CILAMCE 2017
Florianópolis - Santa Catarina - Brazil

Nonlinear Lifting Line Implementation and Validation for Aerodynamic and Stability Analysis

Andre Rezende Dessimoni Carvalho

Pedro Paulo de Carvalho Brito

andre.rdc@hotmail.com

pedropauloengaero@outlook.com

Universidade Federal de Uberlandia

Avenida Joao Naves, 2121, 38408-100, Minas Gerais, Brazil

Abstract.

The well-known Prandtl's Lifting Line Theory, elaborated in 1918, was the first well-succeeded mathematical model capable of describing the flow along the span of a wing. Since then, some modifications implemented by Weissinger, Multhopp, Philips, and Anderson were presented. Compared with other potential flow based methods, such as the Vortex Lattice and Panel Method, the Lifting Line has the advantage of the lowest computational cost and the easiest implementation, making it suitable for fast aerodynamic optimization codes. Furthermore, Anderson's method has the convenience of incorporating both, numerical and experimental, bi-dimensional viscous data. In this paper, we are going to describe the implementation and numerical modifications made in the algorithm proposed by Anderson (2010) to simulate the flow over a finite wing under angles of attack near the stall. A big concern of this work is to show, with enough detail, the effects of the boundary conditions and numerical strategies adopted, over the results and accuracy. Various numerical differentiation, interpolation, and discretization schemes were tested to achieve the values of circulation at the wing tip inside the iterative process. The results obtained were compared with potential flow open source codes, and experimental data, showing good agreement.

Keywords: *Aerodynamics, Nonlinear Lifting Line, Numerical Methods*

1 INTRODUCTION

Fast numerical methods capable of evaluating many configurations during a design process are a big differential during the aerodynamic conceptual phase in the aeronautical industry. The first remarkable achievements date of 1918 when Prandtl developed the classical Lifting Line Theory (LLT). Later, other authors work to extend the method to the nonlinear region of the lift curve, and to accept wings with arbitrary shape, a great accomplishment in the area.

The LLT method is still in use today for preliminary calculations of finite wings because of its low computational cost. The method has a good agreement with the experimental results for predicting the stall characteristics of wings, a difficult task even when more advanced approaches, such as CFD, are applied. The present work shows in a detailed way the capabilities of the Nonlinear Lifting Line (NL-LLT) to predict the near stall characteristics of finite wings and stability parameters.

2 METHODOLOGY

2.1 THEORICAL FUNDAMENTS

Circulation and Lift Generation

A fundamental definition in aerodynamics is the circulation (Γ), because it is how we can mathematically model the lift. It is expressed by Eq. (1), which says that the circulation about a curve C is equal to the vorticity integrated over any open surface bounded by C. In Eq. (1), $\overrightarrow{V}$ is the velocity vector.

$$\Gamma = -\oint_{c} \overrightarrow{V}\, ds = \iint_{s} (\nabla \times \overrightarrow{V})\, ds \tag{1}$$

The importance of the concept of circulation defined above is exposed by the Kutta-Joukowski theorem (1902-1906) that states the lift per unit span (L') on a lifting airfoil or cylinder, is proportional to the circulation, as shown in the Eq. (2), where ρ_∞ is the free stream density, and V_∞ is the free stream velocity.

$$L' = \rho_\infty V_\infty \Gamma \tag{2}$$

Prandtl Classical Lifting Line

The implementation of the classical lifting-line relies on the substitution of a finite wing by a large number of horseshoe vortices (usually 40 is enough). This way, using the Kutta-Joukowski theorem and analyzing carefully the local airfoil section of a wing, we are able to obtain the fundamental equation of Prandtl's lifting-line theory, given by Eq. (3). In this equation, $c(y_0)$ is the local chord at an arbitrary station; y is the spanwise direction coordinate; b is the wing span; and $\alpha_{L=0}$ is the angle of attack for which the lift of the airfoil section is equal to zero.

$$\alpha(y_0) = \left[\frac{\Gamma(y_0)}{\pi V_\infty c(y_0)} + \alpha_{L=0}\right] + \left[-\frac{1}{4\pi}\int_{-b/2}^{b/2} \frac{(d\Gamma/dy)}{y_0 - y}\,dy\right] \tag{3}$$

Equation (3) simply states that the geometric local angle of attack ($\alpha(y_0)$) is the sum of the effective angle of attack (α_{eff}) plus the induced angle of attack (α_i).

2.2 NUMERICAL METHODS

According to the numerical NL-LLT methodology presented by (Anderson, 2010), in order to solve the Eq. (3), we should first divide the wing into k+1 spanwise stations. Then, we assume a preliminary elliptical distribution of Γ at all the stations. Now, using the Simpson's rule to evaluate the integral, the second term in the RHS of Eq. (3) becomes:

$$\alpha_i(y_n) = \frac{1}{4\pi V_\infty}\frac{\Delta y}{3}\sum_{j=2,4,6}^{k} \frac{(d\Gamma/dy)_{j-1}}{y_n - y_{j-1}} + 4\frac{(d\Gamma/dy)_j}{y_n - y_j} + \frac{(d\Gamma/dy)_{j+1}}{y_n - y_{j+1}} \tag{4}$$

In Eq. (4), y_n is an arbitrary station, and Δy is the distance between stations. To perform the derivatives, a second order central difference method may be used. In Eq. (4), when $y_n = y_{j-1},\ y_j$, or y_{j+1}, a singularity occurs. It can be avoided by replacing the term of the sum in which the singularity occurs, by the average value of the same term in the two adjacent sections. As an example, if $y_n = y_{j-1}$, then the first term will be given by:

$$\frac{(d\Gamma/dy)_{j-1}}{y_n - y_{j-1}} = \left[\frac{(d\Gamma/dy)_{j-1}}{y_{n-1} - y_{j-1}} + \frac{(d\Gamma/dy)_{j-1}}{y_{n+1} - y_{j-1}}\right]/2 \tag{5}$$

Once obtained the value of α_i for each station, and assumed that the geometrical angle of attack is known, we can isolate the α_{eff} (first term in the RHS of the Eq. (3)), obtaining the Eq. (6).

$$\alpha_{eff}(y_n) = \alpha - \alpha_i(y_n) \tag{6}$$

With the distribution of α_{eff} calculated, we should obtain the bi-dimensional lift coefficient, (c_l), at each station. These data can be obtained from experimental polars or bi-dimensional codes such as Xfoil (Drela, 1989). Eventually, the value off α_{eff} obtained for some stations can be outside the range of bi-dimensional data. In such cases, the airfoil data should be extrapolated.

To ensure the boundary condition, we should set the local lift coefficient as zero at the tip stations. We also use an unidimensional third order interpolation scheme at the station adjacent to the tip, in order to smooth numerical instabilities associated with the numerical derivative. Figure 1 shows the difference in c_l distribution when using the interpolation for a given wing. As

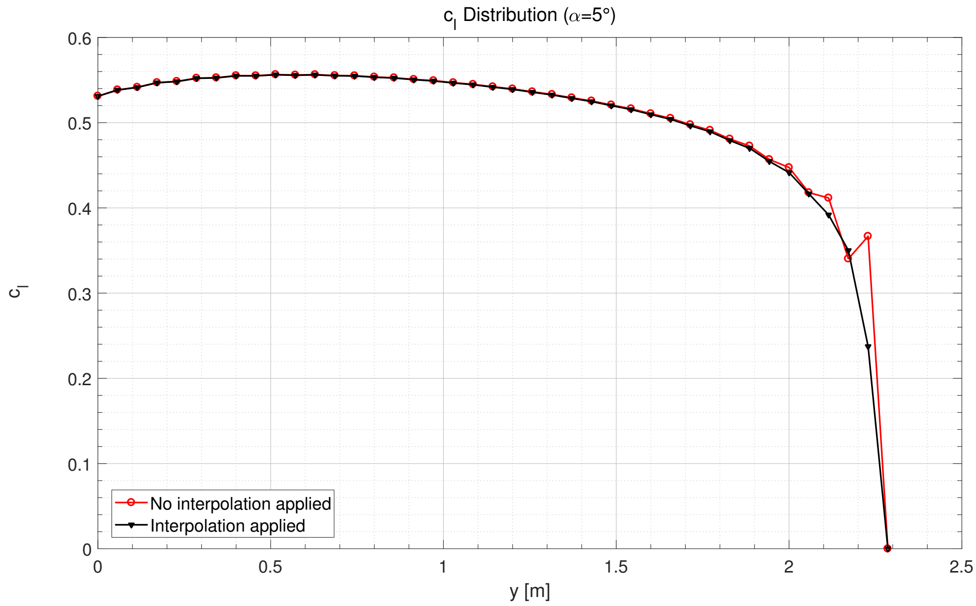

Figure 1: Comparison between c_l distributions ($\alpha = 5°$) according to the use of interpolation in the station adjacent to the tip station.

can be seen, applying the interpolation only in the penultimate station, leaded to a stabilization all over the tip region.

Next, from the Eq. (2) and the definition of lift coefficient, we can calculate a new circulation, given by the Eq. (7).

$$\Gamma(y_n) = \frac{1}{2} V_\infty \, c_n \, c_l(y_n) \tag{7}$$

In Eq. (7), c_n is the chord length at each station. The new distribution of Γ is compared with that assumed previously, starting an iterative process given by the Eq. (8), where Γ_{old} is the circulation calculated in the previous iteration; Γ_{new} is the circulation obtained by Eq. (7); Γ_{input} is the circulation used as input to Eq. (4) in the next iteration; and D is a damping factor, with typical values of 0.05. The iterative process is repeated sufficiently until Γ_{new} and Γ_{old} agree within an acceptable accuracy

$$\Gamma_{input} = \Gamma_{old} + D(\Gamma_{new} - \Gamma_{old}) \tag{8}$$

When performing C_L calculations for various values of α, it is possible to accelerate the convergence by utilizing previous values of converged Γ. The present authors propose substitute the elliptical initial guess in Anderson's methodology (except in the first simulation) by Eq. (9), where $\Gamma_{\alpha+\Delta\alpha}$ is the initial input to be used in the next $C_L(\alpha)$ simulation; Γ_α is the current converged distribution of circulation; $\Delta\alpha$ is the α step between two simulations; and c is the vector of local chords.

$$\Gamma_{\alpha+\Delta\alpha} = \Gamma_\alpha + \frac{1}{2} V_\infty \, c \, 2\pi \, \Delta\alpha \tag{9}$$

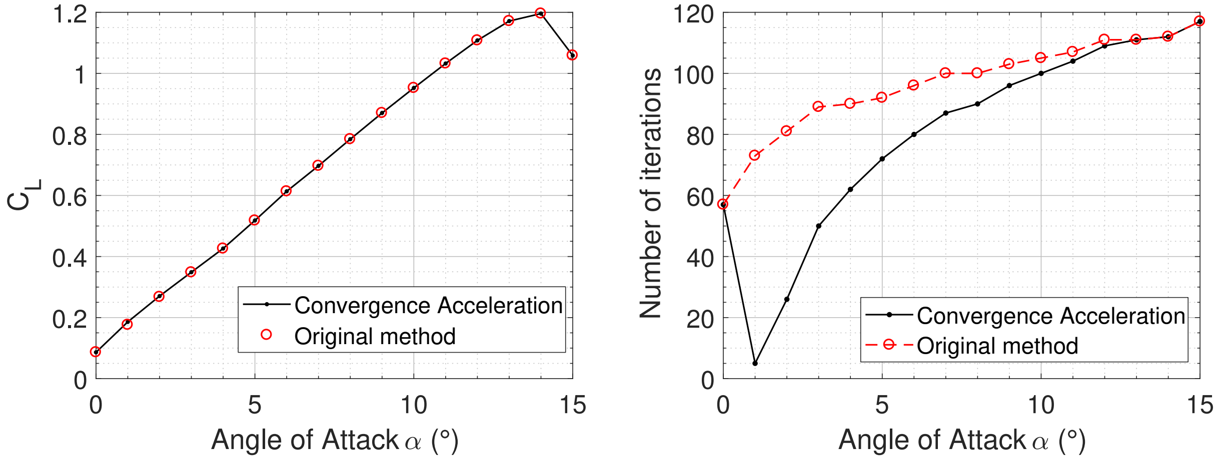

Figure 2: Comparison of results of C_L vs α and number of iterations, according to the use of the acceleration of convergence.

Equation (9) is valid only for the linear region, so if it is necessary to simulate high angles of attack, the initial guess can be weighted linearly, so that, for the second angle of attack in linear region, the initial assumption is given only by Eq. (9), while for the final angles of attack, the initial assumption becomes purely Anderson's method. Figure 2 compares, for a given wing, the number of iterations performed to converge the solutions, and the values of C_L obtained, using the proposed methodology and the original approach.

After the end of the iterative process, the global force and moments coefficients can be calculated. Lift (C_L), induced drag (C_{Di}), and parasite drag (C_{D0}) coefficients are expressed by the equations Eq. (10) to Eq. (12), respectively. In these equations, S is the wing area; c is the local chord; and $c_{d,0}$ is the local airfoil drag, obtained by bi-dimensional drag polars.

$$C_L = \frac{2}{V_\infty\, S} \int_{-b/2}^{b/2} \Gamma(y)\, dy \tag{10}$$

$$C_{D,i} = \frac{2}{V_\infty\, S} \int_{-b/2}^{b/2} \Gamma(y)\, \alpha_i(y)\, dy \tag{11}$$

$$C_{D,0} = \frac{1}{S} \int_{-b/2}^{b/2} c_{d,0}(y)\, c(y)\, dy \tag{12}$$

Beside these coefficients, (Sivells et al., 1947a) also provide the equations Eq. (13) to Eq. (16) to calculate useful coefficients for stability analysis, respectively: pitching-moment (C_m), rolling-moment (C_l), induced-yawing-moment (C_{n_i}) and profile-yawing-moment coefficients ($C_{n,0}$). In these equations, c' is the mean aerodynamic chord, and c_m is the local bi-dimensional pitching-moment.

$$C_m = \frac{1}{S\, c'} \int_{-b/2}^{b/2} c_m(y)\, c(y)^2\, dy \tag{13}$$

$$C_l = -\frac{1}{S\,b}\int_{-b/2}^{b/2} c_l(y)\,c(y)\,y\,dy \tag{14}$$

$$C_{n,i} = \frac{1}{S\,b}\int_{-b/2}^{b/2} \frac{\pi\,c_l(y)\,c(y)\,\alpha_i(y)}{180}\,y\,dy \tag{15}$$

$$C_{n,0} = \frac{1}{S\,b}\int_{-b/2}^{b/2} c_{d,0}(y)\,c(y)\,y\,dy \tag{16}$$

In order to evaluate equations Eq. (13) to Eq. (16) with command deflections, the c_l values over the control surface stations, $c_l(y_{cs})|_{\delta f}$, can be refreshed according to Eq. (17), where $c_l(y_{cs})|_{\delta f=0^\circ}$ is the original c_l over those stations, without considering the control surface; $\partial c_l/\partial \delta_f$ is the derivative of c_l with respect to the surface deflection; and δ_f is the desired angle of deflection. An equivalent equation shall be used for bi-dimensional pitching moment and drag coefficients. Another possibility to take into account the effects of the deflection, is to substitute the airfoil polars over the command surface stations, by the polars of the deflected airfoils.

$$c_l(y_{cs})|_{\delta f} = c_l(y_{cs})|_{\delta f=0^\circ} + \frac{\partial c_l}{\partial \delta_f}\delta_f \tag{17}$$

3 RESULTS

Aiming to compare the results provided by the implemented method, with experimental data and other potential flow based codes, the wing studied by (Sivells, 1947b) was chosen. The case was a straight wing with NACA 64-210 airfoil, aspect ratio 9, taper ratio 0.5, chord root $0.7257\ m$, 2° washout, and 3° dihedral, at Mach 0.17 and Reynolds number $4.4 \cdot 10^6$. Although Anderson's method do not take into account dihedral effects, according to (Barbosa, 2015), 3° is small enough to be negligible in lift calculations. It is worth to note that the dihedral was considered in the other codes.

The other codes used to compare were: a) classical Prandtl's lifting line code; b) Machup (Hunsaker, 2011), a modern lifting line code; c) XFLR5 6.38 (Deperrois, 2017), a code that allows panel method as well as extended lifting line solution using 2D data from Xfoil; and d) AVL 3.35 (Drela et. al, 2017), a vortex lattice code. The bi-dimensional airfoil data used in lifting line based methods (except XFLR5) were extracted from (Abbott, 1959).

Figure 3 shows the curve $C_L\ vs\ \alpha$ with angle step of 0.1° near the stall for the nonlinear methods (the points that did not converge were not considered), and Table 1 points the results of C_L for $\alpha = 5^\circ$, the lift slope at the linear region (C_{L_α}), and the angles of stall. As can be seen, all potential method presented nearly the same deviation from the experimental results, although the present work method has presented the closest value of angle of stall to the experiment. It is also important to note that even XFLR5 using bi-dimensional data from Xfoil, its results were close to the present work using experimental bi-dimensional data.

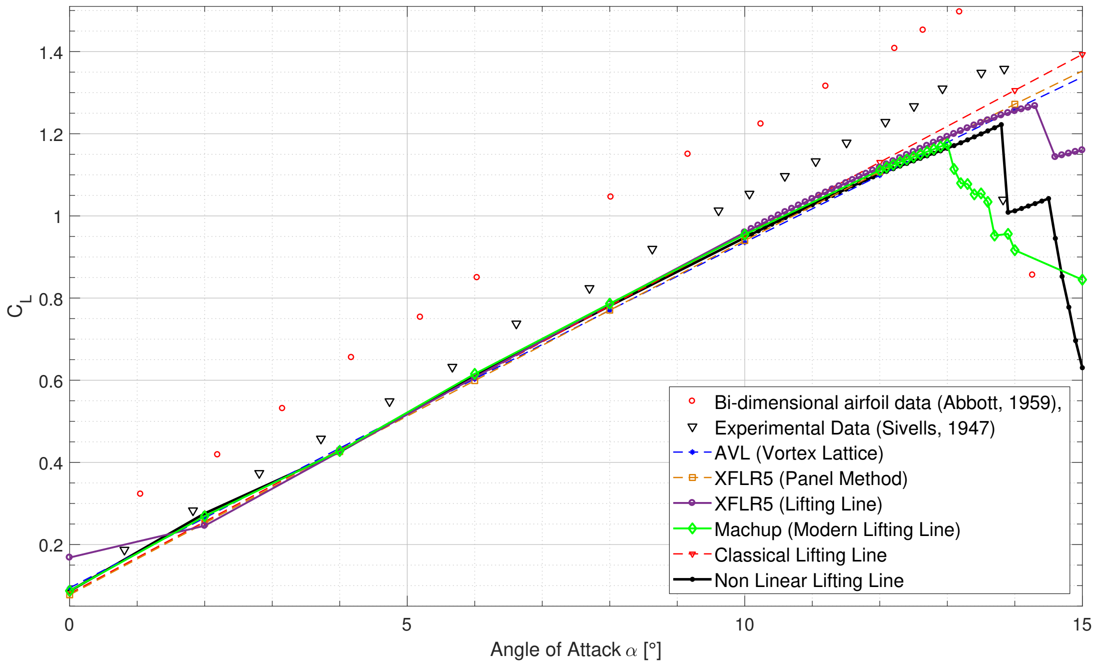

Figure 3: C_L vs α comparison between the Nonlinear Lifting Line, potential codes, experimental data, and 2-D section experimental data.

Table 1: $C_L(\alpha = 5°)$, C_{L_α}, and angle of stall comparison between methods.

	$C_L(\alpha = 5°)$	C_{L_α} (rad^{-1})	Angle of stall
Present work (Nonlinear Lifting Line)	0.5181	4.95	13.7°
Classical Lifting Line	0.5182	5.01	-
Machup (Modern Lifting Line)	0.5196	5.04	13.0°
XFLR5 (Extended Lifting Line)	0.5170	4.86	14.3°
XFLR5 (Panel Method)	0.5130	4.83	-
AVL (Vortex Lattice)	0.5190	4.78	-
(Sivells, 1947b)	0.5721	5.35	13.8°

The c_l distributions for $\alpha = 5°$ are shown in Figure 4. The deviation at the tip for the present method can be explained by the linear discretization proposed by (Anderson, 2010). Other methods use a cosine clustering allowing better resolution near root and tips. This distribution, as (Philips, 2000) suggests, is given by Eq. (18), where n is the desired number of horseshoes on each semispan, and s_i is an arbitrary station position.

$$s_i/b = [1 - cos(i\pi/n)]/4, \quad 0 \leq i \leq n \tag{18}$$

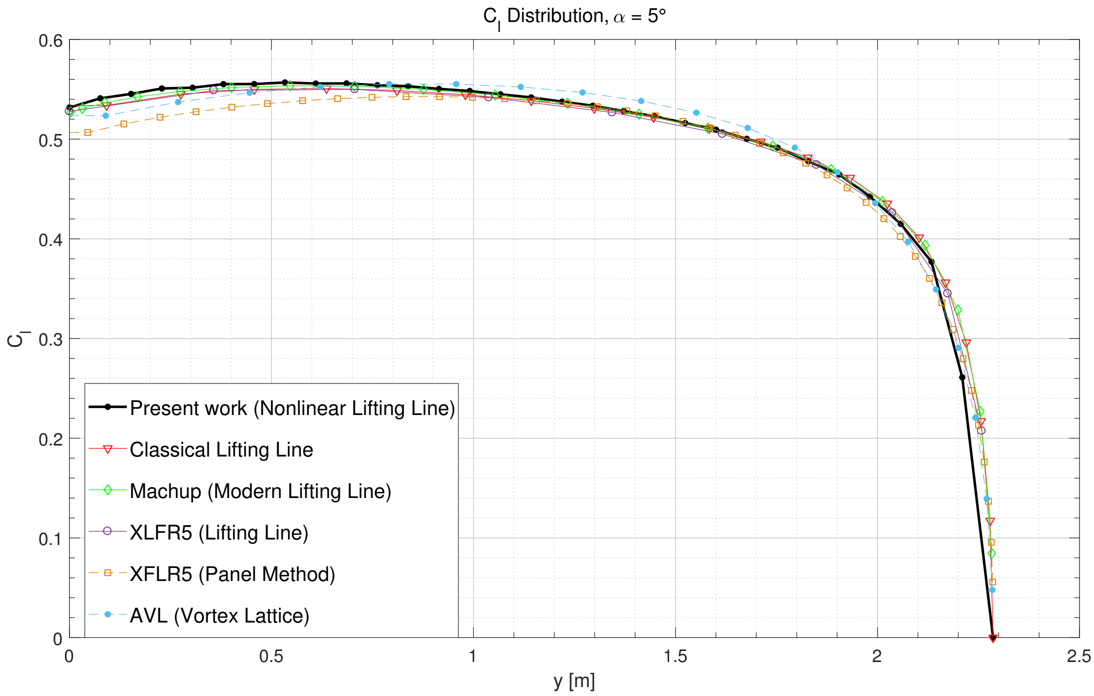

Figure 4: Comparison between c_l distribution over midspan provided by potential methods for $\alpha = 5°$.

In order to evaluate results provided by the equations Eq. (13) to Eq. (17), we implemented a $20°$ asymmetric right deflection of a trailing edge control surface positioned between $0.8 \cdot b/2 \leq y_{cs} \leq 1.0 \cdot b/2$, with chord length of 20% of the local chord. The results were compared with the XFLR5 panel method. Figure 5 compares distributions of c_l and c_d between the methods, and Table 2 compares its global force and moment coefficients.

Although there are numerical instabilities near the inboard section of the control surface for the present work, the global force and moment coefficients agree with XFLR5, while lifting line calculation time was quite faster. It is worth to note that these results should be used carefully, since surface deflections generate high three-dimensional flow, eventually exceeding capabilities of potential flow codes.

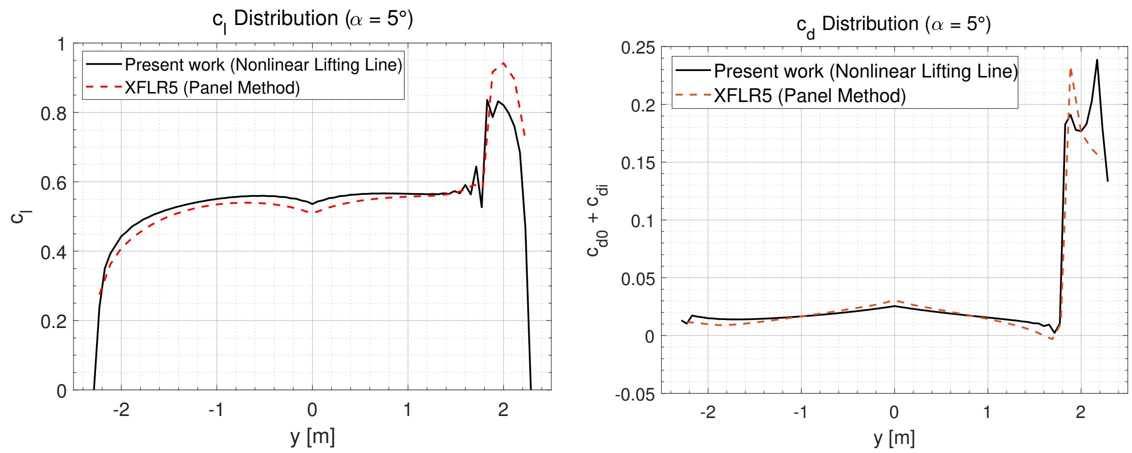

Figure 5: Comparison between c_l and c_d distributions for $20°$ right deflection of a $0.20 \cdot c_n$ trailing edge control surface.

Table 2: Comparison between force and moment coefficients for $20°$ right deflection of a $0.20 \cdot c_n$ trailing edge control surface.

	C_L	C_D	C_n	C_l
Present work	0.552	0.020	0.003	−0.014
XFLR5 (Panel Method)	0.554	0.022	0.002	−0.017

4 Conclusions

We numerically investigated the capabilities of the modified classical NL-LLT for predicting the aerodynamic coefficients in low and near stall angles of attack, where a wing of aspect ratio 9 was simulated.

The use of third order interpolations at the tip stations, and the right boundary conditions, allowed us to obtain smoother lift distribution along the wing span for high angles of attack, and showed to be a reasonable approach to reduce the instabilities at this region. An even better result could be achieved by changing the spacial discretization strategy.

For the case analyzed, the NL-LLT implemented was capable to provide the best prediction for the angle of stall compared with the experimental data from (Sivells, 1947b), with less than 1% error. The method also presented concordance with panel method when simulating effects of deflected surfaces in force and moment coefficients, as well as in lift distribution.

In conclusion, this work confirms that the NL-LLT is a useful approach for predicting the aerodynamic coefficients in small and near stall angles of attack, making it suitable for the conceptual design phase of small subsonic airplanes, when detailed information of the flow field is not required, and low computational cost is needed. However, it is important to pay attention to the limitation of the method in simulating wings with high sweep or dihedral, and low aspect ratio, because of the existence of highly three-dimensional flows in such cases.

REFERENCES

Abbott, I. & Doenhoff A., 1959. Theory of Wing Sections Including a Summary of Airfoil Data. *Dover Publications, Inc.*

Anderson, J., 2010. Fundamentals of Aerodynamics. 5 ed., *McGraw-Hill Education*.

Barbosa, A., 2015. *Influencia da Asa em Gaivota nos Coeficientes Aerodinamicos de uma Aeronave.* Master thesis, Universidade de Sao Paulo

Deperrois. A, 2017. XFLR5 6.38. Retrieved from http://www.xflr5.com/xflr5.htm.

Drela, M. 1989. XFOIL: An Analysis and Design System for Low Reynolds Number Airfoils. *Low Reynolds Number Aerodynamics: Proceedings of the Conference Notre Dame*, pp. 1 to 12, *Springer Berlin Heidelberg*.

Drela, M. & Youngren, H. 2017. AVL 3.35. Retrieved from http://web.mit.edu/drela/Public/web/avl/.

Hunsaker, D., 2011. A Numerical Lifting-Line Method Using Horseshoe Vortex Sheets.

Philips, W., Snyder, D., 2000. Modern Adaptation of Prandtl's Classic Lifting-Line Theory. *Journal of Aircraft*, vol. 37.

Sivells, J. & Neely, R., 1947a. Method for Calculating Wing Characteristics by Lifting-Line Theory Using Nonlinear Section Lift Data. *National Advisory Committee for Aeronautics, Technical Note No 1269*.

Sivells, J., 1947b. Experimental and Calculated Characteristics of Three Wings of NACA 64-210 and 65-210 Airfoil Sections With and Without 2° Washout. *National Advisory Committee for Aeronautics, Technical Note No 1422*.