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ACTIVE NOISE CONTROL IN DUCT USING A CONSTRAINED STATE-SPACE MODEL PREDICTIVE CONTROL

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Abstract. *In this paper, a state-space MPC formulation with constraint handling is proposed for design of Active Noise Control (ANC) in an acoustic duct. The proposed methodology uses as a prediction model a special formulation that uses plant input and output variables as its state variables. The constraint-handling ability of the predictive controller was exploited to ensure that the maximum variation of the control signal is within an acceptable range. The performance of the proposed control loop is demonstrated via numerical simulations and comparison of the results with the classic Filter-x Least Mean Square (FXLMS) algorithm.*

Keywords: *MPC, ANC, Duct, Constraints, State-Space*

1. INTRODUCTION

Noise reduction in industrial and commercial applications has become extremely important in society nowadays in order to take care of human health and improve work conditions (Yoshinobu Kajikawa and Kuo, 2012). As a result, there has been a growing interest of researchers in the active noise control (ANC) techniques development for many different applications (Yang *et al.*, 2014), with example can be cited the control of engine noise inside cars (Gonzalez *et al.*, 2003), in flight cabin interiors (Elliott *et al.*, 1990) and for headphone applications (Kuo *et al.*, 2006). Active noise attenuation is a methodology used for reducing unwanted sound by the addition of a secondary noise with equal amplitude and opposite phase of the primary noise to cancel it out destructively (Lopes and Gerald, 2015). This approach is more efficient way to achieve good noise reduction in a small package or a duct for low frequency, typically below 500 Hz (Zhang and Gan, 2004), (Joël Bordeneuve-Guibé, 2002), where traditional passive noise reduction techniques are inefficient or require relatively large and costly materials (Yoshinobu Kajikawa and Kuo, 2012).

The most common strategy used in the ANC is an adaptive filter known in the literature as filter-x least-mean-square (FXLMS) algorithm in a finite impulse response (FIR) filter (Elliott, 2001). This algorithm is popular because of its relatively low computational load, robustness and simplicity (Zhang *et al.*, 2005). Thereat, several variations of this filter and more sophisticated algorithms have been developed. Thorough reviews of these methods can be found in the literature (Kuo *et al.*, 2010) and (Yoshinobu Kajikawa and Kuo, 2012). However, most of these works do not consider output constraints, which motivated the development of some contributions to address this issue (Qiu and Hansen, 2001), (Rafaely and Elliot, 2000), (Kozacky and Ogunfunmi, 2014a), (Kozacky and Ogunfunmi, 2014b). In some active noise applications, the amplitude of the control signal or the total power of the control signal is limited, either because the actuators used for control have limited driving capability (to prevent output amplifier saturation or nonlinear behavior) or to maintain a specified system power budget (Qiu and Hansen, 2001), (Kozacky and Ogunfunmi, 2014b).

In this context, it may be interesting to consider the potential use of model predictive control (MPC) strategies for ANC. In fact, the MPC has become very popular mainly in industrial applications due to its flexibility, relatively easy of implementation and their ability of handling operational constraints in an explicit manner (Mayne *et al.*, 2000). However, because of the wide frequency range that is encountered in acoustics the practical implementation of this controllers is very challenging (Takács and Rohall'king, 2012). For this reason, very few contributions on the use of MPC for ANC are found in the literature (Joël Bordeneuve-Guibé, 2002), (Yang and Hicks, 2005), (Zhang and Gan, 2004), (Zhang *et al.*, 2005), (Moon *et al.*, 2005), (Moon *et al.*, 2006). In common, such papers did not address important and modern findings of model predictive control theory such as constraint handling issues and state-space models.

However, with the dramatic improvement in hardware power, the continuing advance in real-time computational resources and the development of efficient numerical algorithms for MPC problem added to the fact that in lightly damped structures, as is the acoustic duct, a constrained MPC controller does not require excessive prediction and control horizons to ensure a region of attraction of the feasible set of states (Takács and Rohall'king, 2012) could allow the extension of the use of an constrained MPC to ANC applications.

In the present paper, it is presented a preliminary study of the application of the a state-space MPC formulation with constraint handling for ANC in an acoustic duct. The simulation scenario used a linear, time-invariant analytical model of an acoustic duct. For the prediction model it was considered a special formulation that uses plant input and output variables as its state variables rendering unnecessary the observer design in the implementation of the controller (Wang, 2009). Simulation results evaluate the efficiency of the proposed MPC strategy to attenuate the noise level. In addition, it is presented a comparative performance study between the proposed controller and the FXLMS algorithm.

This paper is organized as follows. In Section 2 the acoustic duct used in this paper is described. Section 3 presents the predictive control formulation used and the special formulation employed for the prediction model. The Section 4 contains numerical results. The conclusions are presented in Section 5.

2. SYSTEM DESCRIPTION

The object of study in this work is the numerical model of the acoustic duct illustrated in Fig. 1. As it can be seen in this figure, it consists in a tubular duct with an primary speaker at one end and an open end. The primary speaker is responsible for producing the noise to be controlled. The control speaker for generating the anti-noise is placed in an intermediate place on the tube. There is also a accelerometer (no acoustic reference sensor) placed on speaker diaphragms to measure the reference signal and one microphone to measure the error signal. The microphone signal error is used to monitor the performance of the controller and the objective of the controller is to minimize this signal. Parameters values for this acoustic duct are: $L = 3.58m$, $x_a = 1.2192m$, and the across-section area is $0.0613m^2$.

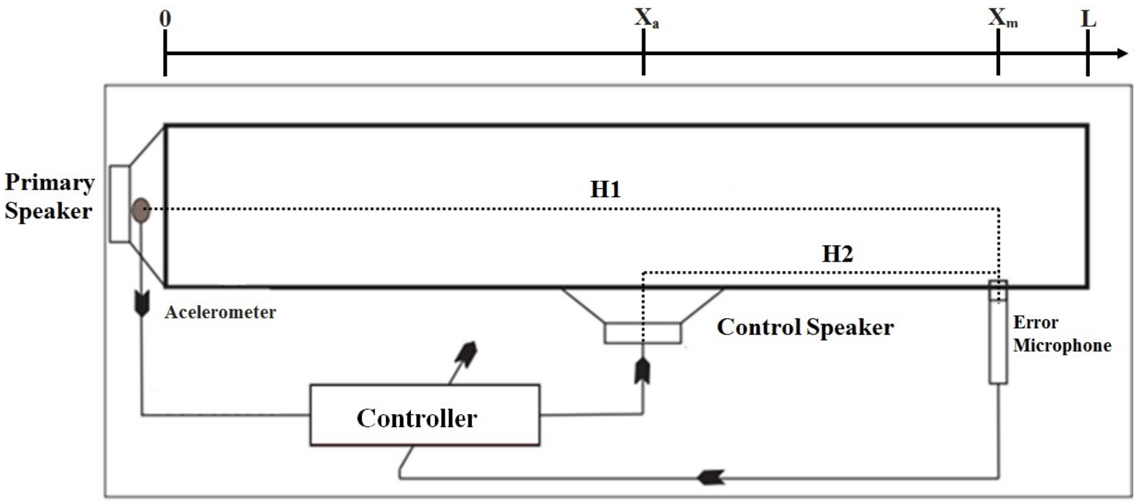

Figure 1. Acoustic duct with actuators and sensors.

As illustrated in Fig. 1 this system has two single-input single-output (SISO) transfer function to be considered. The transfer function between primary speaker and error microphone named as H1 and the transfer function between control speaker and error microphone named as H2. The input of these transfer functions are the signals applied to primary and control speakers, respectively. The outputs of these two transfer functions are added to form the microphone error signal. The manipulated variable is the signal applied to the control speaker. In this paper, these transfer functions were obtained from (Pota and Kelkar, 2000). Owing to the extension of the resulting equations, they are omitted here. For a complete development, please refer to (Pota and Kelkar, 2000).

It is important to emphasize that H1 and H2 are linear, stable and time-invariant models. However, such models present some relevant drawbacks from the viewpoint of control design because they are high-order models (H1 is 42^{th} order model and H2 is 32^{th} order model) and that states are not measurable. Farther, H2 has zeros outside the unit circle (unstable zeros) which leads to a non-minimum phase behaviour.

3. PREDICTIVE CONTROL FORMULATION

The MPC strategy consists of solving a moving-horizon optimal control problem, which a solution is reiterated at each sampling period based on sensor feedback information (Camacho and Bordons, 1999). Basically, this approach consists of three main elements: a prediction model, an optimizer that minimizes a quadratic cost function and the control computation (Maciejowski, 2002). In this work, we used the formulation of a constrained predictive control presented in (Lopes *et al.*, 2006). The following provides a brief description of this formulation. For further details, the reader should consult the original paper.

Initially let us consider that plant's model is described using the following discrete state-space model.

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k), \\ \mathbf{y}(k) &= \mathbf{C}\mathbf{x}(k), \end{aligned} \tag{1}$$

where $\mathbf{x}(k) \in \mathbb{R}^n$ refers to the discrete state and n is order of the model. The vectors $\mathbf{u}(k) \in \mathbb{R}^p$ and $\mathbf{y}(k) \in \mathbb{R}^q$ are, respectively, the input and the output of system at time $k \in \mathbb{Z}$, The real matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ have appropriate dimensions and describe the dynamics system.

The model shown in Eq. (1) is employed to calculate output predictions ($\hat{y}(k+i|k)$) up to N steps in the future based on the information at the current step (k-th sampling instant). In this notation, the hat symbol ($\hat{}$) denotes a predicted variable and N is known in the literature as Prediction Horizon. Such predictions are used in optimization process using the following cost function, which penalizes tracking errors at q plant outputs and control variations at p plant inputs.

$$J(\Delta\hat{U}) = \sum_{j=1}^{q}\sum_{i=1}^{N} \mu_j [\hat{y}_j(k+i|k) - r_j(k+i)]^2 + \sum_{l=1}^{p}\sum_{i=1}^{M} \rho_l [\Delta\hat{u}_l(k-1+i|k)]^2, \tag{2}$$

where $\rho_l > 0$; $l = 1, \ldots, p$ and $\mu_j \geq 0$; $j = 1, \ldots, q$ are design parameters. $\Delta\hat{u}(k+i-1|k)$ term expresses the increment in the control signal for each future step and are defined by $\Delta\hat{u}(k+i-1|k) = \hat{u}(k+i-1|k) - \hat{u}(k+i-2|k)$. The term $r_j(k+i)$ denotes the reference trajectory.

The optimization algorithm is aimed at determining the sequence of future control increments $\Delta\hat{u}(k-1+i|k), i = 1, ..., M$, that minimize this the cost function subject to constraints on the plant inputs and outputs. The value of M (Control Horizon) is typically smaller than N, and it is assumed that $\Delta\hat{u}(k-1+i|k) = 0$ for $M < i \leq N$.

By defining an output weighting matrix $\mathbf{W}_y$ as:

$$\mathbf{W}_y = \begin{bmatrix} \Sigma(\mu) & 0_{q\times q} & \cdots & 0_{q\times q} \\ 0_{q\times q} & \Sigma(\mu) & \cdots & 0_{q\times q} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{q\times q} & 0_{q\times q} & \cdots & \Sigma(\mu) \end{bmatrix}, \text{ where } \Sigma(\mu) = \begin{bmatrix} \mu_1 & 0 & \cdots & 0 \\ 0 & \mu_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu_q \end{bmatrix},$$

and an input weighting matrix $\mathbf{W}_u$ in a similar manner (using weights ρ_l), the cost function can be rewritten in the form:

$$J(\Delta\hat{\mathbf{U}}) = (\hat{\mathbf{Y}} - \mathbf{R})\mathbf{W}_y(\hat{\mathbf{Y}} - \mathbf{R})^T + \Delta\hat{\mathbf{U}}\mathbf{W}_u\Delta\hat{\mathbf{U}}^T, \tag{3}$$

where

$$\mathbf{R} = \begin{bmatrix} \mathbf{r}(k+1) \\ \vdots \\ \mathbf{r}(k+N) \end{bmatrix}, \ \hat{\mathbf{Y}} = \begin{bmatrix} \hat{\mathbf{y}}(k+1|k) \\ \vdots \\ \hat{\mathbf{y}}(k+N|k) \end{bmatrix}, \ \Delta\hat{\mathbf{U}} = \begin{bmatrix} \Delta\hat{\mathbf{u}}(k|k) \\ \vdots \\ \Delta\hat{\mathbf{u}}(k+M-1|k) \end{bmatrix}. \tag{4}$$

Thus, the model presented in Eq. (1) can be rewritten in the form of incremental model as

$$\begin{aligned} \Delta\mathbf{x}(k+1) &= \mathbf{A}\Delta\mathbf{x}(k) + \mathbf{B}\Delta\mathbf{u}(k), \\ \Delta\mathbf{y}(k) &= \mathbf{C}\Delta\mathbf{x}(k). \end{aligned} \tag{5}$$

This model allows you to write the prediction equation in term of $\Delta\hat{\mathbf{Y}}$ as:

$$\Delta\hat{\mathbf{Y}} = \begin{bmatrix} \mathbf{CB} & 0 & \cdots & 0 \\ \mathbf{CAB} & \mathbf{CB} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{CA}^{N-1}\mathbf{B} & \mathbf{CA}^{N-2}\mathbf{B} & \cdots & \mathbf{CA}^{N-M}\mathbf{B} \end{bmatrix} \Delta\hat{\mathbf{U}} + \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^N \end{bmatrix} \Delta\mathbf{x}(k) = \mathbf{P}\Delta\hat{\mathbf{U}} + \mathbf{Q}\Delta\mathbf{x}(k). \tag{6}$$

It can be easily seen that $\Delta\hat{Y}$ and $\hat{Y}$ can be related as:

$$\hat{\mathbf{Y}} = \begin{bmatrix} \mathbf{I}_q & 0 & \cdots & 0 \\ \mathbf{I}_q & \mathbf{I}_q & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{I}_q & \mathbf{I}_q & \mathbf{I}_q & \mathbf{I}_q \end{bmatrix} \Delta\hat{\mathbf{Y}} + \begin{bmatrix} \mathbf{I}_q \\ \mathbf{I}_q \\ \vdots \\ \mathbf{I}_q \end{bmatrix} \mathbf{y}(k) = \mathbf{T}_N^{\mathbf{I}_q}\Delta\hat{\mathbf{Y}} + \Gamma_N^{\mathbf{I}_q}\mathbf{y}(k). \tag{7}$$

By using the identity in (6), it follows that $\hat{\mathbf{Y}} = \mathbf{T}_N^{\mathbf{I}_q}\mathbf{P}\Delta\hat{\mathbf{U}} + \mathbf{T}_N^{\mathbf{I}_q}\mathbf{Q}\Delta\mathbf{x}(k) + \Gamma_N^{\mathbf{I}_q}\mathbf{y}(k) = \mathbf{G}\Delta\hat{\mathbf{U}} + \mathbf{F}$, where $\mathbf{G} = \mathbf{T}_N^{\mathbf{I}_q}\mathbf{P}$ and $\mathbf{F} = \mathbf{T}_N^{\mathbf{I}_q}\mathbf{Q}\Delta\mathbf{x}(k) + \Gamma_N^{\mathbf{I}_q}\,\mathbf{y}(k)$. Therefore, the cost function (3) can be rewritten as:

$$J(\Delta\hat{\mathbf{U}}) = (\mathbf{G}\Delta\hat{\mathbf{U}} + \mathbf{F} - \mathbf{R})^T\mathbf{W}_y(\mathbf{G}\Delta\hat{\mathbf{U}} + \mathbf{F} - \mathbf{R}) + \Delta\hat{\mathbf{U}}^T\mathbf{W}_u\Delta\hat{\mathbf{U}} = \frac{1}{2}\Delta\hat{\mathbf{U}}^T\mathbf{G}\Delta\hat{U} + \mathbf{f}^T\Delta\hat{\mathbf{U}} + \mathbf{c}, \tag{8}$$

where $(1/2)\mathbf{G} = \mathbf{G}^T\mathbf{W}_y\,\mathbf{G} + \mathbf{W}_u$, $\mathbf{f}^T = 2(\mathbf{F} - \mathbf{R})^T\mathbf{W}_y\,\mathbf{G}$ and $\mathbf{c} = (\mathbf{F} - \mathbf{R})^T\mathbf{W}_y\,(\mathbf{F} - \mathbf{R})$.

In the absence of constraints, the control sequence $\Delta\mathbf{U}^*_{\text{without constraints}}$ that minimizes the cost is given by:

$$\Delta\mathbf{U}^*_{\text{without constraints}} = (G^TW_yG + W_u)^{-1}G^TW_y(R - F). \tag{9}$$

If restrictions of the form $\mathbf{u}_{min} \leq \hat{\mathbf{u}}(k-1+i|k) \leq \mathbf{u}_{max}$, $i \in \{1,\ldots,M\}$, and $\mathbf{y}_{min} \leq \hat{\mathbf{y}}(k+i|k) \leq \mathbf{y}_{max}$, $i \in \{1,\ldots,N\}$, are to be satisfied on the manipulated and controlled variables, the minimization of the cost is subject to the following linear constraints on $\Delta\hat{\mathbf{U}}$ (Camacho and Bordons, 1999):

$$\begin{bmatrix} \mathbf{T}_M^{\mathbf{I}_p} \\ -\mathbf{T}_M^{\mathbf{I}_p} \\ \mathbf{G} \\ -\mathbf{G} \end{bmatrix} \Delta\hat{\mathbf{U}} \leq \begin{bmatrix} \mathbf{Bl}_M[\mathbf{u}_{max} - \mathbf{u}(k-1)] \\ \mathbf{Bl}_M[\mathbf{u}(k-1) - \mathbf{u}_{min}] \\ \mathbf{Bl}_N[\mathbf{y}_{max}] - \mathbf{F} \\ \mathbf{F} - \mathbf{Bl}_N[\mathbf{y}_{min}] \end{bmatrix}. \tag{10}$$

where $\mathbf{T}_M^{\mathbf{I}_p}$ is a lower block-triangular matrix of identities and $\mathbf{Bl}_N[\bullet]$ is an operator that stacks N copies of a column vector. The set of constraints (10) can be rewritten as $\mathbf{S}\Delta\hat{\mathbf{U}} \leq \mathbf{b}$. Therefore, the optimization problem to be solved at each sampling period is one of quadratic programming, for which efficient numerical algorithms are available (Maciejowski (2002)).

The control is implemented in a receding horizon manner, where only the first element of the optimized control sequence ($\mathbf{u}^*(k)$) is applied to the plant and the optimization is repeated at the next sampling instant, on the basis of fresh measurements.

3.1 MPC APPLIED FOR ANC

The block diagram shown in Fig. 2 illustrates the proposed control loop for ANC in the acoustic duct (Fig. 1). Therefore, the plant to be controlled is H2. The noise generated by the primary speaker, $\delta(k)$, excites the primary path (H1), whose output is regarded as system disturbance. The objective of the controller is to minimize the error microphone signal $e(k)$. Wherefore, the predictive control reference signal, $r(k)$, is equal to zero.

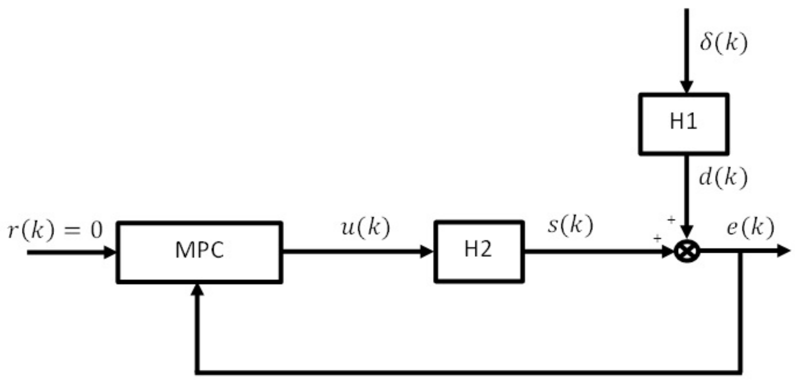

Figure 2. The block diagrama of the MPC control loop.

As shown in section 3, the model to be used in the control system design is taken to be a state-space model. However, the direct conversion of H2 for a state-space model results in a set of states that are not measurable. In this paper, to avoid the design of a state observer it is used the approach proposed in (Wang, 2009) that select the state variables corresponding to the input and output using a special state-space realization. This formulation will be briefly described below.

Suppose that a discrete-time transfer function model H2 is given as

$$H2(z) = \frac{Y(z)}{U(z)} = \frac{a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \ldots + a_0}{b_n z^n + b_{n-1}z^{n-1} + \ldots + b_0}. \quad (11)$$

Using the forward shift operator, the difference equation is expressed as

$$a_{n-1}u(k+n-1) + a_{n-2}u(k+n-2) + \ldots + a_0 u(k) = b_n y(k+n) + b_{n-1}y(k+n-1) + \ldots + b_0 y(k). \quad (12)$$

To avoid the need for future information at the sampling instant k the Eq. (12) is multiplied with a backward shift q^{-n}, leading to

$$y(k) = \frac{a_{n-1}}{b_n}u(k-1) + \frac{a_{n-2}}{b_n}u(k-2) + \ldots + \frac{a_0}{b_n}u(k-n) - \frac{b_{n-1}}{b_n}y(k-1) - \frac{b_{n-2}}{b_n}y(k-2) - \ldots - \frac{b_0}{b_n}y(k-n). \quad (13)$$

Choosing the state variable vector as

$$\mathbf{x}(k) = \begin{bmatrix} y(k-1) & y(k-2) & \cdots & y(k-n) & u(k-1) & u(k-2) & \cdots & u(k-n) \end{bmatrix}^T. \quad (14)$$

Then, the state-space model used to predict the output is

$$\begin{bmatrix} y(k) \\ y(k-1) \\ \vdots \\ y(k-n+1) \\ u(k) \\ u(k-1) \\ \vdots \\ u(k-n+1) \end{bmatrix} = \begin{bmatrix} \frac{-b_{n-1}}{b_n} & \frac{-b_{n-2}}{b_n} & \cdots & -\frac{b_0}{b_n} & \frac{a_{n-1}}{b_n} & \frac{a_{n-2}}{b_n} & \cdots & \frac{a_0}{b_n} \\ 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y(k-1) \\ y(k-2) \\ \vdots \\ y(k-n) \\ u(k-1) \\ u(k-2) \\ \vdots \\ u(k-n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(k). \quad (15)$$

$$y(k) = \begin{bmatrix} \frac{-b_{n-1}}{b_n} & \frac{-b_{n-2}}{b_n} & \cdots & -\frac{b_0}{b_n} & \frac{a_{n-1}}{b_n} & \frac{a_{n-2}}{b_n} & \cdots & \frac{a_0}{b_n} \end{bmatrix} \mathbf{x}(k). \quad (16)$$

The main disadvantage of this formulation is the model order increased to $2n$. However, in this approach is not necessary estimate the state variable $\mathbf{x}(k)$ from the process measurement.

4. SIMULATION RESULTS

In this section are presented some computer simulations to compare the performance of this work's predictive control structure and the classic FXLMS algorithm (Elliott, 2001). All simulations were carried out using the Simulink environment in Matlab® R2014a. The duct dynamics was described using the transfer functions H1 and H2 and the prediction model used the formulation described in the previous section. To implement the predictive control law a specific Matlab S-function was written and the Quadratic Programming problem was solved by using the *quadprog* function in Matlab.

In all simulations the sampling period used was 0.5ms. Additionally, as this work is only an initial study the primary noise was chosen to be only a pure tone sine wave. Exhaustive simulations were executed to determine the best tuning parameters of both algorithms. For the MPC the selected parameters were: $N = 15$, $M = 10$, $\mu = 100$ and $\rho = 0.1$. The FXLMS algorithm used a transversal filter of length 40 and length of the FIR filter that model the estimated secondary path was 80.

To compare control strategies two different simulation scenarios were considered. First, the system were free of constraints. In the second scenario, it was imposed constraints on the maximum amplitude of the control signal.

In the first scenario, four aleatory frequency in range 1 to 140 Hz were chosen for analysis (20, 60, 100 and 140Hz) with unitary amplitude. The efficiency of the ANC system was evaluated using three criteria: the noise reduction (NR) after convergence, the root-mean square error (RMSE) and the convergence rate of the process. The NR is defined as (Qiu and Hansen, 2001):

$$\mathrm{NR} = 10\log_{10}\left(E\left\{d^2(k)\right\}/E\left\{e^2(k)\right\}\right), \quad (17)$$

Table 1. Performance comparison between MPC and FXLMS in terms of NR, RMSE and convergence time.

	MPC				FXLMS			
Frequency (Hz)	20	60	100	140	20	60	100	140
NR (dB)	49.0	23.3	6.3	0.05	106.3	89.8	60.5	47.4
RMSE	0.0091	0.0290	0.0711	0.3048	0.0401	0.0221	0.0244	0.0294
Convergence Time (s)	0.04	0.04	0.04	0.1	0.98	0.98	1.46	2.24

where $E\{\}$ means the expected value and the RMSE is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e(i)^2}. \tag{18}$$

The results are summarized in Table 1. This results reveal that the FXLMS algorithm leads to a much better noise reduction than that MPC in this unconstraints ideal system. Nevertheless the MPC has an acceptable performance noise reduction at low frequencies. It can be seen from Fig. 3.a that, in 20Hz, the MPC can reduce the disturbance to almost zero. The main problem is that this performance is strongly affected by increased frequency. In fact, the controller has not been able to reduce noise in the duct for frequency above 140Hz. We will address this problem in future work reformulating the predictive controller to consider the dynamics of the primary path (H1). Thus, it is believed that the use of the reference signal may lead to a performance gain at higher frequencies.

The results of Table 1 also show that the convergence speed of the proposed MPC algorithm is faster than that of FXLMS algorithm. Indeed, with the MPC was obtained an average reduction of approximately 96% the convergence time compared with the FXLMS algorithm. This quickness of attenuation with the MPC can be observed in Fig. 3.a. The faster convergence explains the lowest value of RMSE obtained with the MPC in 20Hz even with the FXLMS algorithm having a higher noise reduction. Figure 3.b illustrates the comparative performance of the two controllers in frequency domain.

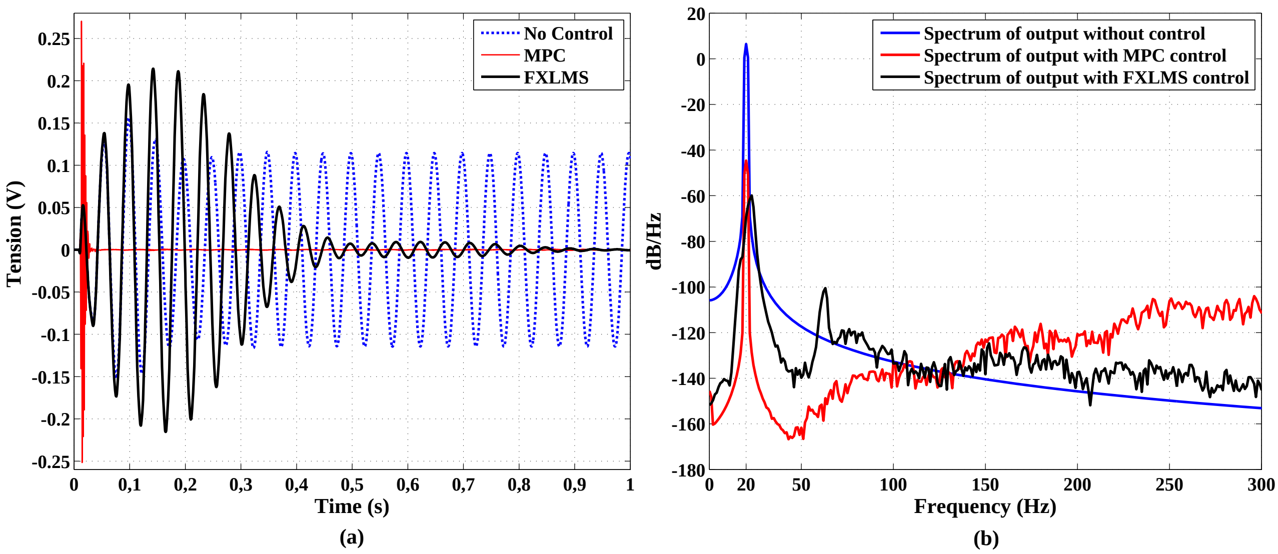

Figure 3. Results obtained for a 20 Hz sinusoidal primary noise. (a) The transient response (error curves) of proposed MPC control and FXLMS algorithm. (b) Power spectrum of active noise cancelling error.

In the second simulation scenario, the constraint-handling ability of the predictive controller was illustrated by imposing restrictions on the amplitude of the control variable. A lower and an upper bound for this variable were set to $[-1, 1]$. This constraint is useful when the controller is used to handle an external actuator, such as a loudspeaker, and can be used to ensure that the actuator is not overloaded (Rafaely and Elliot, 2000). This restriction has been incorporated for both (MPC and FXLMS algorithms) simulations by adding a saturation block preventing values above the specified limits are applied in the plant. For this case, the primary noise was defined as 20 Hz in order to compare with the previous presented case.

The results obtained are shown in Fig. 4. As can be seen in these figures the MPC were able to reduce the error signal while at the same time maintain the constraint. However, as the disturbance amplitude is so large the control variable cannot achieve the required amplitude. As a consequence always remains a certain level of residual error signal (Fig. 4.b). The analysis of performance loss may be carried compared to Fig. 3.a and Fig. 4.b. On the other hand, the FXLMS algorithm not only violates the constraints as the problem is compounded over time (Fig. 4.a.). As can be seen in Fig. 4.b the noise reduction of the FXLMS algorithm is smaller than the MPC for this scenario.

This results suggest that MPC methodologies may be a promising alternative to the ANC when constraints are imposed in the system. However, further study must be done to improve the controller performance at medium and higher frequencies.

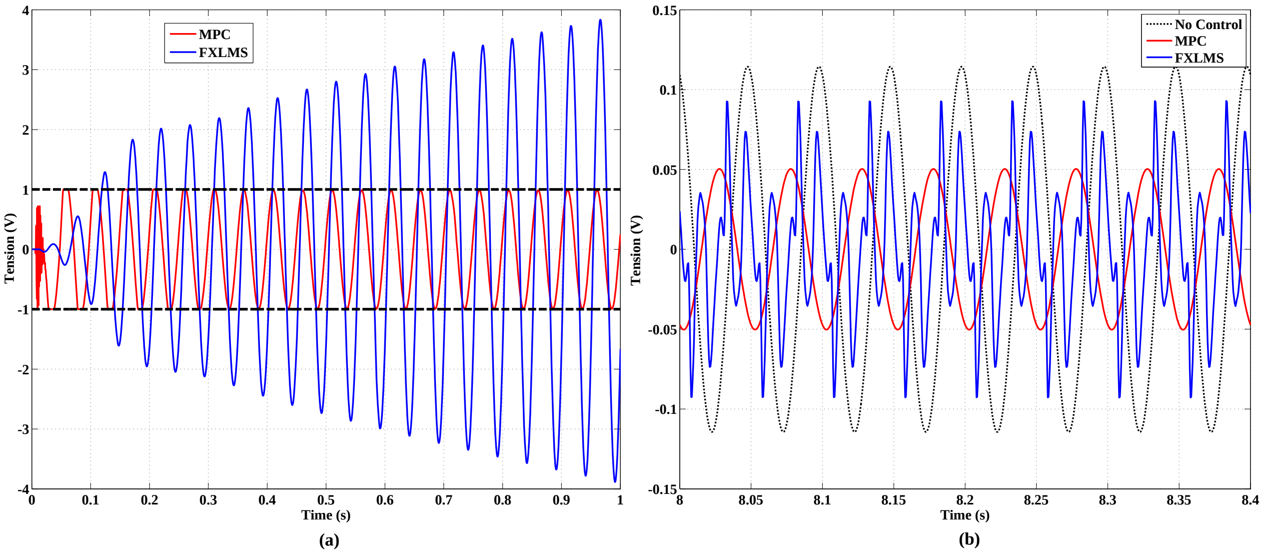

Figure 4. (a) Control signals generated by the MPC and FXLMS algorithm. The dashed thick lines indicate the constraints. (b) The transient response (error curves) of proposed MPC control and FXLMS algorithm with constraint on the control variable. For visualization purposes, the size of the observation window in that figure has been shortened.

5. CONCLUSIONS

In this paper, a constrained state-space MPC formulation was presented to the active noise control in an acoustic duct. The proposed approach was compared to the classic FXLMS algorithm. The results showed that the convergence speed of the proposed algorithm is faster than that of FXLMS algorithm but with a lower noise attenuation for simulations up to 140Hz. Furthermore, the MPC did not show good performance for frequencies above 140Hz. A more detailed investigation into the causes of this poor performance above 140Hz will be carried out in future work. One possible solution to be investigated is to change the formulation of predictive control to take into account the dynamics of the primary path.

The main advantage of MPC was observed in the scenario where restrictions were imposed on the maximum amplitude of the control signal. In this context, the constraint-handling features of MPC was particularly valuable, because the controller were able to reduce the error signal while at the same time maintain the constraint. The results obtained suggest that the approach presented here may be a promising alternative to ANC systems.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

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