



A HYPERELASTIC CONSTITUTIVE MODEL FOR SOFT BIOLOGICAL TISSUES CONSIDERING INDIVIDUAL ASPECTS

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Abstract. *Soft biological tissues are composed mainly by collagen, elastin and ground substance. Due to their structural arrangement, the numerical representation of their mechanical behavior is a nontrivial subject. Moreover, the tissues mechanical behavior can change if taken from different individual or different position of the same individual. Many authors have found experimental evidences that qualitatively correlate the mechanical response of biological tissues with some of their characteristics through individual parameters. The individual parameters used in these works are related to physical, physiological, histological or pathological aspects. The quantification of such correlations can indicate which parameters may be used on material laws in order to improve the mechanical representation of the tissue for different individuals. Aiming to correlate the mechanical behavior of the biological tissue with individual parameters, the scope of the present work is to develop a material model capable to incorporate specific individual variables dependences. The proposed constitutive model is based on classical hiperelasticity, in which the individual parameter dependence is incorporated in energy potentials that modifies the constitutive relation. The individual parameters are taken as external material parameters making possible to employ classical hyperelastic models into the representation of the biological tissue. Numerical examples are presented evidencing the capability of the proposed model to represent the tissues mechanical behavior and the individual characteristics dependences.*

Keywords: *Constitutive model, Individual aspects, Biological tissue*

1 INTRODUCTION

The estimation of the mechanical behavior of biological structures plays an important role in several applications, such as, numerical simulation of artery/stent interaction (Ragkousis et al., 2014), knee prosthesis/muscle skeletal system (Kia et al., 2014), control surgeries (Beyar et al., 2006), among others. The prediction of internal forces and tissue deformation may provide important information in the choice of individualized medical procedures for different patients.

Biological tissues are composed by a cell collection and an extracellular matrix (Junqueira et al., 2004), wherein each one has a specific function. Arteries, tendons and ligaments, due to the presence of fibers highly oriented into a matrix represented by the ground substance, are usually modeled as composites (Holzapfel et al., 2000, Ehret and Itskov, 2009, Vassoler et al., 2012) and shown non-linear and anisotropic mechanical behaviors.

One of the difficulties in applying numerical simulations to real patients resides in to consider individual aspects of the patient to the material model. The same biological tissue may present a distinct mechanical behavior in different individuals or different positions of the same individual (Learoyd and Taylor, 1966, Maher et al., 2012). This aspect is not considered in most material models available in literature (Holzapfel et al., 2000, Ehret et al., 2011). Thus, the objective of this work is present a hyperelastic constitutive model capable to consider individual aspects.

2 CONSTITUTIVE MODEL

Biological tissue can be mathematical described through several assumptions and models, usually employing nonlinear constitutive theories. As examples, one can cite viscoelasticity (Holzapfel and Gasser, 2001, Ciarletta et al., 2006, Vassoler et al., 2012), hyperelasticity (Holzapfel et al., 2000, Fancello et al., 2008), poroelasticity (Tomic et al., 2014), mechanical damage (Hokanson and Yazdani, 1997, Vassoler et al., 2016), among others. In the present work, the hyperelastic structure was studied, employing a phenomenological approach, describing the macroscopic nature of materials as continua.

In order to consider the fibers contributions presented in the tissue, a decomposition of the strain energy function may be made (Holzapfel et al., 2000) as follows:

$$\Psi = \Psi_{iso}(\mathbf{C}) + \Psi_f(\mathbf{C}, \mathbf{A}_f). \quad (1)$$

where Ψ_{iso} consists of an isotropic contribution, Ψ_f consists of an anisotropic contribution provided by the fibers inclusion and $\mathbf{C}$ is the Cauchy-Green tensor.

The structural tensor, $\mathbf{A}_f$, is defined as

$$\mathbf{A}_f = \mathbf{a}_f \otimes \mathbf{a}_f \quad (2)$$

where $\mathbf{a}_f$ is the unit vector defining the fiber direction and is related to the fiber stretch λ_f : $\lambda_f^2 = \mathbf{a}_f \cdot \mathbf{C} \cdot \mathbf{a}_f$.

For the formulation to be sensitive to individual aspects, the potentials must depend on individual parameters. To introduce this dependency, a single parameter dependence (or parameters combinations) is incorporated into the potential. This may be achieved through a function that modifies the constitutive relation and is based on models presented in literature (Avalle et al., 2007, Ehret et al., 2011), which use functions to incorporate certain characteristics into their phenomenological response. Despite the fact that it is a simple modification into the hyperelastic energy function, this assumption is thermodynamically consistent since the free energy of the tissue is different for each individual and may be dependent of an individual material parameter. In this work, the individual parameter is obtained from histological analyses of the tissue (Sokolis, 2010). Physical and physiological parameters may be used as well (Dunkman et al., 2013, Camhi et al., 2013).

In this study, individual aspects are considered only in the fibers contribution, which is defined as

$$\Psi_f = g(\beta)\bar{\varphi}_f(\lambda_f). \quad (3)$$

where $g(\beta)$ is a function of the individual parameter β and $\bar{\varphi}_f$ is a classic hyperelastic energy function. For instance, $g(\beta)$ function can be defined as

$$g(\beta) = \mu\beta^\alpha. \quad (4)$$

where μ and α are material parameters related to an individual parameter β .

In this way it is possible to compute the Piola–Kirchhoff stress tensor as follows

$$\mathbf{P} = 2\mathbf{F} \frac{\partial \Psi_f(\mathbf{C}, \mathbf{A}_f)}{\partial \mathbf{C}} \quad (5)$$

2.1 Results and Discussion

To demonstrate the applicability of the $g(\beta)$ function in a hyperelastic formulation and clarify the models individual parameters influence, some numerical tests were performed. The Holzapfel model (Holzapfel et al., 2000) was used as classical expression

$$\bar{\varphi}_f(\lambda_f) = \frac{k_1}{2k_2} \left\{ \exp \left[k_2 (\lambda_f^2 - 1)^2 \right] - 1 \right\}. \quad (6)$$

where k_1 and k_2 are the materials parameters.

In this investigation, the parameters k_1 and k_2 were obtained from Vassoler et al. (2012), and the individual parameter β , which could represent the volume fiber fraction, varying from 0 to 1, where 0 corresponds to the absence of tendon fibers and 1 corresponds to a tendon formed only by fibers.

The Fig. 1 shows the results for $\mu = 1$ and $\alpha = 1$ of function $g(\beta)$ and Fig. 2 demonstrate the results for $\mu = 1$ and $\alpha = 5$ of function $g(\beta)$.

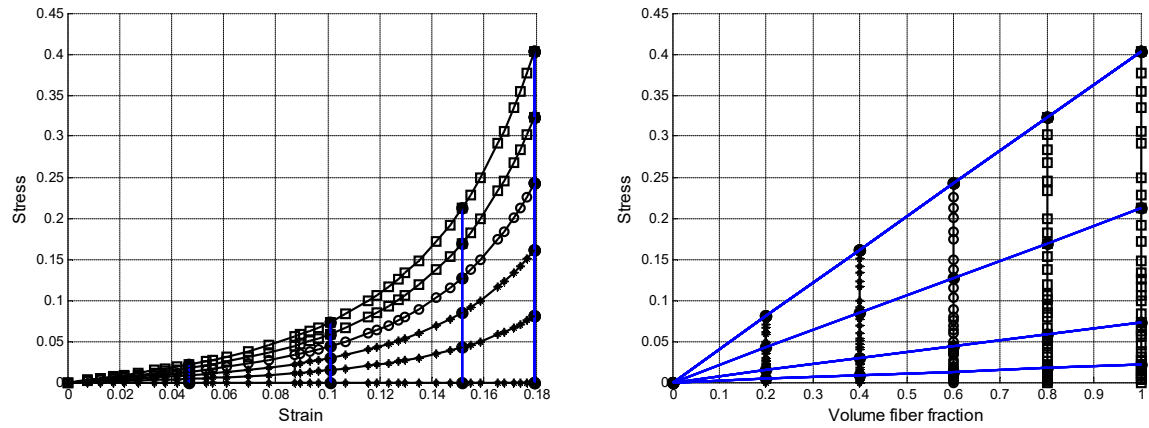


Figure 1. Stress-strain curve for $\mu=1$ and $\alpha=1$.

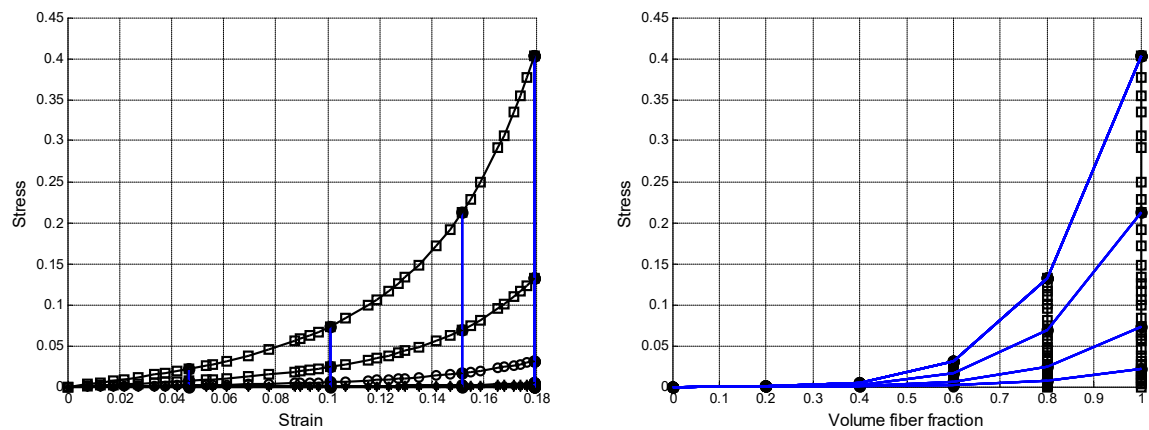


Figure 2. Stress-strain curve for $\mu=1$ and $\alpha=5$.

From Figs. 1 and 2 it is possible to visualize the changes in the stress-strain response with the parameter β (volume fiber fraction) and the material parameters related to the individual aspects. In Fig. 1, as expected, the stress response related to the parameter β (volume fiber fraction) is linear. In the other hand, in Fig. 2 the response is nonlinear due to the choice of the parameter α . The tendencies of mechanical responses obtained from these tests are in agreement with those reported in literature (Ciarletta et al., 2006, Ehret and Itskov, 2009, Ehret et al., 2011, among others). Other tendencies can be obtained using more complex $g(\beta)$ functions.

Besides its simplicity, with the use of the $g(\beta)$, this model demonstrated a great flexibility in the dependence representation of individual parameters. This mathematical framework can be easily combined to more complex constitutive models that consider viscoelastic e mechanical damage effects.

2.2 Conclusions

The constitutive modeling approach proposed in this work incorporate a dependence of individual aspects into a hyperelastic constitutive model presented in literature. Despite that

individual parameters used herein are not related to experimental data of a real biological tissue, the numerical tests are capable to show that a classical hyperelastic model modified by $g(\beta)$ may represent the tissues mechanical behavior considering an individual parameter.

The model representativeness will be better understood with a suitable experimental characterization of the tissue in the further studies that are currently been conducted.

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REFERENCES

- Avalle, M.; Belingardi, G.; Ibba, A., 2007. Mechanical models of cellular solids: parameters identification from experimental tests. *International Journal of Impact Engineering*, vol. 34(1), pp. 3–27.
- Beyar, R.; Gruberg, L.; Deleanu, D.; Roguin, A.; Almagor, Y.; Cohen, S.; Kumar, G.; Wenderow, T., 2006. Remote-control percutaneous coronary interventions. *Journal of the American College of Cardiology*, vol. 47(2), pp. 296–300.
- Camhi, S. M.; Katzmarzyk, P. T.; Broyles, S. T.; Srinivasan, S. R.; Chen, W.; Bouchard, C.; Berenson, G. S., 2011. Subclinical atherosclerosis and metabolic risk: role of body mass index and waist circumference. *Metabolic syndrome and related disorders*, vol. 9(2), pp. 119–125.
- Ciarletta, P.; Micera, S.; Accoto, D.; Dario, P., 2006. A novel microstructural approach in tendon viscoelastic modelling at the fibrillar level. *Journal of biomechanics*, vol. 39(11), pp. 2034–2042.
- Dunkman, A. A.; Buckley, M. R.; Mienaltowski, M. J.; Adams, S. M.; Thomas, S. J.; Satchell, L.; Kumar, A.; Pathmanathan, L.; Beason, D. P.; Iozzo, R. V. et al., 2013. Decorin expression is important for age-related changes in tendon structure and mechanical properties. *Matrix Biology*, vol. 32(1), pp. 3–13.
- Ehret, A. E.; Itskov, M., 2009. Modeling of anisotropic softening phenomena: application to soft biological tissues. *International Journal of Plasticity*, vol. 25(5), pp. 901–919.
- Ehret, A. E.; Böhl, M.; Itskov, M., 2011. A continuum constitutive model for the active behaviour of skeletal muscle. *Journal of the Mechanics and Physics of Solids*, vol. 59(3), pp. 625–636.
- Fancello, E.; Vassoler, J. M.; Stainier, L., 2008. A variational constitutive update algorithm for a set of isotropic hyperelastic–viscoplastic material models. *Computer Methods in Applied Mechanics and Engineering*, vol. 197(49), pp. 4132–4148.
- Hokanson, J.; Yazdani, S., 1997. A constitutive model of the artery with damage. *Mechanics Research Communications*, vol. 24(2), p. 151–159.
- Holzappel, G.A., Gasser, T.C., Ogden, R.W., 2000. A new constitutive framework for arterial wall mechanics and a comparative study of material models. *Journal of Elasticity*, 61:1-48.

- Holzappel, G. A.; Gasser, T. C., 2001. A viscoelastic model for fiber-reinforced composites at finite strains: Continuum basis, computational aspects and applications. *Computer methods in applied mechanics and engineering*, vol. 190(34), pp. 4379–4403.
- Junqueira, L. C. U.; Carneiro, J.; Abrahamsohn, P. A.; Zorn, T. M. T.; Santos, M. F. d. 2004. *Histologia básica: texto e atlas*. In *Histologia básica: texto e atlas*, pp. 488–488.
- Kia, M.; Stylianou, A. P.; Guess, T. M., 2014. Evaluation of a musculoskeletal model with prosthetic knee through six experimental gait trials. *Medical engineering & physics*, vol. 36(3), pp. 335–344.
- Learoyd, B. M.; Taylor, M. G., 1966. Alterations with age in the viscoelastic properties of human arterial walls. *Circulation research*, vol. 18(3), pp. 278–292.
- Maher, E.; Early, M.; Creane, A.; Lally, C.; Kelly, D. J., 2012. Site specific inelasticity of arterial tissue. *Journal of biomechanics*, vol. 45(8), pp. 1393–1399.
- Ragkousis, G. E.; Curzen, N.; Bressloff, N. W., 2014. Simulation of longitudinal stent deformation in a patient-specific coronary artery, *Medical engineering & physics*, vol. 36(4), pp. 467–476, 2014.
- Sokolis, D. P., 2010. A passive strain-energy function for elastic and muscular arteries: correlation of material parameters with histological data. *Medical & biological engineering & computing*, vol. 48(6), pp. 507–518.
- Tomic, A.; Grillo, A.; Federico, S. 2014. Poroelastic materials reinforced by statistically oriented fibres—numerical implementation and application to articular cartilage, *IMA Journal of Applied Mathematics*, vol., pp. hxu039.
- Vassoler, J.; Reips, L.; Fancello, E., 2012. A variational framework for fiber-reinforced viscoelastic soft tissues. *International Journal for Numerical Methods in Engineering*, vol. 89(13), pp. 1691–1706.
- Vassoler, J.; Stainier, L.; Fancello, E. 2016. A variational framework for fiber-reinforced viscoelastic soft tissues including damage. *International Journal for Numerical Methods in Engineering*, vol. 108, pp. 865–884.