A NEW APPROACH AND A COMPARATIVE STUDY OF NUMERICAL SOLUTION STRATEGIES FOR SLIGHTLY COMPRESSIBLE AND IMMISCIBLE TWO-PHASE FLOW IN OIL RESERVOIRS WITH THE FINITE VOLUME METHOD

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Abstract. *We introduced a new approach and performed a comparative study of some of the most well-known strategies for solving the algebraic equations system arising from the space and time discretization of the governing equations for a three-dimensional slightly compressible two-phase (water-oil) in a heterogeneous oil reservoir. The flow is considered to be isothermal, the fluids immiscible and we take into account the compressibility of the fluids and the porous matrix. A model is also used for the well-reservoir coupling considering specified flow rates of injection and production. The resolution strategies considered were the Fully Implicit method (Newton's method), the IMPES method (IMplicit for Pressure and Explicit for Saturation), the Sequential method (implicit for pressure and saturation) and a new Hybrid method (implicit for pressure and the Newton's method for saturation). Verification of the numerical approaches was done considering the Buckley-Leverett problem. As an application example, we considered the problem of production in a Five-Spot well arrangement. For all tests considered in this work the Hybrid method achieved the best computational efficiency.*

Keywords: *Finite volume method, Hybrid method, Oil reservoir, Resolution Strategies, Two-phase flow*

1 INTRODUCTION

When fundamental conservation law equations are applied to the two-phase flow in porous media, we can obtain a balance partial differential equation (PDE) for each phase. The set of governing equations can be represented in different ways, depending on the primitive unknowns to be solved from balance equations and on the variables determined through auxiliary relations, as for example, capillary pressure (Chen, 2007). These nonlinear PDEs are coupled and, except in some special cases, it is not possible to obtain analytical solutions in order to solve them. Thus, numerical methods are applied to get an approximated solution. There are three common approaches that are usually applied on the solution of nonlinear equations (Lu, 2008): the Fully Implicit Method (FIM) (based on the Newton-Raphson); the IMplicit Pressure Explicit Saturation (IMPES) method and the Sequential Method.

The Fully Implicit method (Aziz & Settari, 1979; Collins et al., 1992; Monteagudo & Firoozabadi, 2006; Pacheco et al., 2016) is the most used numerical technique on reservoir simulation in commercial scale (Kardale, 2015). In this method, all nonlinear terms, as for example, capillary pressures and sources (wells) are linearized in a fully implicit way and all unknowns are obtained simultaneously. This numerical strategy is unconditionally stable and allows the choice of large time steps (sometimes, tens of days). However, FIM implies the solution of large linear system of algebraic equations, requiring a great computational effort in order to obtain the solution, particularly for refined meshes. Computational implementation of FIM is also a non-trivial task as compared to IMPES and Sequential methods. When capillary effects are not representative (weakly coupled equations) or small time steps are adopted, better convergence properties can be obtained (Kardale, 2015).

Solution by IMPES method (Sheldon et al., 1959; Stone & Garder, 1961) implies the separate resolution of pressure and saturation equations. For this purpose, the governing equations for two-phase flow are combined in order to obtain an equation in terms of the pressure of one of the two phases, which is implicitly solved. In the same way an equation for the saturation variable is obtained and this equation is explicitly solved (Coats, 1999). In this process capillary pressures and coefficients are evaluated explicitly in time n or $n+1, v$ in a iterative procedure. IMPES method shows the smaller computational effort per iteration because of the pressure-based segregated algorithm and the explicit solution of the saturation equation. Several extensions for the IMPES method have been developed (Watts, 1985; Hurtado et al., 2006; Aziz & Settari, 1979; Chen, 2007; Collins et al., 1992). However, in all IMPES versions the explicit determination of saturation leads to restrictions on the numerical stability of the method, and hence, time step limitations (Coats, 2001).

The Sequential method (Lu, 2008; Kardale, 2015) was firstly applied to the Black Oil model by Watts (1985), using also a pressure-based segregated algorithm as for the IMPES method, but the computation of saturation is implicit. The main objective being the improvement of the numerical stability of the IMPES method, allowing larger time steps.

Recently, Medeiros (2017) proposed an alternative approach in order to numerically solve the two-phase flow equations, here called Hybrid Method. As for the Sequential method, both pressure and saturation equations are computed implicitly using a segregated algorithm. The same pressure-based segregated algorithm of the IMPES method is applied, but now saturation is obtained using a fully implicit linearization (Aziz & Settari, 1979; Ertekin et al., 2001; Kardale, 2015), a Newton's method approximation. The main idea is to improve the numerical

stability without solve simultaneously the pressure and saturation equations.

In this work, we carried out a comparative study of these numerical solution strategies for two-phase flow in oil reservoirs. In all tests considered here the hybrid method showed the best results with respect to computational efficiency.

2 MODELING OF TWO-PHASE FLOW IN POROUS MEDIA

Two-phase flow in oil reservoirs involves simultaneous flow transporting multiple components. Here we considered the immiscible flow of water and oil (without mass transfer between phases (Chen, 2007)). A mass balance equation for flow in a porous medium is given by (Ertekin et al., 2001)

$$\frac{\partial}{\partial t}\left(\phi\rho_\alpha S_\alpha\right) + \nabla\cdot\left(\rho_\alpha \mathbf{v}_\alpha\right) - \dot{q}_{m\alpha} = 0, \tag{1}$$

where ϕ is the porosity, α indicates the phase, ρ_α is the density, S_α the saturation, $\dot{q}_{m\alpha}$ represents a source term (mass per time per volume) and $\mathbf{v}_\alpha$ is the superficial fluid velocity.

Traditionally, Darcy's law is applied for the momentum balance (Whitaker, 1856). This equation was firstly obtained empirically and, posteriorly, it was derived using a rigorous mathematical framework by Whitaker (1999). For multiphase flow, the classical Darcy's law must be modified (Chen, 2007)

$$\mathbf{v}_\alpha = -\frac{k_{r\alpha}}{\mu_\alpha}\mathbf{k}\left(\nabla p_\alpha - \gamma_\alpha \nabla Z\right), \tag{2}$$

where $k_{r\alpha}$ is the relative permeability, $\mathbf{k}$ represents the permeability tensor of the porous media, μ_α is the fluid viscosity, p_α the pressure, Z the depth and $\gamma_{alpha} = \rho_\alpha g$, where g is the gravity acceleration magnitude.

Replacing Eq. (2) in the Eq. (1) we get

$$\nabla\cdot\left[\frac{\rho_\alpha k_{r\alpha}}{\mu_\alpha}\mathbf{k}\left(\nabla p_\alpha - \gamma_\alpha\nabla Z\right)\right] = \frac{\partial}{\partial t}\left(\phi\rho_\alpha S_\alpha\right) - \dot{q}_{m\alpha} \qquad \alpha = w, n \tag{3}$$

where w represents the wetting phase (here, water) and n the non-wetting phase (oil).

Introducing the formation-value-factor (FVF) (Ertekin et al., 2001),

$$B_\alpha = \frac{\rho_{sc_\alpha}}{\rho_\alpha} \tag{4}$$

where ρ_{sc_α} is the density in standard conditions of pressure and temperature (Chen, 2007), writing the source term as

$$\dot{q}_{m\alpha} = \dot{q}_{sc\alpha}\rho_{sc_\alpha} \tag{5}$$

considering standard conditions, $\dot{q}_{sc\alpha}$, we can replace Eqs. (4) and (5) in Eq. (3) and divide all terms by ρ_{sc_α} so that (Medeiros, 2017),

$$\nabla\cdot\left[\frac{k_{r\alpha}}{\mu_\alpha B_\alpha}\mathbf{k}\left(\nabla p_\alpha - \gamma_\alpha\nabla Z\right)\right] = \frac{\partial}{\partial t}\left(\frac{\phi S_\alpha}{B_\alpha}\right) - \dot{q}_{sc\alpha} \qquad \alpha = w, n \tag{6}$$

which can be written for non-wetting and wetting phases, respectively, as

$$\nabla \cdot \left[\frac{k_{rn}}{\mu_n B_n} \mathbf{k} \left(\nabla p_n - \gamma_n \nabla Z \right) \right] = \frac{\partial}{\partial t} \left(\frac{\phi S_n}{B_n} \right) - \dot{q}_{scn} \tag{7}$$

and

$$\nabla \cdot \left[\frac{k_{rw}}{\mu_w B_w} \mathbf{k} \left(\nabla p_w - \gamma_w \nabla Z \right) \right] = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) - \dot{q}_{scw}. \tag{8}$$

Considering fully saturated porous media (Ezekwe, 2010) we have

$$S_w + S_n = 1, \tag{9}$$

where S_w and S_n are the saturations of the wetting and non-wetting phases, respectively.

From capillary pressure definition (Ertekin et al., 2001),

$$p_c = p_n - p_w. \tag{10}$$

Equations (7)-(10) have been used in the numerical simulation of water-oil flow in petroleum reservoirs. Usually we choose the non-wetting phase pressure and the wetting phase saturation as the primitive unknowns (Chen, 2007). Therefore, from Eqs. (7) and (8) (considering that $S_n = 1 - S_w$ and $p_w = p_n - p_c$),

$$\nabla \cdot \left[\frac{k_{rn}}{\mu_n B_n} \mathbf{k} \left(\nabla p_n - \gamma_n \nabla Z \right) \right] = \frac{\partial}{\partial t} \left[\frac{\phi (1 - S_w)}{B_n} \right] - \dot{q}_{scn} \tag{11}$$

and

$$\nabla \cdot \left[\frac{k_{rw}}{\mu_w B_w} \mathbf{k} \left(\nabla p_n - \nabla p_c - \gamma_w \nabla Z \right) \right] = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) - \dot{q}_{scw}. \tag{12}$$

Alternatively, using the chain rule (Kardale, 2015),

$$\nabla \cdot \left[\frac{k_{rw}}{\mu_w B_w} \mathbf{k} \left(\nabla p_n - \frac{dp_c}{dS_w} \nabla S_w - \gamma_w \nabla Z \right) \right] = \frac{\partial}{\partial t} \left(\frac{\phi S_w}{B_w} \right) - \dot{q}_{scw}. \tag{13}$$

Initial and boundary conditions are specified with the purpose of completing the physical-mathematical model, formed by Eqs. (11) and (12) (or (13)). The initial condition is given for an arbitrary time t_0. For instance, for $p = p(x, y, z, t)$ and $S = S(x, y, z, t)$ (Chen, 2007)

$$p(x, y, z, t_0) = p_0(x, y, z) \qquad \text{in } \Omega \tag{14}$$

and

$$S(x, y, z, t_0) = S_0(x, y, z) \qquad \text{in } \Omega \tag{15}$$

where p_0 and S_0 indicate initial pressure and saturation values, respectively, in the resolution domain Ω. In reservoir simulation problems, generally, initial pressures are prescribed at one

reference depth, and the hydrostatic pressure gradient and the capillary pressure are used to determine the remaining values (Ertekin et al., 2001).

When pressure and saturation are specified as a function of position and time at the boundary $\partial\Omega$ (Chen, 2007)

$$p = p_{esp}(x, y, z, t) \text{ in } \partial\Omega \tag{16}$$

and

$$S = S_{esp}(x, y, z, t) \text{ in } \partial\Omega. \tag{17}$$

In this work the production/injection well is treated as a inner boundary. The specification of a pressure value implies in a production/injection at constant pressure at the reservoir sandface. Conversely, this boundary specification at an outer boundary means that the pressure is kept constant in this boundary. This condition occurs when there is a large water influx from a aquifer into a reservoir (Ertekin et al., 2001). With respect to saturation, this condition usually also happens when there is a water influx from an aquifer (Chen, 2007). For a known flow through the boundary (Chen, 2007)

$$\rho\mathbf{v} \cdot \mathbf{n} = f(x, y, z, t), \text{ in } \partial\Omega \tag{18}$$

where $\mathbf{n}$ indicates the normal external unitary vector to the $\partial\Omega$. For a impermeable boundary, $f(x, y, z, t) = 0$. A specification of a normal flow to the boundary corresponds to the prescription of a normal pressure gradient to the boundary (Ertekin et al., 2001). With regard to the outer boundaries, a case of particular interest in reservoir engineering is the no-flow boundary (impermeable boundary), in which pressure gradient is zero (Ertekin et al., 2001).

3 NUMERICAL METHODOLOGY

We used the Finite Volume Method (FVM) as the discretization technique for the PDEs that governing the two-phase water-oil flow in porous media and the initial and boundary conditions. Details of the general numerical aspects considered on the discretization and evaluation of coefficients can be found in Medeiros (2017). For a FVM discretization (Versteeg & Malalasekera, 2007) a numerical grid, as depicted in Fig. 1, can be applied to partition the resolution domain. The domain has dimensions L_x, L_y and L_z and is partitioned using n_x finite volumes in direction x, n_y volumes in direction y and n_z volumes in direction z, respectively, such that

$$\sum_{i=1}^{n_x} \Delta x_i = L_x, \qquad \sum_{j=1}^{n_y} \Delta y_j = L_y, \qquad \sum_{k=1}^{n_z} \Delta z_k = L_z, \tag{19}$$

where indexes i, j and k indicate volumes in directions x, y and z, respectively. The interfaces between the finite volumes are referenced using $i + 1/2, j, k$ for the common boundary that lies between i, j, k and $i + 1, j, k$. A similar procedure is employed for the other spatial directions.

When the FVM is applied (Versteeg & Malalasekera, 2007) we must integrate, over a finite volume and in time, the PDEs that govern the flow through the reservoir. Then, spatial derivatives must be approximated such that we obtain a system of algebraic equations (one for

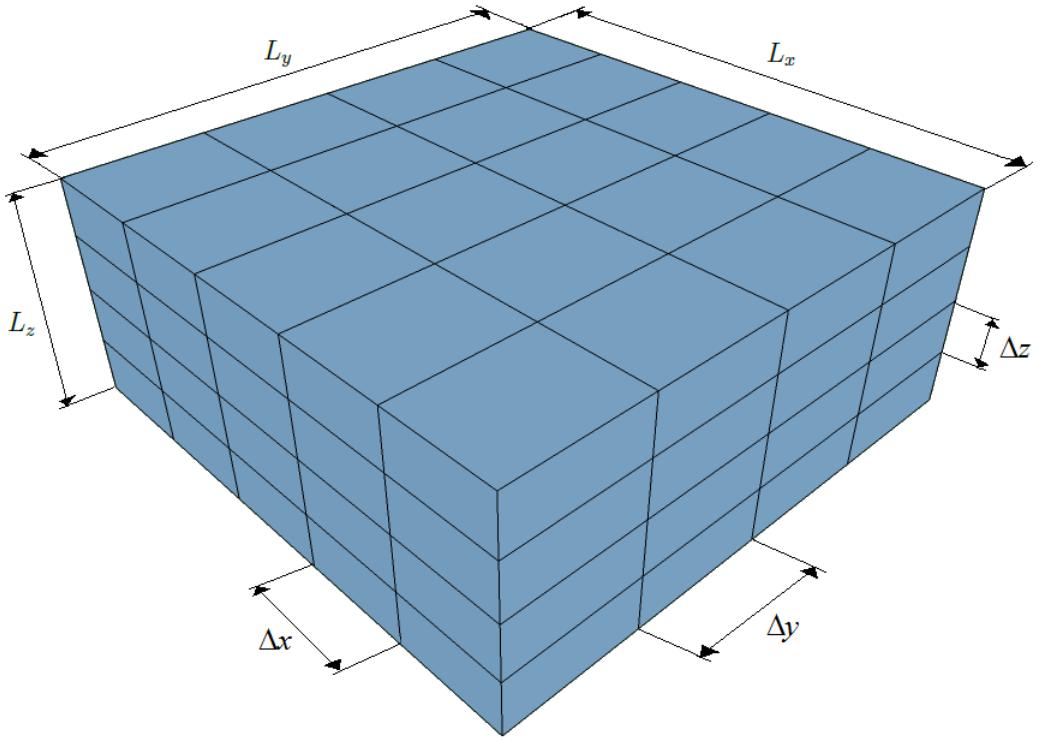


Figure 1: Three-dimensional grid built using the Finite Volume Method.

each cell). A compact notation can be used to indicate terms which are placed at the center of a volume or in an interface (Versteeg & Malalasekera, 2007). Using this notation, interfaces are indicated as: $(i-1/2,j,k)=w$, $(i+1/2,j,k)=e$, $(i,j-1/2,k)=n$, $(i,j+1/2,k)=s$, $(i,j,k-1/2)=a$ and $(i,j,k+1/2)=b$, considering the spatial references west, east, north, south, above and below, respectively. For a given node P located in (i,j,k), their neighbors are $(i-1,j,k)=W$, $(i+1,j,k)=E$, $(i,j-1,k)=N$, $(i,j+1,k)=S$, $(i,j,k-1)=A$ and $(i,j,k+1)=B$. After we have carried out the integration of the PDEs we obtain (Medeiros, 2017),

$$\Delta\left(T_n\Delta p_n\right)_P^{n+1}-\Delta\left(T_n\gamma_n\Delta Z\right)_P^{n+1}=C_{np}\Delta_t p_n+C_{ns}\Delta_t S_w-q_{scn_P}^{n+1} \tag{20}$$

and

$$\Delta\left(T_w\Delta p_n\right)_P^{n+1}-\Delta\left(T_w\Delta p_c\right)_P^{n+1}-\Delta\left(T_w\gamma_w\Delta Z\right)_P^{n+1}=C_{wp}\Delta_t p_n+C_{ws}\Delta_t S_w-q_{scw_P}^{n+1}, \tag{21}$$

where

$$C_{np}=\frac{V_P}{\Delta t}\left\{\left[\phi_P^{n+1}\left(\frac{1}{B_n}\right)_P'+\frac{\phi_P'}{B_{n_P}^n}\right]\left(1-S_{w_P}^n\right)\right\}, \tag{22}$$

$$C_{ns}=-\frac{V_P}{\Delta t}\left(\frac{\phi_P^{n+1}}{B_{n_P}^{n+1}}\right), \tag{23}$$

$$C_{wp}=\frac{V_P}{\Delta t}\left\{\left[\phi_P^{n+1}\left(\frac{1}{B_w}\right)_P'+\frac{\phi_P'}{B_{w_P}^n}\right]S_{w_P}^n\right\} \tag{24}$$

and

$$C_{ws} = \frac{V_{_P}}{\Delta t}\left(\frac{\phi_{_P}^{n+1}}{B_{w_P}^{n+1}}\right), \tag{25}$$

introducing the operators (Ertekin et al., 2001; Medeiros (2017))

$$\Delta_t \varphi = \varphi_{_P}^{n+1} - \varphi_{_P}^{n} \tag{26}$$

and (Ertekin et al., 2001; Medeiros (2017))

$$\begin{aligned}\Delta\left(\xi\Delta\eta\right)_{_P} \equiv\ & \xi_{x_w}\left(\eta_{_W} - \eta_{_P}\right) + \xi_{x_e}\left(\eta_{_E} - \eta_{_P}\right) + \xi_{y_n}\left(\eta_{_N} - \eta_{_P}\right) + \xi_{y_s}\left(\eta_{_S} - \eta_{_P}\right) \\ & + \xi_{z_a}\left(\eta_{_A} - \eta_{_P}\right) + \xi_{z_b}\left(\eta_{_B} - \eta_{_P}\right).\end{aligned} \tag{27}$$

In this work, the so called transmissibilities are defined as

$$T_{lx_e}^{n+1} = \left(\frac{k_x A_x}{\Delta x}\right)_{_e}\left(\frac{1}{\mu_l B_l}\right)_{_e}^{n+1} k_{rl_e}^{n+1}, \tag{28}$$

for the x direction, with similar expressions for other spatial directions. In calculating the transmissibilities at the finite volume interfaces we use harmonical average for rock and geometric proprieties, arithmetic average for fluid properties (pressure dependents) and a first order upwind for properties that are saturation dependents (details can be found in Ertekin et al. (2001)).

For the variable time step we use the empirical criterion given by Minden (2012) for the IMPES method

$$\Delta t_n = \begin{cases} 0.75\Delta t_{n-1} & \text{if} \quad it_{n-1} > 20, \\ 1.05\Delta t_{n-1} & \text{if} \quad it_{n-1} \leq 3, \\ \Delta t_{n-1} & \text{else,} \end{cases} \tag{29}$$

and a similar criterion for the other methods (Medeiros, 2017)

$$\Delta t_n = \begin{cases} n_{decr}\Delta t_{n-1} & \text{if} \quad it_{n-1} > it_{decr}, \\ n_{incr}\Delta t_{n-1} & \text{if} \quad it_{n-1} \leq it_{incr}, \\ \Delta t_{n-1} & \text{else.} \end{cases} \tag{30}$$

where n_{decr} and n_{incr} are the decreasing and increasing time step rates, it_{decr} is the minimum number of iterations for convergence in a previous time step, and it_{incr} is the maximum number of iterations in which it is increased ($it_{incr} < it_{decr}$). The time step increasing is limited to a maximum value $\Delta t_n \leq \Delta t_{max}$. The systems of algebraic equations for unknowns pressure and saturation are solver here using a Preconditioned Bi-conjugate Gradient Stabilized (Saad, 2003).

3.1 Hybrid method

As already mentioned, the Hybrid method (Medeiros, 2017) was introduced to improve the stability characteristics of IMPES and Sequential methods avoiding the high computational effort related to the FIM. Therefore, terms that are wetting phase functions are linearized in a fully implicit way.

From Eq. (20) we can calculate implicitly the pressure at time $n+1$ and iteration $v+1$, and it is no more an unknown in Eq. (21) for the saturation determination. The fully implicit linearization of Eq. (21) leads to a system of algebraic equations that is an approximation of the Newton's method. We stress here the fact that pressure is still an unknown at time level $n+1$ since we only have your estimate in $(n+1, v+1)$. This system of equations can be expressed in the following matrix form

$$\mathbf{J}_w^{n+1,v}\boldsymbol{\delta}\mathbf{S}_w^{n+1,v+1} = -\mathbf{R}_w^{n+1,v}, \tag{31}$$

where $\mathbf{J}_w$ represents the Jacobian matrix and $\boldsymbol{\delta}\mathbf{S}_w^{n+1,v+1} = \mathbf{S}_w^{n+1,v+1} - \mathbf{S}_w^{n+1,v}$.

The unknown vector is is

$$\mathbf{S}_w = (S_{w_1}, S_{w_2}, S_{w_3}, ..., S_{w_N})^T \tag{32}$$

while for residues $\mathbf{R}_w$ we have

$$\mathbf{R}_w = (R_{w_1}, R_{w_2}, R_{w_3}, ..., R_{w_N})^T, \tag{33}$$

where $n = 1, 2, 3, ...N$, for N finite volumes.

Once the linear algebraic equations system represented by Eq. (31) is solved, the saturations can be calculated by

$$\mathbf{S}_w^{n+1,v+1} = \mathbf{S}_w^{n+1,v} + \boldsymbol{\delta}\mathbf{S}_w^{n+1,v+1}, \tag{34}$$

for each cell of the computational domain.

The system (31) can be written for each inner cell as

$$\begin{aligned} -R_{w_P}^{n+1,v} = &\frac{\partial R_{w_P}}{\partial S_{w_A}}\bigg|^{n+1,v} \delta S_{w_A}^{n+1,v+1} + \frac{\partial R_{w_P}}{\partial S_{w_N}}\bigg|^{n+1,v} \delta S_{w_N}^{n+1,v+1} + \frac{\partial R_{w_P}}{\partial S_{w_W}}\bigg|^{n+1,v} \delta S_{w_W}^{n+1,v+1} \\ &+ \frac{\partial R_{w_P}}{\partial S_{w_P}}\bigg|^{n+1,v} \delta S_{w_P}^{n+1,v+1} + \frac{\partial R_{w_P}}{\partial S_{w_E}}\bigg|^{n+1,v} \delta S_{w_E}^{n+1,v+1} + \frac{\partial R_{w_P}}{\partial S_{w_S}}\bigg|^{n+1,v} \delta S_{w_S}^{n+1,v+1} \\ &+ \frac{\partial R_{w_P}}{\partial S_{w_B}}\bigg|^{n+1,v} \delta S_{w_B}^{n+1,v+1}. \end{aligned} \tag{35}$$

where

$$\begin{aligned} R_{w_P}^{n+1,v} = &- T_{wz_a}^{n+1,v}\Delta\Phi_{wz_A}^{n+1,*} - T_{wy_n}^{n+1,v}\Delta\Phi_{wy_N}^{n+1,*} - T_{wx_w}^{n+1,v}\Delta\Phi_{wx_W}^{n+1,*} \\ &- T_{wx_e}^{n+1,v}\Delta\Phi_{wx_E}^{n+1,*} - T_{wy_s}^{n+1,v}\Delta\Phi_{wy_S}^{n+1,*} - T_{wz_b}^{n+1,v}\Delta\Phi_{wz_B}^{n+1,*} \\ &+ C_{wp}(p_{n_P}^{n+1,v} - p_{n_P}^{n}) + C_{ws}(S_{w_P}^{n+1,v} - S_{w_P}^{n}) - q_{scw_P}^{n+1,v} \end{aligned} \tag{36}$$

having been introduced

$$\Delta\Phi_{wwf_c}^{n+1,*} \equiv \left(p_{n_c}^{n+1,v+1} - p_{c_c}^{n+1,v}\right) - \left[p_{wf_{ref}}^{n+1,v+1} + \bar{\gamma}_{w_c}^{n+1,v}\left(Z_c - Z_{ref}\right)\right]. \tag{37}$$

In this formulation we adopt a fully implicit linearization, in terms of the saturation of the wetting phase, also for the source term.

4 RESULTS

In order to contribute to the development of numerical methods for the solution of algebraic systems applied on the reservoir simulation, a comparative study was carried out considering FIM, IMPES, Sequential and Hybrid methods. As practical applications we considered two-phase water-oil flow in a 1/4 five-spot pattern. Homogeneous and heterogeneous cases were studied. The numerical reservoir simulator was developed using the C language. Simulations were performed in a DELL PowerEdge T620 with an Intel Xeon E5-2620 processor, 16 GB of memory, 64-bit and operational system openSUSE 42.1 (x86_64).

4.1 Mesh refinement

For a homogeneous reservoir and a slab geometry we studied the two-phase water injection problem for an initial water saturation S_{w_0} = 0.2 and P_{ref} = 5,800 psi at the top of the reservoir, maintained at a constant temperature of T_{ref} = 150 ^{o}F. Fluid properties are in Table 1 and the correlations used can be found in Medeiros (2017). Table 2 shows rock properties and parameters for capillary pressure and relative permeability. These values are used by default, unless another value is specified. A uniform pressure gradient is imposed ($\partial p/\partial x$ = -0.5 psi/ft) in the yz plan, at the left boundary, and we consider a total injection time $t \cong$ 5.000 days (d), due to the variable time step. For the grid refinement study, three numerical meshes were chosen with different numbers of finite volumes in the x-direction (Table 3), with the purpose of evaluating the computational performance.

Table 1: Fluid properties

γ_{API} = 25.72 ^{o}API	ρ_{sc_l} = 62.37 lbm/ft^3	μ_{wb} = 0.69 cp
p_{ob} = 2,000 psi	p_{wb} = 2,000 psi	μ_{ob}=1.69 cp
B_{ob}= 1.13	S_a = 20 %	B_{wb}= 1.022
c_o = 5 $\times 10^{-6}$ psi^{-1}	$c_{\mu w}$ = 0.52 $\times 10^{-4}$ psi^{-1}	–
c_w = 18 $\times 10^{-6}$ psi^{-1}	$c_{\mu o}$= 1.42 $\times 10^{-4}$ psi^{-1}	–

Table 2: Rock properties and Fluid/rock properties

k_x = 30 mD	S_{iw} = 0.15	$k_{rw_{max}}$ = 0.4
k_y = 30 mD	S_{nrw} = 0.15	$k_{rn_{max}}$ = 0.9
k_z = 7.5 mD	$ew = eow$ = 4	ϕ^0= 0.2
c_ϕ= 4$\times 10^{-6}$ psi^{-1}	epc = 2	$p_{c_{max}}$ = 5.8 psi

Table 3: Dimensions and grids for the slab reservoir

Dimensions (ft)	L_x = 2,000	L_y = 1,000	L_z = 100
Grid 1	$n_x = 125$	$n_y = 50$	$n_z = 5$
Grid 2	$n_x = 250$	$n_y = 50$	$n_z = 5$
Grid 3	$n_x = 500$	$n_y = 50$	$n_z = 5$

Observing Fig. 2, we see that for all grids considered the Hybrid method shows the best computational performance. It is worth noting that only a small number of grids was tested. In this context, we can not guarantee that the same general behavior will be maintained, especially for the more refined meshes. Further studies will be needed to elucidate this issue. However, the Hybrid method appears as a promising strategy.

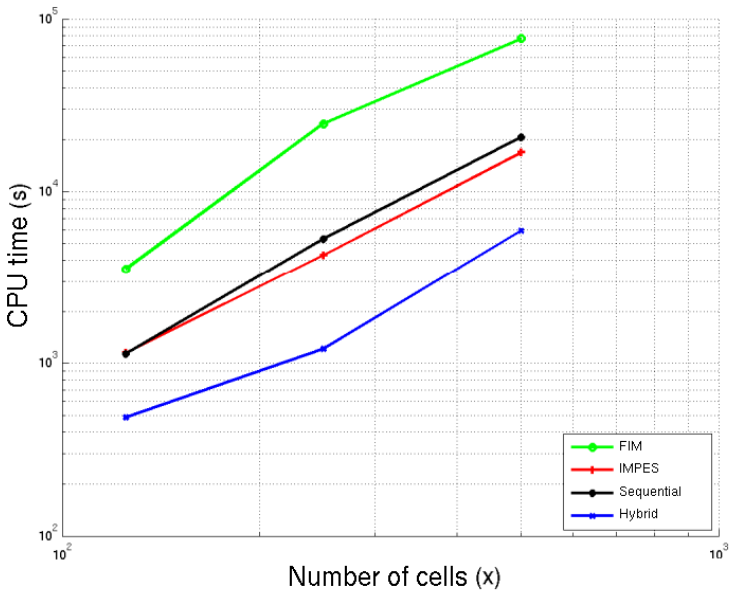


Figure 2: CPU time $\times$ number of cells.

4.2 Buckley-Leverett problem

As a verification test, we now consider the Buckley-Leverett problem, which analytical solution is well known (Ezekwe, 2010). The same geometric and fluid properties of the slab problem are also used here. However, compressibility and capillary effects are neglected. Figure 3 shows the saturation profiles of water for three instants of time, at the center of the yz plan, using a mesh composed by 250 $\times$ 50 $\times$ 5 finite volumes. Results were obtained for the Hybrid, IMPES and Sequential methods with a constant time step Δt = 0.1 d. For this specific problem the Fully Implicit method presented difficulties to achieve convergence, as reported by Kardale (2015), and was not considered.

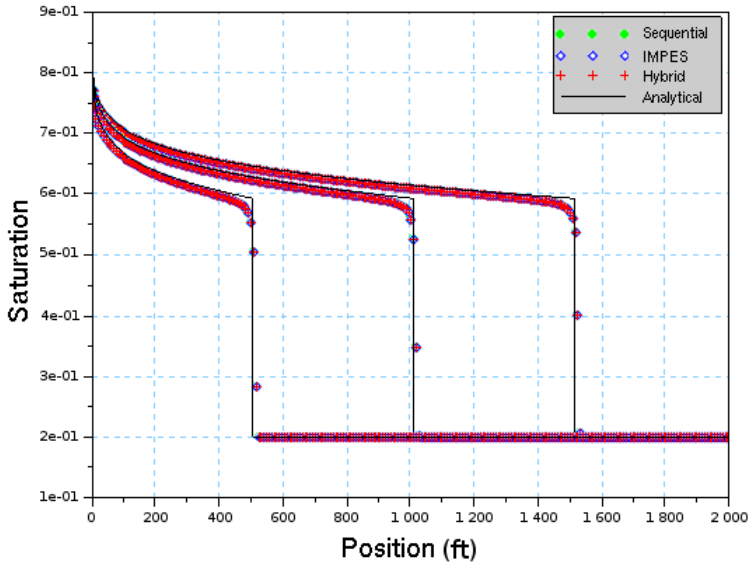


Figure 3: Water saturation for the Buckley-Leverett problem.

As can be seen in Fig. 3, Hybrid, IMPES and Sequential methods provide results that overlap each other (good precision). With respect to the accuracy, when the results are compared with the analytical solution, except for a small numerical diffusion coming from the first order upwind, they are in agreement with the theoretical prediction. There are not spurious oscillations and the numerical solutions are displaced with the correct velocity. Moreover, there are only two points at the shock and we estimate that this provide a good numerical representation.

4.3 Five-spot Pattern

The five-spot arrangement of injection and production is widely used in reservoir simulation as a benchmark test. It consists of four injection wells and one production well. For a square domain, the injection wells are placed at the vertexes of the square and the production well at the center. Due to the symmetry of the problem, only one quarter of the geometry is considered in the numerical simulation, including an injection well and a production well (Fig. 4).

Fluid and rock properties used in this section are the same as those considered for the slab geometry. Geometry and grid data can be found in Table 4 for the flow in a homogeneous medium. Wells for injection and production were placed perpendicularly to the xy plan, in $(x; y)$ = (0;0) ft and $(x; y)$ = (1,000;1,000) ft with lengths equal to Lz. Wellbore rates are equal to $q_{sc_{sp}}$ = -200 STB/d for production and $q_{scw_{sp}}$ = 200 STB/d for injection. The following parameters were used for time step determination: n_{decr} = 0.75; n_{incr} = 1.1; it_{decr} = 25; it_{incr} = 15; Δt_{max} = 10 d and Δt_0 = 0.01 d. Finally, flow was simulated until it reaches approximately 7,000 d of production.

Table 4: Dimensions and grid for 1/4 five-spot (homogeneous case)

Dimensions (ft)	L_x = 1,000	L_y = 1,000	L_z = 100
Grid	n_x = 125	n_y = 125	n_z = 5

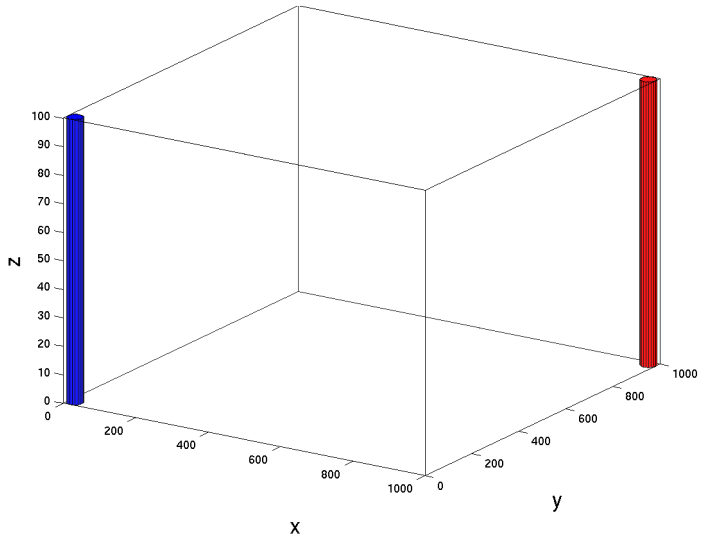


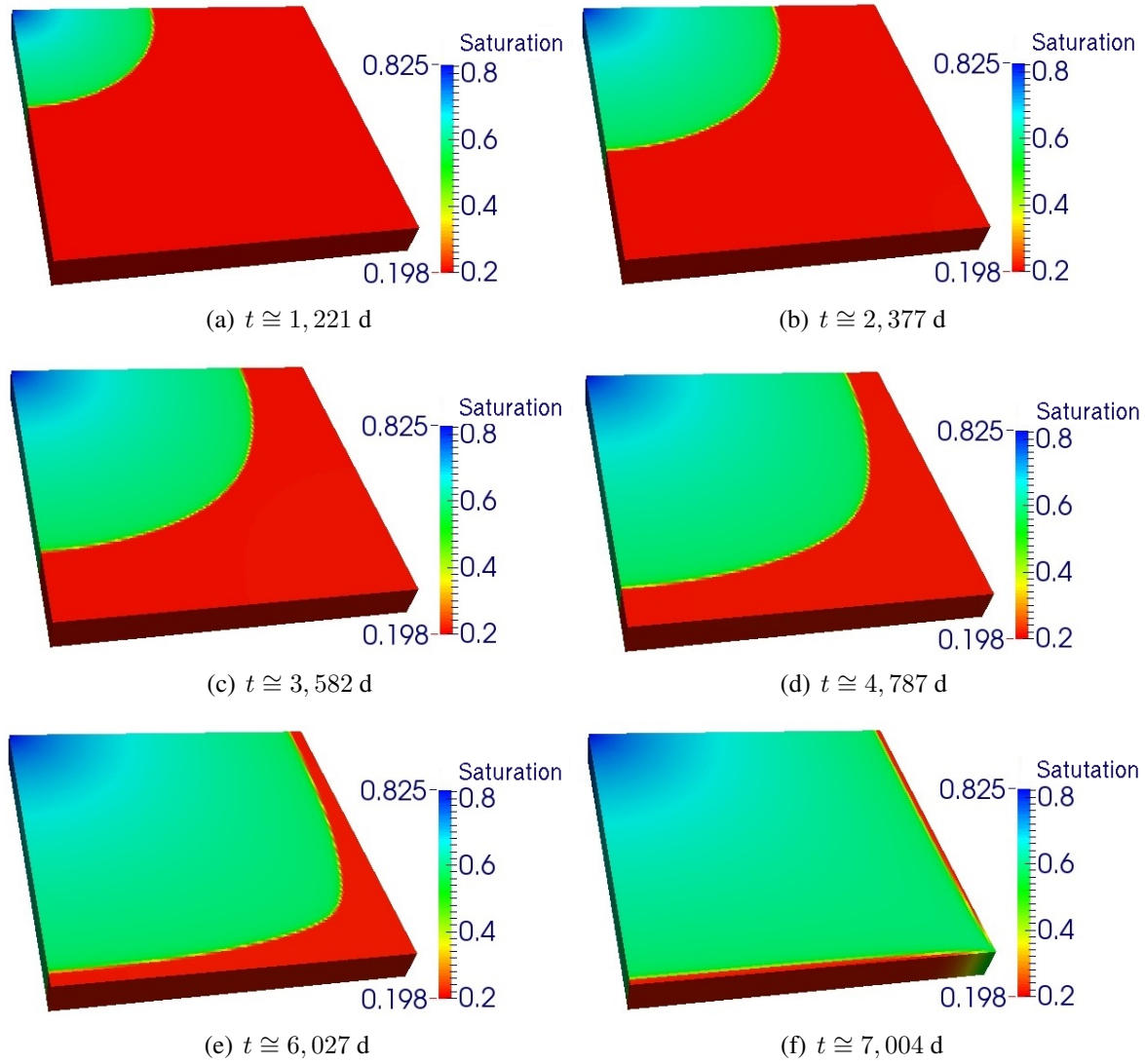
Figure 4: Illustration of a 1/4 five-spot pattern.

Table 5 contain the values obtained and used to determine the best computational performance for the four methods employed here to simulate two-phase flow in a 1/4 five-spot pattern, where Avit is the number of average iterations. Once again, the Hybrid method showed the best performance with regard to CPU time. The advance of the saturation front by the displacing fluid can be seen in Fig. 5, for the Hybrid method. We must emphasize that the water phase flow shows the expected pattern in accordance with technical literature (see, for example, Pacheco et al., 2016). We did not find again significant differences between the saturation values provided by the four methods.

Table 5: Computational performance for 1/4 five-spot (homogeneous case)

Method	Simulation time	Number of time steps	Avit
FIM	17 h 17 min 00 s	4,207	17.30
IMPES	08 h 48 min 10 s	4,542	19.16
Sequential	10 h 16 min 06 s	4,482	20.62
Hybrid	03 h 52 min 20 s	1,796	17.13

The next case considered here is also a two-phase flow in a quarter of five-spot maintaining the same properties and geometric dimensions of the previous case. However, we now introduce two three-dimensional regions (barriers) whose permeabilities are 100 times smaller than those of the rest of the reservoir (Daripa & Dutta, 2017), as depicted in Fig. 6. The origin (vertex) of these barriers are located at the coordinates x_i, y_i and z_i and they have the lengths l_x, l_y and l_z in the direction of the coordinate axes (Table 6).



(a) $t \cong 1,221$ d (b) $t \cong 2,377$ d

(c) $t \cong 3,582$ d (d) $t \cong 4,787$ d

(e) $t \cong 6,027$ d (f) $t \cong 7,004$ d

Figure 5: Water saturation for homogeneous reservoir and 1/4 five-spot.

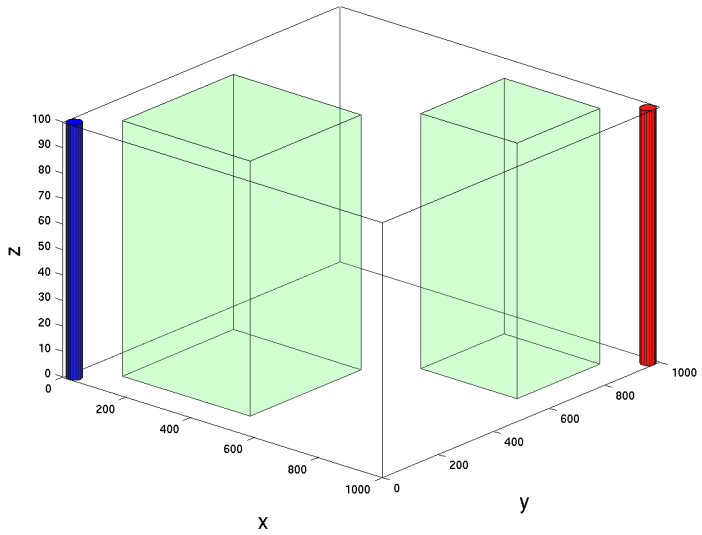


Figure 6: Illustration for 1/4 five-spot (heterogeneous case).

Table 6: Inclusion regions placement

Region	Origin	Dimensions
1	$x_i = 100$, $y_i = 100$, $z_i = 0$	$l_x = 400$, $l_y = 400$, $l_z = 100$
2	$x_i = 600$, $y_i = 600$, $z_i = 0$	$l_x = 300$, $l_y = 300$, $l_z = 100$

The columns in Table 7 contain the CPU time, the number of time steps and the number of average iterations obtained for each method, considering now a two-phase flow in a 1/4 five-spot heterogeneous case. Again, the Hybrid method obtained the best results for computational efficiency. It is worth mentioning that in this specific case the FIM showed better results in terms of CPU time when compared to Sequential and IMPES methods, differently from the previous cases. With respect to the Sequential and IMPES methods, the inclusion of barriers had a significant influence in the CPU time. Figure 7 depicts water saturation field, for different times, obtained using the Hybrid method and the 1/4 five-spot heterogeneous case. We can observe that water flows initially through the preferential paths (higher permeabilities) bypassing the barriers as already expected based on physical evidence. We noticed that even after sufficiently long time, part of the barriers still present a saturation value close to the initial condition (0.2) due to the resistance to flow through these regions.

Table 7: Computational performance for 1/4 five-spot (heterogeneous case)

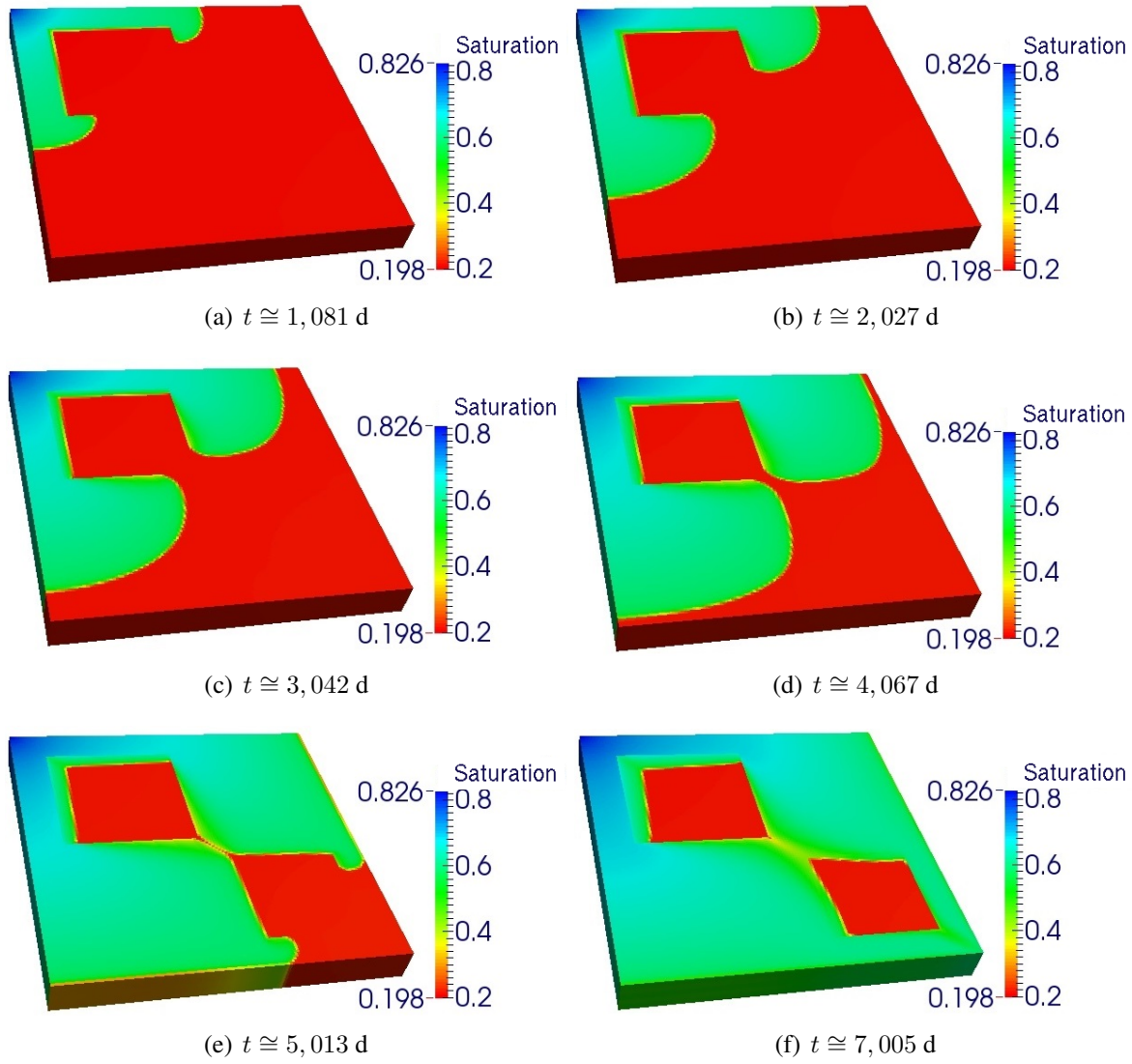
Method	Simulation time	Number of time steps	Avit
FIM	21 h 40 min 29 s	4,398	17.22
IMPES	25 h 44 min 37 s	7,126	11.10
Sequential	32 h 06 min 49 s	7,172	10.44
Hybrid	04 h 03 min 46 s	1,723	17.03

5 CONCLUSIONS

In this work a new approach, called Hybrid Method (Medeiros, 2017), was introduced as an alternative to the numerical resolution of two-phase flow in oil reservoirs. For the slab geometry, homogeneous medium, the Hybrid method showed the best computational performance considering the CPU time, the number of time steps and the number of average iterations. The FIM was the most time consuming method (CPU time) with a high computational effort per iteration. The IMPES and Sequential methods exhibited similar results for the computational performance.

The Hybrid, IMPES and Sequential methods provided the same saturation profiles for the numerical simulation of the Buckley-Leverett problem. The results match the analytical solution except for a small numerical diffusion introduced by the first-order upwind method.

Concerning the two-phase flow simulations in a quarter of five-spot arrangement, we can say that the Hybrid method presented the best numerical performance in both homogeneous and heterogeneous cases, with respect to CPU time and number of time steps. For the heterogeneous



(a) $t \cong 1,081$ d (b) $t \cong 2,027$ d

(c) $t \cong 3,042$ d (d) $t \cong 4,067$ d

(e) $t \cong 5,013$ d (f) $t \cong 7,005$ d

Figure 7: Water saturation for heterogeneous reservoir and 1/4 five-spot.

case, the lowest Avit value was obtained with the Sequential method and the FIM was better than Sequential and IMPES methods when we compare the CPU time and the number of time steps. Contrary to what happened with the IMPES and Sequential methods, inclusion of the barriers did not imply a significant increase in the CPU time for the Hybrid and Fully Implicit methods.

Finally, we concluded that the Hybrid method is an effective alternative to be considered on the solution of nonlinear PDEs for numerical reservoir simulation of two-phase flow. However, this initial study does not intend to exhaust the subject since only a small number of cases were treated.

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