INVERSE ACOUSTIC CHARACTERIZATION OF POROUS MATERIALS IN IMPEDANCE TUBE USING GENETIC OPTIMIZATION

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Abstract. *Porous materials such as foams or fibrous composites are widely used in several applications of noise control, thus the acoustic characterization of these materials is fundamental for an efficient application of resources. Several models have been developed throughout the years looking to acoustically characterize porous materials, usually these models depend on the physical properties of the material. In the model developed by Johnson-Champoux-Allard (JCA) a porous material can be modelled acoustically by five physical parameters: the porosity, the tortuosity, the flow resistivity and the viscous and thermal characteristic lengths. Due to the difficulty in measuring all the necessary properties to characterize the material, at least through the traditional methods, since it is necessary a specific test and apparatus to estimate each property, in recent years researches are being developed and alternative methods for obtaining these properties are being studied. A method that draws attention is the inverse characterization of the properties based on impedance tube measurements, this way it's possible to replace the traditional tests by a single test in impedance tube. The properties needed in the JCA model can be obtained through an optimization process. In this paper a reverse acoustic characterization of porous materials through an optimization process using the differential evolution algorithm is presented. The effectiveness of the presented method is evaluated through an experimentally validated virtual impedance tube numerical model. The experimental validation of the numeric model, which is done through standard samples measurements, is presented and the optimization process is explored and analyzed since the parameters can be controlled and are well known in the numeric model.*

Keywords: *Acoustics, Porous material, Inverse characterization, Impedance tube, Genetic optimization*

1 INTRODUCTION

Material characterization is fundamental in the analysis of a system, since a numerical model needs good input properties in order to be robust in the representation of the physical world and allow evaluation of opportunities for improvement.

However, classical methods for evaluation of the acoustical properties of a material are time consuming, since they demand a different test and apparatus for each property, and are quite sensitive to uncertainties since the quantities are measured indirectly.

With the increase of the computational power in recent years, both numerical models and optimization algorithms are being widely used in engineering. The need to reduce time and cost is making engineers substitute some tests for computational methods.

This lead researchers to evaluate a different approach to the acoustic characterization of materials, based on an inverse optimization method of impedance tube measurement. Atalla & Panneton (2005) presented in their work a methodology for estimation of the acoustical properties of the Johnson-Champoux-Allard (JCA) model for porous materials.

Other works have then been developed as well, such as presented by Doutres et. al (2010) who used a different setup for the impedance tube, or Chazot et. al (2012) who explored a different approach in the inverse estimation.

In more recent works, Zielinski (2015) evaluated uncertainties associated to the measurements in impedance tube, such as the sample positioning, and Zhang & Zhu (2016) evaluated a method for metallic materials.

This paper is part of a bigger work, the aim is to study the uncertainties associated with the optimization process, and develop a methodology that can be used to estimate the physical properties of a material, so that it can be used as input to represent the acoustic behavior of the material in numerical simulations of bigger systems.

2 THEORETICAL BACKGROUND

As explained by Ravindran (2007), the sound propagation in porous media is governed by properties that depends on the geometry of the porous material, the properties of the air in the pores, and on the mechanical properties of the solid.

Five parameters describe the macroscopic physical properties (geometric) of the material and define the complexity of the sound propagation in the porous media. These parameters are: porosity, tortuosity, flow resistivity, characteristic viscous and thermal lengths.

- Porosity is defined as the ratio between the volume of air in the pores and the total volume of the material.
- Tortuosity is the property that describes how the porous material prevents direct flow in the media. Relating the mean length of the fluid path in the material with the material thickness.

- Flow resistivity is the pressure drop necessary to force a unitary flow through the material. Expressing the viscous losses of the sound waves propagating in the porous material.
- The characteristic viscous length is the mean macroscopic dimension of the cells related to the viscous losses. It can be seen as the mean radius of the smaller pores of a porous aggregate.
- The characteristic thermal length is the mean macroscopic dimension of the cells related to the thermal losses. It can be seen as the mean radius of the larger pores of a porous aggregate.

The air in the pores can be considered an ideal gas. The density, speed of sound, viscosity, number of Prandtl and specific heat rate define the properties of the air.

The modified equation for propagation of plane waves in the air contained in rigid porous materials is given by Eq. 1:

$$\frac{\partial^2 p}{dx^2} - \left(\frac{\alpha_\infty \rho_0}{k_{eff}}\right) \cdot \frac{\partial^2 p}{dt^2} - \left(\frac{\sigma \phi}{k_{eff}}\right) \cdot \frac{\partial p}{dt} = 0. \qquad (1)$$

Where p is the sound pressure in the pores of the material, ρ_0 is the air density, k_{eff} is the air effective bulk modulus, α_∞ is the tortuosity, ϕ is the porosity, and σ is the flow resistivity.

The acoustic behavior of a homogeneous porous material can also be determined from its fundamental quantities: the complex wave number and the characteristic impedance. These quantities are the harmonic solution of the plane waves equation and can be used to define the acoustic performance of porous materials, these being the absorption coefficient, the specific impedance and the transmission loss.

3 THE MODEL

3.1 Acoustic Model

The specific impedance can be obtained through standardized measurements in impedance tube. The acoustic model approached by Atalla & Panneton (2005) consists of a porous layer over an impermeable rigid termination in a tube, as illustrated in Fig. 1. The walls of the tube are considered rigid, impermeable and perfectly reflexives. A piston touches the back face of the porous sample, being that the porous layer also touches the walls of the tube around its contour, this one being circular.

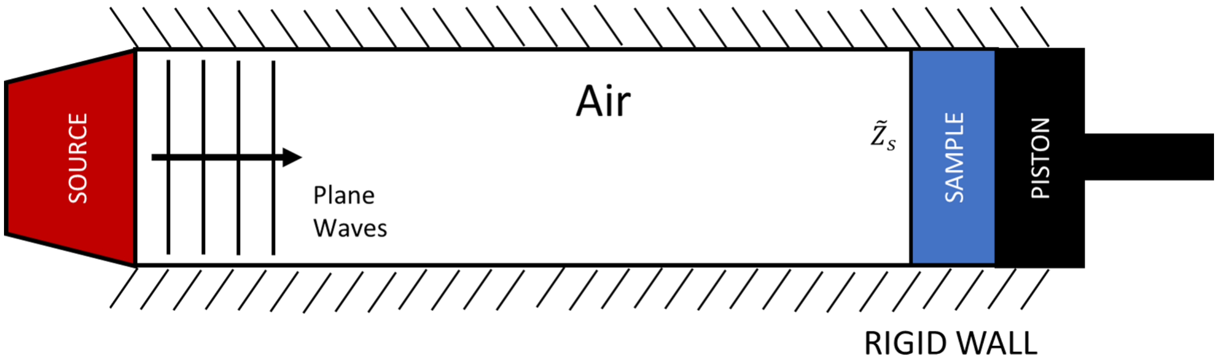

Figure 1. Impedance Tube – Normal Incidence

Normal incidence of acoustic waves excites the frontal face of the porous layer. In the interface between air cavity and porous layer, the specific impedance is given by Eq. 2:

$$\tilde{Z}_s = \frac{\tilde{Z}}{Z_0} = -\frac{j\tilde{Z}_c}{\phi Z_0}\cot(\tilde{k}L). \tag{2}$$

Where $\tilde{Z}$, $\tilde{Z}_c$, $\tilde{k}$ and ϕ are, respectively, the surface impedance (ratio between the acoustic pressure and associated particle velocity), the characteristic impedance of the equivalent fluid, the complex wave number, and the porosity of the porous sample. Z_0 is the characteristic impedance of the air. The symbol ~ indicates that the associated variable has complex component and is frequency dependent, L is the thickness of the sample, and $j = \sqrt{-1}$.

To eliminate the effects of elasticity in the border of the porous sample, the acoustic excitations are in a frequency range in which the porous layer behaves as acoustically rigid, in other words, for frequencies greater than the decoupling frequency.

Since the border of the material is assumed static, only a longitudinal wave of compression in the fluid phase propagates along its axis. Under this supposition of rigid border, the fluid saturating the interconnected cells of the porous media can be macroscopically described as an equivalent homogeneous fluid of density $\tilde{\rho}$ and effective bulk modulus $\widetilde{K}$.

In this case, following the equivalent fluid model based on the works of Johnson et al. (1987) and Champoux & Allard (1991), the characteristic impedance and the complex wave number in the "rigid" porous media are given by:

$$\tilde{Z}_c = \sqrt{\tilde{\rho}\widetilde{K}}. \tag{3}$$

And:

$$\tilde{k} = \omega\sqrt{\frac{\tilde{\rho}}{\tilde{K}}}. \tag{4}$$

With effective density $\tilde{\rho}$:

$$\tilde{\rho} = \rho_0\alpha_\infty\left(1 + \frac{\phi\sigma}{j\omega\rho_0\alpha_\infty}\left(1 + j\,\frac{4\omega\rho_0\eta\alpha_\infty^2}{\sigma^2\phi^2\Lambda^2}\right)^{\frac{1}{2}}\right). \tag{5}$$

And effective bulk modulus $\tilde{K}$:

$$\tilde{K} = \frac{\gamma P_0}{\gamma-(\gamma-1)\left(1+\frac{8\eta}{j\omega B^2{\Lambda'}^2\rho_0}\left(1+j\frac{\omega B^2\rho_0{\lambda'}^2}{16\eta}\right)^{\frac{1}{2}}\right)^{-1}}. \tag{6}$$

In the equations, ω is the angular frequency, P_0 is the barometric pressure, and γ, B^2, ρ_0 and η are the specific heat rate, the Prandtl number, density, and dynamic viscosity of saturated air, respectively.

The five remaining properties are those which define the complexity of the porous sample. They are the porosity ϕ, tortuosity α_∞, flow resistivity σ, characteristic viscous length Λ, and characteristic thermal length Λ'.

The superior frequency limit is defined as the frequency to which normal incidence can be guaranteed. This limit is dependent on the sample diameter (tube internal diameter), being that lower the diameter higher the frequency.

3.2 Optimization Process

The optimization algorithm used in this work is the Differential Evolution, which was developed and presented by Kenneth Price and Rainer Storn.

Differential Evolution (DE) is a method that searches optima of a function iteratively trying to improve a candidate solution with regard to a given measure of quality. Such methods are commonly known as metaheuristics as they make few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions. However, metaheuristics such as DE do not guarantee an optimal solution is ever found.

DE is used for multidimensional functions but does not use the gradient itself, which means DE does not require the optimization function to be differentiable, in contrast with classic optimization methods such as gradient descent and newton methods.

A basic variant of the DE algorithm works by having a population of candidate solutions (called agents). These agents are moved around in the search-space by using simple mathematical formulae to combine the positions of existing agents from the population. If the new position of an agent is an improvement it is accepted and forms part of the population, otherwise the new position is simply discarded. The process is repeated and by doing so it is hoped, but not guaranteed, that a satisfactory solution will eventually be discovered.

Variants of the DE algorithm are continually being developed in an effort to improve optimization performance. Many different schemes for performing crossover and mutation of agents are possible in the basic algorithm.

The crucial idea behind DE is a scheme for generating trial parameter vectors. Basically, DE adds the weighted difference between two population vectors to a third vector. This way no separate probability distribution has to be used which makes the scheme completely self-organizing, as illustrated in Fig. 2.

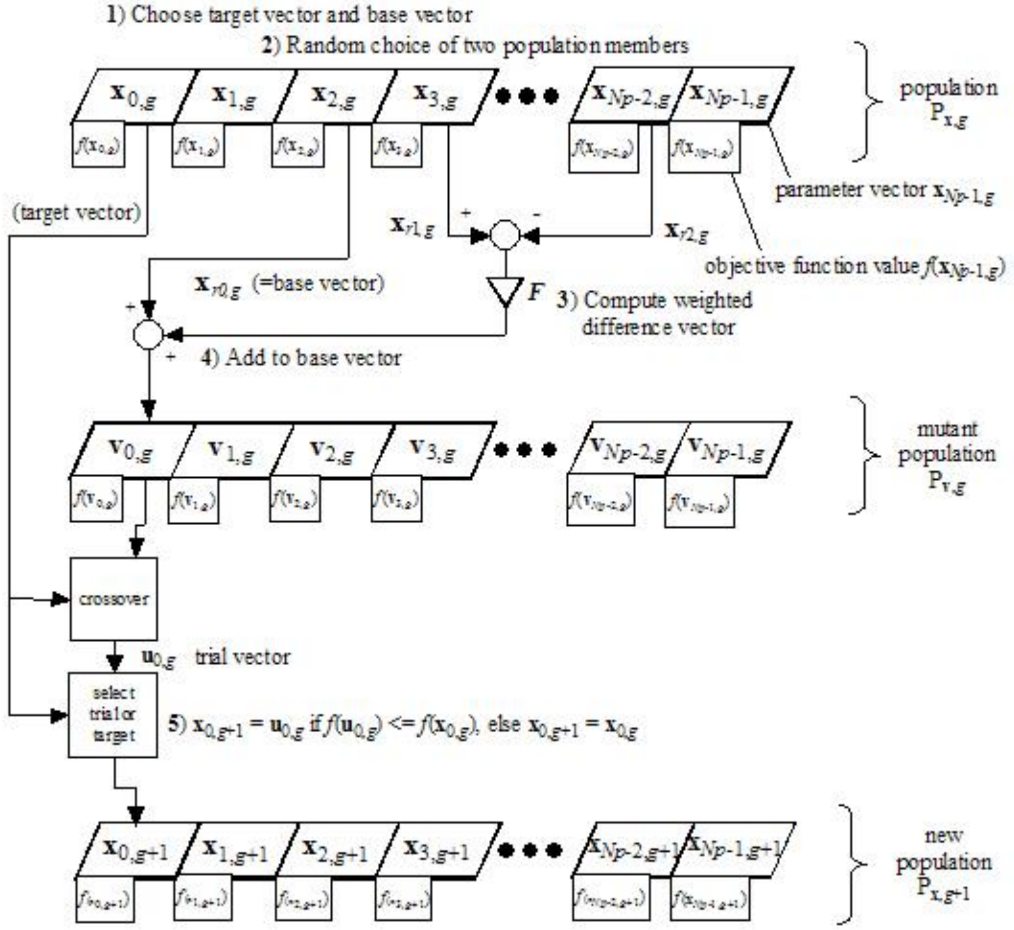

Figure 2. Differential Evolution Algorithm [Source: Price & Storn (1995)]

In this work, a multidimensional optimization function is used to adjust the observed measurements and the acoustic quantities obtained from the physical properties estimated iteratively.

The objective function is defined to minimize the error between the measured impedance and the estimated one (Eq. 7).

$$R(a) = \frac{1}{2}\sum_{i=1}^{N}\left|\tilde{Z}_s^e(a) - \tilde{Z}_s^o\right|^2. \quad (7)$$

Where the superscripts “e” and “o” represent, respectively, the estimated and observed values, “N” is the total number of frequencies calculated in the frequency range of interest. The estimated values are obtained through Eq. 2 for a given parametric vector “a”.

The function is also submitted to the following restrictions during the optimization process:

- $0.1 \leq \phi \leq 1$
- $1 < \alpha_\infty < 10$
- $1000\text{Ns}/m^{-4} \leq \sigma \leq 200000\text{Ns}/m^{-4}$
- $10\mu\text{m} \leq (\Lambda \text{ and } \Lambda') \leq 2000\mu\text{m}$
- $\Lambda < \Lambda'$

Both the inferior and superior boundaries are specified based on definition and also on empiric results.

3.3 Numeric Model

The measurements are made in an impedance tube Bruel & Kjaer Type 4206, in accordance to the ASTM E1050-12 standard. Measurements are made in the frequency range of 50Hz – 1.6kHz, which is related to the 100mm tube diameter, and of 500Hz – 6.4kHz, which is related to the 29mm tube diameter. The schema is shown in Fig. 3 and the tube configurations can be seen on Fig. 4.

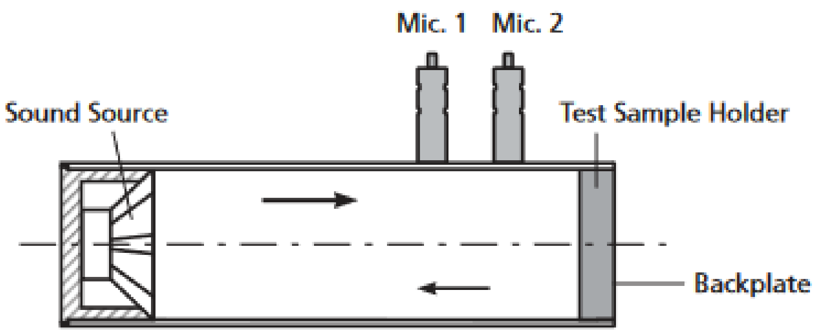

Figure 3. Bruel & Kjaer Type 4206 Impedance Tube Schema [Source: Bruel & Kjaer]

Figure 4. Bruel & Kjaer Type 4206 Impedance Tube [Source: Bruel & Kjaer]

In order to validate the numeric model, the standard calibration sample provided by Bruel & Kjaer, which stands for the foam shown in Fig. 5, was used.

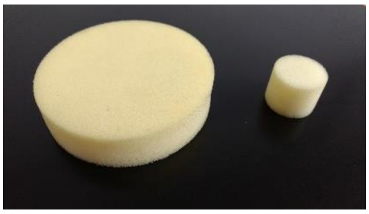

Figure 5. Bruel & Kjaer Standard Calibration Sample

A model of the air cavity of the tube was built with the designed dimensions of the tube (diameter, length from sound source to mic. 1, length from sample to mic 2., and length between mics.).

An acoustic body was defined with the air properties, a unitary velocity was used as excitation representing the sound source, and the measured impedance was prescribed in the sample surface, as illustrated in Fig. 6. The commercial software Ansys Workbench with the Acoustics ACT was used for the numeric simulations.

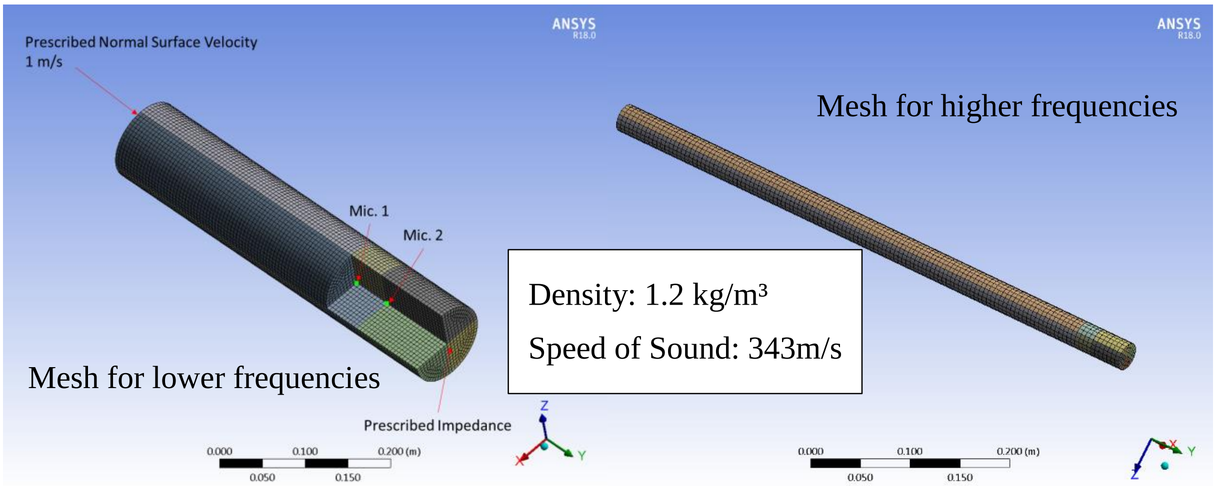

Figure 6. Numeric Impedance Tube Model

The compared result, shown in Fig. 7, is the absorption coefficient, calculated as specified in the ASTM E1050-12 standard from the two microphone method measurement.

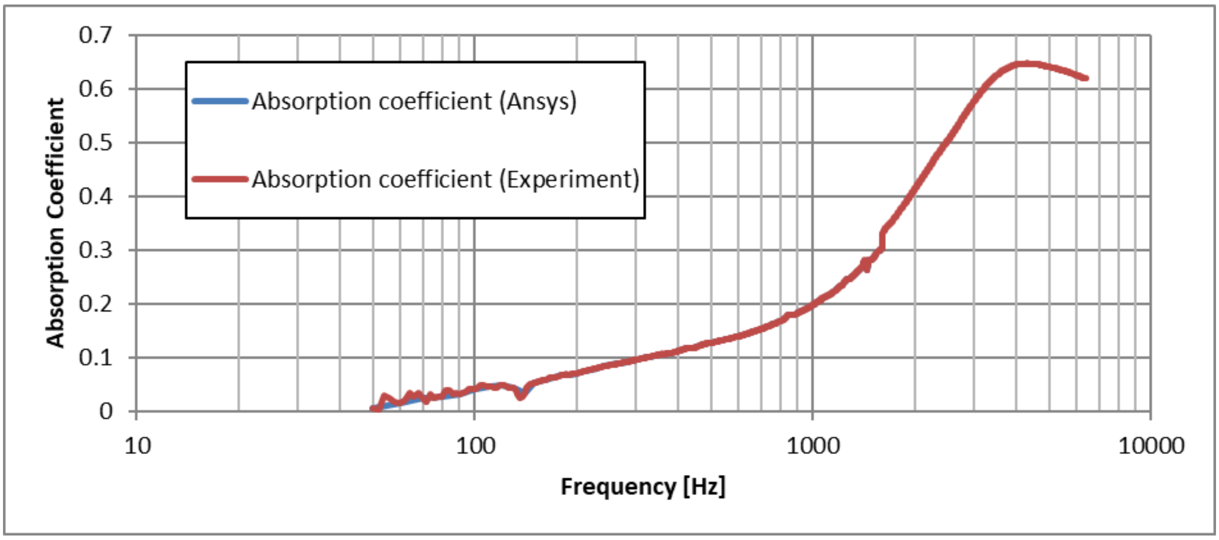

Figure 7. Validation Results

The results show great accordance, indicating that the model is representing well the experiment. Now, from this validated model, another simulation is performed, creating the equivalent acoustic body referring to a generic material, using the JCA formulation as the input properties, as shown in Fig. 8. With these controlled inputs, the optimization step can then be evaluated.

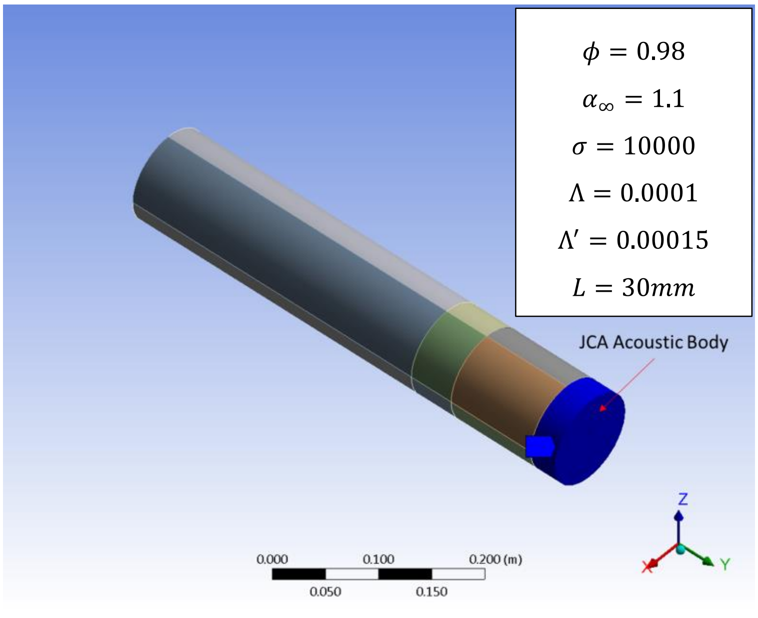

Figure 8. Numeric Impedance Tube Model with JCA Material

4 RESULTS

The optimization algorithm used was the NMinimize function with the Differential Evolution option of the commercial software Wolfram Mathematica.

The equations were written, the objective function was defined and the restrictions imposed. The initial attempt was to minimize the curve in the whole frequency range, however, even though the curve was fit, the parameter results were not good, as shown in Table 1.

Other tests playing with the configuration parameters of the differential evolution were performed, but no improvement in the estimation of the parameters was achieved.

So a second attempt was made optimizing the curve by different smaller frequency ranges, individually, in accordance to the absorption curve shape (Fig. 9), as suggested by other references, such as Atalla & Panneton (2005).

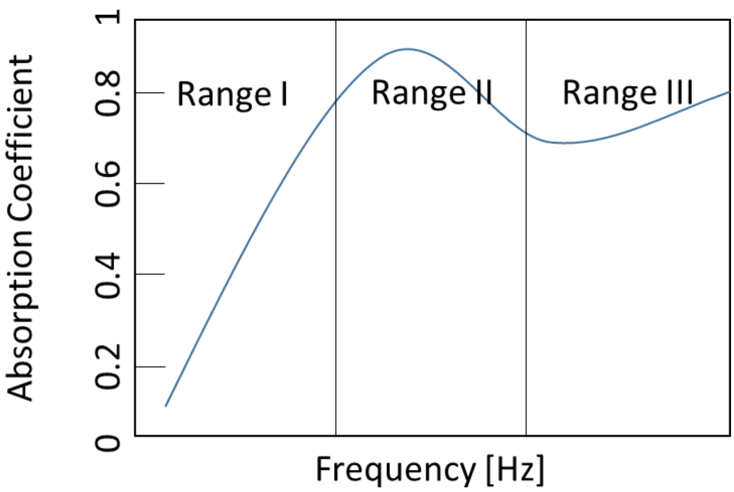

Figure 9. Dominating Parameters Zones

In this attempt different ranges were tested, but in each of these ranges the five parameters were left free to be optimized, trying to find which range fits better each parameter. Even though there was improvement, the results were still not good enough, as shown in Table 1.

The third attempt was to optimize by ranges as before, but for each step the best fitted parameters were prescribed to the next step. In this attempt, the results showed better accordance, as shown in Table 1.

Table 1. Estimated Properties

Properties	Input	1st Attempt (Error)	2nd Attempt (Error)	3rd Attempt (Error)
ϕ	0.98	0.97 (1.0%)	0.98 (0.0%)	0.98 (0.0%)
α_∞	1.1	1.24 (12.7%)	1.20 (9.1%)	1.14 (3.6%)
σ	10000	12860 (28.6%)	10409 (4.1%)	10103 (1.0%)
Λ	0.0001	0.000169 (69.0%)	0.000138 (38.0%)	0.000116 (16.0%)
Λ'	0.00015	0.000169 (12.7%)	0.000138 (8.0%)	0.000164 (9.3%)

The results in the last attempt present errors similar to those usually found by other references.

5 FINAL THOUGHTS AND CONSIDERATIONS

The improvement achieved among the attempts contemplated in this paper is clear, which shows the high sensibility of the methodology. However, the fact that the inputs are known and controlled leaves an open window for further improvement.

Authors from the references agree that subdividing the curve in regions enhances the quality of the estimated results, however there isn't a complete agreement in regard the dominating parameters in each region, showing that it may vary depending on the material behavior.

The numeric model presented is representing very well the acoustic behavior of the material, which allows a wide range of parameters to be tested easily, so further investigation aiming to find a rule of thumb for the optimization process is the next step in this work.

When the procedure is completely defined and revised, further exploration of the numeric model shall be done as well, testing the behavior of the material, with the estimated inputs, as part of a bigger system.

There's also room to evaluate this methodology as a potential tool in material development. However, experimental validations of the parameter estimation must be done.

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