

DIFFERENTIAL EVOLUTION METAHEURISTIC FOR STRUCTURAL OPTIMIZATION IN THE DESIGN OF POWER TRANSMISSION LINE TOWERS

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Abstract. *The present work aims at studying and implementing a metaheuristic Differential Evolution algorithm for the structural optimization in the design of Power Transmission Lines. The implementation of the algorithm is verified and compared in terms of the accuracy of the solution using simple examples available in the literature, like space trusses. The algorithm is briefly reviewed and the criteria for the choice of heuristic parameters are presented. Finally, a more realistic example that optimizes a Power Transmission Line Tower is presented focusing on the minimum mass. Commercial discrete profiles are taken into account as well as loadings originating from dead weight, wind load on the structure, cables and design criteria for load combinations as well as modal (frequencies) and buckling behaviour defined in the NBR 8800, NBR 5422, NBR 6123, NBR 8449 and NBR 8681 standards. The use of the developed algorithm yielded substantial mass reduction in relation to the initial design, where all the proposed constraints (ultimate and serviceability limit state functions) were met.*

Keywords: *Differential Evolution Algorithm, Power Transmission Lines, Integer programming.*

1. INTRODUCTION

The concept of optimization can be interpreted as the search for the best solution of a project in terms of costs, maximization in material strength use and rationalization in the project. In a universe of possible solutions, one wants to maximize positive characteristics, or minimize the negative ones, quantified by a given objective function. In the case of structural optimization, one aims at minimizing the cost function, i.e. getting the configuration that has lower manufacturing costs and still adheres to parameters that ensure safety. In most cases, this feature is represented by mass. An experienced design engineer may develop this task. However, with the increased complexity of problems, constraints and materials, it is desirable to provide tools that seek the solution quickly and efficiently. This study focuses on the recent technique of Differential Evolution (DE) algorithm as a possible answer to this challenge.

The DE has been widely used in optimization problems due to simplicity, robustness and accuracy. The DE, and other evolutionary algorithms are based on the process of natural selection observed by Darwin. This technique uses causal mechanisms as parameters. Just like in nature, living entities are selected according to their fitness, the algorithm checks different individuals as a possible solution to the minimization of the cost function. In the present analysis, the function to be minimized is represented by the mass of a truss structure, which should be reduced to minimize the structural cost with several constraints. However, this process depends on several variables and must comply with building restrictions.

The methodology used in this work passes by the illustration of the use of Differential Evolution Algorithm in engineering problems. Therefore, the adopted approach seeks to be reliable to situations encountered in everyday design offices. By changing the cross sectional areas of the truss structure, the algorithm adapts the search to find available commercial profiles. This, in terms of optimization, represents a discrete optimization.

Especially in the case of power transmission line tower, an elaborate design that makes the rational use of material strength can simultaneously render savings in construction while maintains the safety and integrity of these structural members. With a transmission line mesh that exceeds 107000 km (ABRADE, 2014) and that interconnects a number of regions of the Brazilian country, it is strategic to have projects that are low cost and high reliable in operation. The failure of structural components of transmission lines are just one factor that composes the matrix of failure chain events that can generate direct and indirect costs that come from the interruption of power supply. The event of transmission line towers falling may be consequence of electrical storms effects and has been subject of headlines on newspapers, along with the associated inconvenience and loss (Reuters, 2014). According to Price et al (2006), DE is a stochastic optimization technique, based on populations. This algorithm minimizes an objective function that models the problem to be solved with restrictions in the search space. Studies such as Coello et al. (2007) play an important role in the validation of the algorithm. After comparison with established techniques, convergence analysis and analysis of algorithm stability was possible to demystify the success of DE, allowing specific studies to be performed afterwards. The algorithm proposed by Storn and Price (1995) aimed at solving mono objective problems in continuous space. However, due to ease

implementation and efficiency, DE has been successfully used in multi-objective problems and discrete optimization problems.

Applications in structural steel optimization has been the subject of a large number of works in the technical literature since its development. Kitayama et al. (2011) applied DE in structural optimization of 2D truss structures where the design variables were the cross-sectional areas for each member. In this paper, we used an approach similar to the case of a truss structure, which validates the DE in well-known optimization benchmarks. Based on a population initially random, the differential evolution takes place according to the individual fitness. Control parameters are set to control the evolutionary process: crossover rate (CR), scale factor (F) and population size (NP). DE uses the vector of randomly generated individual. Three individuals are randomly selected and goes through mutation and crossover operations, thus generating a new individual to the next generation. Zahirie (2002) investigated values for the control parameters in order to assure good performance for the algorithm. The author emphasizes the importance of a diverse population and establishes relationships between parameters and population size and diversity. As an additional measure, the author suggests that statistical values based on the population variance to be computed for each component in the population. Thus, it is possible to analyze the influence of the mutation and crossover in the process. Finally, it is concluded that these measures are effective in order to avoid the premature convergence of the algorithm.

In Singh (2009) the behavior of self-supporting steel towers subjected to wind action is studied. The programs are implemented according to the criteria of NBR 6123 and 5422 to calculate the geometry and the static loads on the truss elements. The applied wind load is characterized based on these standards. By modeling the towers, the author uses a non-linear mathematical model for the truss elements and cables and the effect of temperature. Thus, the developed programs could be able to simulate the system behavior. Mendonça et al. (2012) performs a numerical structural analysis of a transmission line tower based on technical information provided by Eletrobras Eletronorte Company. ANSYS platform was used in the computational model. First, it was taken into account equivalent static wind loads. Relevant technical standards were observed to determine the main guidelines. These included the weighting parameters of the self-weight and cables. In a second step, it was observed the dynamic behavior of the structure by a modal analysis. Finally, the results indicated that the static loads were consistent with the allowable limits specified by NBR 6123, and that the structural natural frequencies were above to those excited by the wind.

2. BACKGROUND AND THEORETICAL BASIS

2.1 Introduction

Optimization problems characterizes by searching minimum or maximum points of an objective function. The fundamental concept may be understood by the following definition. Based on a function of n variables to be minimized:

$$\begin{aligned} &\text{Minimize } f(x) \\ &\text{such that } x \in [x_{\min} \ x_{\max}] \end{aligned} \quad (1)$$

where $f(x)$ is the objective function to be optimized, $x = \{x_1 \ x_2, \dots \ x_n\}^T$ is a vector containing the design variables. $x_{\min} = \{x_{\min 1} \ x_{\min 2}, \dots \ x_{\min n}\}^T$ and $x_{\max} = \{x_{\max 1} \ x_{\max 2}, \dots \ x_{\max n}\}^T$ are vectors that defines upper and lower limits for the design variables. These can be continuous, discrete, or a mix of both.

2.2 Differential Evolution algorithm

Storn and Price (1995) developed the differential mutation procedure with the discrete recombination and pair selection that resulted in the Differential Evolution Algorithm. This algorithm initially aimed at the optimization of nonlinear functions in continuous spaces. Like virtually any evolutionary algorithm, Differential Evolution attacks the problem from an initial sample of the objective function composed of multiple randomly generated points. Previously chosen boundary conditions set the domain where the initial population vectors will belong. The vectors are indexed from 0 to the $N_p - 1$, N_p being the number of individuals in the population. Each indexed vector is then perturbed by a weighted difference of two other randomly chosen elements within the population. To produce the test-vector, u_0 DE adds the weighted difference to a third randomly selected vector. In the screening phase, test test-vector competes with the vectors of the population of the same index. The vectors with lower values in the objective function is then stored (in case of minimization). This process is repeated until all N_p vectors have been compared to randomly selected test-vectors. After the last vector be performed, the remaining of the N_p test-vectors become parents of the next generation. There are broad literature for DE and one may cite as essential books to the subject that from Storn and Price (2006). Regarding the open source code of the algorithm, there are also a lot available based on C ++, Java, Scilab, Maple, Labview, Frotran90, Python etc., specially the Matlab toolbox for genetic algorithm that has a modified version of DE.

Many variations of the algorithm DE have been proposed since there and what will be implemented throughout this work in house version of DE/rand/1/bin (DE stands for Differential Evolution, rand stands for random particle assignment for mutation, and bin means binomial recombination. First, upper and lower limits of the vectors should be determined for each vector-search parameter. Once the values have been specified, a random generator determines, according to Eq.(2), for each vector position an initial value in the given domain.

$$x_{i,g} = x_{\min,g} + \text{rand}(0,1) \cdot (x_{\max,g} - x_{\min,g}) \quad (2)$$

where $\text{rand}(0, 1)$ is an operator that returns a random number within the range 0 and 1.

Once initialized, DE will recombine vectors of the population to produce a population of N_p test-vectors. The form this combination takes place depends on the difference between two vectors added to a third vector, all randomly selected. The mutant vector $\mathbf{v}_i$ is by following Equation 4.3 follows:

$$\mathbf{v}_{i,g} = \mathbf{x}_{3,g} + \alpha F(\mathbf{x}_{1,g} - \mathbf{x}_{2,g}) \quad (3)$$

The differential weight $F \in [0, 1]$ is a positive real number that controls the rate at which the population evolves. Although the F value is not restricted, this value rarely exceeds 1.0. An α factor can be used to adjust the algorithm giving more flexibility and robustness in the search. In the case of Transmission Line Tower (LT), it proved be very important. This factor uses the random operator to adjust the difference between the search vectors. To complement the mutation strategy, DE employs the uniform mechanism of crossover. Often recognized as discrete recombination, the binomial crossover arranges two different vectors to form a vector-test. In the case of the vector is arranged with a mutant vector as:

$$\mathbf{u}_{i,g} = \begin{cases} \mathbf{v}_i & ; (\text{rand}(0, 1) \leq Cr) \\ \mathbf{x}_i & \end{cases} \quad (4)$$

The probability of crossover $Cr \in [0, 1]$ is a parameter whose value is selected by the user and controls the fraction of parameters that will be copied from the mutant vector. To determine which parameter will contribute to the vector-test, the crossover engine compares Cr with the value generated by rand operator $[0, 1]$. If the value generated is less than or equal to Cr , the vector-test inherits the mutant vector, $\mathbf{v}_i$, otherwise, the parameter vector is copied from $\mathbf{x}_i$. The selection phase is quite simple. If the test-vector $\mathbf{u}_i$ is equal to or less than the goal vector $\mathbf{x}_i$, this replaces the vector objective in the next generation. Otherwise, the objective vector maintains its position for at least another generation. The comparison of vectors, which gives rise to the evolution of populations, is defined by:

$$\mathbf{x}_{i,g+1} = \begin{cases} \mathbf{u}_{i,g} & ; f(\mathbf{u}_{i,g}) \leq f(\mathbf{x}_{i,g}) \\ \mathbf{x}_{i,g} & \end{cases} \quad (5)$$

2.3 Summary of the algorithm

Setting $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_n)$ as the solution candidate (agent), $f(\mathbf{x})$ as the objective function representing the candidate solution $\mathbf{x}$, F the differential weight, CR the probability of crossover (crossover), one can sketch the following pseudo code in Figure 1.

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Initialize  $N_p$  candidate  $\mathbf{x}$  within the initial limits in the search space for the design variables.
While stopping criteria are not met (population diversity, maximum number of iterations and stabilization of the optimal solution over consecutive iterations), do:
  Increase the number of generations. For each of the  $N_p$  agents  $\mathbf{x}$  from the population, do:
    Take three random agents from the population  $\mathbf{x}_1, \mathbf{x}_2$  and  $\mathbf{x}_3$ .
    Take a random integer  $\mathbb{I} \in \{1, \dots, n\}$  with  $n$  the number of design variables
    Evaluate the new potential positions  $\mathbf{v}_{i,g} = (\mathbf{v}_{1,g}, \dots, \mathbf{v}_{n,g})$ 
    For each  $i$  from a uniform distribution in the range of 0 to 1,  $\mathbf{r}_i = U(0, 1)$ 
      If  $\mathbf{r}_i < CR$  ou  $i = \mathbf{R}$  then do  $\mathbf{u}_{i,g} = \mathbf{v}_{i,g} = \mathbf{x}_{3,g} + \alpha F(\mathbf{x}_{1,g} - \mathbf{x}_{2,g})$ 
      Otherwise, do  $\mathbf{u}_{i,g} = \mathbf{x}_i$ 
      If  $f(\mathbf{u}_{i,g}) < f(\mathbf{x}_{i,g})$  then change individual agent "a" by  $\mathbf{u}_{i,g}$ 
    Select the agent from population that presents lower objective function value in consecutive iterations.
    Evaluate diversity.
  End loop of generations
End of while

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Figure 1. Pseudo code for the DE algorithm.

One of the main problems of evolutionary algorithms is the choice of control parameters. Depending on the choice, the algorithm cannot converge to the global optimum. The very common choice in case there is no information about the problem that will be optimized is to define the number of parents of the population as 10 times the number of design variables, select a weighting factor of $F = 0.8$, and crossover $CR = 0.9$. It was recently found that the selection of F in the range from $[0.5 \ 1.0]$ for each random generation or for each vector difference (called activation technique) significantly improves the convergence behavior, especially for noisy objective functions (Storn and Price, 1995). It was also found that setting CR to a lower value, e.g., $CR = 0.2$ helps in optimization functions that present separable variables since it promotes the search along the coordinate axes, independently.

According Zahirie (2002), premature convergence can be prevented if the likelihood of crossover induces an increase in the population variance. To find the optimal values, the relationship $\sqrt{2F^2 - 2/n + CR/n + 1} > 1$, which was obtained from the analysis of the expected value of the variance of the population of the cost functions, must be followed. The results show that a good behavior during the optimization generation is achieved using this relationship for the heuristic parameters. F is the differential weight that considers the evolution of populations; CR measures the rate of probability

that the crossover operation is applied and n represents the number of agents (candidates) present in the search space. The structural analysis starts based on given boundary conditions, geometric configurations and materials, and finishes evaluating the displacement field, internal and reaction forces. The material failure or standard criteria violation should be verified. In order to reduce the mass of the truss structure the optimization process operates over the structural analysis.

3. TRANSMISSION LINE TOWER (TLT) DESIGN

All transmission line project begins with the choice of geometric parameters of the site and checking the adequacy of the standard geometric design limitations (NBR 5422). The type of TLT capacity must be defined. For instance, they can be cable-stayed or freestanding. There are 3 TL classes: A1, voltage above 230kV, A2, voltage between 88 and 138 kV and A3, for 69kV voltage transmission, which set values defined in NBR 5422 in the design. The TLT are composed along its height by the legs, the extension part, trunk and head. The legs are the supporting elements in contact to the soil; pyramidal modules usually compose the extension part and the head are the elements used to support the insulators and cables as well as the lightning rod. Small variations may occur due to the structural employee type. In TLT design, it is common to employ-continuous bars in modules and locking bars, diagonal and horizontal bars that follows bolted discontinuous profiles, being the U and L profiles, common ones. The supported cables may be of several types and diameters (CA-aluminum cables and CAA- aluminum cables with steel core). Stresses generated by the weight of the cables follow the catenary theory and can be easily evaluated (displacement $= \gamma L^2/8T$, where γ is the weight per unit length, L the span between towers and T the axial cable force).

3.1 Loading

In this work, the cable weight and accessories (insulators, lightning rods, etc.) as well as the dead weight of the truss are taken as permanent actions. The wind action in three directions is taken as a variable action similar to the presence of human elements in the TL maintenance. Disruption of all cables and maintenance of only one stuck to the structure with its own weight by only one side (extreme torsional condition for the TLT) together with the design wind loads is assumed a rare event condition in the design. It should be noted that the actual weight of the structure, as well as the action of wind on their profiles, shall, at each change in project geometry generated by the optimization, be updated accordingly to adequately represent the structure in hand being analyzed as well as the drag forces generated in the cables and bars of the structure.

3.2 Loading combination

NBR 8800 and 8681 specifies the loads combinations (Q) to be used in metal structure designs. Essentially, it is set up combinations for ultimate limit states, service (use) and exceptional states. In NBR 8681 for permanent combinations, one should use:

$$F_d = \sum_{i=1}^m (\gamma_{gi} F_{G1,i}) + \gamma_{qi} F_{Q1,k} + \sum_{j=2}^n (\gamma_{qi} \psi_{0j} F_{Qj,k}) \quad (6)$$

where γ_{gi} , γ_{qi} , are the weights for permanent and variables actions, $F_{G1,i}$, $F_{Q1,k}$ and $F_{Qj,k}$ are the characteristic values of permanent actions and the main variable action and others that may occur together with the dead weight, respectively and ψ_{0j} is the combination factor for variable actions. For service combinations (use), it is recommended:

$$F_{ser} = \sum_{i=1}^m (F_{G1,i}) + \sum_{j=2}^n (\psi_{2j} F_{Qj,k}) \quad (7)$$

where ψ_{2j} is the combination factor for service actions variables. For rare combinations the specification is:

$$F_{rar} = \sum_{i=1}^m F_{G1,i} + F_{Q1,k} + \sum_{j=2}^n (\psi_{1j} F_{Qj,k}) \quad (8)$$

where ψ_{1j} , $F_{Qj,k}$ represents the common values of the variable actions. In this work $\gamma_{gi} = 1,25$, $\gamma_{qi} = 1,4$, $\psi_{0j} = 0,6$, $\psi_{1j} = 0,3$, $\psi_{2j} = 0$. A total number of seven loading combinations were used for displacements, stress and natural frequencies check, namely: Combination (1) dead weight and permanent actions in the structure, Combination(2) dead weight with the wind in the x direction, Combination (3), dead weight with the wind in the y direction, combination(4) dead weight and wind acting at x-y direction 45°, rare Combination(5) dead weight, wind action in x direction and broken main cables, rare Combination(6), dead weight, wind action in y direction and broken main cables, rare Combination(7), dead weight, wind action in x-y (45°) direction and rupture of main cables.

3.3 Wind Loading

The wind action should follow what is specified in NBR 5422 and NBR 6123 despite the basic elements which may provide additional information are present on the last one (for example, most current basic wind speeds basic in Brazil and wind speed variation with ground height are present on both). In general, the drag force due to the wind acting on an effective area (orthogonal projection) A_e and drag coefficient C_a (C_x , C_y , etc) and dynamic pressure q is given by:

$$F_e = C_a q A_e \quad (9)$$

The C_a values are obtained in NBR 6123 tables for metallic profiles and wind angle incidence. The design wind speed should be obtained depending on the height of each tower considered as a module:

$$V_p = K_r K_d (H/10)^{1/n} V_T \quad (10)$$

where ρ is the air density in kg/m³ and V_p is the design wind speed in m/s. The density of the air is assumed $\rho = [1,293 / (1 + 0,00367t)] \cdot [(16000 + 64t^2 - ALT) / (16000 + 64t^2 + ALT)]$, with t , temperature °C and ALT means the elevation of the site. For truss structures with rectangular cross section, the load due to wind action on the trunks of length L , applied at the center of gravity and is given by:

$$A_t = q_0 [1 + 0.2(\sin 2\theta)^2] [S_{T1} C_{xT1} (\sin \theta)^2 + S_{T2} C_{xT2} (\cos \theta)^2] \quad (11)$$

where θ is the wind angle of incidence, S_{T1} and S_{T2} are the orthogonal projected net areas and C_{xT1} and C_{xT2} are the own drag coefficients for faces 1 and 2 and wind acting perpendicular to each face. The calculation can be simplified by using the exposed area ratio in the case of other profiles types.

3.4 Wind Loading on isolated cables

For the wind action at the extended cables with spam z , it is used the equation that provides the loads at the cable attachment points which comes from the perpendicular action of wind on the cable:

$$A_c = q_0 C_{xc} \alpha \frac{z}{2} (\sin \theta)^2 \quad (12)$$

q_0 is the dynamic pressure, C_{xc} is the drag coefficient (1.0), α is the effectiveness factor (Figure 2 of the NBR 5422), z is the length of the gap in hand and θ the wind incidence angle. In the case of insulators, the formulation is somewhat similar to drag forces.

$$A_i = q_0 C_{xi} S_i \quad (13)$$

where C_{xi} is the drag coefficient of the insulation cable and S_i is the orthogonally projected area onto a vertical plane.

3.5 NBR 8800 criteria verifications

NBR 8800 standard uses the method of limit states, so the strengths and deformations must be limited, depending on the used material and the type of structure. Following the implementation of the structural analysis algorithm, standard safety criteria are checked: axial stresses, displacements, and natural frequencies. Usually, steel profiles used in power transmission lines are part of the laminated profiles, so they should follow the guidelines of the corresponding standard.

There are several standard geometric constraints requirements to the profiles. It should be noticed first that tension and compression steel yield strength are not the same, so that bars subjected to axial tensile strength must comply with different criteria of the bars subjected to axial compression force. Therefore, special observations are taken into account to allow for the different modes of buckling. NBR 8800 specifies for compressive bars the effective widths method. This paper adopts the restrictions for tubular profiles defined in NBR 8800:2008.

Regarding the dynamic characteristics, the structure should not present natural frequency similar to that from the exciting wind in order to prevent the resonance phenomenon. NBR8800 specifies limits only for floors in multistory building in order to limit the perception of the occupants. NBR 6123 provides simple formulas to determine the fundamental frequency of typical buildings, but no restrictions to the values. NBR 5422 also does not specify limits. Typical excitations of earthquakes and winds are within 0-5Hz range (around 1 Hz for winds) and therefore it is necessary to avoid TLT with fundamental frequency in this range (Mendonça, 2012).

4. NUMERICAL APPLICATIONS

4.1 72 bar spatial truss (Literature benchmark)

In the following example, a spatial truss structure with 20 nodes and 72 bars is analyzed. This example is used as benchmark in the literature by several authors. This example is intended to validate DE programming in a case that has already been optimized by other heuristic and gradient-based algorithms. Further details regarding the design variables, bar groups, tension and compression limits, etc., can be found in Guerra (2008). Table 1 presents the results obtained using DE and those obtained in the literature.

Table 1. Comparison between best solutions for the 72 bar truss structure problem (mm²).

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	Mass (kg)
(a)	63.8	400.6	175.4	221.9	276.1	338.7	45.16	77.41	745.8	353.5	55.48	58.06	1097.4	399.9	50.32	58.70	170.62
(b)	100.97	352.00	264.77	367.55	337.87	333.61	64.516	64.516	818.32	330.13	64.52	64.52	1216.9	330.52	64.52	64.52	172.20
(c)	103.87	359.35	243.22	326.45	394.19	343.22	64.51	64.51	803.86	338.06	64.51	64.51	1172.9	338.06	64.51	64.51	172.90
(d)	100.20	361.50	237.40	368.10	297.50	359.50	64.60	76.60	806.80	330.00	64.60	64.50	1181.8	334.00	64.50	64.50	172.87

(a) AG (Guerra, 2008), (b) FM Sedaghati, 2005, (c) Venkayya, 1971, (d) DE.

4.2 200 bar spatial truss (Literature benchmark)

The truss shown in Figure 2 was optimized according to NBR 6123, NBR 8800 and NBR 8681 criteria, using commercial tubular aluminum profiles using a genetic algorithm and using a commercial software mCalc3D by Guerra (2008). In this paper, it is compared results obtained using the developed program with those obtained by Guerra (2008) in order to evaluate the reliability of the implementation of NBR standard criteria. Details of load combinations, global and local buckling criteria and allowed displacements can be found in Guerra (2008) (Figure 2). Table 2 shows the results using a commercial software and the developed code. The commercial profile 23 ($D = 33,4\text{mm}$, $t = 3.4\text{ mm}$, $A = 3,19\text{mm}^2$) is the optimal solution found for the upper, lower and diagonal bars, which were grouped for construction reasons. It is noted that the final mass and profiles are identical to those provided by the commercial Software mCalc3D.

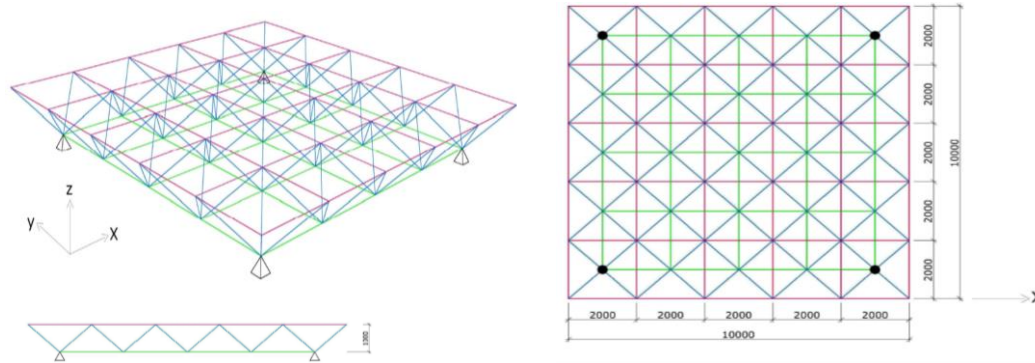


Figure 2. 200 bar spatial truss problem.

Tabela 3 - 200 bar results, Integer design variables.

Design variables	mCalc 3D	GA (Guerra, 2008)	Present work DE
	Profile No.	Profile No.	Profile No.
X_1	23	23	23
X_2	23	25	23
X_3	23	23	23
Mass (kg)	938.79	942.09	938.79

4.3 2 Optimization of TLT - 244 Bar Space Truss

The transmission of energy so that it properly reaches all the country's consumption points is a major challenge in the electric system operation. The TLT have a fundamental role in this process, as support for transmission cables at a safe distance from the ground and other obstacles. Accidents involving TLT cause direct and indirect losses to all that are connected by the damaged lines. In addition, these accidents represent considerable maintenance costs for companies. The main causes that can trigger catastrophic failures of TLT are the loads applied by the wind and disasters such as earthquakes, tornadoes and cyclones. As in Brazil seismic activity is low, it is reasonable to assume that only wind loads bring risks. According to NBR 6123, it is possible to calculate the wind forces applied to buildings, according to the incidence angle and wind speed at each site. The proposed design criteria in this example also follows the parameters defined in the NBR 5422. Due to the likelihood of cascading failures offered by the rupture of a cable, it is also necessary to analyze this condition. The standards defines combination of loads that can be related to the frequency of incidence of the acting loads.

The optimization of the TLT structure shows challenging to the algorithm. The spatial truss structure has 244 bars and 77 nodes. The design variables correspond to the cross sectional areas, assumed to be defined by a list of commercial profiles. The steel material properties as well as the geometry of the structure are intended to represent an actual TLT presented in the literature. The material density is 7850 kg/m^3 and the Young modulus 200 GPa . Nodes 74, 75, 76 and 77 are simply support. Figure 3(a) shows the main geometric dimension of the TLT initial design that will be optimized. In this example, it was used six different loads (dead weight, weight of cables and insulators, x-wind y-wind, x-y wind (45°), rupture of all cables and maintenance of only 1 cable). Seven load combinations were used as previously explained. In this work, the commercial profile table "L" ASTM-36. Aluminum conductor cables with steel core were used, CAA-636MCM 26/7 Grosbeak with diameter $21,9\text{mm}$ dead weight of 13 N/m and ultimate tension load of 31500 N . For the lightning rod it is used EHS 3/8 " cables with 7 galvanized wires with a diameter $9,15\text{mm}$, dead weight of 4.07 N/m and maximum tension load 14 KN . The elements of the structure are divided into groups to save processing time and a make a feasible design, since it would be impractical to specify as design variables for each member cross section.

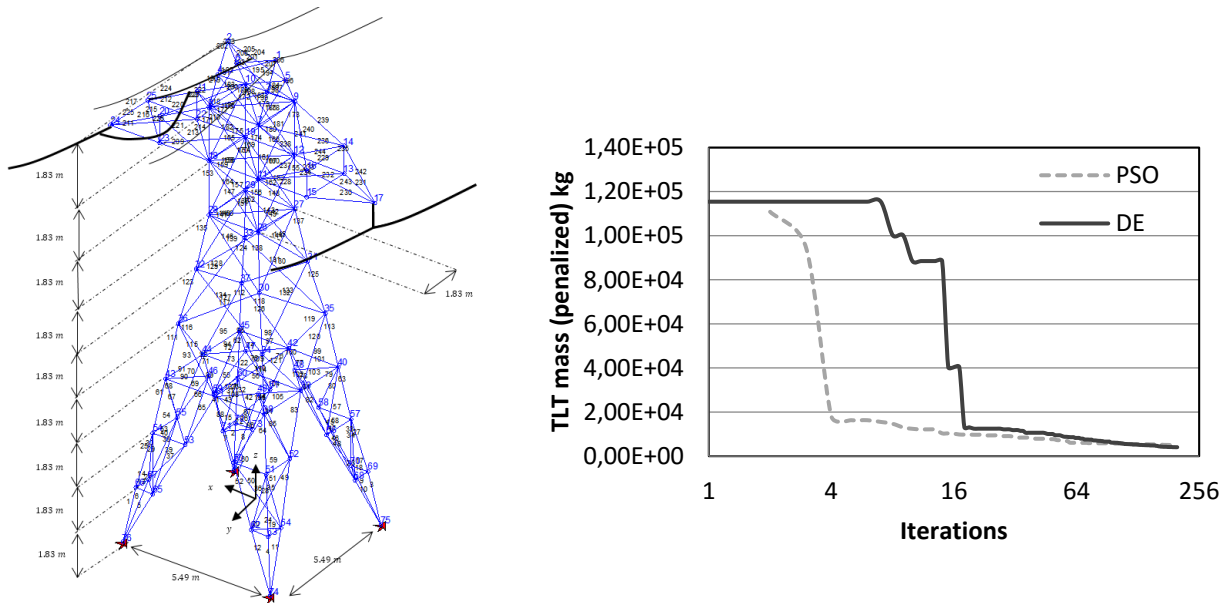


Figure 3. (a) TLT modelled as a 244 bar spatial truss. (b) Truss mass along iterations PSO and DE.

Elements that occupy equivalent positions, considering symmetry are optimized using the same design variable. In this example, the structure was divided into 32 groups namely: 01 to 04, 05 to 12, 13 to 24, 25 to 28, 29 to 36, 37 to 52, 53 to 60, 61 to 64, 65 to 88, 89 to 108, 109 to 110, 111 to 114, 115 to 122, 123 to 126, 127 to 134, 135 to 138, 139 to 146, 147 to 152, 153 to 156, 157 to 170, 171 to 174, 175 to 188, 189 to 201, 202 to 209, 210 to 213, 214 to 218, 219 to 222, 223 to 226, 227 to 228, 229 to 234, 235 to 239, 240 to 244.

It is considered the fundamental natural frequency of the structure as a secondary constraint. In the initial design, the first natural frequency of the structure presented above the prescribed threshold value for the optimization (5 Hz). Once optimized, it was obtained 6.43 Hz as the first natural frequency that is in agreement with the recommendations in the literature (Mendoza, 2012). The critical load combination corresponds to the combination number 7, where there are dead weight (bars and TLT tower elements), rupture of the cables and incidence of wind at 45° . It could be noted that member 111 has the higher tension value $1.4 \cdot 10^5$ N, while the stress limit for this element is $9.43 \cdot 10^5$ N. For compressed members, element 63 presented the higher value of $3.44 \cdot 10^5$ N, while the corresponding design buckling limit is $6.76 \cdot 10^5$ N. Therefore, one concludes that none of the constraints in the final solution of the objective function became active.

The algorithm could search for cross sectional areas that reduced the total mass of the structure without violating standard constraints. The mass value of the structure was minimized to 4012.43 kg by DE, which represents a reduction of 226% for the first feasible solution found (with no penalty).

In order to validate the results and compare DE with other metaheuristic methods, PSO (Particle Swarm Optimization) and the SQP method (Sequential Quadratic Programming) was applied to the optimization problem. For PSO it was used $n=20$; $\omega=0.1$; $c_1=2$ and $c_2=2$. In this case DE obtained results that were better than PSO or SQP. The mass of the optimized structure obtained with SQP was 5668 kg, and that obtained by PSO was 4857 kg. Figure 3(b) shows the evolution of PSO and DE mass values. It is observed that although initially PSO has a faster convergence, lower objective function in 200 iterations is attained by DE. The algorithm quickly converged in few iterations near the best mass, and then presented a less significant reduction in mass over the next iterations. Table 2 shows the cross sectional areas for each selected group of elements for PSO and DE.

Table 2. Best profiles set using DE and PSO for TLT problem.

	A ₁	A ₂	A ₃	A ₄	A ₅	A ₆	A ₇	A ₈	A ₉	A ₁₀	A ₁₁	A ₁₂	A ₁₃	A ₁₄	A ₁₅	A ₁₆	A ₁₇	A ₁₈	A ₁₉	A ₂₀	A ₂₁	A ₂₂	A ₂₃	A ₂₄	A ₂₅	A ₂₆	A ₂₇	A ₂₈	A ₂₉	A ₃₀	A ₃₁	A ₃₂	Mass (kg)
(a)	30	29	18	26	24	19	26	31	20	19	49	27	27	34	23	24	25	18	37	25	17	19	17	21	28	29	27	25	23	48	40	39	4857.23
(b)	34	21	9	26	18	23	6	49	19	14	58	28	25	29	23	18	25	14	31	23	16	23	23	24	32	25	39	28	16	17	22	25	4012.43

(a) PSO, (b) DE.

5. FINAL REMARKS

In this paper, it was coded the Differential Evolution (DE) algorithm in MATLAB language to solve structural optimization problems. The algorithm was developed in order to compare and verify the accuracy of solutions that has been successfully tested in simple examples in the literature (benchmarks). Furthermore, in order to check the generality of the algorithm, it was used in the optimization of spatial truss structures. In these applications, the optimization process showed good behavior and the result was in good agreement with commercial software (mCalc3D). This confirms the accuracy and robustness of the implemented DE as a design tool.

Finally, by applying the DE in the design of a TLT it was demonstrated the versatility of the code. It can be concluded that the optimization problem could be handled due to the few control parameters used by DE. In addition, the DE robustness made it possible to search for a solution in the discrete space, represented in this instance by commercial profile tables. Literature recommendation for the heuristic parameters have been found useful since it was possible to ensure stability and good behavior of the algorithm.

The optimization process was accomplished since it was possible to find feasible solutions within a small number of iterations in a reasonable processing time. The mass reduction of the TLT tower of represented 226% compared to the first feasible configuration. All the NBR 8800 constraints were attained. Moreover, the natural frequency of the structure was also found above limit value, which prevents the structure to resonate when excited by the wind.

The mass reduction of TLT tower structure outperformed the result obtained by other methods like PSO and SQP. DE generated a design of 4012.43 kg, while PSO and SQP converged to 4857.0 kg and 5668.0 kg, respectively.

As future work, the study the DE convergence seems to be promising. In addition, a study of improvement in crossover and mutation mechanisms may enhance the presented result. Finally, another important suggestion for future work is to apply the same methodology for sizing and topology optimization of a TLT.

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