



Numerical Simulation of a Single Bubble Dynamic in Nucleate Pool Boiling by using Finite Difference Method

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Abstract. Heat transfer with phase change is a very effective way to remove high rate heat flux from processes. In this way, nucleate pool boiling is used, for example, to refrigerate heated surfaces, when a fluid boils on the surface. High heat fluxes and high heat transfer coefficient are characteristics of pool boiling. The mechanism of pool boiling is very complex and motivates the interest of many researchers in understanding this process. The nucleation, growth and departure of bubbles from a heated surface have deserved attention and there are many papers in the literature concerning experimental, empirical or semi-empirical and numerical solutions to the problem. Experimental apparatus, generally, is expensive and several other difficulties are associated with this kind of solution. In the last decades, numerical solutions of bubble dynamics have appeared in the literature. Bubble dynamic involves moving contours which difficult the solution of the problem in predicting the form of bubble volume and the velocity of the bubble contour. For simplification, generally, the bubble is considered as spherical or hemispherical. In this work, an analysis is proposed to simulate the growth of a single bubble on a horizontal surface in which the temperature is changing in time. In many works, the temperature of the heater surface is considered constant in time. Considering a hemispherical bubble, the Navier-Stokes equations, in spherical coordinates, are the mathematical model to the problem and they are solved numerically by a finite difference method. Due to the variable temperature field on the surface, the equation of heat conduction in the heater is also considered. The microlayer underneath the bubble is not taking into account in the present model. A first approach considered is a correlation from

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the literature to predict the velocity of the vapor bubble contour and the equations for the velocity and the temperature fields are solved for the adjacent liquid until the time of the bubble departure. After discretization, the algebraic equations are solved and the obtained results are in good agreement with results from the literature. It is observed that is important to consider the effect of the wall-temperature distribution in the numerical simulations of the vapor bubble dynamic.

Keywords: Nucleate pool boiling, bubble dynamics, Navier-Stokes equations, finite difference method.

1 INTRODUCTION

Heat exchangers are used in a wide range of industrial applications including the oil and gas industry, power generation, chemical processing and others. As more refrigerating/heating power is needed, the heat exchangers with monophasic flows became inappropriate. One solution is to use heat exchangers operating with phase change flows, improving the heat transfer performance.

Hal et al. (2017), Quan, Wang e Cheng (2017) and Manetti, Cardoso and Nascimento (2017) are examples of works in which the pool boiling phenomenon was studied to predict the amount of heat that the working fluid can remove from the heated surface. This information is important to better understand the mechanisms responsible for the high heat transfer coefficients observed in this processes and also, to predict the upper-most heat flux limit for the nucleate boiling regime termed critical heat flux (CHF).

According to Mistrya (2014), boiling a fluid on the surface of a heat dissipating device occurs when the surface temperature exceeds the saturation temperature of the fluid. Vapor bubbles will growth and departure from the heated surface to the bulk liquid. Mcfadden and Grassmann (1961) reported that the bubble frequency departure increases proportionally with heat flux applied. As heat flux increases the number of vapor bubbles per unit surface area also increase. The coalesced vapor bubbles on the heating surface form a film that thermally insulates the surface, inhibiting liquid access to the heated surface. The heat transfer is interrupted and the temperature of the heater grows uncontrollably. This condition is the upper heat flux limit for the nucleate boiling regime called *critical heat flux* (CHF).

Among the great number of existing empirical and semi-empirical correlations to predict the boiling heat transfer, Rohsenow (1952) continue to be employed by several research groups and are presented as follows (Carey, 1992):

$$h_{Rohsenow} = \mu_l h_{lv} \left(\frac{g(\rho_l - \rho_v)}{\sigma} \right)^{0.5} \left(\frac{c_{pl}}{C_{sf} h_{lv} Pr_l^s} \right)^3 \Delta T_w^2 \quad (1)$$

where ρ_l , c_{pl} , Pr_l represent the viscosity, the specific heat and the Prandtl number of the liquid. ΔT_w represents the superheating of the heating wall, $\Delta T_w = (T_w - T_{sat})$, and C_{sf} is dependent on the material of the heating wall, the surface roughness and the working fluid and $s = 1.7$ (Carey, 1992).

Nukiyama (1934) was the pioneer in getting of the pool boiling curve shown in Figure 1.

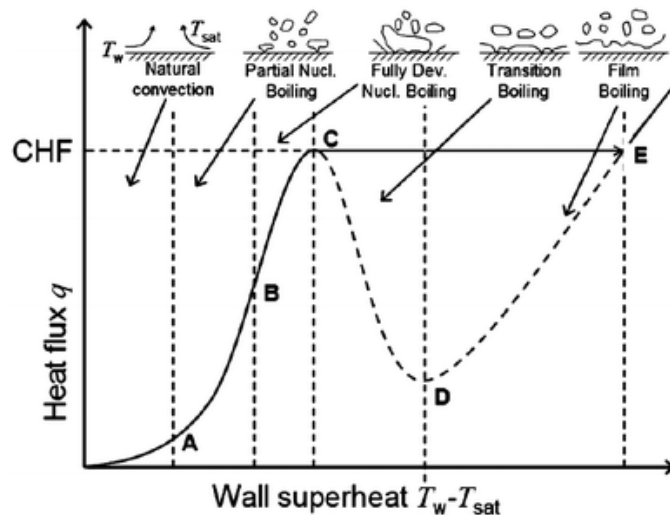


Figure 1. Pool boiling curve (Carey, 1992).

Thus, it is important to clarify the mechanisms responsible for the vapor bubble nucleation, growth and departure, and also, the effect of the interaction between heated surface/working fluid on the boiling heat transfer.

Bubble growth rate is a complex phenomenon that is extremely hard to predict by analytical and exact solution. Therefore, Son and Dir (2007), Sattari et. al. (2014) and Dhir, Warrier and Aktinol (2013) tried to understand the pool boiling phenomenon using computational resources, however they considered the temperature of the heated surface constant in whole process.

Guo and El-Genk (1994) showed, in his numerical studies, that the heated wall thickness influences the vapor bubble growth. Kunkelmann and Stephan (2010) performed a numerical simulation considering the transient condition on the heated surface, but they use empirical correlations to predict the bubble waiting time. Aktinol e Dhir (2014) showed that the temperature of the heated surface depends on the time and position.

This work aims to develop a numerical model of a single vapor bubble dynamic, considering time dependent temperature condition of the heated surface in order to predict the temperature field in the liquid adjacent to the vapor bubble.

2 NUMERICAL MODEL

The numerical model presented in this article is based on second generation formulation from Patil (1991). In that formulation, it is considered:

- laminar flow;
- the flow is considered incompressible despite the phase change process;
- there are not gases dissolved in the liquid;
- only one active nucleation site;
- the vapor bubble is hemispherical;

- thermodynamics properties is considered constant.

The geometry of the problem is shown in Figure 2. This region represents the bubble phase change layer with thickness δ . The domain studied is restricted by the vapor bubble radius $\bar{R}$ and the sum $\bar{R} + \delta$, in r direction; and ϕ angle from zero to $\pi/2$.

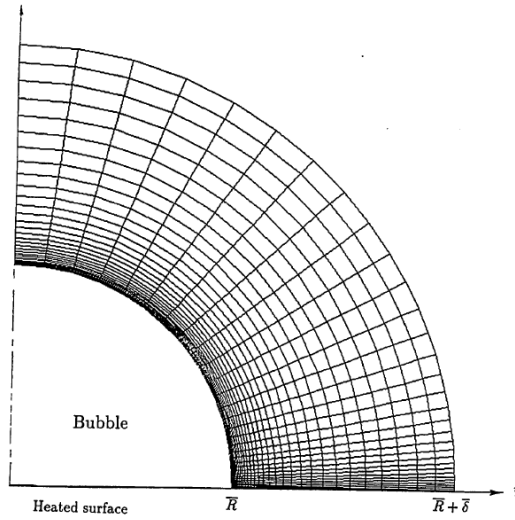


Fig 2. Domain of the fluid adjacent to the vapor bubble (Patil, 1991).

The mathematical formulation to the fluid is the Navier-Stokes equations, in spherical coordinates, as follow:

- continuity equation

$$\frac{1}{\bar{r}^2} \frac{\partial(\bar{r}^2 \bar{u})}{\partial \bar{r}} + \frac{1}{\bar{r} \sin \theta} \frac{\partial(\bar{v} \sin \theta)}{\partial \theta} = 0 \quad (2)$$

- momentum equations

$$\bar{\rho} \left[\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} \frac{\partial \bar{u}}{\partial \theta} - \frac{\bar{v}^2}{\bar{r}} \right] = - \frac{\partial \bar{p}}{\partial \bar{r}} + \bar{\mu} \left(\nabla^2 \bar{u} - \frac{2\bar{u}}{\bar{r}^2} - \frac{2}{\bar{r}^2} \frac{\partial \bar{v}}{\partial \theta} - \frac{2\bar{v} \cot \theta}{\bar{r}^2} \right) \quad (3)$$

$$\bar{\rho} \left[\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} \frac{\partial \bar{v}}{\partial \theta} - \frac{\bar{u}\bar{v}}{\bar{r}} \right] = - \frac{1}{\bar{r}} \frac{\partial \bar{p}}{\partial \theta} + \bar{\mu} \left(\nabla^2 \bar{v} + \frac{2}{\bar{r}^2} \frac{\partial \bar{u}}{\partial \theta} - \frac{\bar{v}}{\bar{r}^2 \sin^2 \theta} \right) \quad (4)$$

- thermal energy equation

$$\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} \frac{\partial \bar{T}}{\partial \theta} = \alpha \nabla^2 \bar{T} \quad (5)$$

where: $\bar{p}$ is the pressure, $\bar{u}$ and $\bar{v}$ are the velocity components in $\bar{r}$ and θ coordinates respectively, and $\bar{T}$ is the temperature field. The Laplacian operator is $\nabla^2 = \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left(\bar{r}^2 \frac{\partial}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right)$. An upper bar indicates dimensional variables.

The pressure field is obtained by the solution of the Poisson equation:

$$\nabla^2 \bar{p} = -\bar{\rho} \left\{ \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left[\bar{r}^2 \left(\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} \frac{\partial \bar{u}}{\partial \theta} - \frac{\bar{v}^2}{\bar{r}} \right) \right] + \frac{1}{\bar{r} \sin \theta} \frac{\partial}{\partial \theta} \left[\sin \theta \left(\bar{u} \frac{\partial \bar{v}}{\partial \bar{r}} + \frac{\bar{v}}{\bar{r}} \frac{\partial \bar{v}}{\partial \theta} + \frac{\bar{u}\bar{v}}{\bar{r}} \right) \right] \right\} \quad (6)$$

The initial and boundary conditions for solving Equations (2) – (6) are

$$\begin{aligned} \bar{v} = \frac{\partial \bar{u}}{\partial \theta} &= 0, \text{ at } \theta = 0 \\ \bar{v} = \bar{u} &= 0, \text{ at } \theta = \pi / 2 \\ \bar{u} = \bar{u}_b, \bar{v} &= 0, \text{ at } \bar{r} = \bar{R} \\ \bar{v} = \bar{u} &= 0, \text{ at } \bar{r} = \bar{R} + \bar{\delta} \\ \bar{v} = \bar{u} &= 0, \text{ at } \bar{t} = 0 \end{aligned} \quad (7)$$

$$\begin{aligned} \bar{p} &= p_{\text{sat}}, \text{ at } \bar{r} = \bar{R}, \bar{r} = \bar{R} + \bar{\delta} \text{ and } \theta = \pi / 2 \\ \frac{\partial \bar{p}}{\partial \theta} &= 0, \text{ at } \theta = 0 \end{aligned} \quad (8)$$

$$\begin{aligned} \bar{T} &= T_v, \text{ at } \bar{r} = \bar{R} \\ \bar{T} &= T_\infty, \text{ at } \bar{r} = \bar{R} + \bar{\delta} \\ \bar{T} &= T_s, \text{ at } \theta = \pi / 2 \\ \frac{\partial \bar{T}}{\partial \theta} &= 0, \text{ at } \theta = 0 \end{aligned} \quad (9)$$

The vapor bubble radius is estimated as shown in Eq. (10a), (Carrey,1992):

$$R(t) = \frac{2Ja\sqrt{3\pi\alpha_l t}}{\pi} \left\{ 1 - \frac{T_w - T_\infty}{T_w - T_{\text{sat}}} \left[\left(1 + \frac{t_w}{t} \right)^{1/2} - \left(\frac{t_w}{t} \right)^{1/2} \right] \right\} \quad (10a)$$

where Ja is the Jakob number and t_w is the waiting time given by:

$$t_w = \frac{1}{4\alpha_l} \left\{ \frac{r_c}{\operatorname{erfc}^{-1} \left[\frac{T_{sat} - T_\infty}{T_w - T_\infty} + \frac{2\sigma T_{sat} (v_v - v_l)}{(T_w - T_{inf}) h_{fg} r_c} \right]} \right\}^2 \quad (10b)$$

The equation for the temperature field in the solid heater is presented below:

$$\frac{\partial^2 T_s}{\partial z^2} = \frac{1}{\alpha_s} \frac{\partial T_s}{\partial t} \quad (11)$$

with the boundary and initial conditions as follow in Eq.(12):

$$\begin{aligned} T_s(z, 0) &= T_{s,0} \\ -k_s \frac{\partial T_s}{\partial z}(0, t) &= q'' \\ k_s \nabla T(H, t) &= k_f \nabla \bar{T} \end{aligned} \quad (12)$$

where k_s and k_f are the thermal conductivities of surface and fluid, respectively, and q'' is the heat flux applied in the solid heater surface.

The model formulation is solved by using the Finite Difference Method. However, to become easier to solve, we introduce some dimensionless variables presented in Eq. (13):

$$\begin{aligned} r &= \frac{\bar{r} - \bar{R}}{\bar{\delta}}; R = \frac{\bar{R}}{R_0}; t = \frac{\alpha \bar{t}}{R_0^2}; \gamma = \delta r + R \\ u_i &= \frac{R_0 \bar{u}_i}{\alpha}; p = \frac{\bar{p} - p_\infty}{\rho \left(\frac{\alpha}{R_0} \right)}; T = \frac{\bar{T} - T_{sat}}{T_{s,0} - T_{sat}} \end{aligned} \quad (13)$$

With this dimensionless radius, the domain of the problem is fixed and the influence of the moving boundary is included in the convective acceleration. The resulting form of the non-dimensionalized equations is:

$$\frac{\partial^2 p}{\partial r^2} + \frac{\delta^2}{\gamma^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{2\delta}{\gamma} \frac{\partial p}{\partial r} + \frac{\delta^2}{\gamma^2} \cot \theta \frac{\partial p}{\partial \theta} = S_p \quad (14a)$$

$$S_p = - \left\{ \frac{1}{\gamma^2} \frac{\partial}{\partial r} \left[\gamma^2 \left(u \frac{\partial u}{\partial r} + \frac{\delta}{\gamma} v \frac{\partial u}{\partial \theta} - \frac{\delta}{\gamma} v^2 \right) \right] + \right. \\ \left. + \frac{\delta}{\gamma} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left[\sin \theta \left(u \frac{\partial v}{\partial r} + \frac{\delta}{\gamma} v \frac{\partial v}{\partial \theta} + \frac{\delta}{\gamma} uv \right) \right] \right\} \quad (14b)$$

$$\delta^2 \frac{\partial u}{\partial t} - U \frac{\partial u}{\partial r} - V \frac{\partial u}{\partial \theta} - Pr \frac{\partial^2 u}{\partial r^2} - Pr \frac{\delta^2}{\gamma^2} \frac{\partial^2 u}{\partial \theta^2} = -\delta \frac{\partial p}{\partial r} + S_u \quad (15a)$$

$$U = \delta \left[\left(\frac{d\delta}{dt} r + \frac{dR}{dt} \right) + \frac{2Pr}{\gamma} - u \right] ; V = \frac{\delta^2}{\gamma^2} [Pr \cot \theta - \gamma v] \quad (15b, c)$$

$$S_u = \frac{\delta^2}{\gamma^2} \left[\gamma v^2 - Pr \left(2u + 2 \frac{\partial v}{\partial \theta} + v \cot \theta \right) \right] \quad (15d)$$

$$\delta^2 \frac{\partial v}{\partial t} - U \frac{\partial v}{\partial r} - V \frac{\partial v}{\partial \theta} - Pr \frac{\partial^2 v}{\partial r^2} - Pr \frac{\delta^2}{\gamma^2} \frac{\partial^2 v}{\partial \theta^2} = -\frac{\delta^2}{\gamma} \frac{\partial p}{\partial \theta} + S_v \quad (16a)$$

$$S_v = \frac{\delta^2}{\gamma^2} \left[Pr \left(2 \frac{\partial u}{\partial \theta} - \frac{v}{\sin^2 \theta} \right) - \gamma uv \right] \quad (16b)$$

$$\delta^2 \frac{\partial T}{\partial t} - U^* \frac{\partial T}{\partial r} - V^* \frac{\partial T}{\partial \theta} - \frac{\partial^2 T}{\partial r^2} - \frac{\delta^2}{\gamma^2} \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (17a)$$

$$U^* = \delta \left[\left(\frac{dR}{dt} + r \frac{d\delta}{dt} \right) + \frac{2}{\gamma} - u \right] ; V^* = \frac{\delta^2}{\gamma^2} (\cot \theta - \gamma v) \quad (17b, c)$$

After the non-dimensionalization of the equations, the discretization is done by Finite Difference Method. The first and second order derivate is approximated as shown in Eq.(18) and (19):

$$\frac{\partial \zeta}{\partial x_i} \approx \frac{\zeta^{x_i+h} - \zeta^{x_i}}{\Delta x_i} \quad (18)$$

$$\frac{\partial^2 \zeta}{\partial x_i^2} \approx \frac{\zeta^{x_i+h} - 2\zeta^{x_i} + \zeta^{x_i-h}}{(\Delta x_i)^2} \quad (19)$$

where ζ is a generic variable, x_i a generic coordinate and h a variation in x_i direction.

After that, using the discretization of the non-dimensional equations, the resulting system is solved by the Gauss-Seidel Method.

3 RESULTS

In this section, we present some results from the simulations. The conditions to run the problem are set as following:

- Water is used as a model liquid
- Saturation temperature: 99°C
- Saturation pressure: 97.85kPa
- Superheating wall degree: 8°C
- Fluid initial temperature: 99°C
- Wall initial temperature: 107°C
- Wall thickness: 2cm
- The heated surface is assumed to be plain copper
- Mesh length (radius, angular, wall thickness and time, respectively): 70 x 70 x 100 x 86
- Fluid at saturation state, in which state physical the proprieties are obtained.

A solver has been developed in Fortran 90 language to simulate the velocity, pressure and the temperature fields in the region corresponding to liquid adjacent to the bubble, as shown in the Figure 2.

In Figures 3 to 6 are shown the results of the simulations. The temperature at interface heated surface-liquid is considered time dependent, since the solution to the temperature field in the heater is also obtained and imposed as boundary condition for solution of the temperature in liquid layer adjacent to the vapor bubble.

Figure 3 and 4 show the u and v components of velocity field, respectively. One may observe that velocities in liquid layer are low. Figure 5 shows the pressure field, while in Figure 6 is shown the temperature field. A simulation considering temperature constant at interface heated surface-liquid has been also performed, in order to compare with the results for time dependent temperature of the heater. The temperature field for the case with temperature constant at the contour is shown in Figure 7.

Figures 3 to 7 shows each variable at 4.3 ms. In Figure 8 one may observe the vapor bubble radius growth during the period considered in the present analysis.

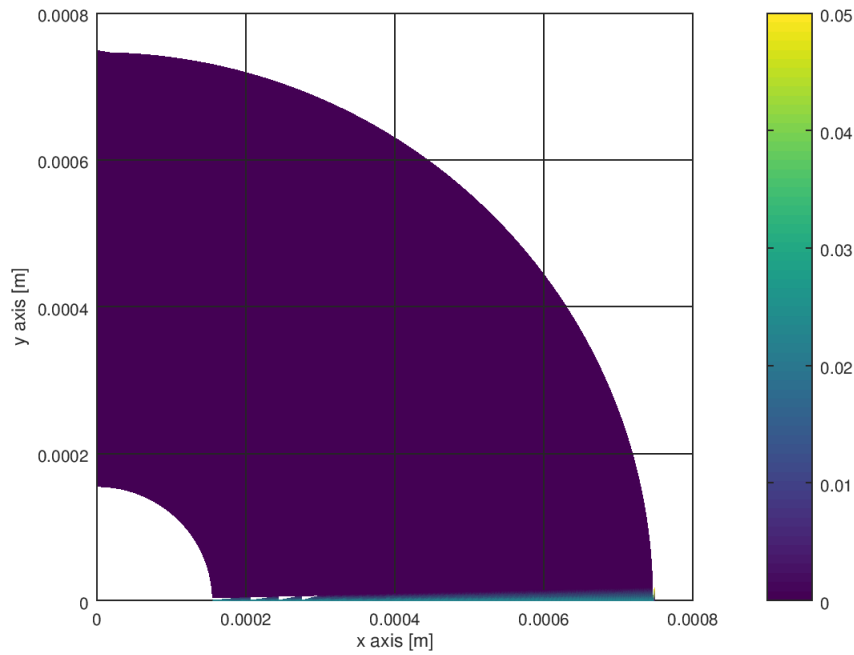


Fig. 3. Dimensionless velocity field in r direction in the liquid layer considering time dependent temperature at interface heated surface-liquid.

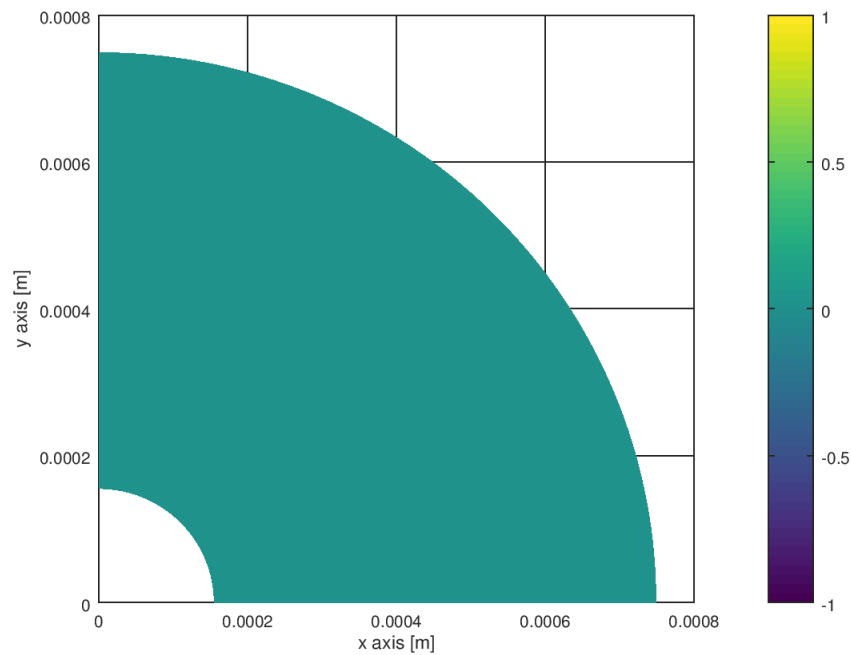


Fig. 4. Dimensionless velocity field in θ direction in the liquid layer considering time dependent temperature at interface heated surface-liquid.

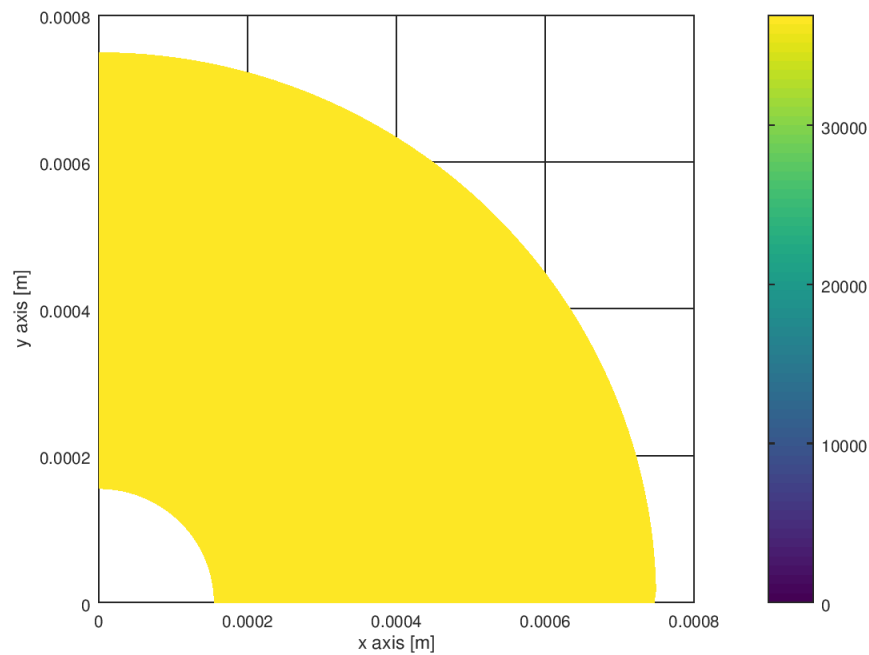


Fig. 5. Dimensionless pressure field in the liquid layer considering time dependent temperature at interface heated surface-liquid.

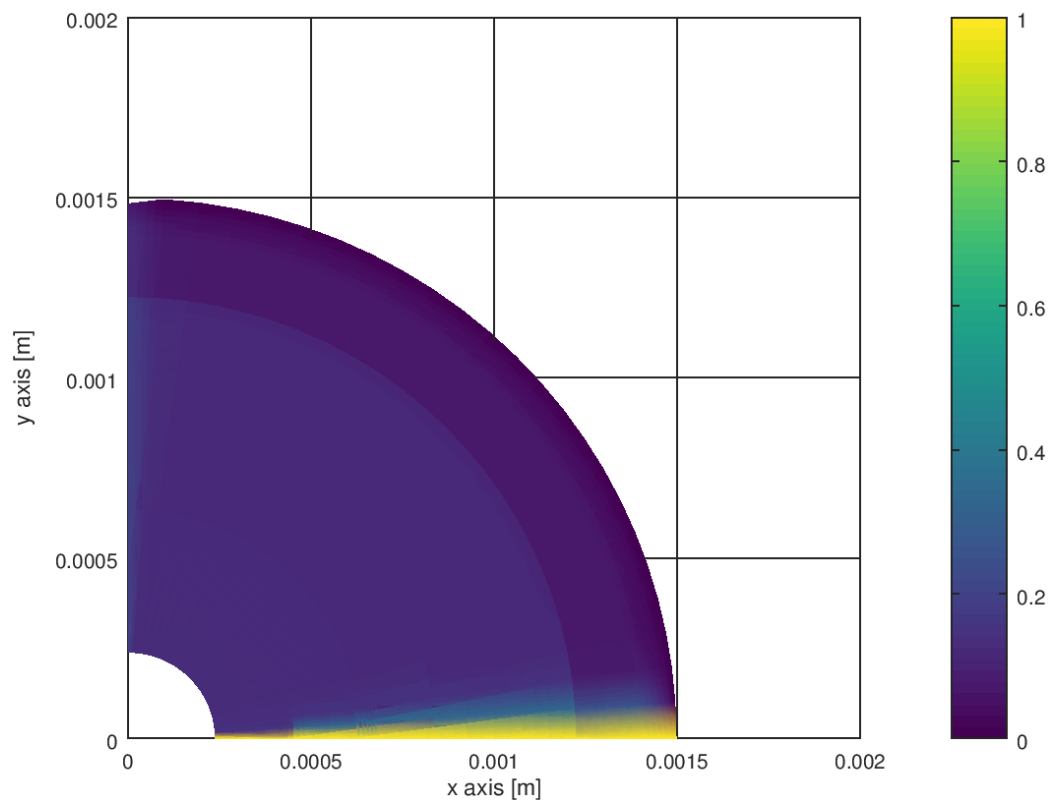


Fig. 6. Dimensionless temperature field in the liquid layer considering time dependent temperature at the interface heated surface-liquid.

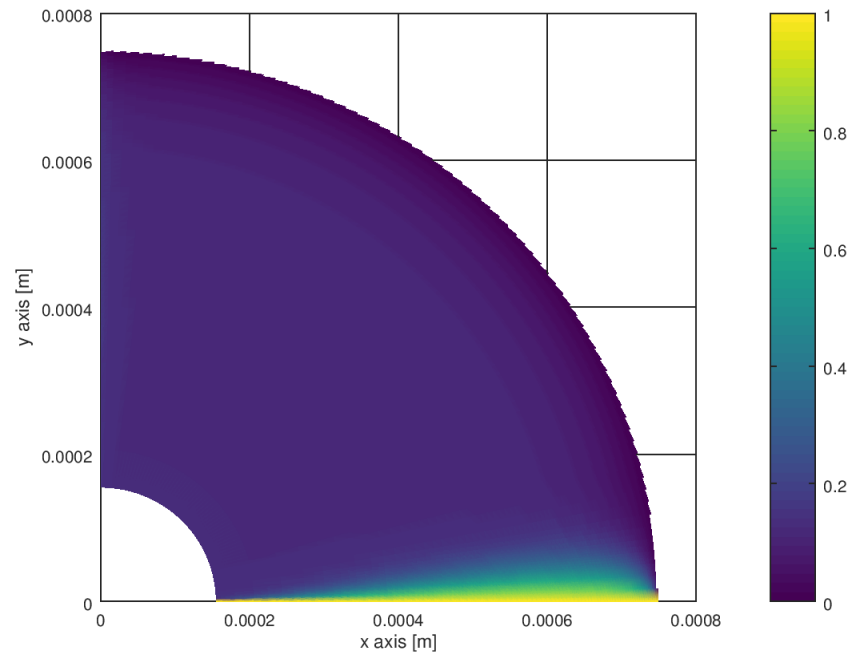


Fig. 7. Dimensionless temperature field in the liquid layer considering heated surface temperature constant.

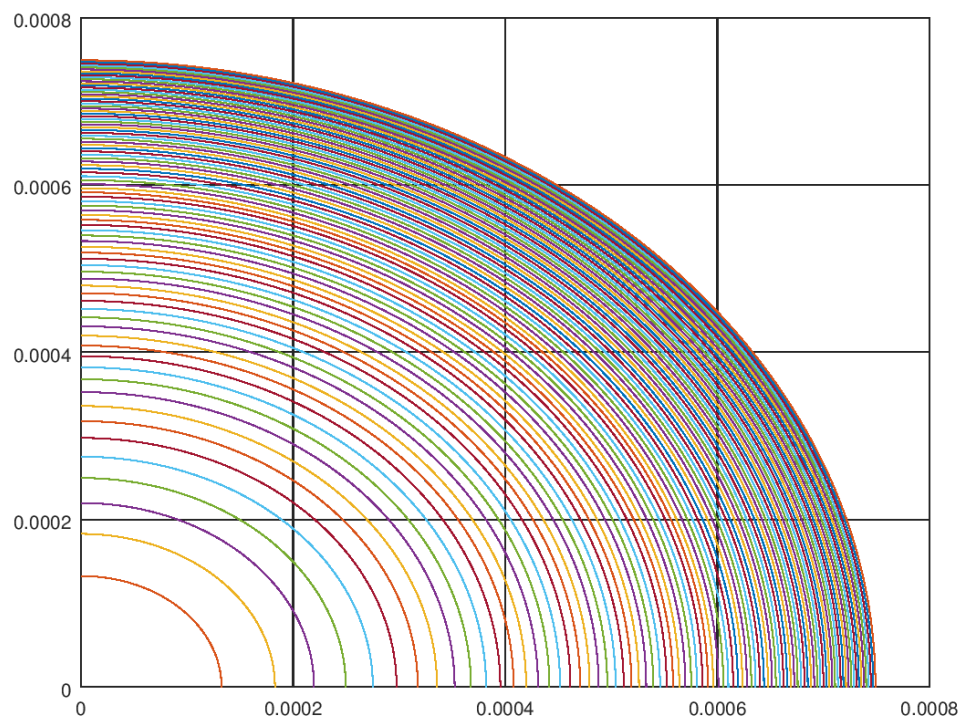


Fig. 8. Vapor bubble radius values for each time step.

The results for constant and time dependent temperature at the contour are quite similar. Both results show that the surface temperature, constant or not, remains near to 107 °C. However, this behavior needs to be better understood and compared with experimental data.

Figures 3 and 4 show that the velocities in the phase change layer are close to zero. Figure 5 shows that the pressure is maintained in saturation pressure. This information implies that all phenomena are guided by heat transfer.

The influence of the mesh refinement is also observed in the present analysis. The results are obtained for a mesh with 70 by 70 points and for a period of time of twelve hours.

4 CONCLUSION

The presented results show that there is no significant difference between consider the heated surface temperature gradient or not. Therefore, taking into accounting that consider non constant temperature we use less computational resources and we solve one less equation, it is better use this deliberation besides the transient wall temperature.

The results show the cooling effect during the vapor bubble growth. The results are in agreement with results of Aktinol and Dhir (2012). Numerical simulation results showed good agreement with data from experiments performed by the group (Kiyomura et al., 2017; Manetti et al., 2017) with respect to time dependent vapor bubble growth rate.

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