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FUZZY INTERVAL AND UNCERTAINTY QUANTIFICATION IN VEHICLE SYSTEM

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Abstract. *The design and analysis of vehicle systems are subject to several sources of uncertainty. Road profile, properties of the suspension parameters, material properties, and vehicle loading conditions are some of the sources of uncertainty, which are present in such systems. Simplifications and model assumptions are also source of uncertainty in the overall dynamic behavior. Many times the determination of statistical parameters representing these variables, within some confidence, is difficult and sometimes very difficult, like in the case of road profile data. The fuzzy interval analysis can prove to be a useful tool to quantify these uncertainties by providing important information such as extreme limits for the system that should be take into account at the design stage. The main purpose of this work is to present results related to the analysis and quantification of uncertainty for a simplified suspension system subjected to different load conditions, uncertain parameters of road profile and suspension parameters.*

Keywords: *fuzzy interval analysis, uncertainty quantification, vehicle system dynamics.*

1. INTRODUCTION

Depending on the degree of accuracy of analysis, the vehicle dynamics can be modeled in quite precise detail. This include rigid body dynamics and deformations of chassis, nonlinear behavior of the tire and the road profile, to name a few. Although precise, these refinements are adversely harmed by its limited ability in predicting vehicle dynamics in the presence of uncertainties. Unfortunately, many sources of uncertainties may shadow some modelling improvements making difficult to measure the degree of importance of the models from one to another approach. Road-tire friction interaction, the road profile itself, the road environmental conditions, the use vehicle condition (loaded, unloaded), detail in the structural model, aerodynamic force loading are just a few commonly occurring sources of uncertainty.

These types of uncertainty are usually separated on epistemic and aleatory. Most of epistemic uncertainty (reducible uncertainty) can be handled by an adequate model of the vehicle that simulates the main dynamics aspects of the vehicle, but the aleatory uncertainty can only be estimated and encompassed by statistics of the process and measured data(irreducible uncertainty).

The ability of propagating and analyzing such uncertainty in vehicle model behavior has importance in the predictions of probable or even extreme behavior scenarios of such systems in the presence of uncertainty. A step forward the understanding of the problem is the model updating of elected system´s uncertain parameters in the presence of experimental scattered data in order to have available a system that not only predicts the behavior of the system but also its variability. Many methodologies have been proposed along recent years to deal with the problem: Stochastic Finite Element Method/Polynomial Chaos (Ghanem and Spanos, 1991), Neuman expansion, Karhunen-Loeve expansion, Monte Carlo Simulations, Interval Analysis (Jaulin *et al.*, 2001) and Bayesian (Beck and Yuen, 2004, Hadjidoukas *et al.*, 2015), Fuzzy Interval Methods (Möller and Beer, 2003), to name a few.

2. BRIEF BIBLIOGRAHICAL REVIEW

The uncertainty analysis on vehicle systems has received increased attention in the last decades. The concern related to the ability of autonomous vehicles to accurately analyze their dynamic response to a given uncertain input, mostly related to terrain and vehicle parameter estimation, is evaluated by Kewlani *et al*. (2011) and Mellinger and Kumar (2010), using polynomial chaos (PC) approaches. They conclude that this approach showed more efficient than pure Monte Carlo simulation and that it could accurately predict vehicle motions in realistic scenarios in the numerically simulation ANVEL environment.

Lara-Molina and Steffen (2014) presented the use of fuzzy logic to uncertainty propagation in a 1-DOF system and a quarter car vehicle model assuming stiffness and mass as uncertain parameters with an alpha-cut formulation. Time responses and FRF envelopes are presented for the time histories and the corresponding spectral responses. They conclude that the methodology can be an acceptable tool in design analysis stage but with reservations related to the computational time spent in the analysis.

Giagopoulos *et al.* (2013) presented an uncertainty quantification and propagation framework to identify nonlinear models of dynamic of a small-scale experimental model of a vehicle with nonlinear wheel and suspension components. The parameters are identified using experimentally obtained response spectra for each of the components tested separately and then effectively propagated to evaluate the uncertainty of output variables of the complete system.

3. FUZZY INTERVAL ANALYSIS

First developed by Zadeh (1965), a fuzzy set defines a class of objects that have some sort of belongness (pertinence) to a group of objects. Differently from traditional Set Theory that have crisp borders, a fuzzy set is composed by a support function (known as membership function) with values that can lie between 0 and 1, thus being possible to deal with vague or incomplete information. In 1978, Zadeh released the Possibilistic Theory using fuzzy sets that played a similar role to the probability theory for definite stochastic distributions. Let us assume a system with input vector ${}^{m}\mathbf{X}=({}^{m}x_1, {}^{m}x_2, \ldots, {}^{m}x_{ni})^T$, where *ni* is the number of input variables, *m* means "measured" and an output vector ${}^{m}\mathbf{Z}=({}^{m}z_1, {}^{m}z_2, \ldots, {}^{m}z_{no})^T$, where *no* is the number of output variables. Let's assume a number of samples of each one of the input vectors (called realizations) ${}_{1}^{m}\mathbf{X}, {}_{2}^{m}\mathbf{X}, \ldots, {}_{ns}^{m}\mathbf{X}$, and the corresponding output vectors ${}_{1}^{m}\mathbf{z}, {}_{2}^{m}\mathbf{z}, \ldots, {}_{ns}^{m}\mathbf{z}$, where *ns* means number of samples. Furthermore, let us assume a numerical model of the system with the same measured inputs ${}^{m}\mathbf{X}=({}^{m}x_1, {}^{m}x_2, \ldots, {}^{m}x_{ni})^T$, adjustable model's parameters vector $\boldsymbol{\theta}=(\theta_1, \theta_2, \ldots, \theta_{np})^T$ with *np* representing the number of parameters and predicted vector outputs. Related to the output vector, it is easy to obtain their upper and lower bounds based on the values from all realizations. These output intervals vector can be gather into a two-column vector representation as:

$$[{}^{m}\underline{\mathbf{z}}, {}^{m}\overline{\mathbf{z}}] = [\min_{i=1,ns}({}_{i}^{m}\mathbf{z}) \quad \max_{i=1,ns}({}_{i}^{m}\mathbf{z})] \tag{1}$$

this represents a measure of model's output uncertainty or dispersion. The same can be stated for the predicted output vector (from the numerical model) and the measured input vector:

$$[{}^{m}\underline{\mathbf{x}}, {}^{m}\overline{\mathbf{x}}] = [\min_{i=1,ns}({}_{i}^{m}\mathbf{x}) \quad \max_{i=1,ns}({}_{i}^{m}\mathbf{x})] \tag{2}$$

$$[\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}] = [\min_{i=1,ns}({}_{i}\boldsymbol{\theta}) \quad \max_{i=1,ns}({}_{i}\boldsymbol{\theta})] \tag{3}$$

$$[{}^{p}\underline{\mathbf{z}}, {}^{p}\overline{\mathbf{z}}] = f([{}^{m}\underline{\mathbf{x}}, {}^{m}\overline{\mathbf{x}}], [\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}]) = [\min_{i=1,ns}({}_{i}^{p}\mathbf{z}([\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}])) \quad \max_{i=1,ns}({}_{i}^{p}\mathbf{z}([\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}]))] \tag{4}$$

What are the output intervals $[{}^{p}\underline{\mathbf{z}}, {}^{p}\overline{\mathbf{z}}]$ for a given set of measured input intervals $[{}^{m}\underline{\mathbf{x}}, {}^{m}\overline{\mathbf{x}}]$ and parameters interval $[\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}]$? This may be thought as finding the variability of the response of the system which means upper and lower bounds of the output behavior. This can also be understood as uncertainty propagation in terms of intervals, since the uncertainty of the inputs and parameters will be propagated to the outputs. This is as a double optimization (minimization and maximization) in each of the hyper-volume quadrants (2^{no}) to find the best input ${}^{m}\mathbf{X}^*$ and parameter $\boldsymbol{\theta}^*$ values that maximizes or minimizes the output norm $\left\|{}^{p}\mathbf{z}\right\|_2$ and that complies with the bounded intervals.

The final output intervals will be the minimum and maximum of each variable for al hyper quadrants stated as:

$$\begin{aligned}
&\text{For each hyper quadrant} \\
&\text{Find vector } \mathbf{x}^*, \boldsymbol{\theta}^* \\
&\text{to minimize } f(\mathbf{z}) = \left\|\mathbf{z}^*\right\|_2 \\
&\text{and maximize } f(\mathbf{z}) = \left\|\mathbf{z}^*\right\|_2 \\
&\text{subject to} \begin{cases} \mathbf{x}^* \in [{}^{m}\underline{\mathbf{x}}, {}^{m}\overline{\mathbf{x}}] \\ \boldsymbol{\theta}^* \in [\underline{\boldsymbol{\theta}}, \overline{\boldsymbol{\theta}}] \\ [\underline{\mathbf{z}}, \overline{\mathbf{z}}] = [\min\limits_{\textit{over all quadrants}}(\mathbf{z}^*) \quad \max\limits_{\textit{over all quadrants}}(\mathbf{z}^*)] \end{cases}
\end{aligned} \tag{5}$$

In order to analyze inputs or model parameters it is usual to represent such values as fuzzy number and the α-cut levels methodology (Möller *et al*., 2000). This methodology consists in multiple interval analyses along membership functions for the output that is itself a fuzzy set composed of fuzzy numbers, as expected. So,

$$\tilde{\mathbf{Z}} = f(\tilde{\mathbf{X}}) \tag{6}$$

where $\tilde{\mathbf{X}}$ means the fuzzy inputs, *f* some deterministic function and $\tilde{\mathbf{Z}}$ the output fuzzy numbers. The fuzzy sets can be discretized in well-defined crisp real intervals with α-levels of belongness or relevance (Fig. 1) and can be defined as:

$$A_{\alpha_k} = \{x \in X \mid \mu a(x) \geq \alpha_k\} \quad \text{or} \quad A_{\alpha_k} = [\underline{x}_{\alpha_k}, \overline{x}_{\alpha_k}] \tag{7}$$

where $\underline{x}$ and $\overline{x}$ represent upper and lower values for a specific α-level, $\underline{x} = \min(A)$ and $\overline{x} = \max(A)$ for a defined interval *A*.

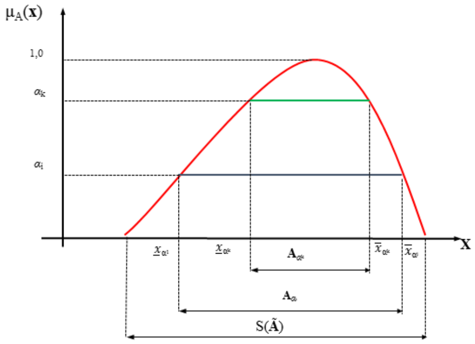

Figure 1. Discrete fuzzy set $\mathbf{A}_{\alpha_k}$ for *i* to *k* α-cut levels.

Once defined the desired discretization for the α-levels, Fig. 2 represent the proposed problem for a two input one output problem $\tilde{\mathbf{X}}_1$, $\tilde{\mathbf{X}}_2$ and $\tilde{\mathbf{Z}}$ as shown:

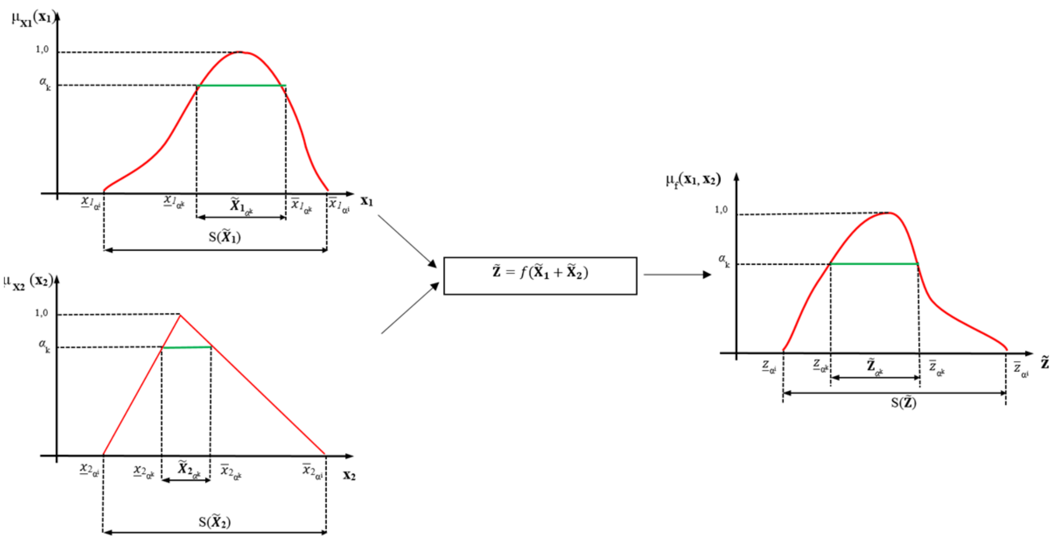

Figure 2. α-cut levels.

4. MODEL EQUATIONS

A 2-DOF quarter-car model is used to obtain the dynamic suspension behavior as shown in Fig. 3. It is composed by an equivalent sprung mass, m_1, which represents ¼ of the car body and its passengers, in kg, an equivalent sprung mass, m_2, which represents a single suspension, wheel and axle, in kg, the suspension stiffness, k_1, in N/m, the tire stiffness, k_2, in N/m and the suspension damping, c_1, in N s/m.

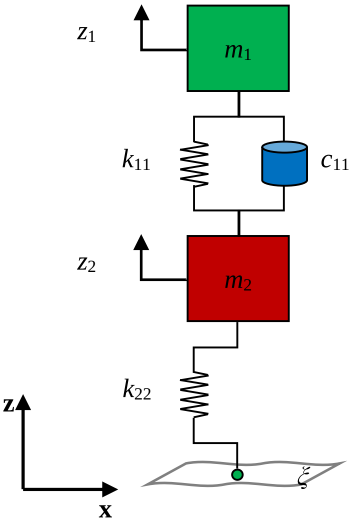

Figure 3. Quarter-car vehicle model.

The related equations regarding to force equilibrium in Fig. 3 in the vertical direction are:

$$\begin{aligned} &m_1\ddot{z}_1 + c_{11}(\dot{z}_1 - \dot{z}_2) + k_{11}(z_1 - z_2) = 0 \\ &m_2\ddot{z}_2 + c_{11}(\dot{z}_2 - \dot{z}_1) + k_{11}(z_2 - z_1) + k_{22}(z_2 - \xi) = 0 \end{aligned} \tag{8}$$

The road irregularities profiles, ξ, are used as input data to study the dynamic behavior of a vehicle which consist in applying the power spectral density (PSD) function. Proposed by Dodds and Robson (1973), the road profile model corresponds to a 2-D isotropic random Gaussian field, where the road is defined as a function with surface properties obtained by experimental measurements applied in one single track along the road. This approach is described by Eq. (9),

$$G_Z(n) = G_0\left(\frac{n}{n_0}\right)^{-w} \tag{9}$$

where w is the exponent of approximation of the PSD curve, n_0 corresponds to the specific wave number according to the standard used, G_0 is the PSD function of a given irregularities profile related to the specified n_0, and n is the wave number measured on the road.

The dynamic behavior of the suspension model uses four common requirements for vehicle design, such as comfort ($a_{min\text{-}max}$), tire force ($tf_{min\text{-}max}$), suspension-working space ($ws_{min\text{-}max}$) and road holding for safety ($rh_{min\text{-}max}$). Each requirement is considered as output variable in the interval analysis.

The objective function, described by Eq. (10), includes these requirements as follows:

$$\begin{aligned} &\text{Find vector } \mathbf{x} = [m_1 \quad k_{22} \quad G_0]^T \\ &\text{to minimize } \; f(\mathbf{x}) = p_1 a(t) + p_2 tf(t) + p_3 ws(t) + p_4 rh(t) \\ &\text{and maximize } \; f(\mathbf{x}) = p_1 a(t) + p_2 tf(t) + p_3 ws(t) + p_4 rh(t) \\ &\text{subject to} \begin{cases} \underline{m_1} \le m_1 \le \overline{m_1} \\ \underline{k_{22}} \le k_{22} \le \overline{k_{22}} \\ \underline{G_0} \le G_0 \le \overline{G_0} \end{cases} \end{aligned} \tag{10}$$

where $\mathbf{p}$ = [1 0 0 0], $\mathbf{p}$ = [0 1 0 0], $\mathbf{p}$ = [0 0 1 0] and $\mathbf{p}$ = [0 0 0 1] to evaluate each requirement.

The input data of the analysis is shown in Tab. 1, which includes all parameters of a typical passenger car suspension model, road profile, interval analysis and spiral optimization algorithm (SOA) as stated by Tamura and Yasuda (2010). The upper and lower values represent the maximum and minimum values of the respective parameter under uncertainty.

Table 1. Input data parameters

Parameter	Unit	Upper Value	Initial Value	Lower Value
Vehicle parameters				
Sprung mass, m_1	kg	280.35	311.5	342.65
Sprung mass (single suspension), m_2	kg		28.0	
Suspension stiffness, k_{11}	N/m		2.1×10^4	
Suspension damping, c_{11}	N s/m		3.0×10^3	
Tire stiffness, k_{22}	N/m	1.35×10^5	1.5×10^5	1.65×10^5
Gravity, g	m/s²		9.81	
SOA parameters				
Number of searching points, n_{sp}			30	
Convergence rate (contraction factor), r			0.99	
Rotation angle (attraction factor), θ_a			$\pi/2$	
Random dispersion, α_{rand}			10.0	
Interval analysis parameters				
Number of input uncertain variables, n_{ip}			3	
Number of output uncertain variables, n_{op}			3	
Number of α-cut levels, n_α			3	
Road profile parameters				
Vehicle velocity, v	m/s		20.0	
Class C road profile spectral density, G_0	m³/cycle	128.0×10^{-6}	256.0×10^{-6}	512.0×10^{-6}
Spatial frequency wave number, n_0	cycle/m		0.1	

5. RESULTS

The results obtained with an interval analysis are shown as follows for each α-cut levels and output variable. It was tested two ways for the uncertainty analysis: (a) interval analysis for each time step and (b) interval analysis for the complete time history. The uncertain input parameters were the sprung mass (u_1), tire stiffness (u_2) and road profile (u_3). Similarly, the uncertain output variable were the maximum and minimum vertical acceleration of sprung mass (z_1), the tire force transmitted to the road (z_2), suspension-working space (z_3) and tire road holding for safety (z_4).

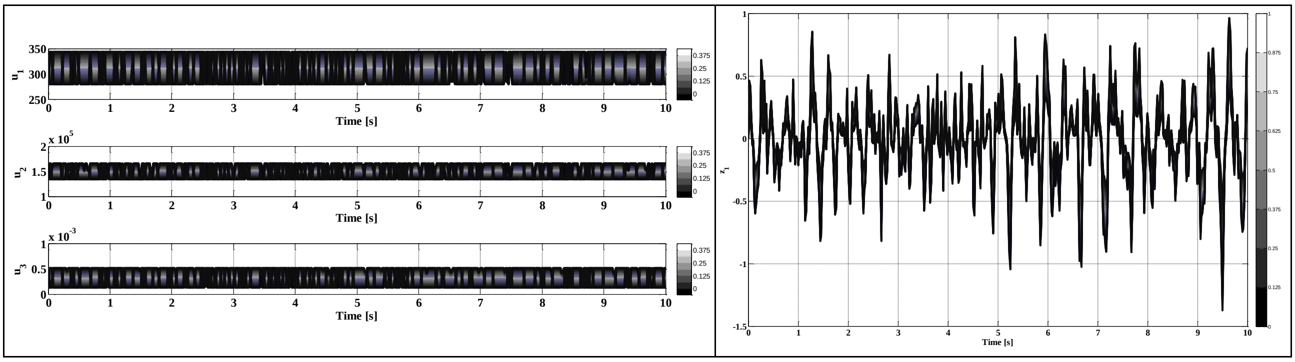

Figure 4. Interval for each input variable along time and corresponding output acceleration envelops.

In Fig. 4, the results in the first analysis is shown. In the left, the interval of each input variable along time is presented and, in the right side, the acceleration envelops of sprung mass is presented. The contour gradient legend (grayscale) represents the α-levels. One can notice that for some time instants, the acceleration is not dependent from the input uncertainty, but, in other ones, it presents an influence. For a sequence of time instants, the input variables can switch in order to generate maximum and minimum values for the acceleration. This means a worst scenario analysis. In this analysis, the right combination of input parameters that maximizes and minimizes the output are available for design purposes. It can be seen that, for the acceleration envelopes, the maximum and minimum values for α = 0.0 are [-1.373, 0.996] m/s² and that they happen in different time instants.

For each presented following figure, the red and the cyan marks are the results obtained using Monte Carlo (MC) simulation and uniform distribution for the uncertain input parameters, showing MC could not reach the best boundaries. This method requires many iterations and computational efforts to achieve these boundaries.

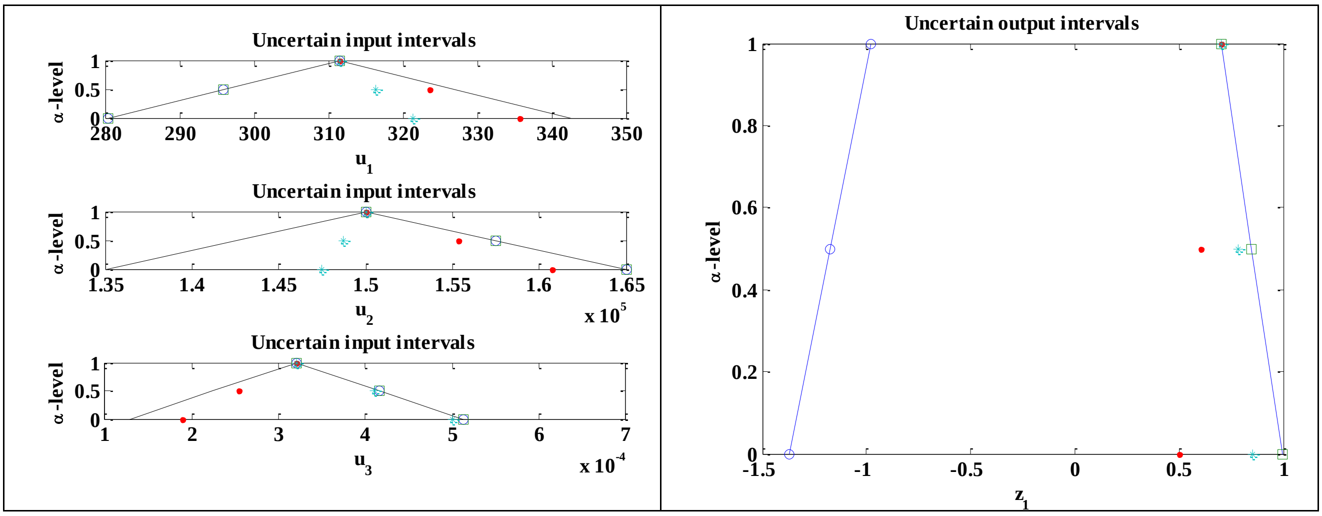

Figure 5. α-cut levels for each input parameters and vertical acceleration of sprung mass [m/s²].

In Fig. 5, for the interval analysis, the upper and lower bounds of uncertain input parameters were the same for each α-level; these input intervals are degenerated, which means that a single real value determines the output interval. A single combination of input parameters generates envelopes containing the minimum and maximum values for the output. It is likely that the combination of minimum sprung mass, maximum tire stiffness and maximum road roughness will result in worst acceleration scenario. For α = 0.0, the presented values for acceleration in Fig. 5 confirm that same results presented in Fig. 4. Differently from the first analysis, in this case, it is not possible to say when the obtained limits will occur. When α = 0.5, the sprung mass (u_1) resulted a single value of 295.9 kg that combines with tire stiffness (u_2) of 1.575 × 10^5 N/m and road profile (u_3) of 4.16 0 × 10^{-4} m³/cycle to obtain the maximum and minimum vertical acceleration of sprung mass (z_1) of [-1.175, 0.8495] m/s².

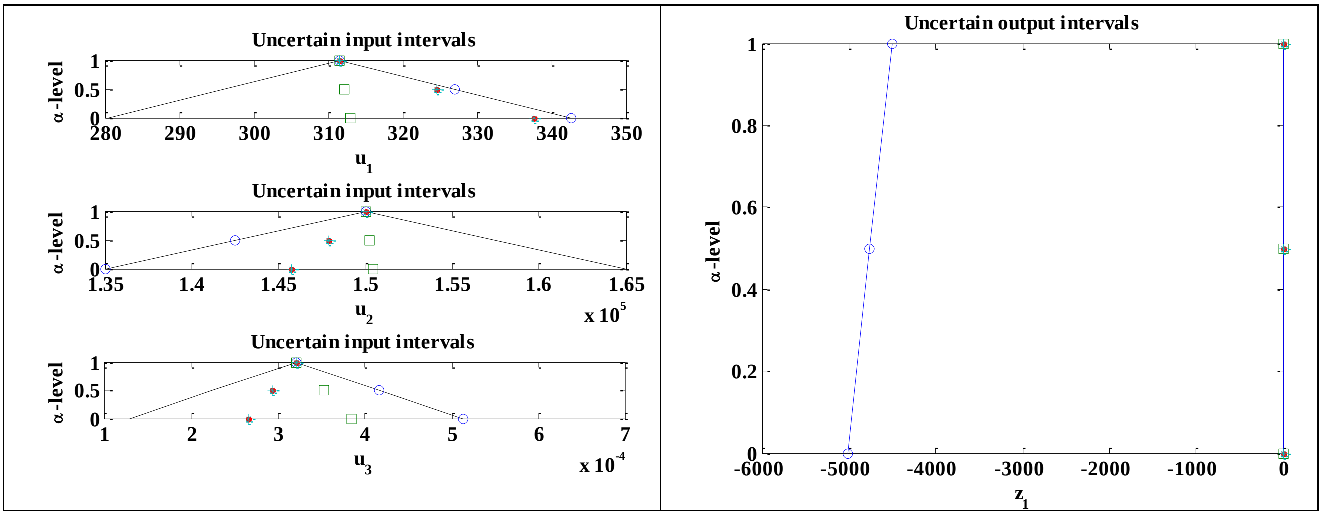

Figure 6. α-cut levels for each input parameters and tire force [N].

Figure 6 shows the upper and lower bounds of uncertain input parameters were different for each α-cut levels. For instance, at α = 0.5, the sprung mass resulted an interval of [312.2, 327.1] kg, the tire stiffness of [1.4250 × 10^5, 1.502 × 10^5] N/m and road roughness of [3.520 × 10^{-4}, 4.16 0 × 10^{-4}] m³/cycle. In order to obtain a tire force of 0 N, which ensures the tire grip with the road profile, it is resulted a lower value of sprung mass (312.2 kg), an upper value of tire stiffness (1.502 × 10^5 N/m) and lower value of road roughness (3.52 0 × 10^{-4} m³/cycle).

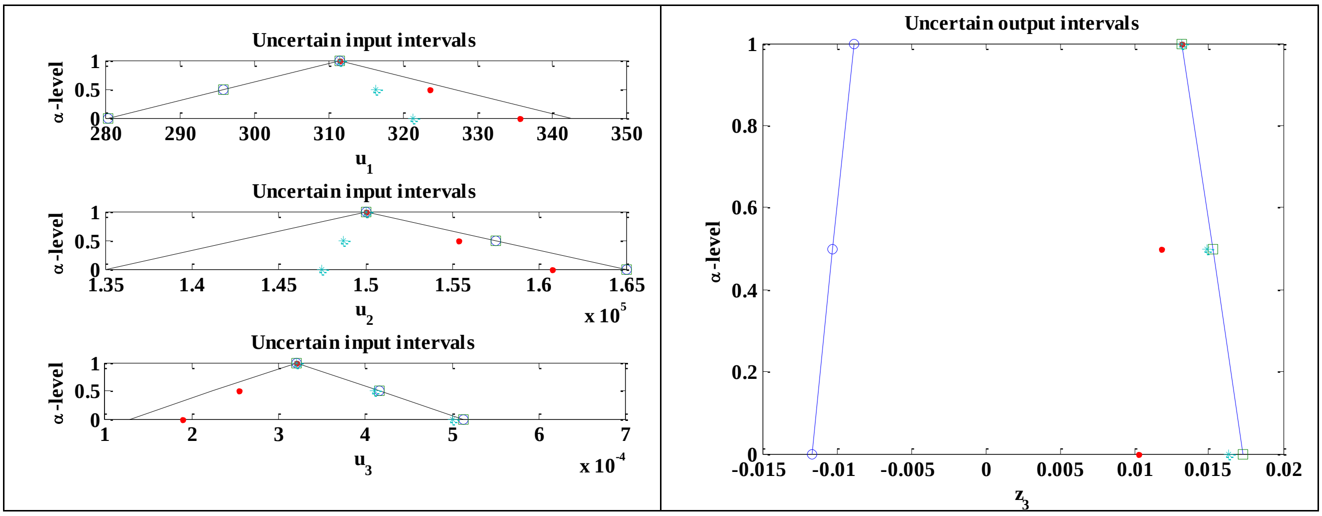

Figure 7. α-cut levels for each input parameters and suspension-working space [m].

In Fig. 7, the upper and lower bounds of uncertain input parameters were the same for each α-level; again, a degenerated intervals, which means that a single real value determines the output interval. Similarly, the uncertainty applied on that input parameters caused same effects in the results of suspension-working space. Generally, in suspension design, if the comfort requirement is mandatory, the suspension-working space requirement can be used only as constraint and not necessarily as one of the conflicting objectives.

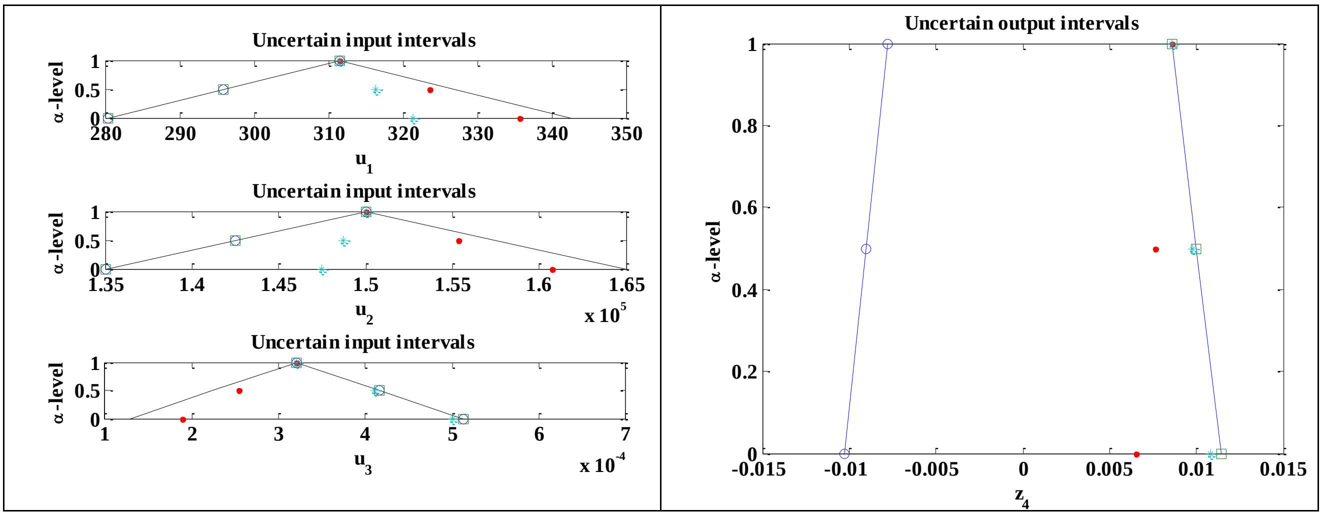

Figure 8. α-cut levels for each input parameters and road holding for safety [m].

In Fig. 8, the upper and lower bounds of uncertain input parameters were the same for each α-cut levels. However, the road holding for safety requirement find the lower bounds for tire stiffness. For instance, in the case of $\alpha = 0.5$, the tire stiffness was 1.575×10^5 N/m for both bounds resulting a road holding interval of $[-9.043 \times 10^{-3}, 1.002 \times 10^{-3}]$ m.

6. CONCLUSIONS

This paper applied the Fuzzy Interval Analysis to improve the design of suspension systems under uncertainties. The suspension system was described as 2-DOF model with uncertainty in sprung mass, tire stiffness and road profile defined by G_0. Four common requirements for suspension design were used, such as comfort, tire force, suspension-working space and road holding for safety. Since the design problem of interval analysis is a non-linear problem, a spiral optimization algorithm was selected based on its capabilities dealing with this kind of problems and low number of heuristic parameters to be fitted.

The Fuzzy Interval Analysis performed during the suspension design can evaluate how the uncertainties affect the whole system. Based on the results of this paper, a robust optimization process should be including these uncertainties in the objective function formulation as boundaries of parameters to be optimized. The interval analysis may be used to obtain extreme values of the input and output parameters that are less likely to occur, representing the worst scenario, which can be difficult to be obtained by other methods, like Monte Carlo simulation. Small perturbations in the parameters that were not considered in the suspension design can result remarkable changes in output variables.

As shown in the previous example, the Fuzzy Interval Analysis could be used in two different ways. The first one based on interval analysis at each time step could result in envelop curves that shows when the uncertainty is large and when it can be neglected. However, this choice means high computational cost since several interval analysis have to be evaluated. The second way showed to be more effective in terms of computational cost since the interval analysis is performed for the completely analyzed time interval.

In engineer point of view, it seems more valuable the second way to analyze the defined system since one is interested in global values for the suspension design requirements, accelerations and tire forces that should comply with specific standard criteria along the service time. By other hand, the first way can be important in case the uncertainty of the output values are pursuit for a specific time.

7. ACKNOWLEDGEMENTS

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