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EIGENSTRUCTURE ASSIGNMENT CONTROL APPLIED TO AIRCRAFT MODEL IN GO-AROUND CONTROLLER DESIGN

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Abstract*. Flight control systems have been designed through classical control methodology since of the early days of the aircraft industry, it happens because using a method that is intuitively visible to the most classical control designers decreases certification time process of the control system. Moreover, nowadays plenty of problems encountered during aircraft design phase by Engineers are unable to be solved by classical control due to its complexity and cross-coupling effects. Having that in mind, it is reasonable that Flight Mechanics Engineers have to know different methodologies to apply in aircraft control systems besides their benefits and disadvantages in comparison with traditional and other modern techniques. It induces revalidation and research of modern techniques, then inside this scenario that the motivation of study of Eigenstructure Assignment (EA) methodology is addressed. In this work, the Eigenstructure Assignment method has been applied to go-around controller design. The method is applied to Stability Augmentation System for longitudinal and lateral axes and then a track controller is designed with auxiliary technique Quadratic Performance Index. Civil and Military requirements are used as guideline during the project.*

Keywords: *Eigenstructure Assignment, Control System, Go-around maneuver*

1. INTRODUCTION

Eigenstructure assignment is one of the basic techniques for designing linear control systems. The eigenstructure assignment problem consists of assigning both a given self- conjugate set of eigenvalues and the corresponding eigenvectors. Assigning the eigenvalues allows one to alter the stability characteristics of the system, while assigning eigenvectors alters the transient response of the system (Afbdelaziz and Valasek, 2004). In order words, this method allows the designer to satisfy directly damping, settling time, and mode decoupling specifications by directly choosing the eigenvalues and eigenvectors (Andry and Shapiro, 1983). Although most of the methodology of eigenstructure assignment is purely mathematical, the requirement for the control system designer to specify a desired eigenstructure, consisting of eigenvalues and eigenvectors corresponding to each of the eigenvalues, introduces an element of subjectivity to the design process.

2. EIGENSTRUCTURE ASSIGNMENT

2.1. Control problem - Eigenvector Assignability

We shall consider a linear time invariant system described by the equations:

$$\dot{x} = Ax + Bu \tag{1}$$
$$y = Cx \tag{2}$$

Exogenous signals and noise do not affect the eigenstructure and thus they are taken to be zero. The following standard assumptions should be made:

- B and C are of full rank
- (A, B) is controllable and (A, C) is observable.

Where $A \in R^{nxn}$, $B \in R^{nxm}$ and $C \in R^{pxn}$ n is the number of aircraft states, p is the number of measured outputs and m is the number of aircraft inputs.

$$u = Ky \tag{3}$$

The constrained output feedback problem can be stated as follows: Given a set of desired eigenvalues λi, i = 1, 2....r, and a corresponding set of desired eigenvectors, *vi*, i =1, 2....r, find, if possible, a real matrix K, some of whose elements are fixed as zero, such that the eigenvalues of A + BKC contain (λi) as a subset, and the corresponding eigenvectors of A + BKC are close to the respective members of the set (*vdi*) (Sobel and Shapiro, 1985).

From this part on, it is discussed the problems of characterizing eigenvectors which can be assigned as closed-loop eigenvectors, and of determining the best possible set of assigned closed-loop eigenvectors in case a desired eigenvector is not assignable (Sobel and Shapiro, 1985).

We begin by considering the closed-loop system:

$$\dot{x} = (Ax + BKC)x(t) \tag{4}$$

Assuming we are given λi, i = 1, 2....r, as the desired closed-loop eigenvalues and assuming *vi* is the corresponding closed-loop eigenvector. Then:

$$(A + BKC)v_i = \lambda_i v_i \tag{5}$$

$$(\lambda_i I - A)v_i = BKCv_i \tag{6}$$

$$v_i = (\lambda_i I - A)^{-1} BKCv_i \tag{7}$$

Thus, since the vector *vi* must lie in the space defined by the span of the columns of

$$(\lambda_i I - A)^{-1} B \tag{8}$$

This matrix can be taken as a vector basis for the closed-loop eigenvectors of the system. The major advantage of using this matrix is that no null space needs to be calculated, saving on computation (Faleiro, 1998). Nonetheless, an inverse has to be calculated, which will cause problems if $(\lambda_i I - A)$ is singular. This will not occur if the designer ensures that none of the desired closed-loop eigenvalues are exactly the same as any of the open-loop system eigenvalues (Faleiro, 1998).

3.2. Controller Synthesis - Gain Computation

In many practical situations, complete specification of *vi* is neither required or known, but rather the designer is interested only in certain elements of the eigenvector (Sobel and Shapiro, 1994). Thus, assuming that *vi* has the following structure (subscript *d* means desirable):

$$v_i^d = [v_{i1}, x, x, x, v_{ij}, x, x, v_{in}]^T \tag{9}$$

In general, however, a desired eigenvector *vdi* will not reside in the prescribed subspace and hence cannot be achieved. Instead a "best possible" choice for an achievable eigenvector is made. This best possible eigenvector is the projection of *vdi* onto the subspace spanned by the columns of $(\lambda_i I - A)^{-1}B$ as depicted in Fig. 1 (Sobel and Shapiro, 1985).

Let λi, *vi* be an eigenvalue-eigenvector pair of the closed-loop system, *u* = *Ky*, then, we define the *Li* matrices and *zi* auxiliary vectors as

$$L_I = (\lambda_i I - A)^{-1} B \tag{10}$$

$$z_i = KCv_i \tag{11}$$

Then the closed-loop eigenvectors comply with

$$v_i = L_i z_i \tag{12}$$

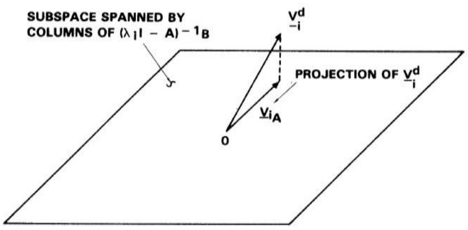

Figure 1. Dim null space in a 3-Dim state space (Sobel and Shapiro, 1985)

To find the value of *zi* corresponding to the projection of *vdi* onto the “achievability subspace” we choose *zi* which minimizes *J*, according to (Sobel and Shapiro, 1985).

$$J = \left\|v_i^d - v_i^a\right\|^2 = \left\|v_i^d - L_i z_i\right\|^2 \qquad \text{Then,} \tag{13}$$

$$\frac{dJ}{dz_i} = 2L_i^T\left(L_i z_i - v_i^d\right) = 0 \tag{14}$$

The optimal *zi* are calculated as

$$z_i = (L_i^T R L_i)^{-1} L_i^T R_i v_i^d; \qquad i = 1, \dots, n \tag{15}$$

As it has been already said, *nu* elements in each eigenvector can be arbitrarily chosen. In many practical situations some components of the desired eigenvectors are not known or required so they should not induce any drawback in the cost function. Therefore, if only *nu* elements in eigenvector *vdi* are specified, then *Ri* is a diagonal matrix with 1 value in the entries corresponding to the specified elements, and zeroes elsewhere (Cruz and Ruiperez, 1997). The achievable eigenvectors *va* are given by

$$v_i^a = L_i z_i; \qquad \mathrm{i} = 1, \dots \mathrm{n} \tag{16}$$

To find the output feedback gain matrix *K* a similarity transformation *T* is chosen so that the B matrix is a lead block identity matrix

$$T = [B\ P] \tag{17}$$

Where *P* is any matrix that makes *T* full rank. Note that this can be always done since we have assumed the B matrix is of full rank (Cruz and Ruiperez, 1997). Applying *T* to the original system, Eq. (1) and Eq. (2), the following transformed system is obtained:

$$\tilde{B} = \begin{bmatrix} I \\ 0 \end{bmatrix} = T^{-1}B \tag{18}$$

$$\tilde{A} = \begin{bmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{bmatrix} = T^{-1}AT \tag{19}$$

$$\tilde{C} = CT \tag{20}$$

Where $\tilde{A}$ has been partitioned into the first nu rows and the last $n - nu$ rows (Cruz and Ruiperez, 1997). The state, eigenvalues and achievable eigenvectors of the transformed system are given by

$$\tilde{x} = T^{-1}x \tag{21}$$

$$\tilde{\lambda_\imath} = \lambda_i \tag{22}$$

$$\widetilde{v_\imath^a} = T^{-1}v_i^a \tag{23}$$

Where *va* has been partitioned into the first nu rows and the last $n - nu$ rows (Cruz and Ruiperez, 1997). We can see that the eigenvalues of the new system are equal to the eigenvalues of the original system.

The gain matrix *K* is then given by:

$$K = \left(\tilde{S} - \tilde{A}_1\tilde{V}\right)\left(\tilde{C}\tilde{V}\right)^{-1} \tag{24}$$

Where

$$\tilde{S} = [\tilde{\lambda}_1 d\tilde{s}_1, \dots, \tilde{\lambda}_{ny} d\tilde{s}_{ny}] \tag{25}$$

$$\tilde{V} = [\widetilde{v_\imath^a}, \dots, \widetilde{v_n^a}] \tag{26}$$

3. THE PROJECT

The Equation (27) shows the longitudinal linear model obtained when it is linearized around the following conditions: Vt = 100 [m/s], h = 915 [m], weight = 255772 [kg], cg =0.29c and γ = −3◦.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \\ \dot{H} \\ \dot{\delta\pi} \\ \dot{\delta e} \end{bmatrix} = \begin{bmatrix} -0.0165 & -0.0314 & -9.7894 & 0.6820 & 0.0001 & 0.2314 & -0.0279 \\ -0.2006 & -0.6342 & 0.5818 & 97.4158 & 0.0010 & 0 & -3.9897 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 & 0 \\ -0.0002 & -0.0083 & 0 & -0.5077 & 0 & 0.0021 & -0.7985 \\ -0.0593 & -0.9982 & 99.8630 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.3000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10.0000 \end{bmatrix} \begin{bmatrix} u \\ w \\ \theta \\ q \\ H \\ \delta\pi \\ \delta e \end{bmatrix} \quad (27)$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 & 0.3 & 0 \end{bmatrix}^T \begin{bmatrix} \delta\pi_c \\ \delta e_c \end{bmatrix} \quad (28)$$

The matrices were augmented by including the throttle dynamic and elevator dynamic.

Seven state variables are available for this controller: forward velocity u, downward velocity w, pitch angle θ, pitch rate q, altitude reference H, throttle dynamic δπ and elevator dynamic δe. Two control variables are available: throttle and elevator deflection (δπc, δec). The measurements of this controller are forward velocity u, downward velocity w, pitch angle θ and pitch rate q. It could be entitled output feedback once H is neglected.

In a typical aircraft model with classical configuration there are two longitudinal modes: the short period and the phugoid. The Tab. 1 gives us the parameters of each mode.

Table 1. Modes and eigenvalues of the open loop system

	Eigenvalues	Damping	Freq (rad/s)	Ts (s)	Tr (s)
Mode 1	-0.5788 ± 0.8894i	0.5455	1.0611	7.9472	3.8008
Mode 2	-0.0002 ± 0.1192i	0.0018	0.1192	∞	∞
Mode 3	-0.3000	1.0000	0.3000	-	-
Mode 4	-10.0000	1.0000	10.0000	-	-

Eigenvectors shall be specified to determine the correct coupling with the desired eigenvalues. The desired eigenvectors are shown below where the symbol “x” means an unspecified component. Short period eigenvectors v1 and v2 are chosen so that they couple w and q while keeping the others unspecified. The phugoid eigenvectors v3 and v4 are chosen as well, so they couple with u and pitch angle θ while keeping the other variables constant and unspecified. Thus we get a satisfactory degree of decoupling between these modes.

Table 2. Desired Eigenvectors

	Short Period		Phugoid		H	throttle	elevator
Eigenvectors	v1	v2	v3	v4	v5	v6	v7
u	x	x	1	x	x	x	x
w	1	x	x	x	x	x	x
θ	x	x	x	1	x	x	x
q	x	1	x	x	x	x	x
H	x	x	x	x	x	x	x
δπ	x	x	x	x	x	1	x
δe	x	x	x	x	x	x	1

With four state feedback we are able to completely assign short period and phugoid modes as desired. The previous choice of eigenvectors have attained the goal due to the decoupling of modes, otherwise one mode would be affected by the other and then the closed-loop eigenvalues would not be assigned perfectly but near the desirable place due to the space spanned achieved.

The gain values for throttle and elevator controls were obtained to achieve the objectives: damping ratio of 0.7 and natural frequency of 1.6 rad/s for short-period mode; damping ratio of 0.5 and natural frequency of 0.29 rad/s for phugoid mode. The gain values might be used in the non-linear time simulation to verify the real behavior of the plant.

Table 3. Modes and eigenvalues of the closed loop system

	Eigenvalues	Damping	Freq (rad/s)	Ts (s)	Tr (s)
Mode 1	-1.1200 ± 1.1426i	0.7000	1.6000	4.1071	1.9643
Mode 2	-0.1450± 0.2511i	0.5000	0.2900	31.7241	15.1724
Mode 3	-0.3	1.0000	0.3	-	-

Mode 4	-8.6271	1.0000	8.6271	-	-

The Equation (29) shows the latero-directional linear model obtained when it is linearized around the following conditions: Vt = 100 [m/s], h = 915 [m], weight = 255772 [kg], cg =0.29c, γ = −3◦ and φ = 0◦

The matrices were augmented by including the aileron and rudder dynamics.

Six states variables are available for this controller: starboard velocity *v*, bank angle φ, roll rate *p*, yaw rate *r*, aileron and rudder dynamics (δa, δr). Aileron and rudder deflection (δac, δrc) are the control variables. The measurements of this controller are starboard velocity *v*, bank angle φ, roll rate *p* and yaw rate *r*.

$$\begin{bmatrix} \dot{v} \\ \dot{\varphi} \\ \dot{\rho} \\ \dot{r} \\ \dot{\delta a} \\ \dot{\delta r} \end{bmatrix} = \begin{bmatrix} -0.1197 & 9.7894 & -0.7001 & -99.9975 & 0 & 1.9838 \\ 0 & 0 & 1.0000 & -0.0594 & 0 & 0 \\ -0.0253 & 0 & -1.3803 & 0.5127 & -0.4852 & -0.0199 \\ 0.0048 & 0 & -0.2187 & -0.3017 & -0.0334 & -0.3279 \\ 0 & 0 & 0 & 0 & -10.0000 & 0 \\ 0 & 0 & 0 & 0 & 0 & -12.0000 \end{bmatrix} \begin{bmatrix} v \\ \varphi \\ \rho \\ r \\ \delta a \\ \delta r \end{bmatrix} \tag{29}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12 \end{bmatrix}^T \begin{bmatrix} \delta a_c \\ \delta r_c \end{bmatrix} \tag{30}$$

As known, most of conventional aircraft models with classical configuration shown three native modes: dutch-roll mode, roll mode and spiral mode. Tab. 4 gives us the parameters of each ones.

Table 4. Modes and eigenvalues of the lateral open-loop system

	Eigenvalue	**Damping**	**Freq. (rad/s)**	**Ts (s)**	**Tr (s)**
Mode 1	-0.0660 ± 0.9533i	0.0691	0.9556	69.6761	33.3233
Mode 2	-1.6310	1.0000	1.6310	2.8203	1.3488
Mode 3	-0.0387	1.0000	0.0387	118.9356	56.8823
Mode 4	-10.0	1.0000	10.0000	-	-
Mode 5	-12.0	1.0000	12.0000	-	-

The open loop has five modes and we are able to identify them easily. The pair- conjugate eigenvalues is related to dutch-roll mode, which is lightly damped and shall be improved, eigenvalue of the mode 2 is far off the imaginary axis so is related to roll mode, the eigenvalue of the mode 3 is near the imaginary axis, then spiral mode. The other modes are related to aileron and rudder dynamics.

Table 5. Desired Eigenvectors

	Dutch-roll		**Roll mode**	**Spiral mode**	**Aileron**	**Rudder**
Eigenvectors	v1	v2	v3	v4	v5	v6
V	1	x	x	x	x	x
φ	x	x	x	1	x	x
p	x	x	1	x	x	x
r	x	1	0	x	x	x
δa	x	x	x	x	1	x
δr	x	x	x	x	x	1

We chose the dutch-roll mode eigenvectors v1, v2 and they are coupled with *v* and *r* while keeping others unspecified. The roll mode eigenvectors v3 are coupled with p and while keeping the other variables unspecified and the spiral mode eigenvectors v4 are coupled with φ. Thus, we get a satisfactory degree of decoupling between these modes.

An output feedback gain matrix is now computed by using eigenstructure assignment in order to achieve the objectives: damping ratio of 0.6 and natural frequency of 1.29 rad/s for dutch-roll mode; damping ratio of 1 and natural frequency of 2 rad/s for roll mode; natural frequency of 1 and natural frequency of 0.01 for spiral mode . Since there are four outputs and two inputs, we are able to modify the four closed loop eigenvalues and to assign two elements to each of the four eigenvectors.

TABLE 6. Modes and eigenvalues of the closed loop system

	Eigenvalues	**Damping**	**Freq (rad/s)**	**Ts (s)**	**Tr (s)**
Mode 1	-0.7800± 1.0399i	0.6001	1.2999	5.8972	2.8204

Mode 2	-2.0000	1.0000	2.0000	2.3000	1.1000
Mode 3	-0.0100	1.0000	0.0100	460.000	220.00
Mode 4	-9.5638	1.0000	9.5638	-	-
Mode 5	-10.6086	1.0000	10.6086	-	-

With four state feedback, we are able to completely assign the dutch-roll, roll and spiral mode as desired. The eigenvector chosen has met the goal with all modes decoupled. The gain values for aileron and rudder controls were obtained and used in non-linear time simulation.

When an aircraft is under manual control (as opposed to autopilot control) the stability augmentation system is, in the most cases, the only automatic flight control systems needed, but in case of high-performance military aircraft or in cases where the pilot may have to maneuver the aircraft to its performance limits and perform a task such as precision tracking, specialized control augmentation systems are useful (Stevens and Lewis, 1992). Once we have planned to design a go-around controller, the control augmentation systems became an interesting feature to improve the accuracy of the task.

The design is composed of a pitch-axis control augmentation, controlling pitch angle and roll-axis control augmentation controlling bank angle. In the inner loop design, the closed-loop state matrix Asas is achieved through the gain matrix Ky calculated by Eigenstructure Assignment method as already commented. Once the final objective is to track a desirable variable, the current system shall be augmented with PI-compensator and then a new closed-loop matrix has to be designed through new gain matrix Kv. In our case this is performed using Quadratic Performance Index method.

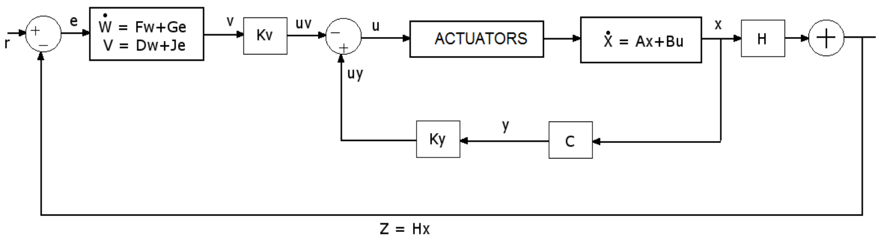

Figure 2. Full Block Diagram

$$\dot{x} = Ax + bu \quad (31)$$
$$y = Cx \quad (32)$$

$$u = uy - uv \quad (33)$$
$$\mathrm{u} = -\mathrm{uv} + \mathrm{KyCx} \quad (34)$$

$$\dot{x} = (Ax + BKyC)x - Buv \quad (35)$$
$$\dot{x} = A_{SAS}x - Buv \quad (36)$$

Once the stabilized state matrix Asas is available, we proceed with the design since the compensator is given:

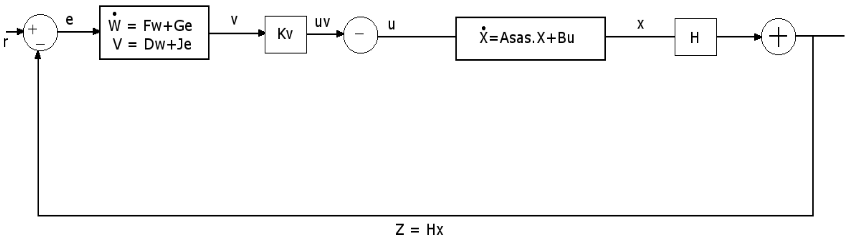

Figure 3. CAS Diagram with SAS

$$\dot{w} = Fw + Ge \quad (37)$$
$$y = Dw + Je \quad (38)$$

Checking the Fig. 3, is given:

$$e = r - z = r - Hx \quad (39)$$

$$\dot{w} = Fw + Ge = Fw + G(r - Hx) = Fw + Gr - GHx \quad (40)$$
$$v = Dw + Je = Dw + J(r - Hx) = Dw + Jr - JHx \quad (41)$$

$$\dot{x_a} = \begin{bmatrix} A_{SAS} & 0 \\ -GH & F \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ G \end{bmatrix} \mathrm{r} \quad (42)$$

$$\dot{x_a} = A_a x_a + B_a u + G_a r \quad (43)$$

New augmented output is defined:

$$y_a = [-JH \quad D] \begin{bmatrix} x \\ w \end{bmatrix} + [J] r = [v] \quad (44)$$

$$y_a = C_a x_a + F_a r \quad (45)$$
$$u = uv = -Kv.v \quad (46)$$

Substituting Eq. (45) in Eq. (46),

$$u = -Kvya = -KvC_a x_a - KvF_a r \quad (47)$$

Finally a new closed-loop matrix is obtained,

$$\dot{x_a} = A_a x_a - B_a KvC_a x_a - B_a KvF_a r + G_a r \quad (48)$$
$$\dot{x_a} = (A_a x_a - B_a KvC_a) x_a + (G_a - B_a KvF_a) r \quad (49)$$

The variable being tracked is identified by matrix H. The minimization of the *Quadratic Performance* requires the resolution of the *Lyapunov* equation - more details might be found on (Stevens and Lewis, 1992). In this model case we have achieved the results with proportional-Integral compensators for longitudinal and latero-directional axes.

During the time-simulation, the aircraft has flown the glide slope path till the decision height when the sink rate is decreased by pitch-up command of 12º and then 8º to perform the go-around. In parallel, after 20 seconds of the first pitch command, the roll command is demanded by 10º in order to simulate a turn. Figure 4 and Fig. 5 exposed graphically the main variables of the situation.

The aircraft reaches the positive gradient of climb therefore the simulation is finished.

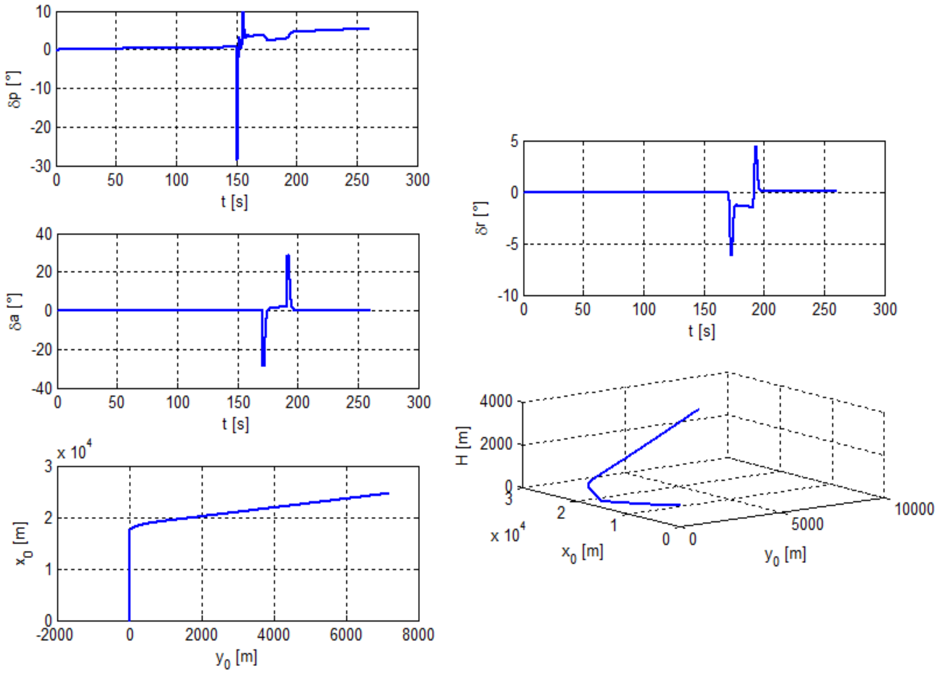

Figure 4. Go-around maneuver without disturbance - Parameters I

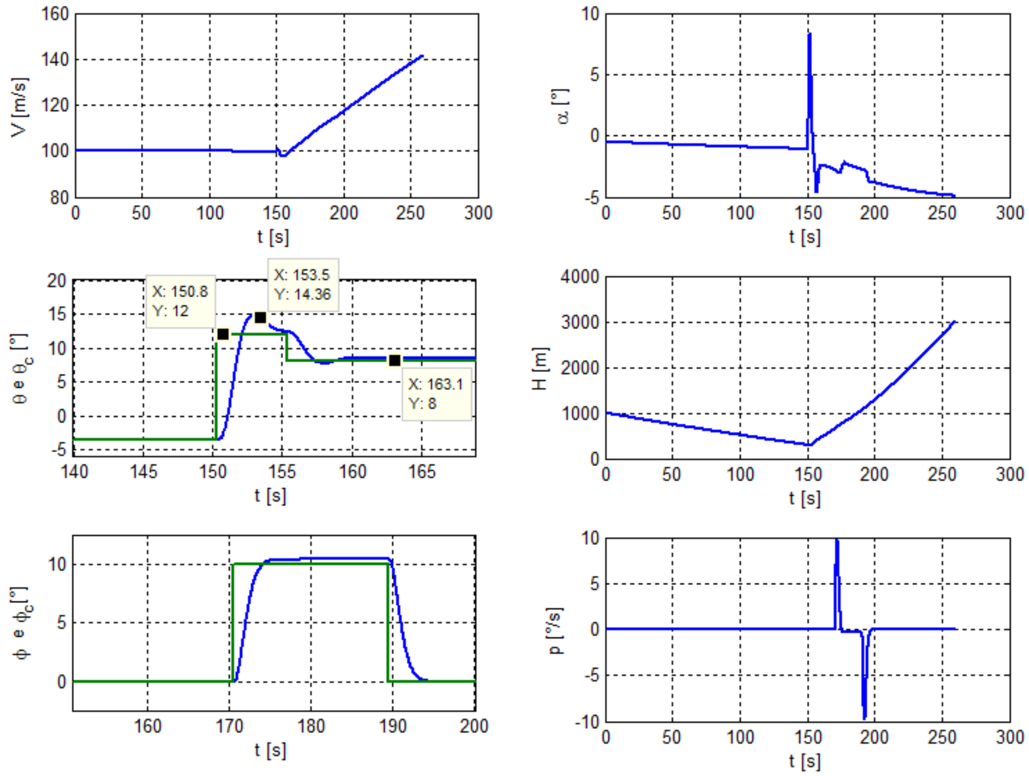

Figure 5. Go-around maneuver without disturbance - Parameters II

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