slender metallic column subject to its own weight under both tension and compression. Ding *et al.* (2014) examined the effect of using an independent finite rotation field in the large displacement analysis of beams, and Lafontaine e*t al.* demonstrated that the formulation of a stable, OSS-based, mixed explicit strain/displacement formulation for the solution of linear and non-linear problems is solid mechanics.

Differences between the experimental study and the Finite Element Method are shown in Table 2. The differences between columns (2), (3) and (4) highlight the influence of the component of the vertical force acting tangentially to the elastic line of the beam, revealing its influence on the calculation of the fundamental vibration frequency. The percentage difference between the results from the linear and nonlinear case, columns (2) and (3), shows a maximum percentage of 20.56 % in the longest system. In this case, the vertical displacement of the deformed configuration differs significantly from the horizontal reference. For that reason, it is necessary to consider the tangential component of the vertical force that acts on the bent beam.

By using a computer procedure it is possible to consider the influence of that component over the bending shape of the beam. This procedure should be used for engineering applications requiring geometric nonlinear analysis, presenting the need for appropriately utilizing computational resources. This means that the eigenvalue analysis by the Finite Element Method (FEM) must be based on the stiffness changed by nonlinear static analysis, which represents the experimental condition that is present in this work. That condition is known as large displacement analysis and considers all of the equilibrium equations in the deformed geometrical configuration of a structure undergoing large deformations. The details of the computational processing that was performed by that method is summarized below and was conducted to serve as a reference to the experimental study.

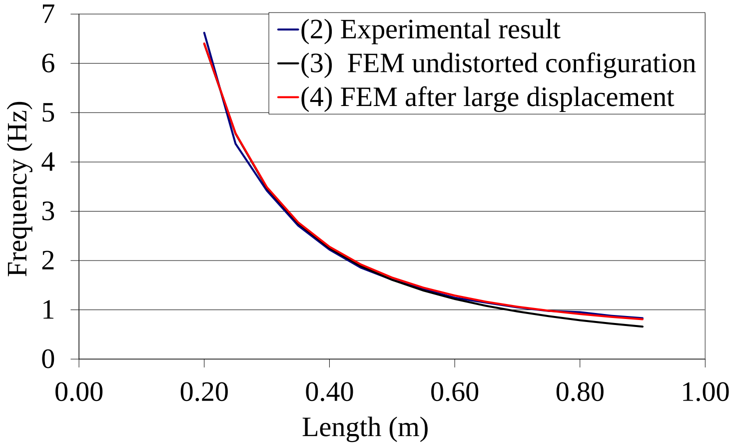


Figure 8. Results from the experiment and FEM

Table 2. Frequencies in Hertz.

(1) L (m)	**(2)** Experimental result	Finite Element Method (FEM)		Differences in relative values		
		(3) Undistorted configuration	**(4)** After large displacement	Column **(2) e (4)**	Column **(2) e (3)**	Column **(3) e (4)**
0.20	6.6230	6.3989	6.4038	3.42%	3.38%	0.08%
0.25	4.3700	4.5733	4.5819	4.62%	4.65%	0.19%
0.30	3.4180	3.4749	3.4884	2.02%	1.67%	0.39%
0.35	2.7100	2.7543	2.7741	2.31%	1.63%	0.72%
0.40	2.2220	2.2517	2.2792	2.51%	1.33%	1.22%
0.45	1.8550	1.8848	1.9214	3.46%	1.60%	1.94%
0.50	***1.6110***	**1.6073**	***1.6544***	2.62%	0.23%	2.93%
0.55	***1.4160***	**1.3915**	***1.4502***	2.36%	1.73%	4.21%
0.60	***1.2450***	**1.2198**	***1.2910***	3.56%	2.02%	5.83%
0.65	***1.1470***	**1.0805**	*1.1648*	1.53%	5.79%	7.80%
0.70	***1.0500***	**0.9657**	*1.0634*	1.26%	8.03%	10.12%
0.75	***0.9770***	**0.8698**	*0.9810*	0.40%	10.98%	12.79%
0.80	***0.9520***	**0.7886**	***0.9130***	4.27%	17.17%	15.78%
0.85	***0.8790***	**0.7192**	***0.8563***	2.65%	18.18%	19.06%
0.90	***0.8300***	**0.6593**	***0.8084***	**2.67%**	**20.56%**	**22.61%**
			Average	2.64%	6.60%	7.04%

In large displacement analysis, the equilibrium equations are written in the deformed configuration of the structureand the solution of the problem may require the completion of an iteration until convergence. As mentioned by Aviram *et al.* (2008), large displacement analysis is sensitive to convergence tolerances that are established by the user. In Table 2, the data in columns (3) and (4) represent values from analysis with eigenvalues that were obtained computationally using FEM assuming an isotropic, homogeneous material with linear elastic behavior with the parameters used in the experimental investigation in addition to a Poisson's ratio equal to 0.3. All of the samples have been modeled as a bar element containing 500 parts. It is important to highlight that the results presented in column (4) refer to the modal analysis that was processed after nonlinear static analysis with the assumption of large displacements by means of a *Lagrangian* solution, as indicated in SAP 2000 (2015). The large displacement effect in SAP 2000 (2015) includes only the effects of large translations and rotations that are processed together with the geometric stiffness matrix. The strains are assumed to be small in all elements, and this analysis considers the equilibrium equations in the deformed configuration of the structure. Large displacements and rotations are accounted for, but strains are assumed to be small. This means that if the position or orientation of an element changes, its effect on the structure is accounted for.

Once nonlinear analysis has been performed, the final stiffness matrix obtained in the process can be used for subsequent linear analyses, including modal analysis.

Modal analysis is a classical method of performing a dynamic analysis of structures, in which enough information about the systems or structures is obtained to reproduce their dynamics. Carrion *et al.* (2014) explained that in the classical modal analysis, this information is related to the natural frequencies of the system (eigenvalues) and the modes of vibration (eigenvectors). In this sense, Shiki *et al.* (2014) confirmed that the mathematical modeling of mechanical structures is an important research area in structural dynamics, wherein, generally, the goal is to obtain a model that accurately predicts the dynamics of the system. However, nonlinear effects are not as well understood as their linear counterparts.

The modal analysis depends on the stiffness of the structural system, for this reason is necessary to update the stiffness to consider nonlinear effects. The matrix obtained with this process is named a tangent matrix and is defined as:

$$\left[K\right] = \frac{\left[dP\right]}{\left[dU\right]} \tag{1}$$

where $[dP]$ and $[dU]$ are column matrices that collect the increment of loads and displacement (dP_i and dU_j).

To update the stiffness matrix to the deformed configuration of a beam, it is necessary to take the equations of strain-displacement nonlinear relations in terms of its partial derivate, as shown in Eq. (2), which are written in terms of the usual displacement components u and v, obtained from *Green Strain*, as indicated in Eq. (3), where ds_0 and ds are the initial and final length of an arbitrary bar element. Thus obtaining the following:

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}^2 + \frac{\partial v}{\partial x}^2\right), \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial u}{\partial y}^2 + \frac{\partial v}{\partial y}^2\right), \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y}\right), \tag{2}$$

$$\varepsilon_G = \frac{1}{2}\left(\frac{ds^2 - ds_0^{\,2}}{ds_0}\right). \tag{3}$$

As proposed by Cook (2004), the set of equations that were obtained after derivation using habitual interpolation functions must be employed together with a co-rotational formulation to solve the large-deflection problem of the beam. After formulation, element matrices are coordinate-transformed so that they operate on the degrees of freedom and are assembled into the global stiffness matrix. Thus, the global stiffness matrix and the global equilibrium equations pertain to the deformed structural geometry. Because this geometry is not known in advance, an iterative process is required.

One of the most recommended iterative processes is the Newton-Rapson Method, in which load increments are used to update the matrix step-by-step to a deformed configuration because it is a function of the displacement that is suffered in pieces. In the local system of the beam, the nodal forces require that the equilibrated element deformation can be computed from the local element stiffness matrix or from element stress. Next, the tangent matrix and the array of the internal node forces of the structure in its current configuration are obtained by the usual FEM process assembly, with the element arrays expanded to the size of the structure. With this, the effect of the normal force over the stiffness of the beam is automatically included. For more details about the specific geometric nonlinear formulations in Finite Element Analysis, Bathe (1975), Clough (1993), Bucalem (2011) and Wang (2015) can be consulted.

4. CONCLUSION

One can conclude that the mathematic computational modeling that is presented in this article predicts the fundamental frequency of a cantilever beam that is largely deformed with an acceptable average of 2.64% of error. Moreover, it is always interesting to note that experimental activities carry uncertainties, such as imperfect support conditions, model clamping pressure, centralizations and uprights, initial deformations of the samples, plastic deformation, and so on. Another valid observation is that while the calculating frequency maintains a downward trend in all cases obtained with the deformed beam, it is comparatively higher than the one obtained in the undistorted configuration, which reveals that a rectifying force improves the stiffness of these elements.

Restricting the conclusion for the case of a clamped beam in just one extremity, it is possible to say that the FEM modeling of modal analysis that is realized after nonlinear static processing by a large displacement shows agreement with the experimental situation that is studied in this work.

5. ACKNOWLEDGEMENTS

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