DEVELOPMENT OF A SIMPLIFIED ANALYTICAL VISCOELASTIC FORMULATION FOR THE ANALYSIS OF LAMINATED COMPOSITES

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Abstract. *Currently several projects pursue the study, modeling and execution of structures with high degree of complexity and in order to be considered all micromechanical characteristics of the materials. In this context, the advanced materials known as composites appear, looking for the best characteristics of each phase. For the study of these materials, some formulations and theories are applied, such as analytical models of micromechanics and classical laminate and numerical theory, such as the finite element method. In CLT the laminate is composed of stacked thin laminas with perfect bonding, which has a constant thickness and is composed by a matrix and unidirectional fibers. Accordingly, this article presents a development of a linear viscoelastic formulation based on classical laminate theory (CLT), originally developed for elastic materials, applying it on glass fibers reinforced ED-6 resin laminas. The linear viscoelastic properties were modeled using the Standard model and included in the matrix material of some laminas. This inclusion was made through the application of the Laplace transform in the constitutive relation of the lamina (Hooke's Law) which allows evaluating elastic expressions of viscoelastic form in the Laplace domain, allowing the evaluation of tensions, deformations or displacements in the time domain.*

Keywords: *Linear viscoelasticity, Laminated composites, Mechanics of materials, Laplace transform.*

INTRODUCTION

Using composites is more efficient in many cases because they offer several vantages over conventional materials once a fundamental characteristic attributed to the union of the two materials and the mechanical behavior that confers the high relation strength / weight and stiffness / weight.

In 1867, James Clerk Maxwell introduced elastic properties in the description of fluid. Boltzmann developed in 1874 the formulation for linear viscoelasticity. Using the superposition of effects, he showed that the strain at time t in response to a general time-dependent stress history $\sigma(t)$ can be written as the sum (or integral) of terms that involve the strain response to a step loading and Vito Volterra developed the theory of functional and integral equations adequate to viscoelastic phenomena model (MARQUES; CREUS, 2012). Several simple models can be deduced, but for this work the chosen model was the generalized Kelvin model.

1 Formulation of elastic properties

Stress and strains are generally related through elastic constants. The effective properties for a ply can be seen in the Eq. (1) (TITA, 2007):

$$\begin{aligned} E_1 &= f_f E_f + f_m E_m \qquad & E_2 &= \frac{E_f E_m}{f_m E_f + f_f E_m} \\ \nu_{12} &= f_m \nu_m + f_f \nu_f \qquad & G_{12} &= \frac{G_f G_m}{f_m G_f + f_f G_m} \end{aligned} \tag{1}$$

Where E and G are the longitudinal and transverse modulus of elasticity, respectively, ν is the Poisson's ratio and f is the volumetric fraction of the fiber f and matrix m. The indices 1 and 2 refer to the local cartesian system of the ply (1 - direction of the fibers and 2 - perpendicular to the fibers).

The general formulation of classical laminate theory starts from the constitutive relation given by Eq. (2):

$$\{\varepsilon\} = [S]\,\{\sigma\} \tag{2}$$

Where S_{ij} are components of the ply flexibility matrix.

1.1 Viscoelastic Formulation

The generalized Kelvin model is characterized by being composed of n + 1 Kelvin models in series, considering that the first damper has zero damping. In this case, the stress in each unit is the same external stress $\sigma(t)$, while the total strain $\varepsilon(t)$ (observable) is the sum of the internal deformations in each element (MARQUES; CREUS, 2012). Thus, we can obtain the equation of the fluence function, given by:

$$D(t-\tau) = \frac{1}{E_0} + \sum_{g=1}^{n} \frac{1}{E_g}\left(1 - e^{\frac{-(t-\tau)}{\theta}}\right) \tag{3}$$

Where θ is the relaxation time, given by $\theta = \eta_g / E_g$.

Stress relaxation is the gradual decrease of stress when the material is held at constant strain (Lakes, 2009). Considering that the matrix of the composite is viscoelastic, we can find the relaxation function of the same as follows in Eq. (6):

$$E_m(t) = E_0\, e^{-t/_T} + \frac{E_0 E_\infty}{E_0 + E_\infty}(1 - e^{-t/_T}) \quad (4)$$

Thus, a Laplace transformation was applied on the effective modulus of the composite (E_1, E_2, ν_{12} and G_{12}) to provide theses effective modulus in time domain for each ply.

1.2 Numerical Methods

The relations of deformation-displacement can be obtained by Kirchhoff's hypothesis:

$$\gamma_{xz} = \gamma_{xz} = \varepsilon_z \quad \varepsilon_x = \varepsilon^0 + z\, k_x \quad \varepsilon_y = \varepsilon^0 + z\, k_y \quad \varepsilon_{xy} = \gamma_{xy}^0 + z\, k_{xy} \quad (5)$$

For ply k, we have:

$$\{\sigma\}^k = [\bar{S}]^k(\{\varepsilon\} + z\{k\}) \quad (6)$$

To find the $\bar{S}$, we need to take S to the Laplace domain, apply in the equations, and invert to the time domain again.

For the elastic properties, the deformation over time can be defined by the following equations:

$$\int_{-t/2}^{t/2} \{\varepsilon\}\, dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \{\varepsilon\}^k\, dz = \sum_{k=1}^{N} [\bar{S}] \int_{z_{k-1}}^{z_k} \{\sigma\}^k\, dz \quad (7)$$

According to Eq. (8) we have:

$$\sum_{k=1}^{N} \left(\{\varepsilon^0\}(z_k - z_{k-1}) + \{k\}\frac{1}{2}(z_k^2 - z_{k-1}^2) \right) = \sum_{k=1}^{N} [\bar{S}]^k \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix}^k \quad (8)$$

Considering that the forces are applied and maintained constant over time we obtain Eq. (12) and performing the same process for the bending moment is shown in Eq. (13):

$$\{\varepsilon^0\} \sum_{k=1}^{N} (z_k - z_{k-1}) \frac{1}{2}\{k\} + \sum_{k=1}^{N} (z_k^2 - z_{k-1}^2) = \{N\} \sum_{k=1}^{N} [\bar{S}]^k \quad (9)$$

$$\frac{1}{2}\{\varepsilon^0\} \sum_{k=1}^{N} (z_k^2 - z_{k-1}^2) + \frac{1}{3}\{k\} \sum_{k=1}^{N} (z_k^3 - z_{k-1}^3) = \{M\} \sum_{k=1}^{N} [\bar{S}]^k \quad (10)$$

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Thus, in time, we have:

$$\{\varepsilon(s)\} = [\bar{S}(s)]\{\sigma\} \qquad\qquad \{\varepsilon(t)\} = [\bar{S}(t)]\{\sigma\} \tag{11}$$

considering σ constant in time. Thus, by applying the integral in z, we obtain:

$$\{\varepsilon^0(t)\}\sum_{k=1}^{N}(z_k - z_{k-1})\frac{1}{2}\{k(t)\} + \sum_{k=1}^{N}(z_k^2 - z_{k-1}^2) = \{N\}_{1x3}^{T}\sum_{k=1}^{N}[\bar{S}(t)]_{3x3}^{k} \tag{12}$$

$$\frac{1}{2}\{\varepsilon^0(t)\}\sum_{k=1}^{N}(z_k^2 - z_{k-1}^2) + \frac{1}{3}\{k(t)\}\sum_{k=1}^{N}(z_k^3 - z_{k-1}^3) = \{M\}_{1x3}^{T}\sum_{k=1}^{N}[\bar{S}(t)]_{3x3}^{k} \tag{13}$$

2 NUMERICAL RESULTS

In this example we tried to present the effective properties of a viscoelastic ply to be studied through the classical laminate theory. This ply has a linear viscoelastic matrix composed of ED-6 resin at room temperature and reinforced by elastic glass fibers, the mechanical properties of which are shown in Table 1.

Table 1 – Elastic and viscoelastic constants of the ply.

Material	**Volume fraction**	E_{el} (GPa)	G_{el} (GPa)	K (GPa)	v_{el}	E_v (GPa)	η_e (GPa.h)	G_v (GPa)	η_g (GPa.h)
Resina ED-6	0,46	8,055	3,2	5,56	0,2586	3,486	532,741	1,8	300
Vidro	0,54	85	35,42	47,22	0,2	-		-	-

The values of the constants $\mathbf{G_{el}}$, K, $\mathbf{G_v}$ and $\boldsymbol{\eta_g}$ are presented in Cai and Sun (2013) and when applying the Laplace transform it was possible to find the values of $\mathbf{E_{el}}$, $\mathbf{v_{el}}$, $\mathbf{E_v}$ and $\boldsymbol{\eta_e}$. For these constants, the following effective functions were obtained as a function of time. Given these values, we obtain the flexibility matrix [S(t)] given by:

$$\begin{aligned} S_{11}(t) &= 0{,}02015492309 + 2{,}880174975 * 10^{-8}e^{-0{,}006754771643t} \\ S_{22}(t) &= 0{,}06330053498 + 0{,}1516113282 * 10^{-5}e^{-0{,}009375000001t} \\ S_{12}(t) &= -0{,}004568456620 + 9{,}247976226 * 10^{-7}e^{-0{,}006754771643t} \\ S_{66}(t) &= 0{,}1585104677 + 0{,}4548339842 * 10^{-5}e^{0{,}009374999998t} \end{aligned} \tag{14}$$

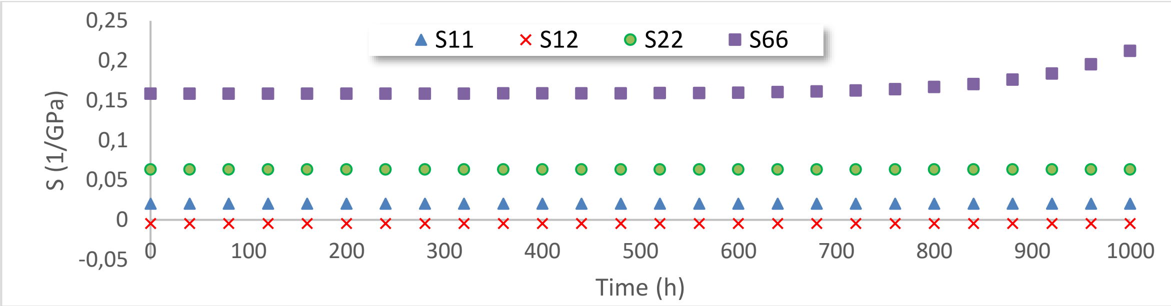

Figure 3 - Time flexibility variation.

It can be seen that for the ply, the flexibility functions over time vary very little. This is due to two aspects: first, the material is composed of 54% fibers, secondly, these fibers are elastic and much more rigid than the matrix. Only the term S_{66} presents considerable variation in time.

3 CONCLUSIONS

The present work presented the development of a formulation that involves classical laminate theory with linear viscoelasticity to analyze the behavior of computational models of laminated composite panels to obtain macroscopic properties, originally developed for elastic materials. Thus, the objective of developing the formulation was reached, since it was possible to implement a plausible formulation from the CLT and viscoelasticity. However, according to the results obtained in this present work, it is understood that in consideration of the chosen material itself, a great elastic stiffness was generated. Thus, the deformation over time does not vary much, but it is noticeable that this variation exists. Thus, the use of other materials is motivated in order to obtain a better deformation curve as a function of time as well as to develop further studies on the subject.

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