



MODEL UPDATING ANALYSIS AND DAMAGE IDENTIFICATION ON A SIMPLY SUPPORTED BEAM

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Abstract. *The main objective of this paper is to present model updating analyses and damage identification. Experimental modal analyses are performed on a simply supported beam. Measurements come from three accelerometers and the input excitation is provided by an impact hammer. The experimental set-up is submitted to a process of assembling and disassembling the beam supports in order to simulate a scenario closer to the real one in which one may face variability of modal data. The Young's modulus and the parameters of a proportional damping model are considered as the updating structural variables in optimization procedure. Data variability is taken into account along the model updating process by means of the likelihood function. The calibrated model is assessed by comparing the model-predicted natural frequencies and the frequency response function (FRF) with their measured counterparts. Finally, the calibrated model is used for damage identification which is simulated by an addition of small masses on the structure. Inverse method is applied to perform this study based on a finite element analysis model updating.*

Keywords: *Model updating, Damage identification, Uncertainties*

1 INTRODUCTION

Numerical models are used to simulate the behaviour of physical systems in all areas of science and engineering. There are several applications in design, decision making and predictive analysis. A key-aspect for these analyses is the reliability of the predictions provided by the adopted model. It is not easy to ensure the compliance with the real system due to the great number of unknowns and uncertainties related to material and physical properties, boundary conditions, load conditions, etc. Simplifying hypotheses are often made which may detract the quality and accuracy of the predictions provided by the computational model.

These issues had led to the development of model updating techniques (Simoen et al, 2015), which aims to calibrate unknown system parameters, based on observed data of the real structure. In a structural mechanics context, measurements of the system of interest (accelerations, natural frequencies, damping ratios, modal shapes, etc.) provide the experimental information required to perform model updating analyses.

Unfortunately, measurements are always subject to some level of variability which diminishes the confidence on models inasmuch as these are calibrated based on measurements. Therefore, it is imperative to determine the reliability of predictions obtained from numerical models. Deterministic model updating strategies have been used to build more reliable computational models. Nevertheless, there are several situations in which they do not provide the robustness necessary to deal with the complexity of unknowns/conditions associated to real structures. More specifically, these techniques in general do not take uncertainties into account.

A noteworthy here is the fact that one may use deterministic approaches and still provide some estimation for the covariance of the unknown parameters, as can be seen in the reference books (Beck & Arnold, 1977) and (Aster et al., 2013). In this perspective, the uncertainty evaluation yields a confidence range of the interest parameter, due to application of probabilistic tools, which can improve the accuracy of numerical results. This aspect is analysed in the present work where a large set of measured modal data allows one to compute the data covariance matrix and subsequently the parameter covariance matrix.

Initially, the deterministic model updating inverse problem is applied to calibrate the computational finite element model. Hereafter there is a uncertainty evaluation based on the covariance matrix of model updating variables with a diagonal variance hypothesis. Damage assessment is performed based on the optimal solution provided by calibration analysis.

2 THEORETICAL APPROACH

The model updating analysis and damage identification on a simply supported beam are performed based on a finite element (FE) computational model implemented in Matlab R2014b. Some pre-defined Matlab functions are used along this study in order to run optimization analyses, to perform statistical inferences as well as some pre-defined functions of CALFEM toolbox (Austrell et al, 2004) are used to construct the FE mesh, assemble global matrices, solve eigenvalue problems, etc. An in-house Particle Swarm Optimization (PSO) (Kiranyaz et al, 2014) algorithm is adopted to solve the damage identification problem.

2.1 Model updating

As earlier stated, one assumes that there are three updating variables which are the Young's modulus (E) and the coefficients of proportional damping model (α and β). In a dynamic

structure context the physical problem is governed by the well known dynamics's ordinary differential equation (Clough & Penzien, 1995) $M \ddot{\mathbf{u}} + D(\dot{\mathbf{u}} + \mathbf{K} \mathbf{u} = \mathbf{F}$.

Where M , D , K and F are respectively the global mass, damping, stiffness and external excitation matrices, and as $\ddot{\mathbf{u}}$, $\dot{\mathbf{u}}$ and $\mathbf{u}$ are the acceleration, velocity and displacement vectors.

The state-space formulation of damped systems is adopted to solve the dynamic's ODE. (Gérardin & Rixen, 2015) show that based on symmetric matrices and state vectors one can join the global matrices in a specific way in order to be able to perform a general solution for an homogeneous or non-homogeneous cases, Eq. 1.

$$\left\langle \mathbf{A} = \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & -M \end{bmatrix}, \mathbf{B} = \begin{bmatrix} D & M \\ M & \mathbf{0} \end{bmatrix}, \mathbf{r} = \begin{bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{bmatrix}, \mathbf{P} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix} \right\rangle \rightarrow \mathbf{A}\mathbf{r} + \mathbf{B}\dot{\mathbf{r}} = \mathbf{P} \quad (1)$$

The frequency response function (FRF) H of a proportional damped structure can be calculated by deriving the state-space formulation Eq. 2, (Gérardin & Rixen, 2015). The variable w represent the frequency, ϕ the eigenvector amplitude, μ the diagonal element of $\boldsymbol{\mu} = \boldsymbol{\phi}^T \mathbf{B} \boldsymbol{\phi}$, i the complex number and λ the eigenvalue. The subscripts k denotes the mode number and $k = 1 \dots N$, p the measurement degree of freedom and q the excitation degree of freedom.

$$H_{pq}(w) = \sum_{k=1}^{2N} \frac{\phi_{kp} \phi_{kq}}{\mu_k (iw - \lambda_k)} \quad (2)$$

The model calibration is formulated as inverse problem (Aster et al., 2013) which is a progressive diminishing misfit between experimental data and FE model predictions, Eq. 3.

$$\mathbf{G}(\boldsymbol{\theta}) = \mathbf{d}, \quad \boldsymbol{\theta} = [E \quad \alpha \quad \beta]^T, \quad \mathbf{d} = [\boldsymbol{\omega}^{exp} \quad \boldsymbol{\zeta}^{exp}]^T, \quad \mathbf{y} = [\boldsymbol{\omega}^{num} \quad \boldsymbol{\zeta}^{num}]^T \quad (3)$$

Where $\mathbf{d}$ denotes the vector of observed data, $\boldsymbol{\theta}$ the vector of parameters, $\mathbf{G}$ the system operator and finally the FE model predictions are denoted by $\mathbf{y}$. Knowing the fact that the operator $\mathbf{G}$ is formulated by the dynamic's ODE, one can infer the Eq. 4.

$$\mathbf{G} \mapsto M \ddot{\mathbf{u}}(t) + D(\boldsymbol{\theta}) \dot{\mathbf{u}}(t) + \mathbf{K}(\boldsymbol{\theta}) \mathbf{u}(t) = \mathbf{F}(t) \quad (4)$$

Thus, based on a progressive diminishing misfit between experimental data and FE model predictions, $\boldsymbol{\epsilon} = \mathbf{d} - \mathbf{y}$ this discrepancy is evaluated by a pre-defined function called cost function, f . The function showed in Eq. 5 was chosen as the cost function to perform model calibration, (Beck & Arnold, 1977).

$$f(\boldsymbol{\theta}) = \sum_{k=1}^N \left[\left(\frac{\omega_k^{exp} - \omega_k^{num}}{\sigma_{k,\omega}} \right)^2 + \left(\frac{\zeta_k^{exp} - \zeta_k^{num}}{\sigma_{k,\zeta}} \right)^2 \right] \quad (5)$$

One should note that the dimension of $\mathbf{d}$, $\mathbf{y}$ and $\boldsymbol{\sigma}_d$ are based on the number of vibration modes performed on modal analysis. It should be remarked that the standard deviation vector $\boldsymbol{\sigma}_d$ is associated with experimental data.

It is essential to observe that model updating here is formulated as an optimization problem where the best solution is the one that leads to the minimum discrepancy between data and model prediction as showed on Eq. 6. A well known pre-defined function of Matlab called *lsqnonlin* is used to solve this problem. For further details refer to (Lsqnonlin MathWorks)

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in D_{\theta}}{\operatorname{argmin}} f(\boldsymbol{\theta}) \quad (6)$$

Where $\hat{\boldsymbol{\theta}}$ represents the optimal deterministic solution and D_{θ} the feasible domain search.

The Eq. 6 provides the deterministic optimization solution in model updating analysis and henceforward one can introduce an initial uncertainty quantification making some assumptions for the residual discrepancy variable ϵ . A key aspect comes from the fact that the solution of dynamic's ODE is a non-linear problem regarding a probability density functions (pdf) in a multivariate parameters analysis context, i.e., the relationship between input and output pdfs is not straightforward. Thus an important conjecture made is that $\epsilon \sim \mathcal{N}(E[\epsilon] = \mathbf{0}, \sigma_{\epsilon}^2 \mathbf{I}_N)$, i.e., ϵ has a Gaussian pdf with zero mean error and unknown variance as well as the updating variable $\boldsymbol{\theta} \sim \mathcal{N}(E[\boldsymbol{\theta}] = \hat{\boldsymbol{\theta}}, \boldsymbol{\Sigma}_{\theta})$ is assumed to be a multivariate Gaussian distribution with known covariance matrix but on the other hand the expected value is assumed to be the optimal solution.

The hypothesis made concerning measurement errors and the set of measured modal data allows one to obtain parameter uncertainty estimates by computing its covariance matrix (Aster et al, 2013) and (Beck & Arnold, 1977). An estimate of the covariance is obtained by means of the linearisation around the optimum solution $\hat{\boldsymbol{\theta}}$. The covariance matrix is required to build the likelihood function.

$$\operatorname{Cov}(\boldsymbol{\theta})_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} = \boldsymbol{\Sigma}_{\theta} = (\mathbf{J}^T \mathbf{J}) \mathbf{J}^{-1} \boldsymbol{\Sigma}_{nn} \mathbf{J} (\mathbf{J}^T \mathbf{J})^{-1} \quad (7)$$

$\mathbf{J}$ is the Jacobian operator, $\mathbf{J} \in \mathbb{R}^{2N \times N_p}$ where N represents the amount of natural frequencies and damping ratios estimates based on vibration modes performed at modal analysis and N_p the number of updating variables, or parameters. It is computed as formulated by Eq. 8 using an in-house finite difference algorithm based on central difference.

$$\mathbf{J}(\boldsymbol{\theta}) = \left. \frac{\partial \mathbf{y}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}} \quad (8)$$

The diagonal variance $\boldsymbol{\sigma}_{diag}$ is an hypothesis made about the variance of the experimental data set which is formulated by Eq. 9. It states that each natural frequency and damping ratio estimated are independent from any other ω_k and ζ_k , where k denotes de k -th vibration mode, i.e., there is no cross variance between any estimation performed for both modal properties.

$$\sigma_{diag} = \begin{bmatrix} \sigma_\omega & \mathbf{0} \\ \mathbf{0} & \sigma_\zeta \end{bmatrix}_{2N \times 2N}, \quad \sigma_\omega = \begin{bmatrix} \sigma_{\omega_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\omega_N} \end{bmatrix}, \quad \sigma_\zeta = \begin{bmatrix} \sigma_{\zeta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_{\zeta_N} \end{bmatrix} \quad (9)$$

After computing the optimal solution $\hat{\theta}$ and the covariance matrix Σ_θ one can assess the uncertainty level based on a probabilistic approach. Moreover, Monte Carlo analysis was chosen to infer about uncertainty propagation in order to create a confidence range that helps the decision making about model calibration quality. 1000 experiments are performed on Matlab with a function designed by Tim Benhman called *rmvnrnd* which generates pseudo-random vectors drawn from a truncated multivariate normal distribution. For further details refer to (Rmvnrnd MathWorks).

2.2 Damage identification

In the present work structural damage is artificially built by placing masses at specific positions along the beam under analyses. The Table 2 shows the addition of four different masses, Figure 5a, on two specific sections of the aluminium beam.

The theoretical approach to identify damage is equivalent to that of model calibration procedure detailed in Section 2.1, although at this stage there is no uncertainty evaluation. Damage assessment is performed by inverse analysis and the earlier updating variable is maintained at its optimal solution along this procedure, so $\theta = \hat{\theta}$.

Experimental data are now based on damage analysis d^{dmg} as well as FE model predictions y^{dmg} , both depending on damage scenario as shown further down.

The optimization analysis continue to be performed on Matlab but in this case an in-house Particle Swarm Optimization (PSO) (Kiranyaz et al, 2014) algorithm with an addition of constriction factors to improve the convergence velocity and stability, suggested by (Clerc & Kennedy, 2002), is adopted to infer about possible damages in the structure.

There are two updating variables, which are: position X_m along the aluminium beam length and magnitude of mass M_m , both related to the respective mass glued on structure, Table 2. Thus one can assume $\eta = [X_m \ M_m]^T$. A key aspect of η 's optimization comes from the exhaustive search by PSO which sorts pseudo-random vectors drawn from the standard uniform distribution for both updating variables and make it less prone to get stuck at local minima of cost function.

Along the optimization procedure the global mass matrix is iteratively modified by PSO where M is the initial global mass matrix and $\Delta M(\eta)$ represents the incremental global mass matrix performed by PSO. For the purposes of dynamic analysis the mass is treated as an infinitesimal mass located at beams surface with only vertical influence and no mass moment of inertia.

Once a mass is glued on the structure the proportional damping hypothesis diminishes its compliance with the real dynamic behaviour. In general the grater becomes the mass magnitude, greater is its non-proportional response. Even so the global damping matrix remains

assembled as governed by proportional damping model. The stiffness matrix is constant during optimization. Eq. 10 summarizes this conjecture.

$$\mathbf{M}(\boldsymbol{\eta}) = \mathbf{M} + \Delta\mathbf{M}(\boldsymbol{\eta}), \quad \mathbf{D}(\hat{\boldsymbol{\theta}}, \boldsymbol{\eta}) = \hat{\alpha} \mathbf{M}(\boldsymbol{\eta}) + \hat{\beta} \mathbf{K}(\hat{E}) \quad (10)$$

As described earlier for *lsqnonlin*, PSO pursuits the optimal solution of $\boldsymbol{\eta}$ by gradually decreasing the residual discrepancy of experimental damage data $\mathbf{d}^{dmg}$ and FE model prediction $\mathbf{y}^{dmg}$ based on a cost function, which at this stage is the same equation described at Eq. 5. Thus, the inverse problem can be stated as Eq. 11.

$$\mathbf{f}(\boldsymbol{\eta})^{dmg} \mapsto \mathbf{G}(\hat{\boldsymbol{\theta}}, \boldsymbol{\eta}) = \mathbf{d}^{dmg} + \boldsymbol{\varepsilon} \quad (11)$$

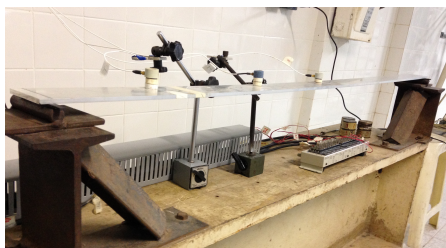
Thus the same conjecture made with ϵ is now applied to $\boldsymbol{\varepsilon}$, which is $\boldsymbol{\varepsilon} \sim \mathcal{N}(0, \sigma_{\boldsymbol{\varepsilon}}^2)$, i.e., $\boldsymbol{\varepsilon}$ has a Gaussian pdf with zero mean error and unknown variance.

After computing the optimal solution $\hat{\boldsymbol{\eta}}$ one can evaluate how effective damage identification was by comparing FE model predictions for natural frequencies and FRF with their measured counterparts.

3 EXPERIMENTAL PROCEDURE

3.1 Modal analysis

A simply supported aluminium beam is taken to perform model updating analyses as shown in Figure 1a. The beam's physical properties are: 1464 mm length; rectangular cross section with 76,2 mm of height and 6 mm width and 2700 kg/m³ mass density. The beam support is detailed in Figure 1b



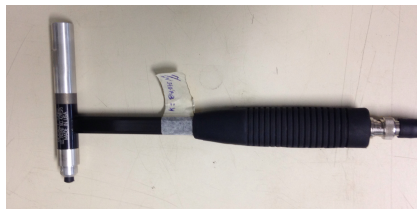
(a) Aluminium beam



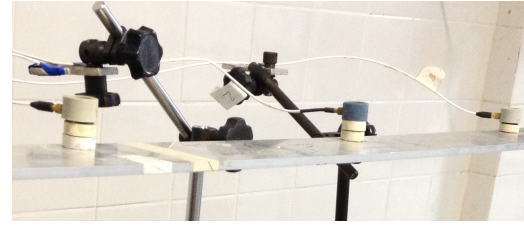
(b) Beam support

Figure 1: Experimental set-up

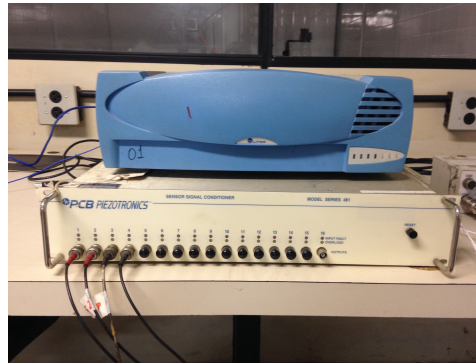
Model 336C and 336C31 PCB Piezotronics accelerometers are used in dynamic modal analysis to collect the acceleration responses of the structure, Figure 2b. An impact hammer, model 086C01 PCB Piezotronics, Figure 2a, provides the excitation to perform the test. The acquisition system is formed by model 481 PCB Piezotronics signal conditioner and model



(a) Impact hammer



(b) Accelerometers



(c) Data acquisition system

Figure 2: Experimental instruments.

APS2000 data acquisition board, Figure 2c. A model AqDados7 Lynx Tecnologia data acquisition software is used to communicate with both equipments. The masses of the presented accelerometers were measured and added into numerical model.

A SIMO analysis (single input multiple output) is adopted to perform the modal test. The excitation is provided with non correlated impact of the hammer. Each analysis considers twenty impacts which are performed with thirty seconds mean gap. Measuring frequency range is set as 0-250 Hz and the signal is low-pass filtered with a cut-off frequency of 500 Hz as well as data acquisition frequency set to 1000 Hz. There were performed seven analysis with approximately ten minutes duration each denoted by "Analysis 1", "Analysis 2", etc.

The accelerometers were placed in order to record until the sixth vibration mode. Figure 3 shows an illustration of the accelerometers and impact positions along the structure.

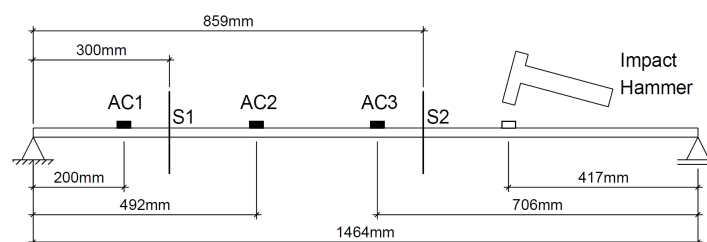


Figure 3: Draw of accelerometers and impact hammer positions along the structure

The vibration data is post-processed by two in-house programs: the "Analizador", (de Andrade, 1997), provides an estimation of natural frequencies and the "STFT", (Bucher, 2001), yields an estimation of damping ratios and natural frequencies.

The large set of measured modal data was analysed with Matlab R2014b. The metric to

perform uncertainty investigation of some measured data ν was the expected value $\bar{v} = E[v_k]$ and its standard deviation $\sigma_\nu^2 = E[(\bar{v} - v_k)^2]$, resulting in $v = \bar{v} \pm \sigma$ for each vibration mode. Table 1 details the modal results.

Table 1: Results of modal data.

Vibration mode	Natural frequency		Damping ratio	
	ω [Hz]	σ [Hz]	ζ [%]	σ [%]
1st	6,78	$\pm 0,01$	0,40	$\pm 0,03$
2nd	26,93	$\pm 0,01$	0,20	$\pm 0,01$
3rd	60,46	$\pm 0,04$	0,27	$\pm 0,04$
4th	106,59	$\pm 0,10$	0,38	$\pm 0,03$
5th	166,46	$\pm 0,24$	0,40	$\pm 0,07$
6th	242,49	$\pm 0,52$	0,44	$\pm 0,08$

Expanding the statistical information showed in Table 1, Figure 4 illustrates every single par of $(\omega, \zeta)_{exp}$ obtained for each vibration mode. For better understanding the results are showed in a matrix way where the principal diagonal represents the histogram (a_{11} for natural frequencies and a_{22} for damping ratios) and the opposite diagonal correspond the scatter plot between variables ($a_{12} = [\omega \times \zeta]$ and $a_{21} = [\zeta \times \omega]$). The matrix is symmetric.

One may look at positions a_{11} , i.e., the histogram of natural frequencies, for all sub-figures of Figure 4 and notice that although it is not a probability density function (pdf) these distributions are very similar to the Gaussian one. On the other hand if one analyse the damping ratio's histogram, i.e. positions a_{22} this conclusion does not fit as well as for natural frequencies where sometimes it may look like a bimodal distribution (Figure 4b), a poisson distribution (Figure 4e) or even seems not to be any distribution (Figure 4a). For further details of pdfs refer to (Papoulis, 1990).

According to (Johnson & Wichern, 2007), the analysis of the scatter plot, positions a_{12} or a_{21} , allows one to infer about the relationship between variables. For example, the specific value of 0,4% of first vibration mode (position a_{21} of Figure 4a) seems be associated with almost all natural frequencies estimated in this case, i.e., (6,76 - 6,8) Hz range of values. On the other hand, greater values of ω of 5th mode provides lower values of ζ and vice versa, (position a_{21} of Figure 4e). Thus, at this point, "independently" means that one variable cannot be predicted with any accuracy from the second measurement.

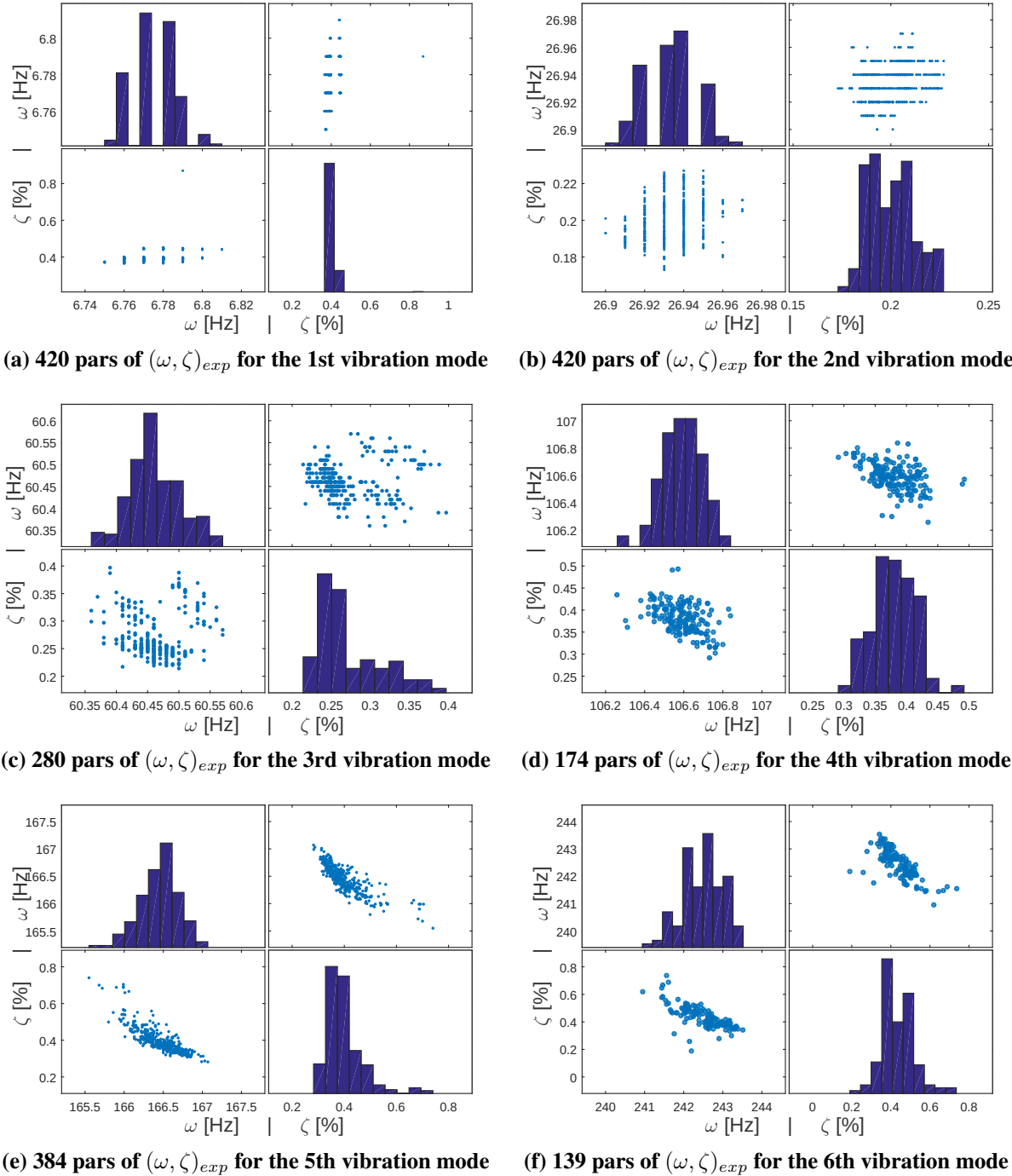


Figure 4: Histogram and scatter plot of natural frequencies and damping ratios for each vibration mode.

Expanding this concept we can suggest that 5th and 6th vibration modes have dependent modal properties and the 1st, 2nd and 3rd independent ones. The last one seems indicate some dependency, but it is not an easy definition. Therefore, the scatter plot of 4th mode is initially considered not conclusive.

One must state that these conclusions were made only considering the observation of experimental data behaviour. In order to state robust conclusions it is necessary go further in

multivariate analysis aspects such as hypotheses testing and euclidean distance.

3.2 Damage analysis

In this study, damage identification analyses follow the same standard procedure described at Section 3.1, i.e., the SIMO analysis, experimental set-up, instruments and the procedure of post-processing were performed as earlier detailed.

Damage is simulated by the addition of four different masses, Figure 5a, on two specific sections of the structure as shown in Table 2. Figure 5a shows the added masses and Figure 5b the damage scenario A1.

One should remark that each case has only one lumped mass placed over the structure. The sections S1 and S2 are illustrated in Figure 3.

Table 2: Damage scenarios

Damage index	Added mass	Position	Section
Scenario A1	158,4 g	300 mm	S1
Scenario A2	49,6 g	300 mm	S1
Scenario B1	94,5 g	859 mm	S2
Scenario B2	36,9 g	859 mm	S2



(a) Added masses. 158,4 g, 94,5 g, 49,6 g and 36,9 g.



(b) Added mass at S1 section.

Figure 5: Experimental set-up.

After data's post-processing the variation of natural frequencies of damaged structure are assessed by comparing both frequencies (damaged and undamaged ones) according with the

following formula: $dif = \left(1 - \frac{\omega^{dmg}}{\omega^{udmg}}\right) \times 100\%$. Damping ratios were not evaluated in this stage of work. Table 3 details the natural frequencies for both situations.

Table 3: Natural frequencies of undamaged and damaged structure. Units: ω [Hz] and dif [%].

Mode	Undamaged	Scenario A1		Scenario A2		Scenario B1		Scenario B2	
	ω^{udmg}	ω^{dmg}	dif	ω^{dmg}	dif	ω^{dmg}	dif	ω^{dmg}	dif
1st	6,78	6,59	2,7	6,71	0,9	6,50	4,1	6,66	1,7
2nd	26,93	25,19	6,5	26,31	2,3	26,57	1,3	26,76	0,6
3rd	60,46	57,16	5,5	59,14	2,2	59,16	2,2	59,84	1,0
4th	106,59	104,32	2,1	105,21	1,3	102,61	3,7	104,48	2,0
5th	166,46	165,25	0,7	165,78	0,4	165,46	0,6	165,87	0,4
6th	242,49	234,54	3,3	238,73	1,5	232,9	4,0	237,82	1,9

The analysis of Table 3 allows one to conclude that all natural frequencies damaged ω^{dmg} are lower than the ω^{udmg} , i.e., all dif values are positive. This behaviour was already expected based on the fact that if we look at theoretical natural frequency formula for one degree of freedom problem (1DoF): $\omega_{1DoF} = \sqrt{\frac{K_{1DoF}}{M_{1DoF}}}$, if $\uparrow M_{1DoF} \sim \downarrow \omega_{1DoF}$ because K_{1DoF} is constant. The multi degree of freedom (MDoF) conclusion is straightforward.

4 MODEL UPDATING

After performing the experimental and computational procedures described in Section 3 and Section 2 respectively, the results from model calibration are detailed in Section 4.1 and the damage assessment responses are described in Section 4.2.

4.1 Model calibration

There are three outputs for model calibration, as follows:

- (i) Damping curve ($\zeta \times \omega$) plot.
- (ii) Scatter plot of model updating variables $\theta = [E \ \alpha \ \beta]^T \sim \mathcal{N}(\hat{\theta}, \Sigma_{\theta})$.
- (iii) FRF for accelerometer's position denoted by AC1.

The uncertainty propagation of model updating variables was performed by Monte Carlo analysis with 1000 realizations. It demonstrates the influence of unknown structural variables, denoted in this work as $\theta = [E \ \alpha \ \beta]^T$, on FE model predictions. As earlier stated this procedure helps the decision making about model calibration quality. For further details refer to Section 2.1.

The deterministic model updating analysis was performed with *lsqnonlin*, a pre-defined function of Matlab, and yields the optimal solution $\hat{\theta}$ detailed in Table 4

Table 4: Optimal solution $\hat{\theta}$ performed at deterministic model updating analysis for model calibration

E [GPa]	α	β
76,7	3,14e-1	9,14e-6

One can see in Figure 6 that up to the fifth vibration mode their natural frequencies, damping ratios and its respective standard deviations are within the 98% credibility interval. On the other hand the last computed vibration mode is outside the simulated region.

It is interesting to remark that the optimal solution (red line) and the mean solution of confidence range (dashed black line) are almost identical. Despite the fact that one of the hypotheses of the present study is $\theta \sim \mathcal{N}(\hat{\theta}, \Sigma_{\theta})$, which is highlighted by Figure 6, it is not always true, i.e., $E[\theta] = \hat{\theta}$. Refer to Figure 8.

This behaviour can be explained by the fact that the map from the parameter space to model response is non-linear. It modifies the straightforward relationship between the output and input pdfs.

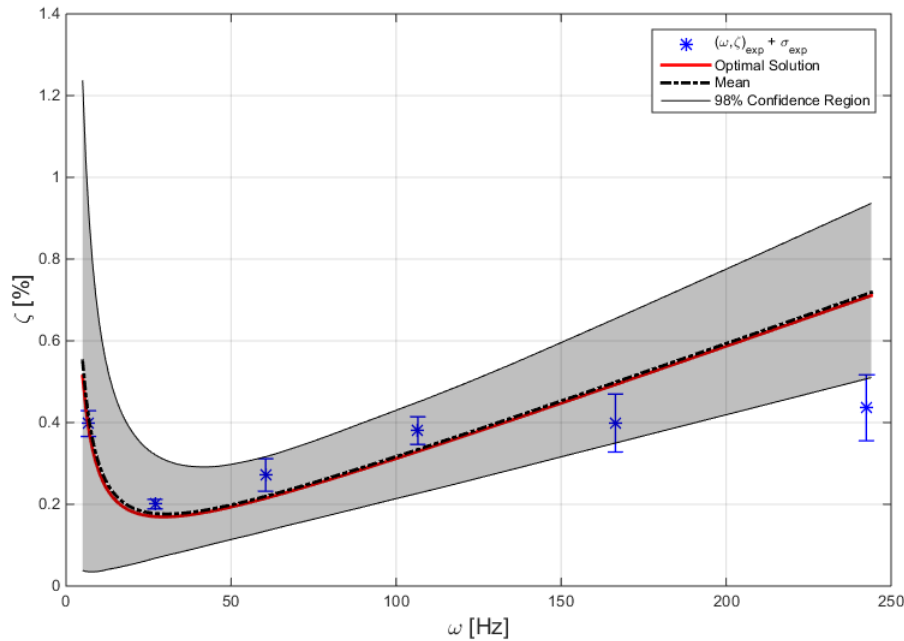


Figure 6: Damping curve ($\zeta \times \omega$) plot

Figure 6 allows one to suggest that, for this specific case, the proportional damping hypothesis does not represent the best formulation for high order frequencies because proportional damping model provides a continuously increasing function and the experimental data seems to be approximately constant for 4th up to 6th vibration modes.

It should be remarked that this conclusion is based on observations concerning the damping behaviour. It would be necessary go further on damping model formulations and maybe sensitivity analyses in order to state that this damping model does not fit high order frequencies in this case.

Thus, looking towards model calibration one can conclude that FE model prediction provides good results for lower natural frequency modes, i.e., up to the fifth vibration mode.

Figure 7 details the scatter plot of θ . As earlier stated, this is a matrix plot, which shows the histogram of variables in the main diagonal and its correlation outside the diagonal, i.e., positions a_{11} , a_{22} and a_{33} represent respectively the E , α and β histograms; a_{21} demonstrates the correlation between $(E \times \alpha)$, a_{31} illustrates the correlation between $(E \times \beta)$ and finally a_{32} shows the correlation between $(\alpha \times \beta)$. The matrix is symmetric.

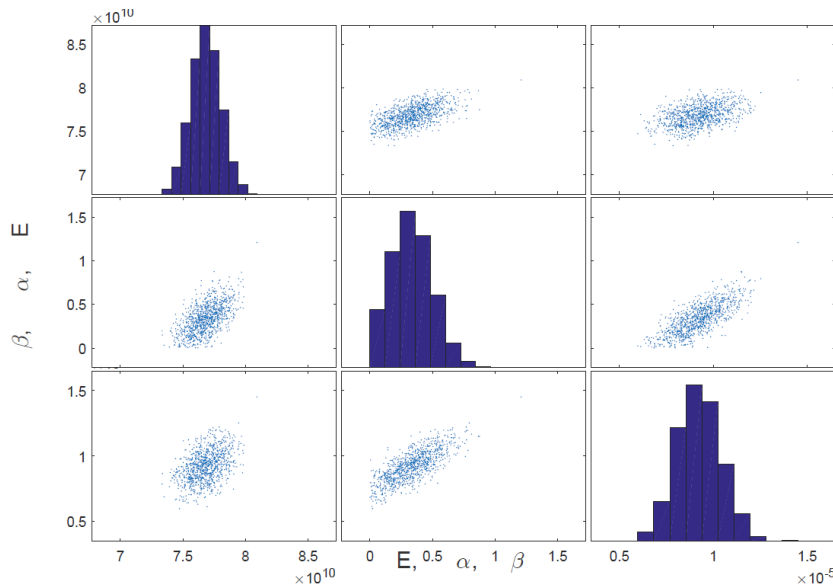


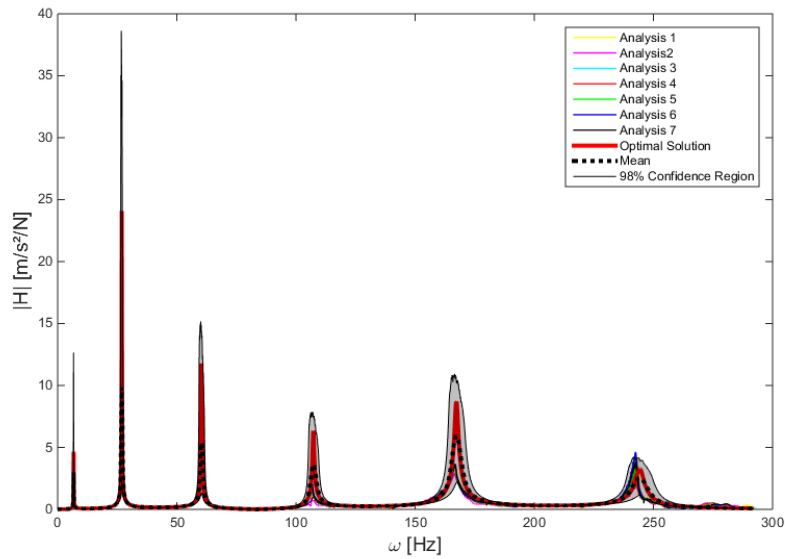
Figure 7: Scatter plot of model updating variables $\theta \sim \mathcal{N}(\hat{\theta}, \Sigma_{\theta})$

In this perspective the Young's modulus and the α and β coefficients of proportional damping model have a range of possible values. One can see that the mean values presented in Figure 7 are exactly the same values detailed in Table 4. It stems from the fact that one assumes the hypothesis: $\theta = [E \ \alpha \ \beta]^T \sim \mathcal{N}(\hat{\theta}, \Sigma_{\theta})$ where we impose that the mean value must be the optimal solution achieved by model updating analysis.

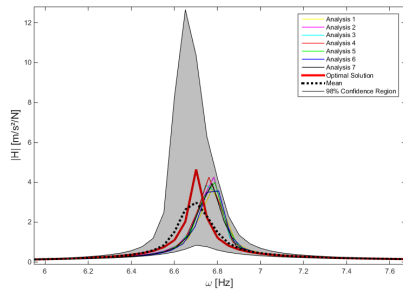
Moreover, Figure 7 allows one to infer about the relationship between variables. We can note that there is an increasing behaviour of (position a_{32} of Figure 7), where greater values of α provides greater values of β . It suggests that these variables have a dependent relationship.

The relation between $(E \times \alpha)$ and $(E \times \beta)$ seems to indicate some level of dependency.

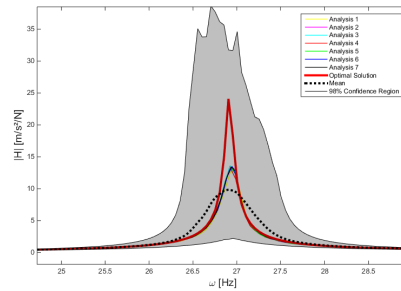
Here again "independently" means that one variable cannot be predicted with any accuracy from the second measurement.



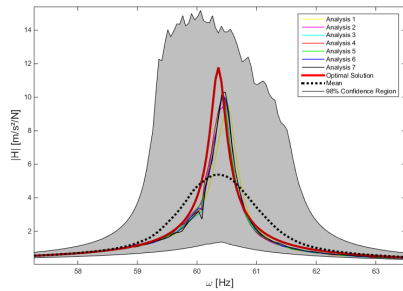
(a) AC1 FRF



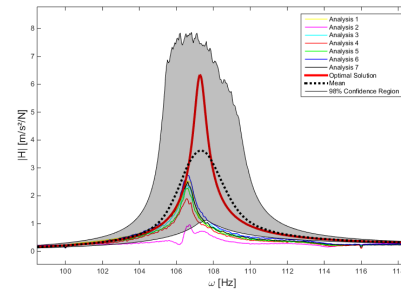
(b) Detail of 1st vibration mode region of AC1 FRF



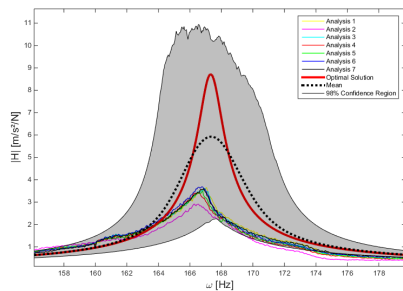
(c) Detail of 2nd vibration mode region of AC1 FRF



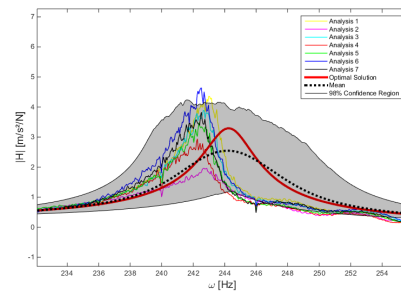
(d) Detail of 3rd vibration mode region of AC1 FRF



(e) Detail of 4th vibration mode region of AC1 FRF



(f) Detail of 5th vibration mode region of AC1 FRF



(g) Detail of 6th vibration mode region of AC1 FRF

Figure 8: Frequency response function for AC1 accelerometer

It is necessary remark that the negative range of probability distribution was ignored because the coefficients α and β must be positive values. The Monte Carlo simulation was performed with a Matlab function called *rmvnrnd* designed by Tim Benham (Rmvnrnd Math-Works). For further details refer to Section 2.1.

The FRF for AC1's position is shown in Figure 8 with some detailed information about the resonance range of each vibration mode in Figure 8b up to Figure 8g.

Figure 8a allows one to see that all experimental FRF data are within the credibility interval. Moreover, even if there are some shifting at the FRF's amplitude or natural frequency for a specific mode, in general the FE model simulated the experimental data by approximations either with the optimal solution or the mean one.

The behaviour of uncertainty quantification of FRF is similar to the one evaluated at damping curve, Figure 6. As greater becomes the frequency, increases the uncertainty levels of FE model predictions. This is reflected in the credibility interval shown in both figures.

A key aspect about this study is that one could produce good results with the simplest statistical uncertainty inference. It was able to enlarge the FE simulation scenario due to the fact that one can take model uncertainties into account for model calibration analyses.

4.2 Damage identification

The damage system here is artificially built considering the placement of masses at some specific locations. Therefore, the damage identification process is phrased as an optimization problem in which the position X_m and mass M_m are parameters of interest. Thus, $\eta = [X_m \ M_m]^T$.

There are two outputs for damage assessment, as follows:

- (i) Deterministic solution for model updating variables η .
- (ii) FRF for accelerometer position denoted by AC1.

The optimization analysis for damage identification was performed on Matlab with an in-house Particle Swarm Optimization algorithm (PSO) (Kiranyaz et al, 2014) with an addition of constriction factors to improve the convergence velocity and stability suggested by (Clerc & Kennedy, 2002).

PSO performs an exhaustive search by sorting pseudo-random vectors η from an uniform pdf for each iteration which makes it less prone to get stuck at local minima of the cost function. Thus, it is hardly like that exist two identical values after perform PSO algorithm two times. In this perspective the response should be understood as a non-unique solution, which allows small fluctuations on their values if we run the PSO algorithm several times repeatedly.

Considering the stochastic properties of PSO described above, one of possibles optimal solutions for damage identification is detailed in Table 5 depending on damage scenario and according with the following formula: $diff = \left(1 - \frac{v^{num}}{v^{exp}}\right) \times 100\%$, where v denotes the parameter of interest.

Generally speaking, one notices that despite the fact that the model frequencies are quite close to the measured ones the estimated parameters X_m and M_m present very large levels

Table 5: Optimal solution $\hat{\eta}$ performed at deterministic model updating analysis for damage assessment in scenario A1. Units: X_m [mm], M_m [g], $diff$ [%] and ω [Hz].

Damage Scenario	Position X			Magnitude M		
	X_m^{exp}	X_m^{num}	$diff_{X_m}$	M_m^{exp}	M_m^{num}	$diff_{M_m}$
A1	300	315	+5,0	158,4	147,5	-6,9
A2	300	231	-23,0	49,6	44,2	-10,9
B1	859	835	-2,8	94,5	96,1	+1,8
B2	<i>False Positive</i>					

of discrepancies when compared to the true values. The damping ration predictions are not consistent with measured data. This indicates that a revision of the damping model is required here.

One can see that the best damage assessments did occur where the magnitude of masses were greater, scenarios A1 and B1. The relative errors $diff$ for case B1 are less than the ones for case A1. It seems that the greater is the magnitude of mass, better its convergence becomes, although the best approximations of natural frequencies were performed in scenario A2.

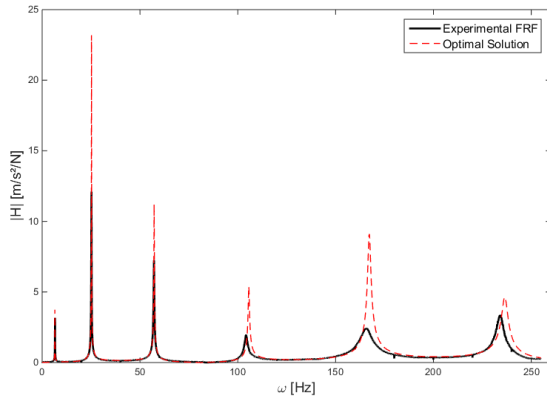
It should be remarked that PSO provided a false positive response during the optimization procedure of damage scenario B2, i.e., the algorithm found a different position for the same mass. In order to avoid symmetrical results regarding the simply supported beam, the position domain for identification of X_m was earlier stated to be the second half of its length. Nevertheless, the algorithm yielded the following response: $X_m = 1320 \text{ mm}$ and $M_m = 40 \text{ g}$, where it was expected: $X_m = 859 \text{ mm}$ and $M_m = 36,9 \text{ g}$. PSO did not predict the real damage scenario for B2 configuration.

One must alert that there are no assurance, with the hypotheses made in Section 2, that the other damage scenarios do not have false positives. To achieve the presented results PSO was performed ten times per analysis. This procedure does not have the robustness to state that any solution is a unique response. It would be necessary go further on damage assessment hypotheses with sensitivity analyses and probabilistic tools in order to avoid false positives and false negatives.

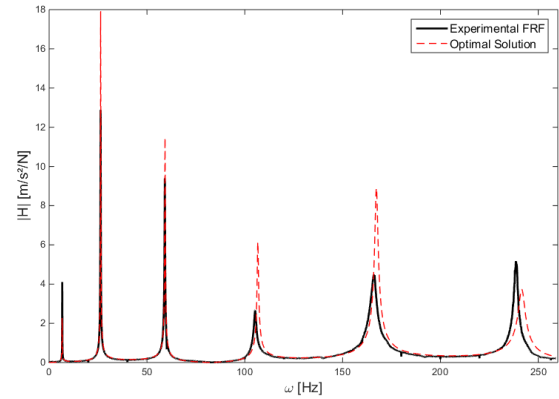
Finally, Figure 9a up to Figure 9c present the FRFs for the accelerometer position denoted by AC1. The behaviour of FRF of damage scenario B2 is very similar to the other ones but it is not shown because it is a false positive.

The resonance regions of FRF graphs confirm the natural frequencies approximation behaviour stated by Table 5. Even if there are some shifting at the FRF's natural frequency peak for some specific mode, in general the FE model presents good approximations between experimental and computational FRFs for all damage scenarios.

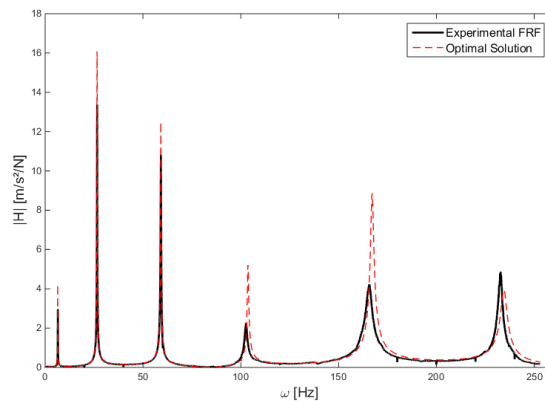
Nevertheless the FRF's amplitude at the resonance range and its peak were poorly fitted for all damage scenarios. The computational model provides higher amplitudes than their measured counterparts. This phenomena can be explained by the inability of optimization procedure



(a) Damage scenario A1 FRF for AC1 position



(b) Damage scenario A2 FRF for AC1 position



(c) Damage scenario B1 FRF for AC1 position

Figure 9: Frequency response function for AC1 accelerometer in damage scenarios

predict the real scenario considering the hypotheses made. It seems be necessary go further in some theoretical hypothesis in order to achieve accurate results.

5 CONCLUSION

Although deterministic model updating strategies are of great value, they cannot provide information about the parameter uncertainties. Nevertheless, model linearisation around the maximum likelihood inference allows one to determine an estimate for the covariance matrix of the unknown parameters. This work uses the most basic strategy in which an estimator for covariance of the system parameters is also an output of the model updating analysis.

The covariance of model parameters can be naturally used for uncertainty quantification analysis to evaluate the uncertainties in response predictions. This is particularly useful when dealing with modal damping factors which are known to present large variability.

One could produce good results with the simplest statistical uncertainty inference. The applied method was able to enlarge the FE simulation scenario due to the fact that one can take model uncertainties into account for model calibration analyses.

In the perspective of damage identification the FE model could not achieve accurate estimations for all damage scenarios, although the maximum difference between experimental

and numerical natural frequencies were less than 7%. For scenarios A1 and B1, the ones with greater magnitude of mass, the FE model provides good results with a maximum misfit between experimental and numerical values around 7% and 5% for magnitude and position predictions respectively.

The FE model achieved an accurate estimation of natural frequencies for almost all vibration modes, although their amplitude peaks were poorly fitted. The computed FRF provided always a higher amplitude when compared with its experimental measurement. One of many explanation can be that the cost function used to perform damage identification did not provide the robustness to achieve accurate results during the optimization procedure.

PSO provided a false positive response during the optimization procedure of damage scenario B2. Based on the fact that the FRF analysis did not increase the level of information in order to differentiate the false positive, it seems be necessary go further on damage assessment hypotheses with sensitivity analyses and probabilistic tools in order to avoid false predictions.

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