DAMAGE IDENTIFICATION IN A MULTI-DOF SYSTEM UNDER UNCERTAINTIES USING OPTIMIZATION ALGORITHMS

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Abstract. *In this article four optimization algorithms are applied to identify damage in a multi-DOF dynamical system, composed by masses, springs and dampers. The damage is introduced artificially by choosing a lower value for the stiffness. The algorithms applied are: Nelder-Mead Simplex, BFGS Quasi-Newton, interior point, and sequential quadratic programming - SQP. In addition, some different strategies to identify damage are applied. First, a deterministic analysis is performed to identify the damage; i.e, the values of the stiffnesses. Then, ramdom forces are considered, and after, the stiffness values are considered uncertain. The difference among the results are analyzed.*

Keywords: *Damage identification, Uncertainties, Optimization*

1 INTRODUCTION

In recent years, the main industries that use components and mechanical structures in their production line (automotive, aeronautics, building construction, naval, nuclear, oil and gas, etc.) have increased their investments in research and technological development in order to obtain efficient methods to analyze the integrity of structures and prevent catastrophes and/or accidents from occurring, ensuring people's lives and avoiding economic losses.

Structural integrity monitoring consists of detecting failures in initial states, intervening in their propagation and consequently preventing any accidents from occurring (causing the stopping or damaging to the structure). This current and important line of research (Hall, 1999) is called Structural Health Monitoring (SHM).

The failures in mechanical components lead to damage to the mechanical structure. Damage is defined as changes introduced into a system that negatively affect your current or future performance. A damage is significant only by comparing two states of the system: the current state and the initial (or undamaged) state.

In mechanical vibration's area the system modeling is characterized by the knowledge of the geometry of the structures, the properties of the materials, the initial and boundary conditions and the force terms that excite the structure. The output (response) of the model, such as displacement, accelerations, vibration modes, etc. depends on the physical properties of the structures. Changes in these properties can be caused by cracks, loosening of connections or any other damages and may be detectable through modal properties.

The process for structural damage identification can be classified based on mechanical performance, attending to five levels (Ritter, 1993 and Worden et al., 2004):

1. Detection: the presence of any damage is verified.
2. Location: it determines where the damage is located.
3. Classification: it determines the type of damage.
4. Extension: it gives an estimate of the extension and severity of the damage.
5. Prognosis: ít attempts to predict the remaining life of the structure.

According to Ooijevaar (2014) the majority of works found in the literature meet the first two levels of performance (detection and location of damages).

In order to enrich and to improve the studies related to the damage identification in mechanical systems, the present work aims to contribute through a stochastic approach to this class of problems. In addition, four optimization techniques will be compared for solving the identification problem.

2 METHODOLOGY

In this section it will be presented the methodology used for the damage identification in mechanical structures from the study of dynamics of linear systems. In short, the methodology consists in using optimization techniques to identify stifiness parameters of a mechanical system acted by a given force. Such analysis will be conducted in the frequency domain and uncertainties in the model and response will be considered.

Aiming for a better understanding of the reader, this section has been subdivided into three subsections. The first two subsections will address the theoretical foundations with the main concepts used in the base of this methodology, namely: **Optimizations techniques** and **Analysis of Uncertainties**.

The last section is responsible for compiling all the knowledge presented in the previous sections and structuring the methodology itself.

2.1 Optimizations techniques

Next, the optimization techniques used in the damage identification analyzes will be presented. In all techniques the following stopping criteria were used:

- Maximum iteration number - 1000;
- Objective function's tolerance - $1 \cdot 10^{-6}$.

Nelder-Mead Simplex

The Nelder-Mead Simplex algorithm, published for the first time in 1965, is an extremely popular direct search method for the minimization of non-linear and unrestricted multidimensional problems (Lagarias et al., 1998).

This method seeks to minimize a nonlinear scalar function of n real variables using only function values, without any information about the derivative (explicit or implicit). The method maintains at each step a nondegenerate *simplex* - a geometric figure in $\Re^n$ of nonzero volume which is the convex envelope of $n+1$ vertices (a triangle formed by three non-collinear points, in $\Re^3$ a tetrahedron formed by four non-coplanar points, and so on).

Four scalar parameters must be specified to define the Nelder-Mead method completely. They are the coefficients of: reflection (ρ); Expansion (χ); Contraction (γ) and shrinkage (σ). According to the original work of the authors of the method (Nelder et al., 1965), these parameters must present the following relations:

$$\rho > 0 \quad \chi > 1 \quad \chi > \rho \quad 0 < \gamma < 1 \quad 0 < \sigma < 1 \tag{1}$$

In the present work, the following values will be used for the parameters (usually found in the literature):

$$\rho = 1 \quad \chi = 2 \quad \gamma = \frac{1}{2} \quad \sigma = \frac{1}{2} \tag{2}$$

Thus for the k-th iteration, we have a nondegenerate *simplex* Δ_k consisting of $n+1$ vertices). These vertices $x_i^{(k)}$ are ordered in the form $x_1^{(k)}$,...,$x_{n+1}^{(k)}$ such that

$$f_1^{(k)} \leq f_2^{(k)} \leq \cdots \leq f_{n+1}^{(k)} \tag{3}$$

where $f_i^{(k)} = f(x_i^{(k)})$. At the end of this k-th iteration, a new nondegenerate *simplex* Δ_{k+1} (consisting by new $n+1$ vertices) will be obtained such that $\Delta_{k+1} \neq \Delta_k$.

The Nelder-Mead Simplex method is constituted by five steps: ordering; reflection; expansion; contraction and shrinkage.

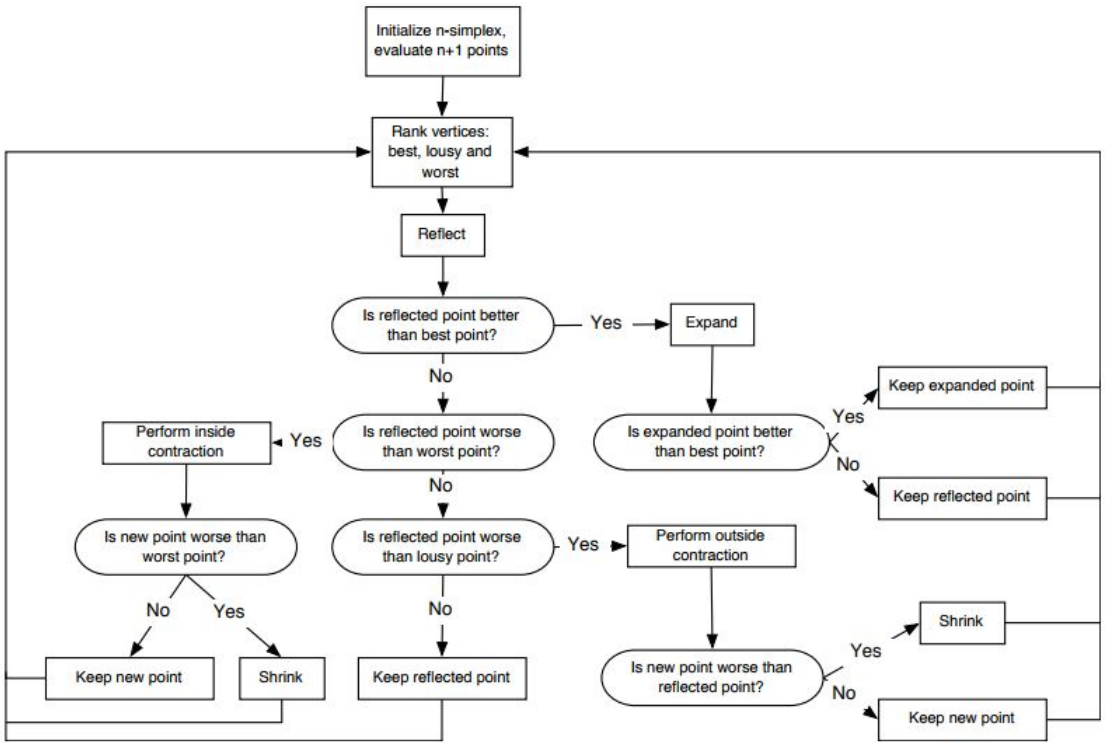

Figure 1: Flow chart for the Nelder-Mead algorithm (Hicken et al., 2012)

BFGS Quasi-Newton

Of the methods that use information from the gradient of the objective function, the most usual are the Quasi-Newton methods. These methods construct curvature information at each iteration to formulate a quadratic problem of shape

$$\min_x (\frac{1}{2} x^T H x + c^T x + b) \tag{4}$$

Where the Hessian matrix H is a symmetric positive definite matrix, c is a constant vector, and b is a constant. The optimal solution to this problem occurs when the gradient vector (taken as objective function) becomes zero, that is,

$$\nabla f(x^*) = Hx^* + c = 0 \tag{5}$$

In this way, the optimal solution is given by

$$x^* = -H^{-1} c \tag{6}$$

Traditional Newton methods (as opposed to quasi-Newton methods) calculate H directly and proceed in a downward direction to find the minimum after a number of iterations. Calculating H numerically involves a large number of numerical operations. In this way, Quasi-Newton methods avoid this by using the observed behavior of $f(x)$ and $\nabla f(x)$ to construct curvature information to approximate H using an appropriate update technique.

Several updating techniques of the Hessian matrix were developed. However, the Broyden-Fletcher-Goldfarb-Shanno algorithm (BFGS) was used because it was considered an effective method for general purposes. The update of the Hessian matrix by the BFGS algorithm is given by

$$H_{k+1} = H_k + \frac{q_k q_k^T}{q_k^T s_k} - \frac{H_k s_k s_k^T H_k^T}{s^T H_k s_k} \tag{7}$$

Where,

$$\begin{aligned} s_k &= x_{k+1} - x_k \\ q_k &= \nabla f(x_{k+1}) - \nabla f(x_k) \end{aligned} \tag{8}$$

At each iteration k, a linear search is performed in the direction $d = -H_k^{-1} \cdot \nabla f(x_k)$.

And therefore, for a given α to be determined, the new value for the iteration x_{k+1} is $x_{k+1} = x_k + \alpha d_k$.

Sequential Quadratic Programming - SQP

In constrained optimization problems, the overall goal is to transform the problem into a simpler subproblem that can soon be solved and used as the basis of an iterative process. One of the methods used for this type of simplification uses the Karush-Khun-Tuker (KKT) equations. The KKT equations are necessary conditions for optimality in a constrained optimization problem. If the optimization problem is said convex, then the KKT equations are both necessary and sufficient for an overall solution of the problem.

The solution of the KKT equations is the starting point for many nonlinear programming algorithms. The quasi-Newton methods guarantee superlinear convergence by accumulating second order information with respect to the KKT equations using a quasi-Newton update procedure. These methods are commonly referred to as Sequential Quadratic Programming (SQP) methods, since a QP subproblem is solved in each main interaction. The SQP process is widely used in solving nonlinear problems in several areas, as it is one of the methods that offers the best efficiency and precision.

In the main iterations of the SQP method an approximation of the Hessian H of the Lagrangean L is obtained using a quasi-Newton update method. Hessian is then used to generate a QP subproblem whose solution serves to form a search direction in a one-dimensional search procedure. This methodology used in the algorithm was initially proposed by (Wilson, 1963) and interpreted by (Beale, 1967).

In this way, a problem of the type

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x) \\ \text{subject to} \quad & g_i(x) \leq 0 \\ \text{and} \quad & h_i(x) = 0 \end{aligned} \tag{9}$$

It is solved as follows, for an iteration k:

- **1st Step**

A constant $x^{(k)}$ is fixed and the associated quadratic programming problem is solved for the direction d:

$$\begin{aligned} \underset{d}{\text{minimize}} \quad & \frac{1}{2} d^t S d + \nabla f^t(x^{(k)}) d \\ \text{subject to} \quad & \nabla g^t(x^{(k)}) d + g(x) \leq 0 \\ \text{and} \quad & \nabla h^t(x^{(k)}) d + h(x) = 0 \end{aligned} \tag{10}$$

- **2nd Step**

Update $x^{(k+1)} = x^{(k)} + \alpha_k d$ and return to step 1.

Interior Point

The approach used by this method to solve constrained minimization problems is to solve a sequence of approximate minimization problems.

For each $\mu > 0$, the approximate problem is given by:

$$\min_{x,s} f_\mu(x, s) = \min_{x,s} f(x, s) - \mu \sum_i ln(s_i) \quad \text{sujeito a } h(x) = 0 \text{ e } g(x) + s = 0 \tag{11}$$

For the approximate problem there are as many s_i variables as the inequality constraints g. The s_i are limited to being positive to keep $ln(s_i)$ limited. Since μ decreases to zero, the minimum of f_μ must approach the minimum of f. The logarithmic term added is called the barrier function. This method is described in (Byrd et al., 1999).

The approximate problem is a sequence of problems with equality constraints. To solve the approximate problem, the algorithm uses a direct step in (x, s). This step attempts to solve the KKT equations for the approximate problem by a linear approximation. This is also called Newton's step.

In this way, the direct step (Δx, Δs) to be given in each iteration is determined by solving

the following system of equations:

$$\begin{bmatrix} H & 0 & J_h^T & J_g^T \\ 0 & S\Lambda & 0 & -S \\ J_h & 0 & I & 0 \\ J_g & -S & 0 & I \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta s \\ -\Delta y \\ -\Delta \lambda \end{bmatrix} = - \begin{bmatrix} \nabla f - J_h^T y - J_g^T \lambda \\ S\lambda - \mu e \\ h \\ g + s \end{bmatrix} \tag{12}$$

Where,

- H denotes the Hessian of the Lagrangean of f $\therefore$ $H = \nabla^2 f(x) + \sum_i \lambda_i \nabla^2 g_i(x) + \sum_j \lambda_j \nabla^2 h_j(x)$;
- $J_g \equiv$ Jacobian of the function g; $J_h \equiv$ Jacobian of the function h;
- $S = diag(s)$; $\Lambda = diag(\lambda)$;
- $\lambda, y \equiv$ Lagrange multipliers associated with constraints g, h, respectively;
- $e \equiv$ vector of 1's of the same size of g;

2.2 Analysis of Uncertainties

In the present work, two classes of results will be presented referring to two strategies to identify parameters (Ritto et al., 2016): deterministic and stochastic. In short, the deterministic strategy does not involve probabilistic models, while the stochastic strategy considers some probabilistic model (eg parameter and/or measurement noises are modeled as random variables). Considering the deterministic strategy, we can construct a point estimator, which provides as an estimate a single numerical value for the parameter to be estimated p^*. In this case, the level of knowledge about p is encoded in a single value p^*. In addition, it should be noted that this estimator is a random variable since for each set of experimental data, a different value is observed for p^*. Although a point estimator is a random variable, this strategy is called deterministic identification because there are no probabilistic models involved. For example, given a set of experimental data $\mathbf{x}_{exp}$ and a predictive model $\mathbf{x}(p)$, which depends on a parameter p, we can, for example, estimate the value of this parameter by minimizing the metric $p^* = argmin \|\mathbf{x}_{exp} - \mathbf{x}(p)\|^2$.

That is, we search for the value of p that makes the response of the model $\mathbf{x}(p)$ as close as possible to the available experimental data $\mathbf{x}_{exp}$. In this case there is no hypothesis related to probability distributions. In the stochastic strategy, the parameters of interest and measurement noises are modeled as random variables and they are assigned probabilistic models. From experimental data the parameters of the probabilistic models (mean, variance, ...) can be identified from point estimators. In this case, the probability distribution does not change, only the values of its parameters that are estimated.

At this point, it is worth mentioning that in the present work the identification process is performed using the same computational model that was used to generate the synthetic data required for identification (data generation was performed using the Monte Carlo method). This

is known in the literature as an inverse crime and often generates extremely optimistic estimates. However, it was decided to work in this way to simplify the understanding of the methodology studied.

2.3 Methodology - Mathematical modelling

The set of the equations of motion referring to a generic problem of linear dynamics of mechanical structures, written in the time domain, can be summarized in the following equation:

$$[M]\{\ddot{x}\}(t) + [C]\{\dot{x}\}(t) + [K]\{x\}(t) = \{f\}(t) \tag{13}$$

Where $x(t) \equiv$ position vector, $x \in \Re^n$; $[M] \equiv$ mass matrix; $[C] \equiv$ damper matrix; $[K] \equiv$ stifness matrix, $[M], [C, [K] \in \Re^{n \times n}$.

At this point, it is worth noting that the stiffness matrix will be the only "mathematical entity" representative of the model that will contain the parameter information to be determined in the damage identification process. By way of illustration, the "format" of this "mathematical entity" applied to the model problem will be presented:

$$[K] = [K](k_1, k_2, ..., k_n) = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & : \\ 0 & -k_3 & \ddots & -k_{n-1} \\ 0 & \cdots & -k_{n-1} & k_{n-1} + k_n \end{bmatrix} \tag{14}$$

Given the system of differential equations that model the dynamic behavior of the system, the Fourier transform is applied to it in order to work in the frequency domain. In this way, we rewrite the Eq. (13) in the frequency domain:

$$-\omega^2[M]\{X\}(\omega) + i\omega[C]\{X\}(\omega) + [K]\{X\}(\omega) = \{\hat{f}\}(\omega) \tag{15}$$

In this way,

$$(-\omega^2[M] + i\omega[C] + [K])X(\omega) = \hat{f}(\omega) \therefore \boxed{X(\omega) = (-\omega^2[M] + i\omega[C] + [K])^{-1}\hat{f}(\omega)} \tag{16}$$

At this point, it is valid again to emphasize the dependence of the stiffness matrix $[K]$ with the parameters of interest, according to Eq. (14). Therefore, this dependency in the position vector $X(\omega)$ is reiterated; for simplicity we rewrite the set of parameters $(k_1, k_2, ..., k_n)$ as a vector $\vec{k}$ and thus we have:

$$X(\omega, \vec{k}) = (-\omega^2[M] + i\omega[C] + [K](\vec{k}))^{-1}\hat{f}(\omega) \tag{17}$$

Once the response function of the system is determined in the frequency domain, and considering that the same function will generate the experimental results, four analyzes will be carried out that will base the methodology of this work.

Before explaining such analyzes, it is worth mentioning that the system response in the frequency domain is a vector $X(\omega) \in \Re^n$. The importance of this observation is that in real physical systems it is not always possible to collect all types of information, either due to the complexity of the system (difficulty of access, physical limitations, etc.) or the unavailability of sensors and measuring equipment . Thus, since the mechanical system is studied under a discrete approach (presenting n degrees of freedom), only three components of the vector $X(\omega) \in \Re^n$ will be used in the analyzes.

Therefore in the following detail of the performed analyzes, the vector $\tilde{\mathbf{X}}(\omega)$ will be adopted as being the one that contains the components of the observed vector $X(\omega)$.

Deterministic Analysis

This analysis is presented only to illustrate the efficiency and robustness of each optimization method, as well as to compare the computational time required in each of the methods. No uncertainty shall be considered in this analysis.

The damage identification process is summarized in obtaining a set of parameters $\vec{k}_* = (k_1, k_2, ..., k_n)$ from an optimization problem involving the comparison of a model $\tilde{X}(\omega, \vec{k})$ with the experimental results $\tilde{X}_{exp}(\omega)$, and compare such set $\vec{k}_*$ with a set of nominal parameters of a healthy structure $\vec{k}_{NOM}$.

In this way, the metric used in the optimization problem is related to the *least mean square method*, that is, given a set of experimental data $\tilde{X}_{exp}(\omega)$ of a damaged structure, it is desired to obtain a set of parameters $\vec{k}_*$ such that the quadratic difference between $\tilde{X}_{exp}(\omega)$ and model $\tilde{X}(\omega, \vec{k})$ is minimal. That is,

$$\vec{k}_* = \underset{\vec{k}}{\text{minimize}} \quad \|\tilde{X}_{exp}(\omega) - \tilde{X}(\omega, \vec{k})\|^2, \quad \vec{k} \in \Re^n \tag{18}$$

At this point, a concept for damage will be defined. This will be calculated as a function of each of the parameters of the vector $\vec{k}$ as follows:

$$D_i = 1 - \frac{(\vec{k}_*)_i}{(\vec{k}_{NOM})_i} \tag{19}$$

Stochastic Analysis - Uncertainty on force

This analysis could be considered identical to the previous one except for the concern that the estimated value for the force is uncertain. In this way, uncertainty is considered to be an additive term following a normal distribution $\aleph(0, \sigma_F^2)$.

Since the term $\hat{f}(\omega)$ of the Eq. 17 is modeled as a random variable, therefore the vector $\tilde{X}(\omega, \vec{k})$ will also be a random variable and then the vector $\vec{k}_*$ will be a random variable.

In this way, 1000 Monte Carlo simulations will be performed for the problem given by the Eq. 18 and therefore statistics as means and standard deviations will be determined for the parameter vector $\vec{k}_*$ and the damages D_i.

The goal of this analysis is to evaluate the sensitivity of the optimization methods to variations of uncertain nature in the model.

Stochastic Analysis - Uncertainty on response

Analogously to the previous section, the vector $\tilde{X}(\omega, \vec{k})$ will be considered a random variable. The difference is that now the uncertainty will be modeled directly in this term.

In this way, the uncertainty of the random variable $\tilde{X}(\omega, \vec{k})$ is an additive term that follows a normal distribution $\aleph(0, \sigma^2_{exp})$.

Again, 1000 Monte Carlo simulations will be performed in order to determine statistics for the parameter vector $\vec{k}_*$ and damages D_i.

Stochastic Analysis - Uncertainties on parameters

This analysis can be considered the most important of the present work. It consists of considering a stochastic model for the variables to be determined in the optimization process related to the damage identification.

Thus let's consider the parameter vector $\vec{k} = (k_1, k_2, ..., k_n)$. At this point, it will be modeled as a random vector $\vec{K} \sim \Psi(\vec{\mu}, [\Sigma])$, that is, a random vector that follows a generic distribution Ψ of mean $\vec{\mu}$ and covariance matrix $[\Sigma]$.

The central idea of this analysis is to determine which parameters $\vec{\mu}^*$ and $[\Sigma]^*$ best fit the stochastic model for the random variable $\vec{K}$ in the sense of optimization of the damage identification process.

For this type of analysis, only one experimental output will be considered $\tilde{X}_{exp}(\omega)$.

Thus, denoting by Θ the ordered pair $(\vec{\mu}, \Sigma)$, the optimization problem associated with the damage identification is written as:

$$\vec{\Theta}^* = \underset{\vec{\Theta}}{\text{minimize}} \quad \alpha E\{\|\tilde{X}_{exp}(\omega) - \tilde{X}(\omega, \vec{\Theta})\|^2\} + (1-\alpha) Desv_\sigma\{\|\tilde{X}_{exp}(\omega) - \tilde{X}(\omega, \vec{\Theta})\|^2\} \tag{20}$$

Where α is a parameter for weighting between the information regarding the mean and the standard deviation $Desv_\sigma$.

That is, for each iteration k of the optimization process, Monte Carlo simulations will be performed with the random vector $\vec{K} \sim \Psi(\Omega^{(k)})$ and thus several samples of the mean square difference $\|\tilde{X}_{exp}(\omega) - \tilde{X}(\omega, \vec{\Theta})\|^2$ will be collected. With these samples, the value of their expectation $E\{\cdot\}^{(k)}$ and its standard deviation $Desv_\sigma\{\cdot\}^{(k)}$ are calculated; thus, it proceeds to the next iteration in order to minimize the weighting between the values of $E\{\cdot\}^{(k)}$ and $Desv_\sigma\{\cdot\}^{(k)}$ calculated.

At the end of the analysis, the best values (in the sense of optimization) will be estimated for the parameters $\vec{\mu}$ and $[\Sigma]$ from the random vector $\vec{K} \sim \Psi(\vec{\mu}, [\Sigma])$ related to the damage

identification process.

3 EXAMPLE MODEL

The mechanical system to be studied consists of a set of mass elements interconnected in series by elastic springs and dissipating elements. An illustration for n interconnected masses (and consequently n degrees of freedom - dof) can be visualized in Fig. 2:

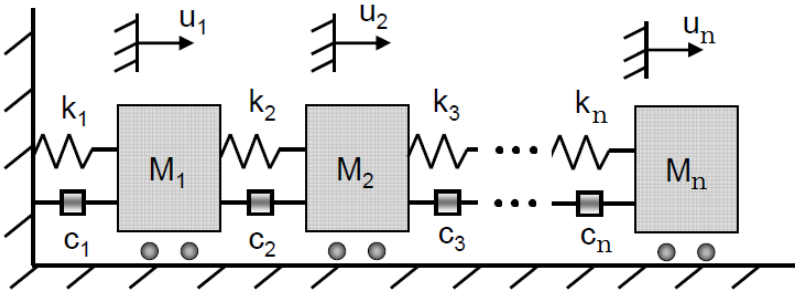

Figure 2: Mechanical system

For the present study a system with six degrees of freedom was considered. In addition, the application of an instantaneous force (modeled as a *Delta de Dirac*) was considered in the second and fourth mass elements of magnitudes $5N$ and $8N$ respectively. In this way, the representation of the system is illustrated in Fig. 3:

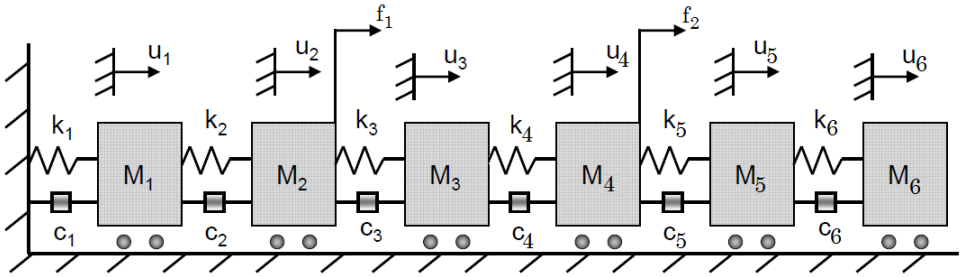

Figure 3: Mechanical system - 6 dof

In addition it is valid to note that the simulated damage in the mechanical system was a 25% reduction in the stiffness of spring 1 ($k_1^D = 0.75k_1^S$) and 65% in the stiffness of spring 5 ($k_5^D = 0.35k_5^S$).

The components of the $X(\omega)$ vector that will form the vector $\tilde{X}(\omega)$ are the components X_1, X_5 and X_6.

In addition, for this problem, the following nominal values were considered for the set of parameters: $m_1 = 20kg$; $m_2 = 12kg$; $m_3 = 24kg$; $m_4 = 18kg$; $m_5 = 20kg$; $m_6 = 15kg$; $c_1 = 25\frac{Ns}{m}$; $c_2 = 24\frac{Ns}{m}$; $c_3 = 23\frac{Ns}{m}$; $c_4 = 26\frac{Ns}{m}$; $c_5 = 20\frac{Ns}{m}$; $c_6 = 21\frac{Ns}{m}$; $k_1 = 40\frac{N}{m}$; $k_2 = 28\frac{N}{m}$; $k_3 = 13\frac{N}{m}$; $k_4 = 31\frac{N}{m}$; $k_5 = 23\frac{N}{m}$; $k_6 = 24\frac{N}{m}$.

Thus, the matrices that constitute the system are:

$$[M] = \begin{bmatrix} 20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15 \end{bmatrix}, [C] = \begin{bmatrix} 49 & -24 & 0 & 0 & 0 & 0 \\ -24 & 47 & -23 & 0 & 0 & 0 \\ 0 & -23 & 49 & -26 & 0 & 0 \\ 0 & 0 & -26 & 46 & -20 & 0 \\ 0 & 0 & 0 & -20 & 41 & -21 \\ 0 & 0 & 0 & 0 & -21 & 21 \end{bmatrix},$$

$$[K] = \begin{bmatrix} 68 & -28 & 0 & 0 & 0 & 0 \\ -28 & 41 & -13 & 0 & 0 & 0 \\ 0 & -13 & 44 & -31 & 0 & 0 \\ 0 & 0 & -31 & 54 & -23 & 0 \\ 0 & 0 & 0 & -23 & 47 & -24 \\ 0 & 0 & 0 & 0 & -24 & 24 \end{bmatrix} \tag{21}$$

For the analyzes involving uncertainties the following values were used: $\sigma_F = 0.3$; $\sigma_{exp} = 10\%$ from maximum value of the vector $\tilde{X}_{exp}(\omega)$.

For the stochastic analyzes with uncertainties in the parameters, it was assumed that the components of the random vector $\vec{K}$ are independent and uncorrelated. In this way two types of probability distributions were used as marginal distributions according to the characteristics of the problem (the parameters k_i s must be positive): Gamma distribution and Normal Truncated distribution.

The Gamma distribution probability density function can be written as a function of the parameters α and θ through the following equation (where $\alpha = \frac{\mu_G^2}{Var_G}$ and $\theta = \frac{Var_G}{\mu_G}$):

$$f(x) = x^{\alpha-1} \cdot e^{\frac{-x}{\theta}} \cdot \begin{cases} \frac{1}{\Gamma(\alpha)\theta^\alpha} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{22}$$

The Truncated Normal distribution probability density function can be written as a function of its mean μ_{NT} and variance σ_{NT}^2 by the following equation:

$$f(x) = \begin{cases} \frac{\frac{1}{\sqrt{2\pi\sigma_{NT}^2}} exp(-\frac{(x-\mu_{NT})^2}{2\sigma_{NT}^2})}{1 - G(0)} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

Where $G(y) = \int_{-\infty}^{y}(\frac{1}{\sqrt{2\pi\sigma_{NT}^2}}exp(-\frac{(x-\mu_{NT})^2}{2\sigma_{NT}^2}))dy$.

4 RESULTS

In this chapter the results for the analyzes described in the previous subsection will be presented.

All analysis were made using Matlab R2015a and a computer with the following systems characteristics: *AMD Quad Core A10-4600M with Radeon(tm) HD Graphics 2.30 GHz, 8GB RAM, Windows 7 64 bits*.

In the results presented below, the following nomenclature was used for the optimization methods:

- Method 1 $\longleftrightarrow$ Nelder-Mead Simplex;
- Method 2 $\longleftrightarrow$ BFGS Quasi-Newton;
- Method 3 $\longleftrightarrow$ Interior Point;
- Method 4 $\longleftrightarrow$ Sequential Quadratic Programming - SQP;

4.1 Deterministic Analysis

The damages identified in this analysis are presented in the following table:

Table 1: Results - Damage Identification

Damage	Method 1	Method 2	Method 3	Method 4
Element 1	**24,78%**	**25%**	**25%**	**25%**
Element 2	**5,96%**	0%	0%	0%
Element 3	**0,95%**	0%	0%	0%
Element 4	**-15,84%**	0%	0%	0%
Element 5	**64,31%**	**65%**	**65%**	**65%**
Element 6	**-1,23%**	0%	0%	0%

The computational costs in each method are presented below:

Table 2: Computational cost (in seconds)

Method 1	Method 2	Method 3	Method 4
14.57	6.60	4.14	5.81

In Fig. 4 we can see the difference between the healthy structure's response and the damaged structure's response. We can compare the results obtained by the four methods also. It was plotted only the first component of the $X(\omega)$ vector in Fig. 4.

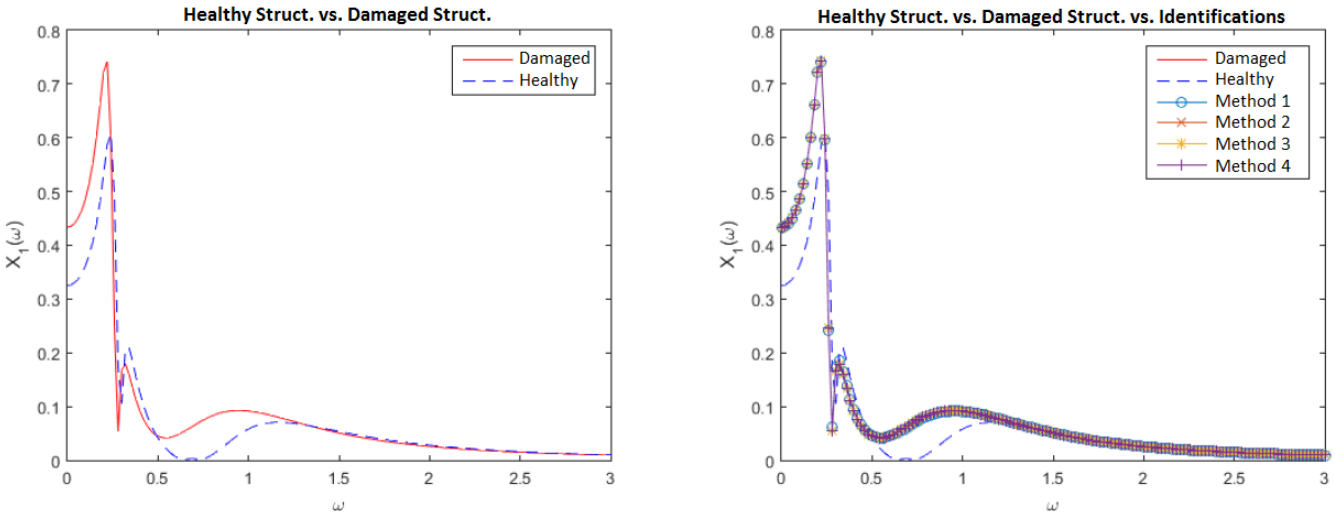

Figure 4: Healthy structure's response vs. Damaged structure's response

4.2 Stochastic Analysis - Uncertainty on force

The sensitivity of the model of a healthy structure to the variations imposed on the force can be considered as null, since all the algorithms converged to the solution of a healthy structure, that is, no damage was identified in a healthy structure subject to uncertainties in the force imposed to the studied mechanical system.

The results found for the identified damages are presented in the following table:

Table 3: Damages - Uncertainty on force

Element		Method 1	Method 2	Method 3	Method 4
Elem. 1	Mean	**24,98%**	**25,00%**	**25,00%**	**25,00%**
	Desv.	0,15%	0,00%	0,00%	0,00%
Elem. 2	Mean	**0,43%**	0,00%	0,00%	0,00%
	Desv.	2,16%	0,00%	0,00%	0,00%
Elem. 3	Mean	**0,04%**	0,00%	0,00%	0,00%
	Desv.	0,40%	0,00%	0,00%	0,00%
Elem. 4	Mean	**-1,27%**	0,00%	0,00%	0,00%
	Desv.	6,54%	0,00%	0,00%	0,00%
Elem. 5	Mean	**64,95%**	**65,00%**	**65,00%**	**65,00%**
	Desv.	0,32%	0,00%	0,00%	0,00%
Elem. 6	Mean	**-0,18%**	0,00%	0,00%	0,00%
	Desv.	0,86%	0,00%	0,00%	0,00%

The computational costs in each method are presented below:

Table 4: Computational costs (in seconds)

Method 1	Method 2	Method 3	Method 4
2966	1278	1268	1103

4.3 Stochastic Analysis - Uncertainty on response

Healthy structure

The results found for the damages identified in a healthy structure are presented in Tab. 5. These results show the sensitivity of the methodology adopted to the uncertainties in the measurements of the experimental responses.

Table 5: Damages in healthy structure - Uncertainty on response

Element		Method 1	Method 2	Method 3	Method 4
Elem. 1	Mean	-1,64%	1,24%	0,06%	0,06%
	Desv.	4,01%	3,89%	3,88%	3,88%
Elem. 2	Mean	0,37%	-4,00%	1,08%	1,08%
	Desv.	6,17%	6,38%	6,19%	6,19%
Elem. 3	Mean	-3,25%	-4,16%	-3,48%	-3,48%
	Desv.	9,71%	10,07%	9,99%	9,99%
Elem. 4	Mean	5,79%	2,33%	1,90%	1,90%
	Desv.	6,25%	4,51%	5,46%	5,46%
Elem. 5	Mean	-3,78%	-1,50%	-6,13%	-6,13%
	Desv.	7,56%	7,06%	7,61%	7,61%
Elem. 6	Mean	1,12%	2,75%	2,39%	2,39%
	Desv.	6,36%	6,32%	7,96%	7,96%

Os tempos computacionais gastos em cada método estão apresentados a seguir:

Table 6: Tempo computacional (em segundos)

Método 1	Método 2	Método 3	Método 4
2272	1445	1400	1278

Estrutura danificada

Os resultados encontrados para os danos identificados numa estrutura danificada são apresentados na tabela a seguir:

Table 7: Danos em estrutura danificada - Incerteza na resposta

Elemento		Método 1	Método 2	Método 3	Método 4
Elem. 1	Média	24,10%	24,00%	24,95%	24,52%
	Desv.	3,50%	3,50%	3,48%	3,53%
Elem. 2	Média	1,93%	-1,08%	1,47%	0,33%
	Desv.	5,64%	5,85%	5,15%	5,98%
Elem. 3	Média	-6,67%	-4,94%	-7,44%	-5,78%
	Desv.	10,38%	10,18%	10,55%	10,02%
Elem. 4	Média	-6,68%	-0,57%	-8,75%	-0,55%
	Desv.	7,94%	6,66%	7,72%	7,13%
Elem. 5	Média	62,23%	60,86%	60,93%	60,77%
	Desv.	5,18%	5,47%	5,47%	5,37%
Elem. 6	Média	-3,16%	-2,53%	-3,92%	-1,19%
	Desv.	8,13%	5,86%	8,82%	7,84%

The computational costs in each method are presented below:

Table 8: Computational costs (in seconds)

Method 1	Method 2	Method 3	Method 4
3086	1479	1690	1427

4.4 Stochastic Analysis - Uncertainty on parameters

As described in section 2.3 the central idea of this analysis is to determine the parameters that best describe a given probability distribution prescribed for a set of variables.

Therefore, the values found are presented in the following tables:

Table 9: Means and Variances - Gama distribution

Stifness		Method 1	Method 2	Method 3	Method 4	**Nominal with Damage**
k_1	μ	32,8	28,4	29,0	29,1	**30**
	Var.	3,01	2,88	2,59	1,98	
k_2	μ	29,3	28,3	28,3	28,1	**28**
	Var.	3,31	2,75	2,39	2,12	
k_3	μ	14,1	13,5	13,4	13,4	**13**
	Var.	3,65	2,55	2,40	2,08	
k_4	μ	33,7	32,9	32,8	32,3	**31**
	Var.	3,26	3,18	2,79	2,28	
k_5	μ	9,2	8,8	8,6	8,3	**8,05**
	Var.	3,03	2,78	2,37	1,88	
k_6	μ	25,2	24,7	24,6	24,6	**24**
	Var.	2,91	2,82	2,48	1,94	

Table 10: Means and Variances - Normal Truncated distribution

Stifness		Method 1	Method 2	Method 3	Method 4	**Nominal with Damage**
k_1	μ	32,2	29,2	29,2	29,3	**30**
	Var.	2,91	2,76	2,32	1,78	
k_2	μ	28,7	28,3	27,8	27,5	**28**
	Var.	2,87	2,56	2,19	1,96	
k_3	μ	13,8	13,5	12,6	12,4	**13**
	Var.	2,65	2,51	2,30	1,98	
k_4	μ	32,6	32,1	30,8	30,3	**31**
	Var.	3,16	2,28	2,19	1,99	
k_5	μ	8,6	8,5	8,2	7,9	**8,05**
	Var.	2,63	2,44	2,27	1,90	
k_6	μ	24,8	24,6	23,4	22,6	**24**
	Var.	2,61	2,37	2,18	1,89	

The computational costs for each method are given below for each marginal probability distribution:

- **Gamma distribution**

Table 11: Computational costs (in hours)

Method 1	Method 2	Method 3	Method 4
12.6	2.6	2.0	2.3

- **Normal Truncated distribution**

Table 12: Computational costs (in hours)

Method 1	Method 2	Method 3	Method 4
12.7	2.7	2.0	2.3

5 FINAL REMARKS

The great contribution that we hope to obtain with this work and its subsequent continuity is to produce a material that adds knowledge of techniques of optimization and stochastic modeling applied in processes of damage identification in mechanical systems.

Thus, with the results obtained in the previous analyses, a detailed analysis of the results found in each section is carried out.

5.1 Deterministic Analysis

By observing the results of the section 4.1 it is noted that each of the optimization techniques were able to predict the damage imposed to the structure.

Besides it is observed that the Nelder-Mead Simplex method has identified non-existent damages in the structure. Such observation was expected since such a technique consists of a metaheuristic method. In this way, it is verified that such technique can be suggested as an initial search for a solution that will feed some other linear programming algorithm, for example.

Another conclusion reached is that the Nelder-Mead Simplex method is more computationally costly than the other methods used - its computational cost is more than double the costs of the other techniques studied.

5.2 Stochastic Analysises

If we look at the results of the sections 4.2 and 4.3 we find that, firstly, for the type of force modeled, the model response is insensitive to the uncertainties considered for the force. The Quasi-Newton BFGS, Interior Point and Sequential Quadratic Programming - SQP methods proved to be quite accurate in the process of damage identification.

Again it is concluded that the Nelder-Mead Simplex method is more computationally costly than the other methods used in the analyzes - *Note: at this point, in order to avoid repetitions it is noted that such a conclusion will be the same for the analyzes presented subsequently*.

Regarding the sensitivity of the model to the uncertainties in the response, it is noticed that this was sensitive to such uncertainties, since non-existent damages were identified in the healthy structure. This observation suggests a greater investigation of the subject in two aspects:

- To impose restrictions on the values of the parameters in order to limit them to a range, since in the Interior Point and Sequential Quadratic Programming methods only the optimization parameters were limited - the idea is to limit them superiorly;
- To evaluate techniques of regularization of the objective function in order to minimize the sensitivity to uncertainties.

Despite the sensitivity of the healthy structure to the uncertainties in the measurement of the experimental response, all techniques achieved a good approximation of the damage in the damaged structure (Table 7).

5.3 Stochastic Analysis - Uncertainty on parameters

In this analysis, the main conclusion reached is that this type of analysis is very costly computationally (according to Tables 11 and 12). The motivation for the continuity of this work is to look for another technique for the propagation of uncertainties other than the Monte Carlo technique, such as expansion techniques based on chaos polynomials and the method of stochastic collocation.

REFERENCES

Beale, E. M. L., 1967. *Numerical Methods*. In: Nonlinear programming, J. Abadie ed, North - Holland.

Byrd, R. H., Hribar, M. E., Nocedal, J., 1999. "An Interior Point Algorithm for Large-Scale Nonlinear Programming", *SIAM Journal on Optimization*, v. 9, pp. 877-900.

Hall, S. R., 1999. "The effective management and use of structural health data". In:*INTERNATIONAL WORKSHOP ON STRUCTURAL HEALTH MONITORING*, pp. 265-275.

Hicken, J. E., Alonso, J. J., 2012. Lecture notes from the course *AA222 - Introduction to Multidisciplinary Design Optimization*. Stanford.

Lagarias, J. C., Reeds, J. A., Wright, M. H., Wright, P. E., 1998. "Convergence Properties of the Nelder-Mead Simplex Algorithm in Low Dimensions", *SIAM Journal on Optimization*, v. 9, pp. 112-147.

Nelder, J. A., Mead, R., 1965. A simplex method for function minimization, *Computer Journal* 7, 308?313.

Ooijevaar, T. H., 2014. *Vibration based structural health monitoring of composite skin-stiffener structures*. Tese de D.Sc., University of Twente.

Ritter, A., 1993. *Vibrational Based Inspection of Civil Engineering Structures*. Tese de D.Sc., Aalborg University.

Ritto, T. G., Castello, D. A., 2016. *Quantificação de incertezas e estimação de parâmetros em dinâmica estrutural: uma introdução a partir de exemplos computacionais*. 1 ed. São Carlos, SBMAC.

Wilson, R. B., 1963. *A Simplicial Algorithm for Concave Programming*. Tese de D.Sc., Harvard University Graduate School of Business Administration.

Worden, K., Dulieu-Barton, J. M., 2004. "An overview of intelligent fault detection in systems and structures", *Struct. Heal. Monit.*, v. 3, n. 1, pp. 85-98.