CILAMCE 2017
Florianópolis - Santa Catarina - Brazil

EVALUATION OF ELASTIC PROPERTIES OF ULTRA HIGH PERFORMANCE CONCRETE REINFORCED WITH STEEL FIBER: MEAN-FIELD HOMOGENIZATION VERSUS FE HOMOGENIZATION

Marcelo Silva Medeiros Júnior, Ph.D.

marcelomedeiros@ufc.br

Departamento de Engenharia Estrutural e Construção Civil, Universidade Federal do Ceará

Campus do Pici - Bloco 728, 60440-900, Fortaleza, CE - Brasil

Evandro Parente Júnior, D.Sc.

evandro@ufc.br

Laboratório de Mecânica Computacional e Visualização, Universidade Federal do Ceará.

Campus do Pici, 60440-900, Fortaleza, CE, Brasil.

Abstract. *Ultra-High Performance Concrete Fiber-Reinforced Concrete (UHPFRC) is emerging as a key component in the next generation infrastructure due to its capacity to fulfill higher demands on strength, durability, toughness, corrosion resistance, and energy absorption. Progress in reinforcing fibers design, cementitious matrices, and enhancements in the bonding between fiber and matrix has played a crucial role in achieving performance characteristics that are unprecedented in conventional concretes. The improved ductility and energy absorption capacity of UHPFRC is mainly attained through fiber bridging at cracked planes, therefore the fiber-matrix interactions at a microscopic level must be taken into account during its design phase. In this paper an elastic micromechanical analysis of steel-fiber reinforced concrete was carried out using analytical homogenization schemes. The results were later compared to a finite element model (FEM) that considered the actual fiber geometry with its curved ends (hooked-end fiber). The results from the FEM displayed strong agreement with the micromechanics models developed for elliptical inclusions. The results pointed out to the feasibility of using elliptical inclusions with an equivalent aspect ratio in lieu of the actual fiber geometry when calculating strain concentration tensors and their relationship to the overall composite stiffness.*

Keywords: *UHPFRC, Hooked-end fibers, Micromechanics, Eshelby Tensors.*

1 INTRODUCTION

The idea of using discontinuous, short, ductile fibers to reinforce brittle materials is not new, yet the usage of steel fibers to reinforce cementitious composites has presented steady growth in the past 30 years (Yudenfreund et al., 1972). The current uses of steel fibers in concrete or shotcrete include shear reinforcement in structural elements, blast-resistant structures, precast products and tilt-up construction, tunnel linings and slope stabilization works, repairs and rehabilitation, and slab-on-grade applications (Yoo and Banthia, 2016).

Ultra-High Performance Fiber-Reinforced Concrete (UHPFRC), formerly known as Reactive Powder Concrete, is emerging as a key component in the next generation infrastructure granted that it can fulfill higher demands on strength, durability, toughness, corrosion resistance, and energy absorption. This type of concrete develops an extremely high compressive strength (in excess of 150 MPa) and fracture energies,(G_f), of up to 40 kJ/m^2 (Richard and Cheyrezy, 1995). To achieve such a high strength material, a low water-to-binder ratio (usually W/B = 0.2) is employed additionally to ultra-fine admixtures and sometimes heat treatment during curing. This new class of concrete is designed by close packing density and use of pozzolanic mineral admixtures. In consequence of that, it has very low porosity, especially continuous porosity. Studies using Scanning Electron Microscope (SEM) indicated that the structure of hardened paste was very dense due to very low W/B, hydration of cement and the pozzolanic effect of silica fume and mineral admixtures (Van Tuan et al. 2011).

Progress in reinforcing fibers design, and enhancements in the bonding between fiber and matrix has played a crucial role in achieving performance characteristics that are unprecedented in conventional concretes (Naaman, 2003). The improved ductility and added energy absorption capacity is mainly attained through fiber bridging at cracked planes, therefore the fiber-matrix interactions at a microscopic level must be taken into account during its design phase.

Compared with other types of deformed steel fibers commercially available in the market (such as the corrugated steel fiber), the hooked-end steel fiber has a relatively simple geometry, consisting of one straight shaft and two hooks at both ends as seen in Figure 1. As it happens in any fiber reinforced composite, the fibers inside the UHPFRC will be invariably misoriented and will follow different orientation distributions depending on the geometry of the molded part, pouring procedure and materials used, amongst other factors (Barnett et al., 2010; Suuronen et al., 2013). However, aligned-fiber properties are always calculated as a prelude to modeling the effects of misorientation and misalignment in the fiber distribution (Doghri and Tinel, 2005).

In this paper an elastic micromechanical analysis of steel-fiber reinforced concrete was carried out using analytical homogenization schemes. The results were later compared to a finite element model (FEM) that considered the actual hooked-end fiber geometry. The results from the FEM displayed strong agreement with the micromechanics models developed for elliptical inclusions. Based on these promising results, the conclusion was drawn that elliptical inclusions with an equivalent aspect ratio could potentially replace the actual fiber geometry when calculating strain concentration tensors and their relationship to composite stiffness.

Figure 1: Hooked-end Steel Fiber (source: www.bekaert.com)

2 THEORETICAL BACKGROUND

UHPFRC can be taken as a macroscopically homogeneous material. However, it is clear that on a microscopic level it presents a heterogeneous microstructure. Hence, it's important to investigate how this microstructure affects the material behavior on the macroscopic level. For an explicit description of the material's heterogeneity it is common to employ the idealized defects or inhomogeneities, such as inclusions, cracks or micro-voids. In order to achieve a homogeneous macroscale representation of the material with spatially constant effective properties, it is necessary to homogenize the heterogeneous microstructure.

In a micro-macro homogenization approach, a material point on the macroscopic level can be related to a representative volume element (RVE) V on the microscopic level where stresses and strains prevail as varying micro-fields. The macro-stresses and macro-strains which characterize the mechanical state of the macroscopic material point can be seen as volumetric averages of the microscopic fields. V must be chosen large enough to contain many fibers, but small compared to any length scale over which the average loading or deformation of the composite varies. Denoting the position vector by $\boldsymbol{x}$, the volume-average stresses and strains are defined as the average of the pointwise stress $\sigma_{ij}(\boldsymbol{x})$ and $\epsilon_{ij}(\boldsymbol{x})$ over the volume V.

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_V \sigma_{ij}(\boldsymbol{x})dV \tag{1}$$

$$\bar{\epsilon}_{ij} = \frac{1}{V}\int_V \epsilon_{ij}(\boldsymbol{x})dV \tag{2}$$

It is convenient to define volume-averaged stresses and strain for the fiber and matrix phases. Considering a two-phase composite consisting of an isotropic elastic matrix that occupies a volume V_m and an inclusion phase (representing the fibers) that occupies a volume V_f where the sum of these two volumes corresponds to the volume V. It is also postulated that

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

these two phases are perfectly bonded at their interfaces. The corresponding volume fractions of matrix and fibers are given by $v_m = V_m/V$ and $v_f = V_f/V = 1 - v_m$ respectively.

It follows that the average fiber and matrix micro-fields (stress or strain) over the respective domains V_f and V_m are related by:

$$\bar{\sigma}_{ij}^{f} = \frac{1}{V_f}\int_{V_f} \sigma_{ij}(\boldsymbol{x})dV \ ; \quad \bar{\sigma}_{ij}^{m} = \frac{1}{V_m}\int_{V_m} \sigma_{ij}(\boldsymbol{x})dV \tag{3}$$

$$\bar{\epsilon}_{ij}^{f} = \frac{1}{V_f}\int_{V_f} \epsilon_{ij}(\boldsymbol{x})dV; \quad \bar{\epsilon}_{ij}^{m} = \frac{1}{V_m}\int_{V_m} \epsilon_{ij}(\boldsymbol{x})dV \tag{4}$$

The relationships between the matrix and fiber averages and the overall averages can be defined in terms of the preceding definitions as:

$$\bar{\sigma}_{ij} = v_f\bar{\sigma}_{ij}^{f} + v_m\bar{\sigma}_{ij}^{m} = v_f\bar{\sigma}_{ij}^{f} + (1 - v_f)\bar{\sigma}_{ij}^{m} \tag{5}$$

$$\bar{\epsilon}_{ij} = v_f\bar{\epsilon}_{ij}^{f} + v_m\bar{\epsilon}_{ij}^{m} = v_f\bar{\epsilon}_{ij}^{f} + (1 - v_f)\bar{\epsilon}_{ij}^{m} \tag{6}$$

The Gauss' divergence theorem can be used to express these two macroscopic quantities by integrals over the boundary ∂V of the averaging domain V. It is assumed that the microscopic fields are differentiable throughout the entire domain, which doesn't hold true for heterogeneous materials with discontinuously varying properties. Considering the equilibrium condition $\sigma_{ik,k} = 0$ and $x_{j,k} = \delta_{jk}$ (where δ_{jk} is the Kronecker delta) the following expression for the pointwise stress is achieved.

$$\sigma_{ij} = \left(x_j\sigma_{ik}\right), k = x_{j,k}\sigma_{ik} + x_j\sigma_{ik,k} \tag{7}$$

It is then possible to obtain a representation of the macro stresses by substituting equation 7 into equation 1. The results is shown as follows,

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_{V} \left(x_j\sigma_{ik}\right), k\, dV = \frac{1}{V}\int_{\partial V} x_j\sigma_{ik}n_k dA = \frac{1}{V}\int_{\partial V} t_i x_j dA \tag{8}$$

where $\boldsymbol{n}$ is a unit vector normal to dA and $\boldsymbol{t}$ is a traction vector. A similar expression for strain can also be developed if the strain tensor ϵ_{ij} is substituted by the corresponding displacement vector $\boldsymbol{u}$ into the definition of macrostrains (equation 2). If the domain V is subjected to a surface displacement $\boldsymbol{u}(\boldsymbol{x})$ the divergence theorem can be applied, resulting in Equation 9. This is called the *average strain theorem* (Hill, 1964):

$$\bar{\epsilon}_{ij} = \frac{1}{2V}\int_{V} \left(u_{i,j} + u_{j,i}\right) dV = \frac{1}{2V}\int_{\partial V}\left(u_i n_j + u_j n_i\right) dA \tag{9}$$

Assuming that this surface displacement boundary condition corresponds to the macro strain $\bar{\epsilon}_{ij}$, it's possible to relate the strain average per phase by means of the strain concentration tensor. This tensor is essentially the ratio between the average fiber strain and the corresponding average in the matrix phase.

$$\bar{\epsilon}_{kl}^{f} = B_{ijkl}^{\epsilon} : \bar{\epsilon}_{kl}^{m} \tag{10}$$

The per-phase average strains are related to the overall macrostrain by:

$$\bar{\epsilon}_{kl}^{m} = [v_f B_{ijkl}^{\epsilon} + \left(1 - v_f\right) I_{ijkl}]^{-1} : \bar{\epsilon}_{kl} \tag{11}$$

$$\bar{\epsilon}_{kl}^{f} = B_{ijkl}^{\epsilon} : [v_f B_{ijkl}^{\epsilon} + \left(1 - v_f\right) I_{ijkl}]^{-1} : \bar{\epsilon}_{kl} \tag{12}$$

Where I designates the fourth order symmetric identity tensor. Different homogenization schemes will differ by the expression of B_{ijkl}^{ϵ} , but in all of them the macro-stiffness tensor of the two phase composite is written as:

$$\bar{C}_{ijkl} = \left[v_f C_{ijkl}^{f} : B_{klmn}^{\epsilon} + \left(1 - v_f\right) C_{ijkl}^{m}\right] : \left[v_f B_{klmn}^{\epsilon} + \left(1 - v_f\right) I_{ijkl}\right]^{-1} \tag{13}$$

Where C_{ijkl}^{f} and C_{ijkl}^{m} are the fiber and matrix stiffness's respectively.

2.1 Mean-Field Homogenization

The multiscale method adopted in this work is the so called hierarchical method (also known as sequential method), where information is transmitted from the microscale to the macroscale (Belytschko and Song, 2010). In a hierarchical approach, the micro-scale is modeled using an RVE, or unit cell, and is homogenized to determine the effective properties for use in an upper scale (meso or macro-scale) analysis.

A plethora of sound work exists for determining the effective properties of composite structures through the use of analytical micromechanics (Benveniste, 1987; Eshelby, 1957; Mori and Tanaka, 1973; Peng et al., 2016) and other semi-analytical, and finite element (FE) approaches (Doghri and Tinel, 2005; Huang et al., 2012; Tian et al., 2015). These micromechanics models provide the necessary input to perform structural FE analysis based on the micro-scale properties using a "bottom-up" approach.

A well-known analytical homogenization scheme is the Mean-Field method which is built upon the concept of an imaginary RVE. The inhomogeneities are treated as separate homogeneous domains in an aggregate of domains. The fields over the subdomains are represented by their averaged values. The complementary problem then is to solve for the interaction of the subdomains and find the new averages that satisfy the boundary conditions (Perdahcıoğlu and Geijselaers 2011). As is typical for mean field micromechanics models, fourth-order concentration tensors relate the average stress or average strain tensors in inhomogeneities and matrix to the average macroscopic stress or strain tensor, respectively. Mean field homogenization schemes are relatively simple to implement in a numerical code and are very efficient from a computational standpoint. The homogenization schemes used in this work are briefly described as follows.

2.1.1 Mori-Tanaka Model

The problem of finding the strain concentration tensor due to a single inhomogeneity in an infinitely large matrix was first derived by Eshelby (Eshelby, 1957). He proposed that the stress concentrated in an inhomogeneity can be equally represented as the stress concentrated by an inclusion undergoing an arbitrary type of transformation strain produced without external forces (Eigenstrain). In this problem, the equilibrium strain and stress are to be determined in the case where a strain is prescribed in a certain volume within an infinitely large homogeneous material. The work of Toshio Mura is indicated as a further reading on the theory behind this model (Mura, 1982).

Except for very simple models (Reuss and Voigt models), all the other mean field homogenization schemes are based on the fundamental work of Eshelby (Eshelby, 1957). The Mori-Tanaka model (Mori and Tanaka, 1973) considers the strain concentration tensor to be equal to that of a single inclusion problem, which in turn means that the inclusions in the RVE, experience the matrix strain as the far-field strain in the Eshelby theory. The strain concentration tensor in Mori-Tanaka's model is given by:

$$H_{ijkl}^{MT} = B_{ijkl}^{\epsilon} = (I_{ijkl} + S_{klmn}\colon [{C_{ijkl}^{m}}^{-1}\colon C_{klmn}^{f} - I_{ijkl}])^{-1} \tag{14}$$

Where S_{klmn} is the Eshelby Tensor (defined in Appendix A). The homogenized macro stiffness is obtained by substituting equation 14 into 13.

2.1.2 Lielens's Model

This model is based on an interpolation of the Mori-Tanaka scheme as a function of the volume fraction of the phases (Lielens, 1999). This approach increases the accuracy for intermediate volume fractions. The strain concentration tensor in Lielens is defined by an interpolative homogenization based on a smooth function $\xi(v_f)$ that satisfies the following conditions:

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

$$\xi(v_f) > 0, \quad \frac{d\xi(v_f)}{dv_f} > 0, \quad \lim_{v_f \to 0} \xi(v_f) = 0, \quad \lim_{v_f \to 1} \xi(v_f) = 1 \tag{15}$$

Consequently Lielens proposed a simple quadratic expression as the interpolation function.

$$\xi(v_f) = \frac{1}{2} v_f (1 + v_f) \tag{16}$$

Lielens strain concentration tensor can be interpreted as a properly chosen interpolation between the upper and lower boundaries estimates of the Hashin-Shtrikman model (Hashin and Shtrikman, 1963). These boundaries can also be defined in terms of Mori-Tanaka concentration tensor as:

$$H_{lower}^{HS} = H_{ijkl}^{MT} \tag{17}$$

$$H_{upper}^{HS} = {H_{ijkl}^{MT}}^{-1} \tag{18}$$

The interpolated strain concentration tensor is given by:

$$H_{ijkl}^{Lielens} = B_{ijkl}^{\epsilon} = \left[\left(1 - \xi(v_f)\right) {H_{lower}^{HS}}^{-1} + \xi(v_f) {H_{upper}^{HS}}^{-1} \right]^{-1} \tag{19}$$

2.1.3 Self-Consistent Model

The Self-Consistent homogenization scheme has been originally developed to provide the equivalent stiffness of polycristals and take into account the interaction of the matrix and the grains following Eshelby's formulation. This approach was originally proposed by Hill (Hill 1964, 1965) and focused on spherical particles and continuous aligned fibers. The application to short-fiber composites was later proposed by Laws and McLaughlin (Laws and McLaughlin, 1979).

Self-Consistent models are also referred to as embedding models, because the properties of the composite are assumed to be the same as the averaged properties of a single particle embedded into an infinite matrix with a given stiffnes C_{ijkl}. Of course these properties are initially unknown; however, numerical solutions are easily obtained by an iterative algorithm. The Self-Consistent strain concentration tensor is given by:

$$H_{ijkl}^{SC} = B_{ijkl}^{\epsilon} = \left[I_{ijkl} + S_{ijkl} \colon \left(C_{klmn}^{-1} \colon C_{mnop}^{f} - I_{ijkl} \right) + I_{ijkl} \right]^{-1} \tag{20}$$

3 FINITE ELEMENT MODEL

A Finite Element analysis was carried out to model a periodic, three-dimensional array of fibers as seen in Figure 2. Seven unit cells (54mm x 28mm x 28mm) were created in which the number of fibers was appropriately chosen to correspond to seven different volume fractions ranging from 1% to 30%. Twenty-node isoparametric elements were used. To determine E_{11} the boundary conditions (normal displacement) of the back, bottom and left faces were set to be fixed. The tangential displacements on all faces were left unconstrained.

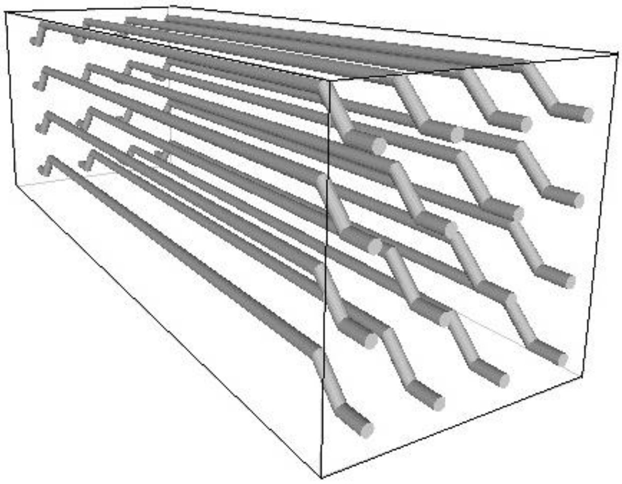


Figure 2 Representative Volume Element used in the FE analysis

The applied strain was very low to maintain the results in the overall linear region. The average stress was computed from the reaction force in the loading direction, divided by the corresponding area of the unit cell. The material properties used in the Finite Element model are shown in Table 1 and are typical of Hooked-end steel fibers and Ultra-High Performance Concrete (matrix).

Table 1: Material Properties

Phase	Young's Modulus [GPa]	Poisson's Ratio
Matrix (UHPC)	60	0.18
Steel Fiber	210	0.30

4 RESULTS AND DISCUSSION

To verify the accuracy of the approach proposed herein, the results from the finite element simulations are compared against the three aforementioned micromechanics models. Figure 3 shows the stress distribution in the unit cell. Due to the fiber array arrangement, the periodicity of the stress field becomes evident.

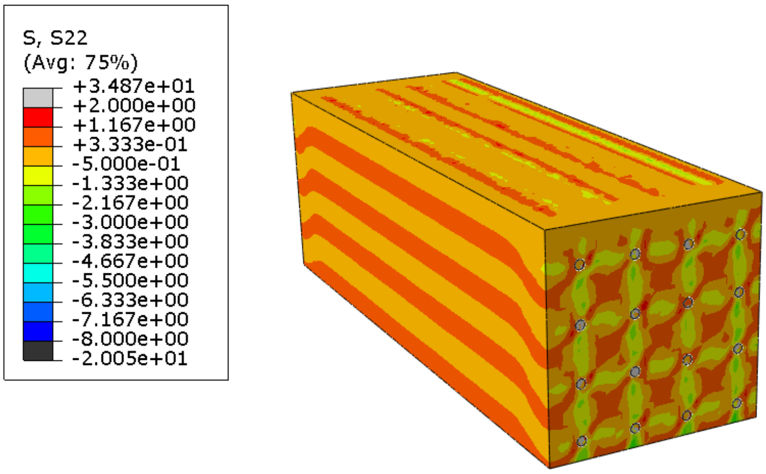


Figure 3 Finite Element results of one of the unit cells

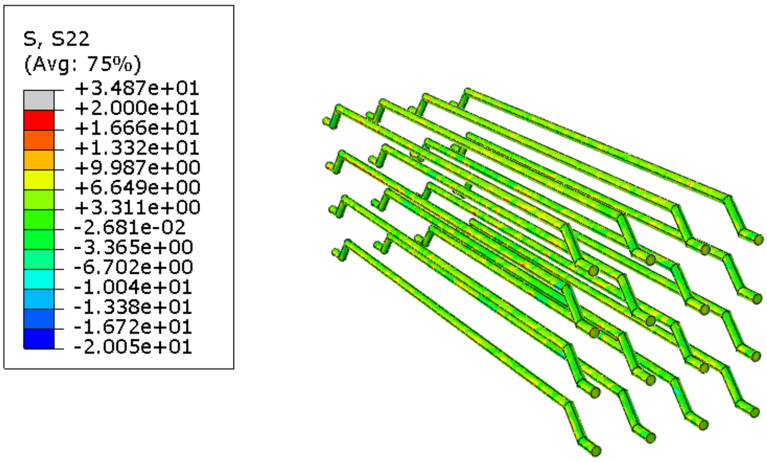
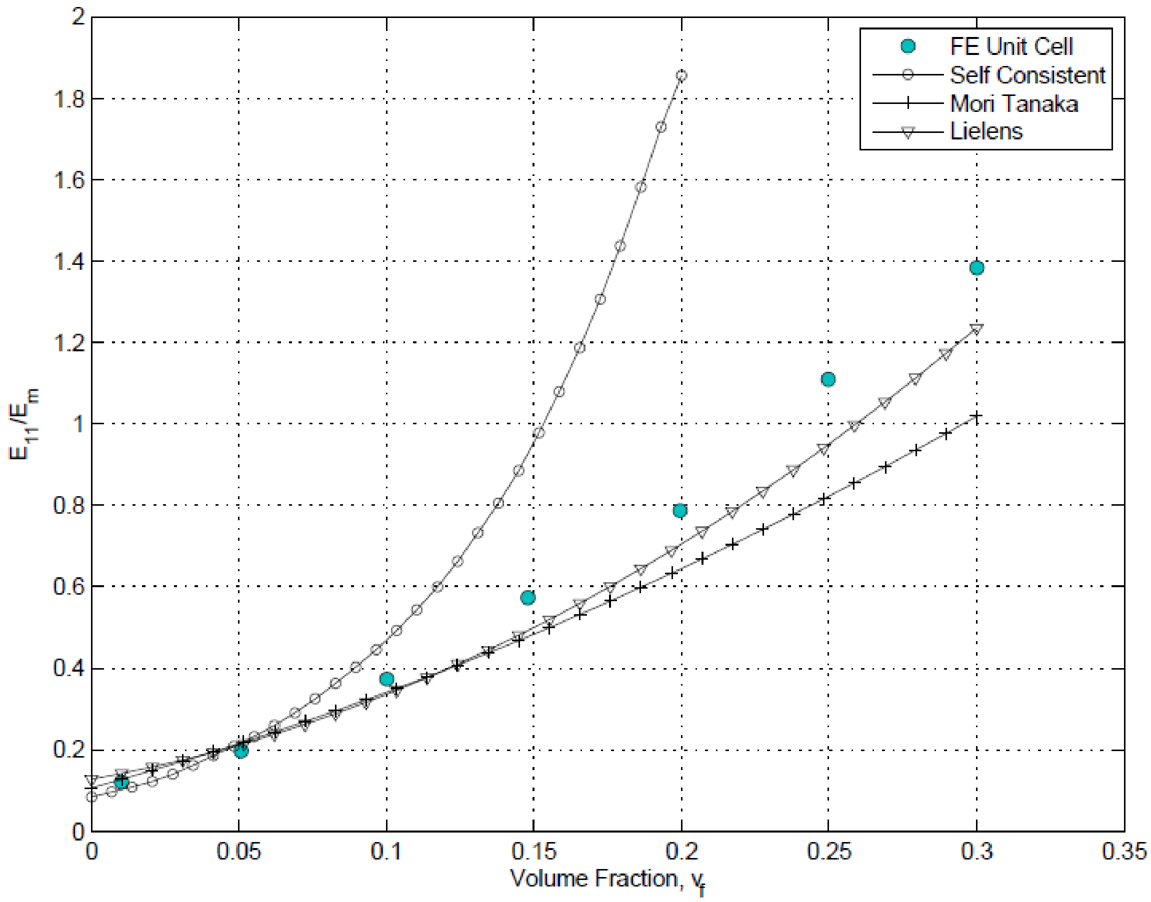


Figure 4 Corresponding FE Results on the fiber array

Figure 5 shows the results for E_{11} versus volume fraction. The data shows that the numerical simulations got closer to Lielens Model, even though it didn't get too far from the Mori-Tanaka's model. The results might indicate also a dependency on the packing arrangement of the fibers. In this work the unit cells were designed to have a somewhat uniform inter-fiber spacing.



The overall numerical results seem to be within the range of the analytical models, especially at lower volume fractions, which is the case for most UHPFRC (usually 1% to 2.5%). It points out to the feasibility of using elliptical inclusions with an equivalent aspect ratio instead of the actual fiber geometry when calculating strain concentration tensors and the corresponding homogenized composite stiffness.

This work presents the preliminary results of a bigger research effort where the effects of the random orientation of the fibers will be taken into consideration. The work presented in this paper is the first step towards a two-step homogenization scheme. This scheme consists of creating an RVE that that will be able to represent the UHPC reinforced with steel hooked-end fibers in terms of overall homogenized behavior. In this RVE the randomly oriented fibers will be decomposed into a set of pseudo grains containing all the fibers aligned in one specific direction each. Therefore the importance of this work consist in answering the question of whether analytical solutions for elliptical inclusions apply to inclusions with the shape of the fibers used in the production of UHPFRC.

APPENDIX A – Components of Eshelby's Tensor

For a fiber-like ellipsoidal inclusion, the components of the Eshelby Tensor, S_{ijkl}, are given by:

$$S_{1111} = \frac{1}{2(1-v_m)}\left(\frac{4\alpha^2-2}{\alpha^2-1} - 2v_m - \left(1-2v_m+\frac{3\alpha^2}{\alpha^2-1}\right)g\right) \tag{A1}$$

$$S_{2222} = S_{3333} = \frac{1}{4(1-v_m)}\left(\frac{3\alpha^2}{2(\alpha^2-1)} + \left(1-2v_m-\frac{9}{4(\alpha^2-1)}\right)g\right) \tag{A2}$$

$$S_{1122} = S_{1133} = \frac{1}{2(1-v_m)}\left(-\frac{\alpha^2}{\alpha^2-1} + 2v_m + \left(1-2v_m+\frac{3}{2(\alpha^2-1)}\right)g\right) \tag{A3}$$

$$S_{2211} = S_{3311} = \frac{1}{2(1-v_m)}\left(-\frac{\alpha^2}{\alpha^2-1} + \frac{g}{2}\left(\frac{3\alpha^2}{\alpha^2-1} - (1-2v_m)\right)\right) \tag{A4}$$

$$S_{2233} = S_{3322} = \frac{1}{4(1-v_m)}\left(\frac{\alpha^2}{2(\alpha^2-1)} - \left(1-2v_m+\frac{3}{4(\alpha^2-1)}\right)g\right) \tag{A5}$$

$$S_{1212} = S_{1313} = \frac{1}{4(1-v_m)}\left(-\frac{2}{\alpha^2-1} - 2v_m - \frac{g}{2}\left(1-2v_m-\frac{3(\alpha^2+1)}{\alpha^2-1}\right)\right) \tag{A6}$$

$$S_{2323} = \frac{S_{2222} - S_{2233}}{2} \tag{A7}$$

Where α is the inclusion's aspect ratio (length/diameter). All the remaining components are nil. For prolate inclusions g is a function given by:

$$g = \frac{\alpha^2}{(\alpha^2-1)^{3/2}}\left[\alpha(\alpha^2-1)^{1/2} - \frac{1}{\cosh(\alpha)}\right] \tag{A8}$$

REFERENCES

Barnett, S. J., Lataste, J.-F., Parry, T., Millard, S. G., and Soutsos, M. N. (2010). "Assessment of fibre orientation in ultra high performance fibre reinforced concrete and its effect on flexural strength." *Materials and Structures*, 43(7), 1009–1023.

Belytschko, T., and Song, J.-H. (2010). "Coarse-graining of multiscale crack propagation." *International Journal for Numerical Methods in Engineering*, 81(5), 537–563.

Benveniste, Y. (1987). "A new approach to the application of Mori-Tanaka's theory in composite materials." *Mechanics of Materials*, 6(2), 147–157.

Doghri, I., and Tinel, L. (2005). "Micromechanical modeling and computation of elasto-plastic materials reinforced with distributed-orientation fibers." *International Journal of Plasticity*, 21(10), 1919–1940.

Eshelby, J. D. (1957). "The Determination of the Elastic Field of an Ellipsoidal Inclusion, and Related Problems." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 241(1226), 376–396.

Hashin, Z., and Shtrikman, S. (1963). "A variational approach to the theory of the elastic behaviour of multiphase materials." *Journal of the Mechanics and Physics of Solids,* 11(2), 127–140.

Hill, R. (1964). "Theory of mechanical properties of fibre-strengthened materials: I. Elastic behaviour." *Journal of the Mechanics and Physics of Solids*, 12(4), 199–212.

Hill, R. (1965). "Theory of mechanical properties of fibre-strengthened materials—III. self-consistent model." *Journal of the Mechanics and Physics of Solids*, 13(4), 189–198.

Huang, Y., Xu, L., and Kyu Ha, S. (2012). "Prediction of three-dimensional composite laminate response using micromechanics of failure." *Journal of Composite Materials*, 46(19–20), 2431–2442.

Laws, N., and McLaughlin, R. (1979). "The effect of fibre length on the overall moduli of composite materials." *Journal of the Mechanics and Physics of Solids*, 27(1), 1–13.

Lielens, G. (1999). "Micro-Macro Modeling of Structured Materials, Ph.D. Dissertation." Université de Louvain-La-Neuve.

Mori, T., and Tanaka, K. (1973). "Average stress in matrix and average elastic energy of materials with misfitting inclusions." *Acta Metallurgica*, 21(5), 571–574.

Mura, T. (1982). *Micromechanics of defects in solids*. Springer Netherlands, Dordrecht.

Naaman, A. E. (2003). "Engineered Steel Fibers with Optimal Properties for Reinforcement of Cement Composites." *Journal of Advanced Concrete Technology*, 1(3), 241–252.

Peng, X., Tang, S., Hu, N., and Han, J. (2016). "Determination of the Eshelby tensor in mean-field schemes for evaluation of mechanical properties of elastoplastic composites." *International Journal of Plasticity*, 76, 147–165.

Perdahcıoğlu, E. S., and Geijselaers, H. J. M. (2011). "Constitutive modeling of two phase materials using the mean field method for homogenization." *International Journal of Material Forming*, 4(2), 93–102.

Richard, P., and Cheyrezy, M. (1995). "Composition of reactive powder concretes." *Cement and Concrete Research*, 25(7), 1501–1511.

Suuronen, J. P., Kallonen, A., Eik, M., Puttonen, J., Serimaa, R., and Herrmann, H. (2013). "Analysis of short fibres orientation in steel fibre-reinforced concrete (SFRC) by X-ray tomography." *Journal of Materials Science*, 48(3), 1358–1367.

Tian, W., Qi, L., Zhou, J., Liang, J., and Ma, Y. (2015). "Representative volume element for composites reinforced by spatially randomly distributed discontinuous fibers and its applications." *Composite Structures*, 131, 366–373.

Van Tuan, N., Ye, G., van Breugel, K., and Copuroglu, O. (2011). "Hydration and microstructure of ultra high performance concrete incorporating rice husk ash." *Cement and Concrete Research*, 41(11), 1104–1111.

Yoo, D.-Y., and Banthia, N. (2016). "Mechanical properties of ultra-high-performance fiber-reinforced concrete: A review." *Cement and Concrete Composites*, Elsevier Ltd, 73, 267–280.

Yudenfreund, M., Odler, I., and Brunauer, S. (1972). "Hardened portland cement pastes of low porosity I. Materials and experimental methods." *Cement and Concrete Research*, 2(3), 313–330.