CHECKERBOARD CONTROL IN 3D ANALYSIS OF BONE REMODELING

Bárbara Belisa Soffiatti

José Eduardo Gubaua

Gabriela Wessling Oening Dicati

Jucélio Tomás Pereira

bah.soffiatti@gmail.com

jose.gubaua@yahoo.com.br

gabyoening@hotmail.com

Prof_Jucelio_tomas@hotmail.com

Federal University of Paraná

Av. Cel. Francisco H. dos Santos, 210, Jardim das Américas, 81.531-970, Paraná, Curitiba, Brazil.

Abstract. *Bone is a living tissue that adapts according to external stimuli and internal factors, in a process called bone remodeling. Mechanical stimuli generate stress and strain fields that induce bone cells to rebuild and remodel the bone tissue. By using mathematical and computational tools, engineering can contribute on this type of analysis. Simulations using Finite Element Method (FEM) coupled with math models of bone remodeling present results of bone behavior similar to those observed in clinical analysis. However, these simulations show a numerical problem called checkerboard, which is a tendency of the algorithm to intercalate the highest and the lowest densities values in some regions. The purpose of this paper is to evaluate different types of numerical smoothing treatment in a tridimensional model of a human femur to solve the problem of checkerboard pattern. In this paper two approaches are discussed according to the proposed objectives: density smoothing and stress smoothing. Together with the application of stress smoothing treatment, it is necessary to study the mechanical stimulus parameter. The results show that density smoothing generates a densities field that does not characterize femur morphology, while stress smoothing keeps the morphology and almost eliminate the problem.*

Keywords: *Smoothing Treatment, Checkerboard, Finite Element Method, Bone Remodeling, Abaqus.*

INTRODUCTION

Bone remodeling (B.R.) is an adaptation process of the bone provided by internal and external stimuli of the body. The anatomist Julius Wolff (1836-1902) related the change of bone form with external loadings applied on it. Because human skeleton is responsible for sustaining the body and help muscles to execute movements, it receives constant loads on individual daily activities, which generate strain and stress along the bone tissue and act as mechanical stimuli for B.R.. The process to renew the old bone for a new one has three phases. On the first phase, the bone cells called osteoclasts resorb the vestiges of old bone. Then, osteoblasts, cells responsible for bone deposition, work on the formation of the new bone in a transition phase that takes the longest time to be completed (Roesler, 2006). The last phase is the finalization of the bone tissue layer and its maintenance.

Technology advance has contributed a lot on bone remodeling researches. Through mathematical and computational tools is possible to simulate the behavior of bones subjected to loadings that represent a human activity. One way to make this analysis is by finite element method (FEM), which divides the bone into elements in a mesh that determines stress and strain fields and generates a densities field that is qualitatively comparable to the morphology of human structure.

Many different models of bone remodeling were developed over the years, which can consider one or more assumption of its behavior such as, biologicals aspects of an individual and external mechanical stimuli. As an example of the first, there are the impact of hormones and metabolic action on bone adaptation (Bougherara et al., 2010). For the last, some examples of external agents are strain density energy (Beaupré et al., 1990; Jacobs et al., 1995; Jacobs et al., 1997), mass strain density energy (Weinans et al., 1992), damage (Doblaré and García, 2001) or strain (McNamara and Prendergast, 2007). Moreover, there are analysis of bone in a macro scale or micro scale (Mercuri, 2013), where it assume different properties for each approach.

The remodeling process on FEM analysis characterizes a bone by changing its density according to loading application. Hence, in the end of the process we note two different aspects on femur structure (Jacobs et al., 1995). One happens on the region far from loading application, where it shows a structure with high density on the boarding and low density on the center of the bone (Chen et al., 2007). This aspect is similar to bone morphology, where there is a cortical bone (compact bone with high density) and a cancellous bone (more porous and with low density), relative to the extremity of femoral diaphysis and the center of femoral head, respectively (Figure 1). The other aspect is a numerical problem that occurs in places close to loading application, in areas where the highest and lowest density intercalate in a chess board aspect. This problem is called checkerboard and do not characterize the morphology of a bone structure. It is a tendency of the B.R. algorithm that causes a poor result. Therefore, there is the need to reduce or eliminate checkerboard since the aim of B.R. simulation is to show how bone structure is in real life.

Checkerboard happens due to the relation between the density and Young's Modulus for cancellous bone (Silva et al., 2015). To reduce checkerboard different methods can be applied, for example, smoothing treatment of the image on a post-processing phase (Sigmund e Petersson, 1998), which improves the quality of the results on the end of the process, but does not improve the FEM or B.R. calculations. Another way to reduce checkerboard is by using elements of higher order, however, it increases the time of data processing (CHEN et al., 2007).

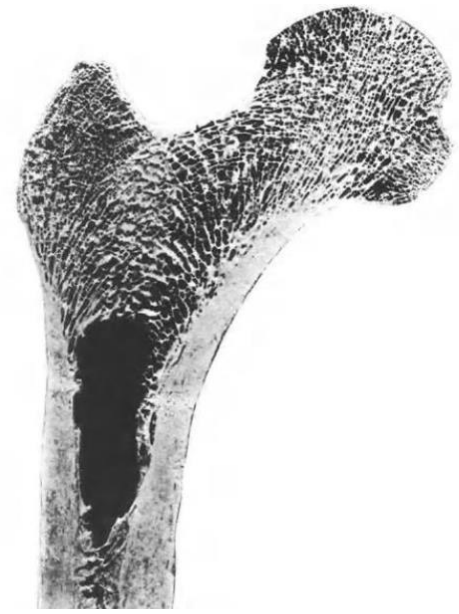


Figure 1. Coronal section of the right femur (Wolff, 1986)

The aim of this paper is to analyze what method improves the numerical results on the same way that does not lose the bone morphology that is seen in a real femur. The analysis uses the software Abaqus to run FEM problem coupled with a subroutine in Fortran language. The techniques to solve checkerboard and the final calculation are made on the software Matlab. This work uses the isotropic model of Stanford bone remodeling, which takes on bone structure in a macro scale. That being so, the simulation on this paper considers the mechanical stimuli that it is subjected to in a material that has the same behavior for all directions. The solid femur model is divided into linear tetrahedral elements with constant density.

This paper uses two methods during bone remodeling process to reduce checkerboard in a 3D femur model. The first method is a density smoothing treatment, applied in densities after B.R. process. This method is commonly used as a post-processing tool, even so, we applied it during the analysis for every step. The second is a stress smoothing treatment, which improves the stress field that is applied for every step before bone remodeling. These methods are better explained in sections 1.3 and 1.4, respectively.

1 METHODS

This section presents the methodology used on this research.

1.1 Isotropic model of Stanford bone remodeling

The model of bone remodeling used on this paper takes bone tissue as an isotropic, linear and elastic material, based on Jacobs et al. (1995) which used the formulation of Beaupré et al., (1990). This remodeling process is characterized by the updating of density for every element in FEM analysis, according to the daily tissue stress stimulus that it is subjected to, ψ_t. The density changes for every step of simulation through the Euler's method as

$$\rho_{n+1} = \rho_n + \dot{\rho}\Delta t, \qquad (1)$$

where ρ_{n+1} is the new bone density, ρ_n the old density, Δt is the bone remodeling frequency, that is, the time interval in which the process happens and $\dot{\rho}$ is the density rate, calculated by

$$\dot{\rho} = k\, S_V\, \dot{r}\, \rho_c. \qquad (2)$$

where $\dot{r}$ is the remodeling rate, ρ_c is the cortical bone density, k is the active surface and S_V is the surface area available for bone remodeling in each step. Surface area (Eq. 3) is calculated in function of porosity by a fifth order polynomial equation according to Martin (1984) and improved by Aznar et al, (1999). However, Corso (2006) implemented a fifth order polynomial in function of the apparent density, ρ, and it is determined as

$$S_V = -0.07 + 8.1\rho - 7.2\rho^2 + 5.1\rho^3 - 2.1\rho^4 + 0.23\rho^5. \qquad (3)$$

According to this model, there are three regions that determine the bone tissue behavior, in function of daily tissue stress stimulus, ψ_t. The first happens when the daily tissue stress stimulus is lower than the reference value (ψ_t^*), what results on bone resorption. The second region is the dead zone, which is bounded by a parameter W and with ψ_t^* defined as the constant level of daily stress stimulus that will not change bone density. In this area, as shown in Fig. 2, the remodeling rate is zero. The third region is where the daily tissue stress stimulus is over the reference value, resulting in bone formation.

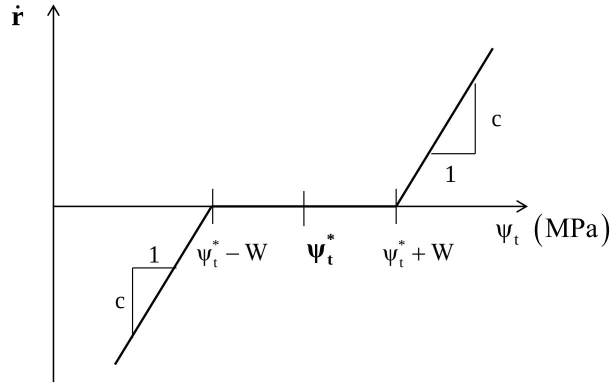


Figure 2. Dead Zone applied on bone remodeling model.

That being so, we can calculate the remodeling rate, $\dot{r}$, by

$$\dot{r} = \begin{cases} c\left[\psi_t - \left(\psi_t^* - W\right)\right] & if\ \psi_t < \left(\psi_t^* - W\right) \\ 0 & if\ \left(\psi_t^* - W\right) \le \psi_t \le \left(\psi_t^* + W\right), \\ c\left[\psi_t - \left(\psi_t^* + W\right)\right] & if\ \psi_t > \left(\psi_t^* + W\right) \end{cases} \qquad (4)$$

where $\dot{r}$ depends on the remodeling speed, c, the limits of dead zone interval, W, the reference value, ψ_t^* and the daily tissue stress stimulus, ψ_t, which is calculated by

$$\psi_t = \left(\sum_{days} n_i * \bar{\sigma}_{t_i}^m\right)^{1/m}, \qquad (5)$$

with $\bar{\sigma}_{t_i}$ as the effective stress in the region, n_i as the number of daily cycles of i loadings and m an empirical constant of the material. The effective stress is related with the stress in a continuum level, $\bar{\sigma}(\rho)$, its apparent density, ρ, and cortical bone density, ρ_c, (Beaupré et al. 1990) as the Eq. 6 describes.

$$\bar{\sigma}(\rho) = \left(\frac{\rho}{\rho_c}\right)^2 \bar{\sigma}_t . \tag{6}$$

The stress in a continuum level, $\bar{\sigma}(\rho)$, is determined in function of Young's modulus, E, and strain energy density, U, as

$$\bar{\sigma} = \sqrt{2\mathrm{EU}}, \tag{7}$$

with E calculated by

$$E = b(\rho)\rho^{\beta(\rho)}. \tag{8}$$

In this Equation b and β are empirical constants in function of material density, as shown in Table 1. The same way, Poisson's ratio changes in function of bone density.

Table 1. Constants values of b, β and ν in function of density

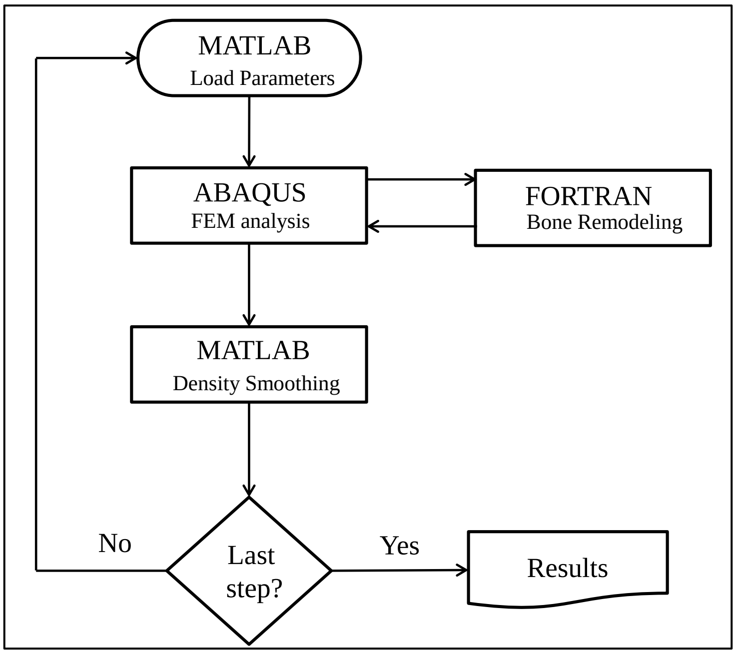
Constants	$\rho \le 1.2e-6$ $[kg/mm^3]$	$\rho > 1.2e-6$ $[kg/mm^3]$
b	2014	1763
β	2.5	3.2
ν	0.2	0.32

1.2 Computational implementation

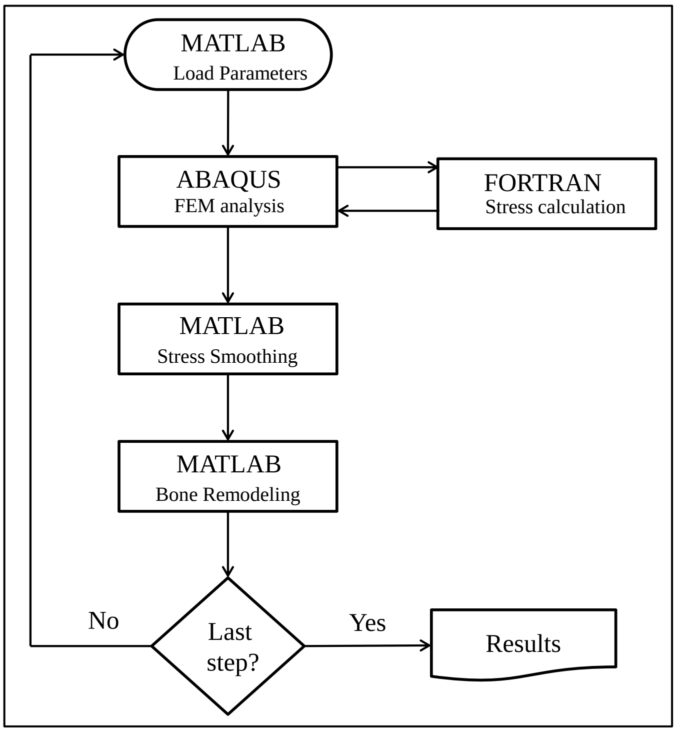
For this study, we implemented two different computational analyzes according to the smoothing process applied. Figures 1a and 1b display how the software Matlab, Abaqus and Fortran are related for density and stress smoothing methods, respectively.

For the simulation with density smoothing treatment, first the software Matlab loads the model initial parameters and calls Abaqus that executes FEM analysis, coupled with Fortran which contains the Stanford bone remodeling model. After this, Abaqus exports the updated densities field for Matlab to process the density smoothing treatment. Finally, we compute the total mass variance and Eulerian norm of mass variance vector for every step in order to compare the results.

Likewise, the simulation with stress smoothing treatment has the same beginning as the previous one. The difference is that Fortran does not solve bone remodeling process. It makes the stress calculation and then, Abaqus exports a stress field to Matlab, which realizes the stress smoothing treatment and, after that, the remodeling process, where the updating of density occurs. Matlab computes the results of mass variance, completing the cycle.



(a)



(b)

Figure 3. Processes Flowchart for (a) Density and (b) Stress Smoothing processes.

The remodeling process on Fortran, for the first analysis, uses a subroutine called User Material (UMAT), whereas the second analysis brings the remodeling parameters on a function in Matlab. For both methods, the parameters of isotropic Stanford bone remodeling model are the same, as shown on Table 2.

Table 2. Parameters of isotropic model of Stanford Bone Remodeling

Parameter	Value	Unit
n_i	3000	Cycles/day
ψ_t^*	50.0	MPa
W	$0.125 \times \psi_t^*$	MPa
m	4.0	-
c	2.0e-5	(mm/day)(MPa/day)
ρ_c	2.0e-6	kg/mm³
Δt	7	days
k	1.0	-

On the beginning of the simulation the densities field is homogenous, that is, all the elements of the mesh have the same density of $1.0e^{-6}$ kg/mm³. The range of maximum and minimum acceptable densities is $2.0e^{-6}$ kg/mm³, which is the cortical bone density (Jacobs et al., 1995) and $0.2e^{-6}$ kg/mm³ for 90% of porosity (Hall, 2011).

On this paper, we analyses how a human femur remodels during a gait, that is, the adaptation of a femur from a person that walks approximately 3km per day on a period of 1000 days. The gait of an individual is divided into three movements, one when the foot touches the floor, the abduction of the lower limbs and the adduction (Gubaua, 2016; Dicati, 2015). For each movement we apply two loadings: a compression force on the head of femur due to its contact with acetabulum, and a traction force on the greater trochanter, which represents muscle response. Table 3 and Figure 2 display the areas (Greenwald and Haypes, 1972), intensity (Jacobs et al, 1995) and directions (Bagge, 1999) of the six loadings for every movement.

The simulation runs with a mesh of 147,638 tetrahedral linear elements, called C3D4 (continuum, three-dimensional and solid with 4 nodes) and 27,819 nodes on Abaqus. The homogenous boundary conditions are applied on a solid with isotropic, linear and elastic behavior fixed on femoral diaphysis. The solid added on femoral diaphysis (Figure 2) has the aim of reducing the stress concentrators of the analysis.

Table 3. Intensity and directions of loadings

Loads	Intensity [N]	Direction (x,y,z)	Area [mm²]
Compression 1	2317	Surface pressure	1541.05
Compression 2	1158	Surface pressure	1159.28
Compression 3	1548	Surface pressure	1315.61
Traction 1	703	(0.149,-0.06241,0.9869)	809.90
Traction 2	351	(-0.3232,0.3878,0.8632)	597.29
Traction 3	468	(-0.3827,-0.4073,0.8292)	707.21

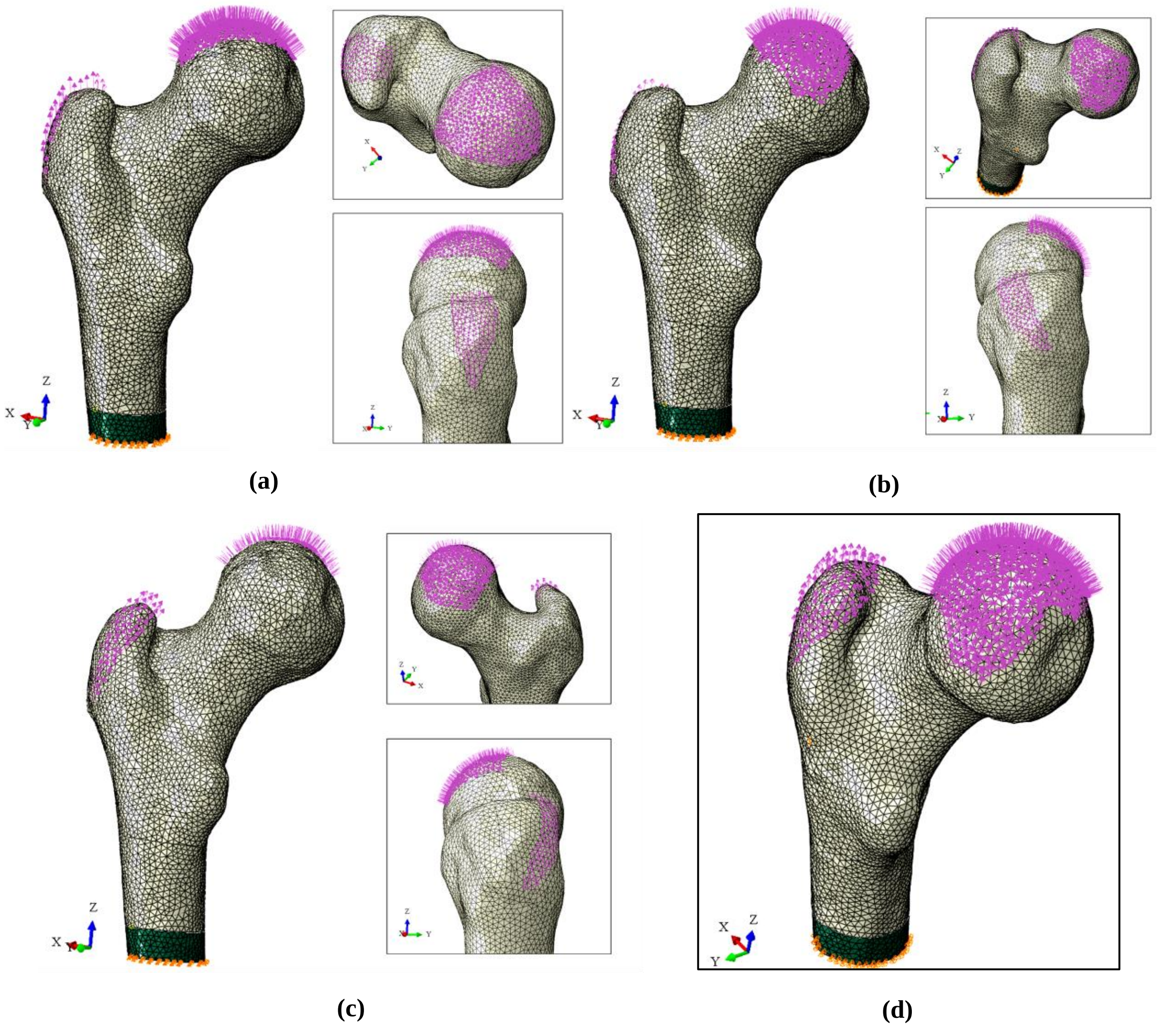


Figure 4. Loading application areas and boundary conditions for (a) Compression and traction 1, (b) Compression and traction 2 and (c) Compression and traction 3. (d) 3D view of femur model.

The total mass of the model after the analysis is calculated using the updated densities and the volume of the elements. The mass variance is made by the difference of densities field between actual and previous steps, multiplied by the volume vector of the femur model. We prefer to use the Eulerian norm of the vector of mass variance, because without it, the curve crosses the axis of days, suggesting that in a step there are none mass variance.

1.3 Density Smoothing

One of the methods used to reduce the final checkerboard in FEM analysis is the density smoothing treatment. It improves the density values of the elements after bone remodeling process has been executed. We apply density smoothing for every step and it is basically composed by two averages. The first for every node of the mesh, and then for every element.

Thus, this method takes the density values of all elements connected to a certain node, No_i, and makes a simple average, aiming to obtain a nodal average value, $\rho\left(No\right)_i$, as

$$\rho_{No} = \frac{\sum_{j=1}^{Nel} \rho_{el_j}}{Nel}, \tag{9}$$

where Nel_j is the number of elements connected to the node No_i, and $\rho\left(el\right)_j$ are the densities of these elements. Then, every element assumes a density value from the simple average of the densities from the four nodes belong to the element, and it is represented by ρ_{el}^* as

$$\rho_{el}^* = \frac{\sum_{k=1}^{4} \rho_{No_k}}{4}. \tag{10}$$

1.4 Stress Smoothing

Another method to reduce checkerboard on densities is by a stress smoothing treatment. It works in the middle of FEM analysis, by improving the stress field of the model. That is, after Abaqus processes the mesh and applies the loadings, the stress field is exported and its values for every element are improved.

Instead of using the elementary stress field, $\hat{\sigma}$, obtained from FEM, on bone remodeling calculation, we smooth it before the application. Thus, for a node *No* that connects a total of *Nel* elements, there are *Nel* different stress vectors $\hat{\sigma}$. Therefore, the nodal stress vector $\left(\sigma_{No}^*\right)$ is estimated, through simple nodal average of the elementary values, as shown in Eq. 11.

For every element there are six stress components that are storage on a matrix of stress. First, it is necessary to make a nodal stress average for every component of the stress vector, as it can be seen below

$$\sigma^*_{No} = \frac{\sum_{i=1}^{Nel} (\sigma_i)_j}{Nel}, \tag{11}$$

where $(\sigma_i)_j$ is the stress vector obtained from FEM for *i-th* node of the *j-th* element of the set. After that, we calculate the new stress for every element, σ^*_{el}, through a simple average of the stress values of the four nodes of the element.

$$\sigma^*_{el} = \frac{\sum_{i=1}^{4} \sigma_{No_i}}{4}. \tag{12}$$

Note that this method makes an adjustment on stress values before the process of bone remodeling occurs, thus, according to the results, we will have to analyze and adjust ψ^*_t, which is the value on Stanford bone remodeling model that defines the level of daily stress stimulus that bone will not remodels.

2 RESULTS AND DISCUSSION

Figure 5 presents the bone densities distribution for original approach (Fig. 5a), density (Fig. 5b) and stress (Fig. 5c) smoothing treatment.

After the conclusion of bone remodeling process, we note difference between densities fields of the simulations with the two methods of smoothing treatment and the original approach. This last one shows, as expected, a configuration with checkerboard on regions close to loadings and the femur morphology on regions far from the loadings (femoral diaphysis), with the cortical and trabecular bone well defined.

On the other hand, the densities field from density smoothing treatment presents a distribution that is not similar to the real femur, due to the great amount of cortical bone on femoral diaphysis that covers the medullary canal, and the behavior near to femoral head. This treatment smooths the final values of the process in every step and because density is a main parameter for bone remodeling, its impact on the distribution is greater. Smoothing process works as a “filter”, so when it is applied on densities field, it homogenizes the field and does not allow the correct characterization.

However, with the stress smoothing treatment we note a structure that keeps femur morphology and reduces checkerboard dramatically. We observe the formation of lateral and medial corticals, the medullary cavity, the distribution of trabecular density in proximal epiphyses and the Ward’s triangle in femoral neck. If we compare Figure 5 (a) with Figure 5(c), we see a similar density distribution. Yet, Figure 5(c) shows greater density on femoral diaphysis (medullary cavity) than what is expected on a real femur. This can happen as a result of change on stress values, once stress stimulus, ψ^*_t, controls the rate of bone remodeling. That being so, we made a sensitivity analysis, where the stress stimulus, ψ^*_t, is adjusted.

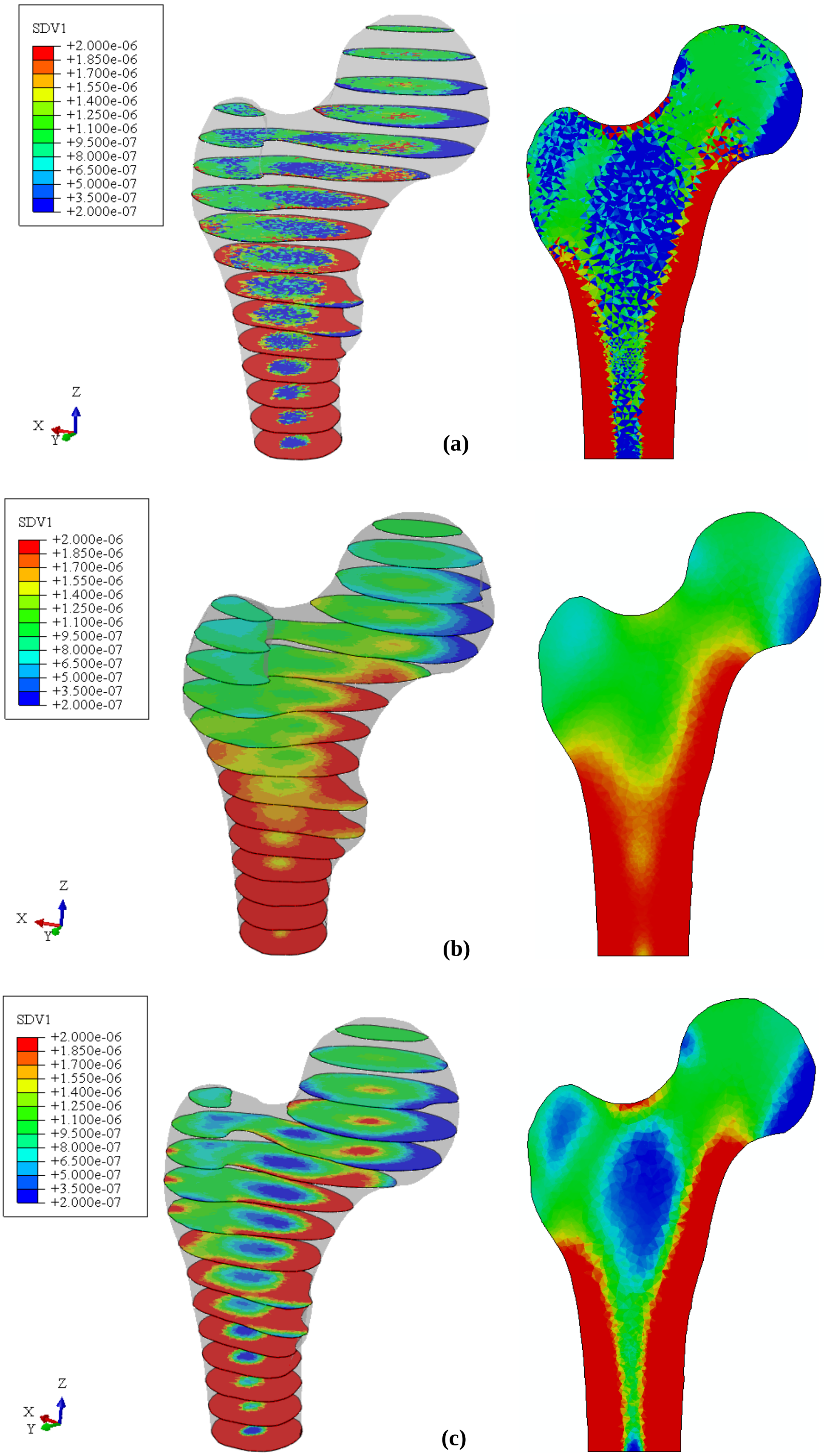


Figure 5. Densities fields after 1000 days of loading [SDV1 in kg/mm³] (a) with checkerboard; (b) density smoothing treatment; (c) stress smoothing treatment.

Originally, the stress stimulus, ψ_t^*, in which bone does not remodel, that is, does not resorb or form new bone, is 50.0 MPa. By the analysis of the daily stimulus, ψ_t, for every step on the simulation with stress smoothing treatment we found that the averages of the values oscillate between 56.0 and 58.0 MPa, which means that, with the originally ψ_t^*, the daily stimulus, ψ_t, stays mostly on the area of bone formation from the dead zone (Figure 2). Thus, we adjust the value of stress stimulus for 60.0 MPa in order to study its behavior and after that compare the results for 40.0 and 70.0 MPa, as it is shown in Fig. 6.

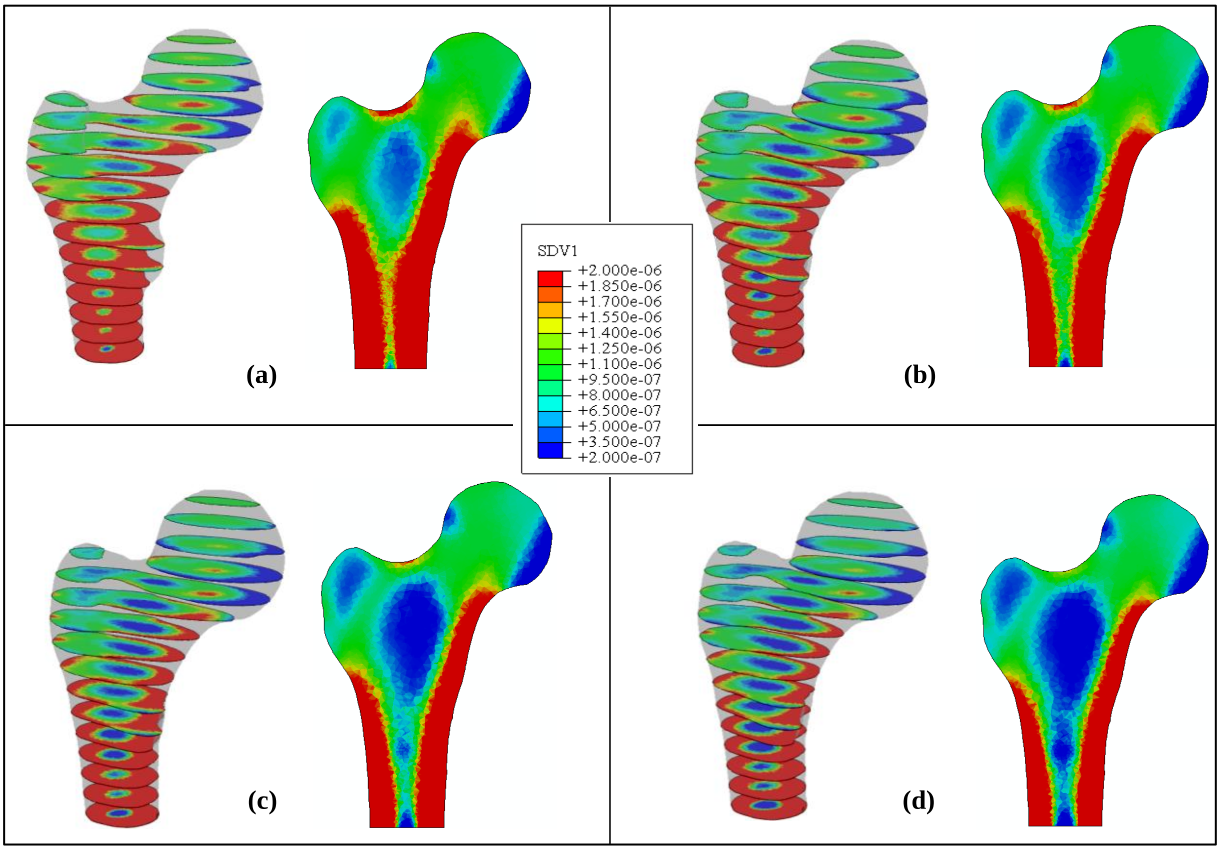


Figure 6. Final densities field [SDV1 in kg/mm³] with stress smoothing for (a) ψ_t^* =40.0 MPa; (b) ψ_t^* =50.0 MPa; (c) ψ_t^* =60.0 MPa; (d) ψ_t^* =70.0 MPa.

According to Fig. 6, the higher stress stimulus values provide densities distributions more similar to bone morphology, with a cortical thickness and medullary canal close to the real bone distribution (Fig. 1). By comparing the total mass variation (Fig. 7) of the simulations with stress smoothing treatment (SS) for different stress stimulus, ψ_t^*, and with the original approach (OA), we note a decrease on the total mass when ψ_t^* increases, because of the relation between dead zone and rate of bone remodeling. Furthermore, the curve of original approach shows a bigger inclination compared to the curves with smoothing treatment due to checkerboard. Another important point is the tendency to stabilize the densities distribution, that is, bone mass variation is minimum or has a small variation in the end of the analysis. Note that, the curve of original approach on the graph of Fig. 7 does not stabilize as the others because the presence of checkerboard.

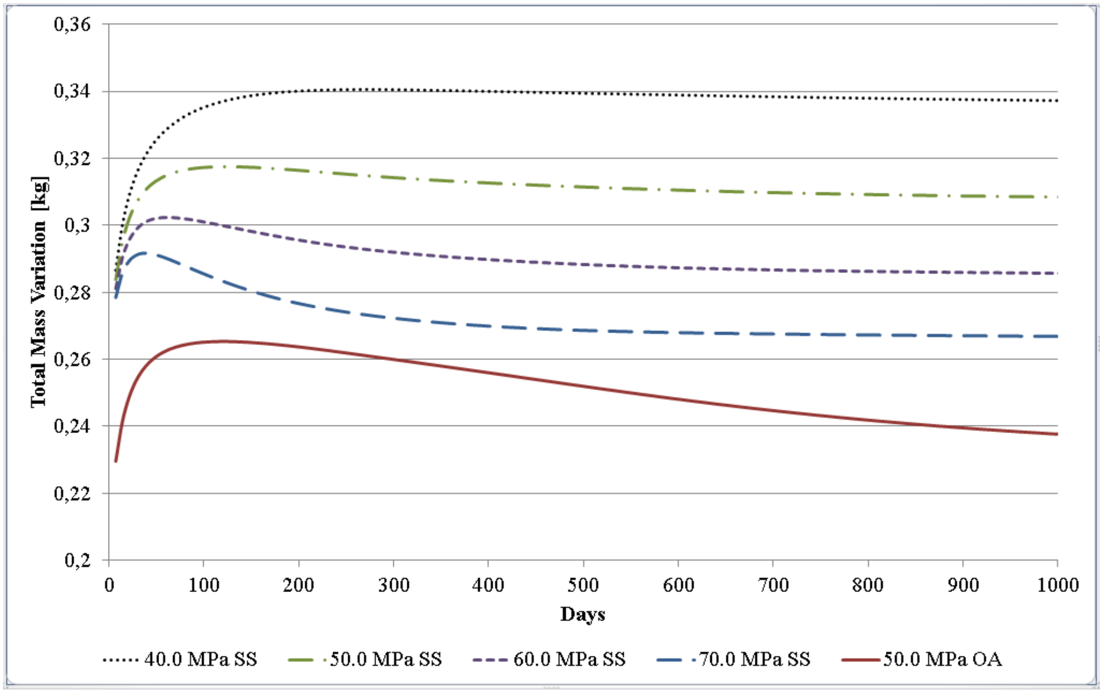


Figure 7. Comparative graph of total mass variation for different values of stress stimulus, ψ_t^*.

3 CONCLUSION

In general, checkerboard is a numerical problem presents in 2D (Silva et al., 2015) or 3D (Jacobs et al., 1995; Dicati, 2015) analysis of bone remodeling, near to the loadings application. There are some alternatives on bibliography to reduce it. However, we found that smoothing treatments made during the process of bone remodeling would give us good results and would not consume a great time of data processing. Therefore, through the results of density smoothing treatment, we understand that it cannot be applied for every step due to it loses the aspects of bone morphology. This method shows better results as a post-processing treatment, where it smooths the densities field of the last step and keeps bone characteristics. On the other hand, the stress smoothing treatment shows very good results, mainly when we adjust the values of stress stimulus. Because this method makes an adjustment on a parameter that in FEM analysis accumulate bigger errors, the stress smoothing is a more reliable treatment to solve checkerboard. It is worth mentioning that the methods used to reduce this type of numerical problems depends on the aspects and particularities of each simulation.

REFERENCES

Aznar, J. M. G., 1999. Modelo de Remodelación Ósea: Análisis Numérico y Aplicaciones al Diseño de Fijaciones de Fracturas del Fêmur Proximal. PhD thesis, University of Zaragoza.

Bagge, M., 1999. *Remodeling of bone structures*. PhD Thesis,Technical University of Denmark.

Beaupré, G. S., Orr, T. E., Carter, D. R., 1990. *An Approach for Time Dependent Bone Modeling and Remodeling – Theoretical Development*. Journal of Orthopaedic Research 8, pp. 651-661.

Bougherara H , Klika V , Maršík F , Ma ˇrík IA , Yahia L ., 2010. *New predictive model for monitoring bone remodeling.* J Biomedical Materials Research v. 95A, pp. 9–24.

Chen, G., Pettet, G., Pearcy, M., McElwain, D. L. S., 2007. *Comparison of two numerical approaches for bone remodeling.* Medical Engineering & Physics, vol. 29, pp. 134-139.

Corso, L. L., 2006. *Aplicação de Procedimentos de otimização e remodelamento ósseo na simulação e análise em problemas de biomecânica.* Master thesis, Federal University of Rio Grande do Sul.

Dicati, G. W. O., 2015. *Análise comparativa entre diferentes abordagens de remodelação de tecidos ósseos em modelos de elementos finitos bidimensionais e tridimensionais*. Master thesis, Federal University of Paraná.

Garijo, N., Fernández, J. R., Pérez, M. A., García-Aznar, J. M., 2013. *Numerical stability and convergence analysis of bone remodeling model.* Comput. Methods Appl. Mech. Engrg, 271, pp 253-268.

Greenwald, A.S., Haynes, D.W., 1972. *Weight-bearing areas in the human hip joint.* The Journal of Bone and Joint Surgery, v. 54, p. 157-163.

Gubaua, J.E., 2016. *Análise numérica da influência da rigidez de próteses femorais no processo de remodelação óssea associado à técnica cirúrgica de artroplastia total de quadril*. Master thesis, Federal University of Paraná.

Hall, S. J., 2011. *Basic Biomechanics*. 6.ed. New York: Mc Graw Hill.

Jacobs, C. R., Leveston, M. E., Beaupré, G. S., Simo, J. C., Carter, D. R., 1995. *Numerical instabilities in bone remodeling simulations: the advantages of a node-based finite element approach*. J. Biomechanics, vol 28, pp 449-459.

Martin, R.B., 1984. *Porosity and specific surface of bone*. Critical Reviews in Biomedical Engineering, vol. 8, pp. 651-661.

Mercuri, E. G. F., 2012. *Modelagem multiescala de tecidos mineralizados considerando a micromecânica da dinâmica celular.* Master thesis, Federal University of Paraná.

McNamara, M. L.; Prendergast, P. J. *Bone remodeling algorithms incorporating both strain and microdamage stimuli*. Journal of Biomechanics, v. 40, p. 1381-1391, 2007.

Robling, A. G., Castillo, A. B., Turner, C. H., 2006. *Biomechanical and Molecular Regulation of Bone Remodeling.* Annual Review of Biomedical Engineering 2006.8: pp. 455-498.

Roesler, C. R. M., 2006. *Adaptação mecânica do osso em torno de implantes ortopédicos.* PhD thesis, Federal University of Santa Catarina.

Silva, J., Gubaua, J. E., Dicati, G. W. O., Pereira, J.T., 2015. *Análise de diferentes metodologias para evitar a formação do padrão similar de checkerboard em uma análise de remodelação óssea bidimensional.* XXXVI Iberian Latin-American Congress on Computational Methods in Engineering (CILAMCE 2015).

Sigmund, O., Petersson, J., 1998. *Numerical instabilities in topology optimization: A survey on procedures dealing with checkerboard, mesh-dependencies and local minima*. Structural Optimization, vol. 16, pp. 68-75.

Wolff, Julius, 1986. Book *The Law of Bone Remodeling*, pp8.