A SIMPLE APPROACH FOR STABILIZATION OF LINEAR SECOND-ORDER ASYMMETRIC SYSTEMS

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Abstract. *Linear second-order systems with an asymmetric structure on the damping and stiffness matrices arise from the modeling of dynamic vibrating systems with aeroelastic or friction-induced phenomena, such as flutter in wings of spacecraft, attrition in belt conveyors, among others. Asymmetry can induce complex eigenvalues in the RHP, that is, unstable, dangerous vibrations appear with crescent amplitude. The central goal of feedback control in such systems is stabilizing these undesired eigenpairs. Meanwhile, the others can be kept unchanged. This last property is known as no spillover, and several researchers have devoted attention on this in the last two decades. Regarding the solution of no spillover for asymmetric systems, the contributions on the problem solution are seldom. By applying no spillover for asymmetric systems with the well-known receptance approach, we proposed in this work a technique for stabilization of unstable eigenpairs with a single control input and fixed influence matrix, using state or derivative feedbacks. Extra degree-of-freedom for the feedback gains is provided by assigning a set of 2k-1 out of 2k unstable or ill damped complex eigenvalues to the LHP with adequate considerations. Two numerical experiments based on real-world models borrowed from the literature are given to illustrate the merits of the proposed approach.*

Keywords: *Eigenvalue assignment, Second order systems, Asymmetric Systems, Receptances, Feedback*

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1 INTRODUCTION

Second-order models for linear dynamic systems rise from several physical phenomena, such as mechanical vibrations (Inman 2001; J. E. Mottershead and Ram 2006), acoustics and wave propagation (Li and Yu 2016), electrical oscillations (Misrikhanov and Ryabchenko 2006), among other examples. Systems that are modeled as second-order exhibit behaviors that can be satisfactorily explained by this approach rather than a first-order model, as resonance and flutter, two type of dangerous vibrations. In most cases, the models obtained from mechanical vibrating structures is given as a finite element model - FEM (Betcke et al. 2013) with higher orders, but lumped parameters systems also can be modeled using the second-order approach, with lower orders in general. Models obtained with FEM approach and even lumped parameters systems very often exhibit some exploitable properties in their matrices, such as symmetry or antisymmetry and definiteness (B. N. Datta, Elhay, and Ram 1997; B. N. Datta 2010). Moreover, symmetric systems in which the mass and stiffness matrices are positive (semi)definite, the spectrum is stable, such that feedback control is applied in general to avoid the resonance phenomenon or for improve the damping characteristic of the system. This approach is known as active vibration control - AVC, and several works have described their advantages and advances over the years (J. E. Mottershead and Ram 2006; Fuller 2008; Gudarzi 2015; Palazzolo 2016). Another relevant techniques that take advantage of the system matrices properties are the finite-element model update - FEMUP and eigenvalue embedding (Carvalho 2002). For models in which some of the matrices are asymmetric, a dangerous behavior can arise due to the presence of unstable eigenvalues. Friction induced vibrations (H. Ouyang 2010; Singh and H. Ouyang 2013) and aeroelasticity (Tisseur and Meerbergen 2001) are some real-world examples of such systems. In such cases, the primary goal for a feedback control system is to stabilize these eigenvalues rather than modify natural frequencies as in the symmetric case. In the literature, state or derivative feedback control strategies are found to be successfully applied in the stabilization of a wing under air-stream (Henrion, Sebek, and Kucera 2005; Abdelaziz 2013) and also in a belt conveyor under friction by external slider contact (Liang, Yamaura, and H. Ouyang 2017). Recently, the paper Liang, H. J. Ouyang, and Yamaura (2016) have addressed the problem of partial eigenvalue assignment for stabilization of asymmetric systems by state-feedback and the well-known receptance method (J. E. Mottershead and Ram 2007; Tehrani and J. Mottershead 2012), a case that is rarely explored in the literature. The unstable and even some ill damped eigenvalues are re-assigned to prescribed position in the LHP by the designer using a single control input. In the present proposal, that method is modified to give additional degree-of-freedom in state or derivative feedback gains computation by solving an optimization problem. To achieve this, all but one of the undesirable eigenvalues is reassigned to a given position. This free eigenvalue is then constrained to lie at LHP. Such extra degree-of-freedom can be used, as an instance, to assign robustness or minimum norm of the feedback matrices. The rest of the paper is organized as the preliminary concepts and statement of the problem; the description of the proposed modified optimization approach for partial eigenvalue assignment; two numerical experiments using models presented at literature; and the concluding remarks and perspectives in future researches.

2 PRELIMINARIES AND STATMENT OF THE PROBLEM

2.1 Second-Order Dynamic Linear Models

Let be the second-order linear dynamic system given by the following equation:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are, respectively, the mass, damping and stiffness matrices, each of order n, $\mathbf{x}(t)$ is the displacement vector and $\mathbf{f}(t)$ is a external vector force. For feedback control purposes, the force vector is given by an m-dimensional actuator assembly $\mathbf{u}(t)$, that reaches the system through an influence matrix $\mathbf{B} \in \mathbb{R}^{n\times m}$:

$$\mathbf{f}(t) = \mathbf{B}\mathbf{u}(t), \tag{2}$$

The solution $\mathbf{x}(t)$ for the homogeneous system, that is, when $\mathbf{u}(t) \equiv 0$, is given as:

$$\mathbf{x}(t) = \mathbf{y}e^{\lambda t}, \tag{3}$$

where $\mathbf{y}$ and λ are, respectively, an eigenvector and the associated eigenvalue, that solves the eigenvalue-eigenvector problem:

$$\mathbf{Q}(\lambda_k)\mathbf{y}_k = 0,\ k = 1, 2..., 2n \tag{4}$$

The pencil

$$\mathbf{Q}(\lambda) = \lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K} \tag{5}$$

is the so-called quadratic pencil. The set of all solutions of the characteristic equation

$$\Delta(\lambda) = det[\mathbf{Q}(\lambda)] \equiv 0 \tag{6}$$

is known as the spectrum of (4)-(5). System matrices are symmetric in general, but there are some cases in which damping or stiffness can exhibit asymmetric components. Dangerous locations within a small portion of the spectrum can occur. In such cases, AVC by state or derivative feedback is a viable alternative to mitigate the instability by reassigning these unwanted eigenvalues to a safe location complex plane. Moreover, ill damped eigenvalues can also be reassigned in the same way.

2.2 State Feedback, Derivative Feedback and Receptance Method for Active Vibration Control

By applying Eq(2), the designer can use two well-known linear techniques: state or derivative feedback. We consider here the single input case, i.e., $m = 1$. State feedback control is given as:

$$\mathbf{u}(t) = -\mathbf{f}_s\dot{\mathbf{x}}(t) - \mathbf{g}_s\mathbf{x}(t), \tag{7}$$

where $\mathbf{f}_s$ and $\mathbf{g}_s$ are two unknown velocity and displacement feedback matrices. By it hand, the derivative feedback is implemented by:

$$\mathbf{u}(t) = -\mathbf{f}_d\dot{\mathbf{x}}(t) - \mathbf{g}_d\ddot{\mathbf{x}}(t). \tag{8}$$

In several works, state feedback and derivative feedback have been reported to solve vibration control problems with full or partial spectrum assignment for symmetric and asymmetric second-order systems (B.N. Datta et al. 2000; Henrion, Sebek, and Kucera 2005; Bai, B. N. Datta, and Wang 2010; Araújo et al. 2016).

The receptance approach for feedback design is introduced in (J. E. Mottershead and Ram 2007). The open-loop equation for receptance representation of the system (1) is given in frequency domain by taking Laplace Transform:

$$\mathbf{X}(s) = \mathbf{H}(s)\mathbf{B}\mathbf{U}(s) \tag{9}$$

where $\mathbf{H}(s) = \left(\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K}\right)^{-1}$ is the open-loop receptance. A remarkable advantage of some methods based upon receptance is the need of only measured data to construct it, avoiding system matrices, in general, affected by uncertainties. See Tehrani and J. Mottershead (2012) and references therein. Other works consider hybrid approaches with repentance and system matrices, as that of Bai, Yang, and B. N. Datta (2016). Those works explore the closed-loop receptances as a function of the open-loop one by the Shermann-Morrison formula, for state feedback and state derivative feedback. By replacing (7) in (1) and taking again Laplace transform, the Shermman-Morrison formula results in:

$$\mathbf{H}_{cls}(s) = \mathbf{H}(s) - \frac{\mathbf{H}(s)\mathbf{B}(\mathbf{g}_s + \mathbf{f}_s s)\mathbf{H}(s)}{1 + (\mathbf{g}_s + \mathbf{f}_s s)\mathbf{H}(s)\mathbf{B}} \tag{10}$$

Following the same steps with (8) being replaced in (1), one obtains:

$$\mathbf{H}_{cld}(s) = \mathbf{H}(s) - \frac{\mathbf{H}(s)\mathbf{B}(\mathbf{g}_d s^2 + \mathbf{f}_d s)\mathbf{H}(s)}{1 + (\mathbf{g}_d s^2 + \mathbf{f}_d s)\mathbf{H}(s)\mathbf{B}} \tag{11}$$

In case of asymmetric systems, the receptance measurement can be more involved, since that the tests to be conducted experimentally require more sophisticate apparatus. As an instance, to the measurement of full receptance in aircraft engines, wind tunnel must be employed. In its hand, the structural receptance for the aircraft is based in symmetric part of the model, and its measurement is easy and simpler than the former. In a recent paper (Liang, H. J. Ouyang, and Yamaura 2016), the knowledge of structural receptance, that of the symmetric part of the model, and also of the symmetric and asymmetric part of the system matrices have been applied to solve the problem of partial eigenvalue assignment in single input control systems. In that work, asymmetry is considered in the damping and stiffness the characteristic equation of the full receptance for the closed-loop system is computed by using:

$$det\left[\mathbf{I} + \mathbf{H}_{cl}(s)\left(\mathbf{C}_{as}s + \mathbf{K}_{as}\right)\right] = 0 \tag{12}$$

where $\mathbf{H}_{cl}$ is given as (10) or (11); $\mathbf{C}_{as}$ and $\mathbf{K}_{as}$ are the asymmetric components of the damping and stiffness matrices, according with:

$$\mathbf{C} = \mathbf{C}_s + \mathbf{C}_{as} \tag{13}$$

$$\mathbf{K} = \mathbf{K}_s + \mathbf{K}_{as} \tag{14}$$

Notice that in general, the mass (or inertia) matrix $\mathbf{M}$ not contains asymmetric components. Equation (12) is then imposed to assign some closed-loop eigenvalues in prescribed locations. To assure that only the unwanted eigenvalues will be reassigned, an unobservability condition

(B. N. Datta, Elhay, and Ram 1997) is imposed to the set of eigenvalues $\mathfrak{N}$ that must be unchanged, by setting the constraint:

$$\mathbf{U}\Gamma = 0 \tag{15}$$

in which

$$\Gamma = \begin{bmatrix} \mathbf{g}_s \\ \mathbf{f}_s \end{bmatrix}$$

in the design of state feedback (7) or

$$\Gamma = \begin{bmatrix} \mathbf{g}_d \\ \mathbf{f}_d \end{bmatrix}$$

in case of derivative feedback (8). The matrix $\mathbf{U}$ is a real representation for the matrix whose rows is composed by the vectors $\begin{bmatrix} \mathbf{y}_j^T & \lambda_j \mathbf{y}_j^T \end{bmatrix}$ in design of (7) or $\begin{bmatrix} \lambda_j^2 \mathbf{y}_j^T & \lambda_j \mathbf{y}_j^T \end{bmatrix}$ for design (8), with $j \in \mathfrak{N}$.

Partial eigenvalue assignment according with (Liang, H. J. Ouyang, and Yamaura 2016) is conducted by prescribing the set of all new eigenvalues, and then there is no degree-of-freedom to attain other performance indexes. Following up, the problem to be solved is stated.

2.3 Statement of the Problem

Given the system (1) with asymmetric matrices of damping and stiffness, compute the feedback matrices $\mathbf{g}_s,\ \mathbf{f}_s$ - or $\mathbf{g}_d,\ \mathbf{f}_d$, such that the set of eigenvalues $\mathfrak{N}$ remain unchanged and a performance index - as robustness or minimum norm - be minimized.

To solve this problem, the unwanted eigenvalues cannot be all assigned to a prescribed position. In the solution proposed in the next section, a degree-of-freedom to achieve the solution is detailed.

3 THE APPROACH

Systems with asymmetric damping and stiffness matrices often exhibit some unstable eigenpairs, and, additionally, some ill damped ones. Let this set of unwanted eigenpairs with $2k$ elements, closed under complex conjugation, says, $\lambda_1, \bar{\lambda}_1 ... \lambda_k, \bar{\lambda}_k$, and a set of new locations to exactly $2k-1$ of them, given by $\mu_1, \mu_2, ..., \mu_{2k-1}$. Define a performance index $J(\mathbf{g}, \mathbf{f})$, e.g., for robustness or minimum norm of the feedback matrices. The proposed approach to solve the partial eigenvalues assignment is then given as:

$$\min_{\mathbf{g},\mathbf{f}} J(\mathbf{g}, \mathbf{f})$$
$$s.t. \begin{cases} \det\left[\mathbf{I} + \mathbf{H}_{cl}(\mu_j)\left(\mathbf{C}_{as}\mu_j + \mathbf{K}_{as}\right)\right] = 0,\ j = 1, .., 2k-1 \\ \mathbf{U}\begin{bmatrix} \mathbf{g} \\ \mathbf{f} \end{bmatrix} = 0 \\ \det(\mathbf{M}_{cl}^{-1}\mathbf{K}_{cl}) > 0 \end{cases}, \tag{16}$$

in which $\mathbf{M}_{cl}$ and $\mathbf{K}_{cl}$ are the closed-loop mass and stiffness matrices respectively. For state feedback, one has $\mathbf{M}_{cl} \equiv \mathbf{M}$ and $\mathbf{K}_{cl} \equiv \mathbf{K} + \mathbf{B}\mathbf{g}_s$, while for derivative feedback, they result $\mathbf{M}_{cl} \equiv \mathbf{M} + \mathbf{B}\mathbf{g}_d$ and $\mathbf{K}_{cl} \equiv \mathbf{K}$ This optimization problem will be nonlinear in general due to the first and third constraint. Some particular cases, in which the asymmetric components $\mathbf{C}_{as}$ and $\mathbf{K}_{as}$ are sparse matrices, the fisrt constraint can become linear. The third constraint acts as a penalty term for the position of the free eigenvalue. Since that the product of all eigenvalues obeys:

$$\prod \lambda_k = \det(\mathbf{M}_{cl}^{-1}\mathbf{K}_{cl}) \tag{17}$$

it assures then that the free eigenvalue lies in the complex LHP.

4 NUMERICAL EXPERIMENTS

In order to illustrate the proposed methodology to solve the stabilization with additional performance in asymmetric second-order system, two real world-based examples is then detailed in the next subsections.

4.1 Wing in an Airstream - The Flutter Phenomenon

In this example, we take the matrices from a 2×2 study on aeroelastic flutter of a wing in an air-stream, that can be seen in classical book (Frazer, Duncan, and Collar 1938):

$$\mathbf{M} = \begin{bmatrix} 44.7000 & -1.1500 \\ -1.1500 & 0.7450 \end{bmatrix}, \quad \mathbf{C}_s = 0, \quad \mathbf{K}_s = \begin{bmatrix} 3.3700e+04 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_{as} = \begin{bmatrix} 460.2000 & -48.3600 \\ 10.6600 & 8.8400 \end{bmatrix}, \quad \mathbf{K}_{as} = \begin{bmatrix} 0 & -6.8276e+03 \\ 0 & 0.2420e+03 \end{bmatrix}$$

The open-loop eigenvalues are $0.6868 \pm 25.9749i$, $-11.5477 \pm 15.6328i$. It is evident that the first eigenpair is unstable, and a crescent amplitude, dangerous oscillation then can occur. Now, considering the influence matrix:

$$\mathbf{B} = 100 \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

the first eigenpair is to be assigned in new position. One eigenvalue is setting to target -1, while the other is kept free under the third constraint of (16). The unobservability condition for the second constraint is attained as $\mathbf{U}_s$ for state feedback and $\mathbf{U}_d$ for derivative feedback:

$$\mathbf{U}_s = \begin{bmatrix} -0.1833 & -0.4292. & -0.2616 & -8.6617 \\ -0.1522 & -0.8712 & 4.6233 & 16.7709 \end{bmatrix},$$

$$\mathbf{U}_d = \begin{bmatrix} 75.2964 & 362.1993 & -0.2616 & -8.6618 \\ -49.2990 & -58.2564 & 4.6233 & 16.7709 \end{bmatrix}.$$

Table 1: Closed-loop comparison in the Example 4.1

Feedback Matrices	
State Feedback	Derivative Feedback
$\mathbf{f}_s^T = \begin{bmatrix}1.3432 & -0.2603\end{bmatrix}$ $\mathbf{g}_s^T = \begin{bmatrix}9.1773 & 0.5142\end{bmatrix}$	$\mathbf{f}_d^T = \begin{bmatrix}23.3888 & -7.0694\end{bmatrix}$ $\mathbf{g}_d^T = \begin{bmatrix}-0.0420 & -0.1434\end{bmatrix}$
Closed-loop eigenvalues	
State Feedback	Derivative Feedback
$-1,\ -31.3745,\ \ -11.5477 \pm 15.6328i$	$-1,\ -32.7953,\ \ -11.5477 \pm 15.6328i$
Sensitivity Matrices Norms	
State Feedback	Derivative Feedback
$\|\|\mathbf{S}_{\mathbf{K}s}\|\|_F = 1.1034e+03$	$\|\|\mathbf{S}_{\mathbf{K}d}\|\|_F = 52.2291$
$\|\|\mathbf{S}_{\mathbf{C}s}\|\|_F = 1.3989$	$\|\|\mathbf{S}_{\mathbf{C}s}\|\|_F = 0.0699$

Finally, the performance index is setting as the condition number of closed-loop eigenvectors of the equivalent first order system (B. Datta 2003):

$$J(\mathbf{g}, \mathbf{f}) = \kappa_2\left(\mathbf{Y}_{cl}\right),$$

with

$$\mathbf{Y}_{cl} = \begin{bmatrix} \mathbf{y}_{cl}^{(1)} & \mathbf{y}_{cl}^{(2)} & \cdots & \mathbf{y}_{cl}^{(2n)} \\ \lambda_{cl}^{(1)}\mathbf{y}_{cl}^{(1)} & \lambda_{cl}^{(2)}\mathbf{y}_{cl}^{(2)} & \cdots & \lambda_{cl}^{(2n)}\mathbf{y}_{cl}^{(2n)} \end{bmatrix}$$

Taking these into account to solve the problem (16) leads to the results in the Table 1. For the sake of comparison of the two feedback strategies, the spectral sensitivities to uncertainties on damping and stiffness matrices were computed according with (Araújo et al. 2016):

$$\mathbf{S}_{\mathbf{K}s} = \frac{\det\ (\mathbf{K} + \mathbf{B}\mathbf{g}_s)}{\det\ \mathbf{M}}(\mathbf{K} + \mathbf{B}\mathbf{g}_s)^{-T}$$

$$\mathbf{S}_{\mathbf{K}d} = \frac{\det\ \mathbf{K}}{\det\ (\mathbf{M} + \mathbf{B}\mathbf{g}_d)}\mathbf{K}^{-T}$$

$$\mathbf{S}_{\mathbf{C}s} = -\mathbf{M}^{-T}$$

$$\mathbf{S}_{\mathbf{C}d} = -(\mathbf{M} + \mathbf{B}\mathbf{g}_d)^{-T}$$

For both feedbacks employed, the condition number corresponding to solution of (16) gives $\kappa_2\left(\mathbf{Y}_{cl}\right) \approx 728$. However, the robustness performance of derivative feedback can be claimed to be better, since that the sensitivities against uncertainties on damping and stiffness matrices are very lower in comparison with state feedback design. A Monte-Carlo simulation with 10,000 points considering uncertainties of 0.1% on $\mathbf{K}$ and 1% on $\mathbf{C}$ (on norms) is performed to validate this conclusion. The results in Figure 1 confirms that claim. Although the same condition number achieved to both the designs, derivative feedback presents a better performance.

4.2 Friction-Induced Vibration in a Belt-Conveyor System

In this experiment, the 4×4 model presented in (Liang, H. J. Ouyang, and Yamaura 2016; H. Ouyang 2010) is approached. The system matrices represent a friction induced vibration model of a belt conveyor system with a contact point, and they are given as:

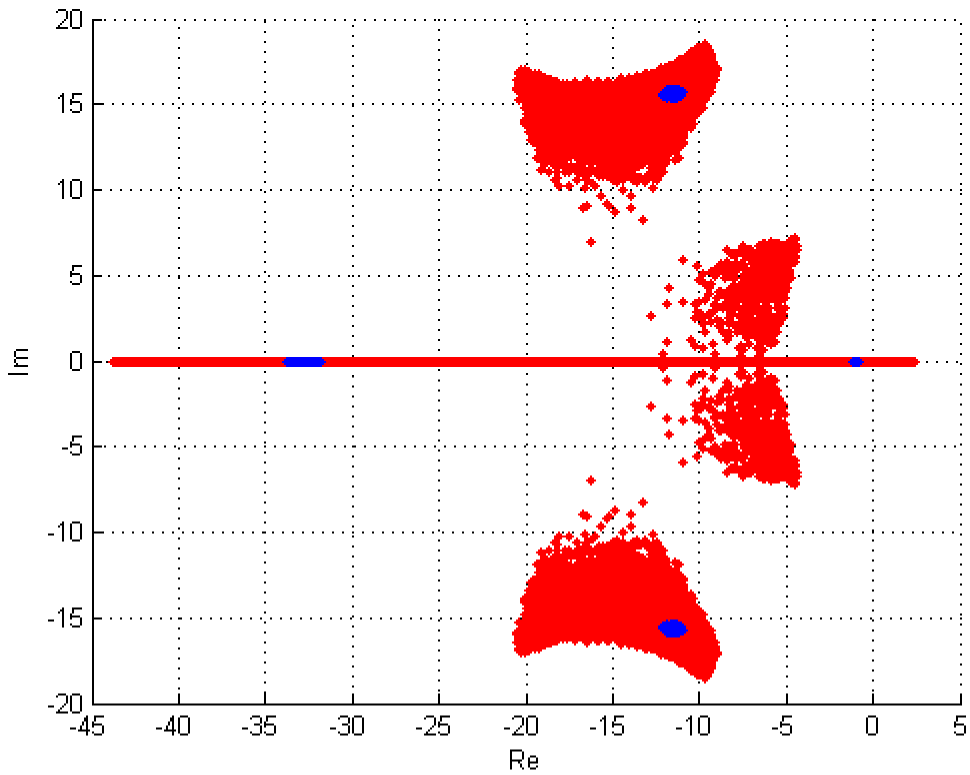

Figure 1: Eigenvalue locus for the wing in an air stream with perturbation of the damping and stiffness matrices in the Example 4.1: state feedback (red points) and state derivative feedback (blue points).

$$
\mathbf{M} = \mathbf{I}_4, \quad \mathbf{C}_s = \begin{bmatrix} 0.1 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \quad \mathbf{K}_s = \begin{bmatrix} 200 & 0 & -100 & 0 \\ 0 & 200 & 0 & -100 \\ -100 & 0 & 150 & -50 \\ 0 & -100 & -50 & 350 \end{bmatrix},
$$

$$
\mathbf{C}_{as} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.05 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{K}_{as} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

The open-loop eigenpairs are $0.0028 \pm 8.9464i$, $-0.0538 \pm 12.1336i$, $-0.1702 \pm 19.7343i$ and $-0.5288 \pm 16.8222i$. The first eigenpair is obviously unstable, as well as the second one is poorly damped, and closing of the imaginary axis, and then will be reassigned by the proposed feedback design. The other must remain unchanged. The target location to the new position of a single eigenvalue is -0.5, and the other eigenpair will be reassigned to the positions $-0.5 \pm 12.1336i$; the fourth eigenvalue is kept free under the third constraint in (16). The remaining

Table 2: Closed-loop comparison in the Example 4.2

Feedback Matrices	
State Feedback	Derivative Feedback
$\mathbf{f}_s^T = \begin{bmatrix} 6.2290 & 2.8309 & 8.3903 & 0.0627 \end{bmatrix}$	$\mathbf{f}_d^T = \begin{bmatrix} 138.8731 & 64.1231 & 192.2721 & -2.8726 \end{bmatrix}$
$\mathbf{g}_s^T = \begin{bmatrix} -109.1061 & 63.9078 & -120.1432 & 52.1052 \end{bmatrix}$	$\mathbf{g}_d^T = \begin{bmatrix} 29.1559 & -13.2112 & 33.5978 & -11.9465 \end{bmatrix}$
Closed-loop eigenvalues	
State Feedback	Derivative Feedback
$-0.5,\ -7.0551,\ -0.5 \pm 12.1336i,$	$-0.5,\ -7.0551,\ -0.5 \pm 12.1336i,$
$-0.5288 \pm 16.8222i,\ \ -0.1702 \pm 19.7343i$	$-0.5288 \pm 16.8222i,\ \ -0.1702 \pm 19.7343i$
Sensitivity Matrices Norms	
State Feedback	Derivative Feedback
$\|\|\mathbf{S}_{\mathbf{K}s}\|\| = 2.8193e+07$	$\|\|\mathbf{S}_{\mathbf{K}d}\|\| = 8.8185e+05$
$\|\|\mathbf{S}_{\mathbf{C}s}\|\| = 2$	$\|\|\mathbf{S}_{\mathbf{C}d}\|\| = 3.3224$

spectrum must be kept unperturbed. Using the given matrices and the goals of the design, the first constraint in (16), for both state feedback and derivative feedback can be written in the following linear form:

$$\begin{bmatrix} 0.0038 & 0.0023 & 0.0077 & 0.0046 & -0.0019 & -0.0011 & -0.0038 & -0.0023 \\ 0.0091 & 0.0304 & 0.0055 & 0.0164 & -0.1034 & -0.0455 & -0.0443 & 0.0205 \\ -0.0081 & -0.0025 & -0.0034 & 0.0024 & -0.1064 & -0.3677 & -0.0653 & -0.2005 \end{bmatrix} \begin{bmatrix} \mathbf{g} \\ \mathbf{f} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

The matrix of the second equality constraint in state feedback case is given as:

$$\mathbf{U}_s = \begin{bmatrix} 0.1161 & 0.4508 & -0.2246 & -0.8533 & -0.6503 & -0.2695 & 0.7242 & 1.1079 \\ -0.0320 & -0.0098 & 0.0348 & 0.0488 & -2.2849 & -8.8955 & 4.4256 & 16.8311 \\ -0.0474 & -0.0122 & 0.0219 & -0.0249 & 12.4800 & 3.3125 & -10.4166 & -2.7576 \\ 0.7404 & 0.1965 & -0.6185 & -0.1647 & 0.4052 & 0.1012 & -0.0418 & 0.5056 \end{bmatrix},$$

while for derivative feedback results in:

$$\mathbf{U}_d = \begin{bmatrix} -44.9808 & -175.5006 & 87.2127 & 331.9606 & -0.6503 & -0.2695 & 0.7242 & 1.1079 \\ 13.2219 & 6.8314 & -15.0456 & -24.7275 & -2.2849 & -8.8955 & 4.4256 & 16.8311 \\ 0.2168 & -0.0495 & 4.8047 & 9.9641 & 12.4800 & 3.3125 & -10.4166 & -2.7576 \\ -210.1544 & -55.7763 & 175.2522 & 46.1211 & 0.4052 & 0.1012 & -0.0418 & 0.5056 \end{bmatrix}$$

The results for the problem (16) considering the same cost function as in example 4.1 can be seen in Table 2. Again, the results for the two feedback is consistent, with the prescribed eigenvalues being correctly allocated. A better robustness is seen for derivative feedback by analyzing the computed sensitivities. Figure 3 illustrates this conclusion when a more concentrated cloud for derivative feedback is obtained when the matrices of stiffness and damping are perturbed in norm by 1% and 10% respectively.

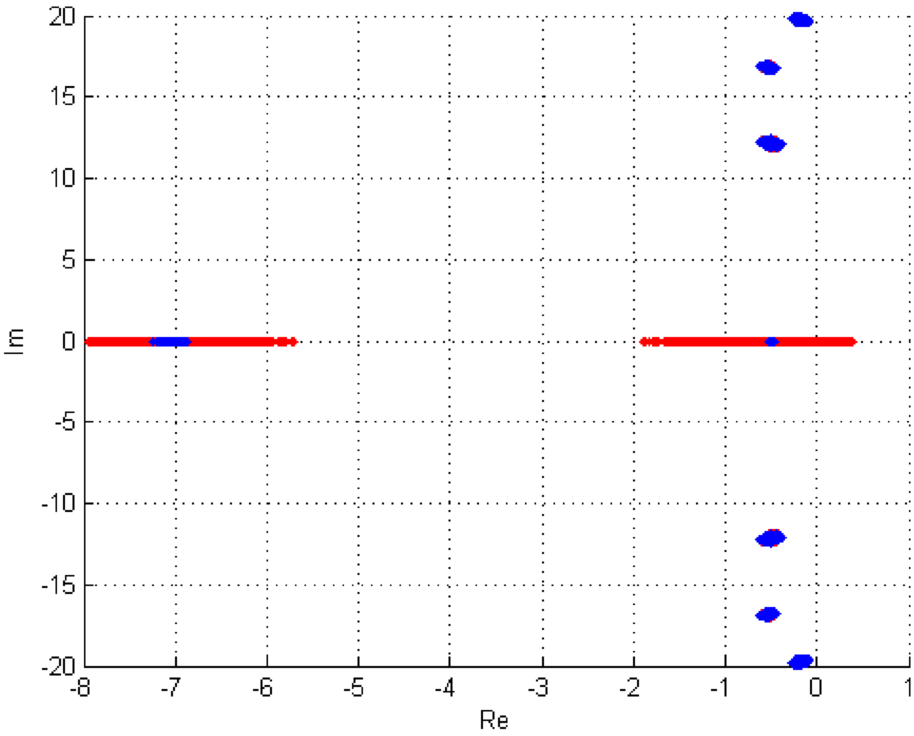

Figure 2: Eigenvalue locus for the friction-induced vibration with perturbation of the damping and stiffness matrices in the Example 4.2: state feedback (red points) and state derivative feedback (blue points).

5 CONCLUDING REMARKS

We show in this paper a methodology to stabilization of second-order linear systems with asymmetric matrices of damping and stiffness. Such matrices rise in systems with nonconservative forces and lead to unstable or dangerous vibrations. The method is an improvement of that in which the structural receptances of the symmetric part of the systems are used to assign new safe positions for the undesired eigenvalues, while the other part of the spectrum is kept unchanged. It's enough to let one single eigenvalue free in the complex LHP and thus assign, as an instance, robustness in the design for the optimal feedback matrices. Two numerical examples using real world situations are given, and the results confirm the potential of the proposed approach.

Future works include the use of other possibilities of turns up the eigenvalue assignment problem flexible regarding the feedback matrices, and also include other performance requisites in the cost function.

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