A MODEL TO ANALYZE STRUCTURES SUBJECTED TO CHEMICAL-MECHANICAL STRESS

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Abstract. *Corrosion is one of the main pathological manifestations that the reinforced concrete is susceptible. However, there is no suitable methodology - at structural level - that includes the evolution of the damage caused by corrosion. The evaluation of the mechanical behavior of reinforced concrete can be performed based on the lumped damage mechanics through the implementation of plastic hinges and damage variables that describe the cracking, together with the principle of conservation of energy of thermodynamics, which allows the analysis of damage due to corrosion. Thus, the present work combined the lumped damage mechanics with the thermodynamics to analyze structures subjected to chemical-mechanical stress from the creation of a constitutive law of corrosion coupled to mechanical damage.*

Keywords: *Lumped damage, Reinforced concrete, Corrosion*

1 INTRODUCTION

The reinforcement concrete (RC) is the most used material in construction due to its durability, facility of access and execution. However, it is susceptible to pathological manifestations when exposed to certain environments. The corrosion is one of the main pathologies of reinforcement concrete, but there is no suitable methodology - at the structural level - that includes the evolution of the damage caused by corrosion (Andrade and Mancini, 2011).

In the reinforcement concrete, the steel is initially protected from corrosion because the concrete is alkaline, i.e. contains microscopic pores with high concentrations of soluble calcium, sodium and potassium oxides, which in the presence of water form hydroxides and leads to passive layer in the steel (Broomfield, 2006). However, this protection lasts until the corrosion develop and the structural consequences are: reduction of steel ductility; loss of the reinforcement/concrete bond; and concrete cracking (Andrade, 2012).

Therefore, we can name three principal reasons of the importance of corrosion's studies, savings, safety and conservation (Revie and Uhlig, 2008). Corrosion is a huge problem faced by the government and engineers, because of economy loss caused by corrosion's damage which there are billion dollars spent in safety for year (Broomfield, 2006).

Lumped damage mechanic (LDM) combines fracture mechanics and the lumped plasticity model, assuming that the cracks are concentrate in plastics hinges (Flórez-López, Marante and Picón, 2015). In this paper, LDM will be used to couple corrosion to the analysis of the frame's behavior. The use of LDM for modelling corrosion if RC frames was first proposed by Coelho (2017). LDM reduces the computational cost of the analyses making possible reliability studies of complex structures.

2 MODEL OF DAMAGE COUPLED TO CORROSION

2.1 Initial concepts

The frame analysis considers the generalized displacements of the nodes and the equivalents external forces applied to each node. In a planar frame, each node has three generalized displacements (Fig. 1), generating the displacements matrix $\{U\}^T = [u_1, w_1, \theta_1, u_2, \cdots, u_n, w_n, \theta_n]$, where n is the number of nodes, u_n are the horizontal displacements, w_n are the vertical displacements and θ_n are the rotations.

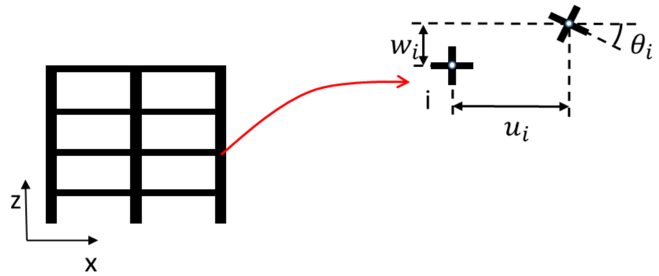

Figure 1. Planar frame and generalized displacements

The matrix of external forces $\{F\}^T = [Ru_1, \mathrm{R}w_1, M_1, Ru_2, \cdots, Ru_n, Rw_n, M_n]$, where Ru_n are the horizontal forces, Rw_n are the vertical forces and M_n are the external moments. The displacements matrix and the matrix of external forces are defined with their respective boundary conditions.

Generalized deformations and stresses are defined for each element of the frame. Thus, considering an element b of a frame (Fig. 2), the deformations $\left(\{\Phi\}_b^T\right)$ and the stresses $\left(\{M\}_b^T\right)$, are defined by the matrices: $\{\Phi\}_b^T = [\phi_i^b, \phi_j^b, \delta_b]$ and $\{M\}_b^T = [m_i^b, m_j^b, n_b]$.

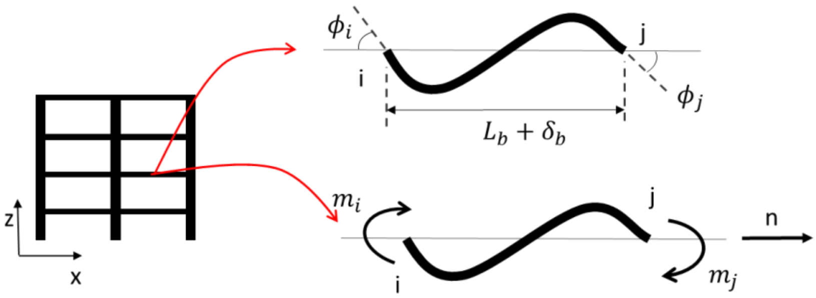

Figure 2. Planar frame, generalized strains and stresses

The kinematic equations relate deformations and displacements of the frame, according to the equations below (Flórez-López, Marante and Picón, 2015).

$$\phi_i = \frac{\sin\alpha}{L_b}u_i - \frac{\cos\alpha}{L_b}w_i + \theta_i - \frac{\sin\alpha}{L_b}u_j + \frac{\cos\alpha}{L_b}w_j$$

$$\phi_j = \frac{\sin\alpha}{L_b}u_i - \frac{\cos\alpha}{L_b}w_i - \frac{\sin\alpha}{L_b}u_j + \frac{\cos\alpha}{L_b}w_j + \theta_j \tag{1}$$

$$\delta = -\cos\alpha\, u_i - \sin\alpha\, w_i + \cos\alpha\, u_j + \sin\alpha\, w_j$$

In matrix notation the kinematic equations can be written as: $\{\Phi\}_b = [B]_b\{U\}_b$, where $[B]_b$ corresponds to the transformation kinematic matrix. The equilibrium equations, represents the relation between internal stresses and external forces:

$$\sum [B]^T\{M\} = \{P\} \tag{2}$$

The relation between stresses and deformations are obtained through constitutive equations, that characterize the behavior of the elements.

LDM assumes that cracking is lumped at the plastic hinges (Fig. 3). Then, the plastic hinges become inelastic hinges. The cracking state is represented by the internal variable $(D) = (d_i, d_j)$ for each element.

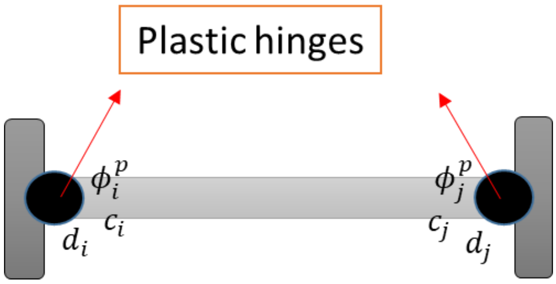

Figure 3. Plastic hinges

Similarly, the corrosion is lumped at the plastic hinges, defined by a new internal variable $(C) = (c_i, c_j)$ for each element. In the analysis of pitting corrosion, we assumed that the only effect of corrosion in reinforcement concrete is the reduction of reinforcement's effective area. This reduction is represented by pit depth $(p(t)$, as shows the Fig. 4, that represents the corroded area induced concrete cracking.

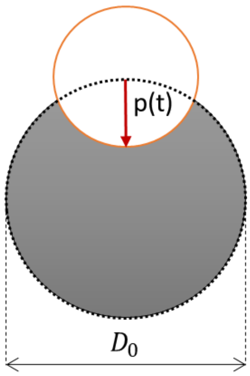

Figure 4. Pit depth

Consequently, it is obtained new values for the proprieties of the section, i.e., reduces for example, plastic moment and the ultimate moment. So, the pit depth, can be defined as a function of time as (Stewart, 2004):

$$p(t) = 0{,}0116 * i_{cor} * R * t \tag{3}$$

Where, R is a statistical parameter and by simplification is equal to 5,08 (Coelho, 2017), i_{cor} is the corrosion rate (in $\mu A/cm^2$) and t is the time in years (Stewart, 2004). The corrosion rate is defined empirically for typical environmental condition as (Vu and Stewart, 2000):

$$i_{cor} = \frac{37{,}8 * (1 - {}^{w}/_{c})^{-1{,}64}}{d} * 0{,}85 * (t - t_{ini})^{-0{,}29} \tag{4}$$

Which, ${}^{w}/_{c}$ is the water-cement ratio, d is the cover, t is the time and t_{ini} is the corrosion start time. This way, the state variable of corrosion (c), represents a rate of corrosion, i.e., assume values between zero and one, that means, no corrosion until total destruction of the bar.

$$c = \frac{1}{D_0} * p(t) \tag{5}$$

Where, D_0 is the diameter of the bar. Therefore, can be calculated the reinforcement's effective area (A_s) using the corrosion rate:

$$A_s = \frac{A_0}{\pi} \begin{cases} -4c_i^2 \arcsin\sqrt{1-c_i^2} + 2c_i\sqrt{1-c_i^2} - \arcsin\left(2c_i\sqrt{1-c_i^2}\right) + \pi & if\ c_i < \frac{\sqrt{2}}{2} \\ -4c_i^2 \arcsin\sqrt{1-c_i^2} + 2c_i\sqrt{1-c_i^2} + \arcsin\left(2c_i\sqrt{1-c_i^2}\right) & otherwise \end{cases} \tag{6}$$

Where, A_0 is the initial steel area.

The deformations in an element ($\{\Phi\}_b$) demonstrate in Eq. 7 are composed by elasticity deformations ($\{\Phi^e\}_b$), plastic deformations ($\{\Phi^p\}_b$) and damage rotations ($\{\Phi^d\}_b$). And when the corrosion starts the damage rotations is increase by the change in the proprieties of the section for the reduction of the reinforcement's effective area.

$$\{\Phi\}_b = \{\Phi^e\}_b + \{\Phi^p\}_b + \{\Phi^d\}_b \tag{7}$$

Hence, they are divided into a law of elasticity, a law of plasticity, a law of damage and law of corrosion. (Coelho, 2017). Where the law of elasticity is:

$$\{\Phi - \Phi^p\} = [F(D)]\{M\} \tag{8}$$

$[F(D)]$ corresponds to the flexibility matrix. And for the plasticity law, initially, it's considered the plastic rotations are zero, in other words, there's only elastics strain. For the calculation, it's necessary to define a yield function (f):

$$f = \left|\frac{m}{1-d} - h(c)\dot{\phi}^p\right| - k_0(c) \leq 0 \tag{9}$$

Where m is the moment, d is the damage, $h(c)$ is the term of linear kinematic hardening as a function of corrosion and $k_0(c)$ is the moment due to corrosion. These parameters are determinate with the plastic damage, i.e., the plastic rotations is zero, the moment k_0 is defined as (Coelho, 2017):

$$k_0 = \frac{M_p}{1-d_p} \tag{10}$$

Where M_p is the plastic moment, d_p is the plastic damage. The plasticity function is zero in the ultimate moment, so, the kinematic hardening is obtained as a function of ultimate plastic rotation.

$$h(c) = \frac{1}{\phi_{p_u}}\left(\frac{M_u}{1-d_u} - \frac{M_p}{1-d_p}\right) \tag{11}$$

Where M_u is the ultimate moment, d_u is the ultimate damage.

The yield function can assume values less than or equal zero, so, while this function is less than zero, no plastic rotations have been generated yet, and when zero is equal, the ball joint is active, that is, there is a plastic deformation in the strain. Thus, the law of plasticity (Eq. 12) defines whether the label is active or blocked (Flórez-López, Marante and Picón, 2015).

$$\begin{cases} f < 0 \Rightarrow \dot{\phi}^p = 0 & (locked\ hinge) \\ \dot{\phi}^p \neq 0 \Rightarrow f = 0 & (active\ hinge) \end{cases} \tag{12}$$

For the law of damage, It's first necessary to define the function of resistance:

$$R(d,c) = R_0(\mathrm{c}) + q(c)\frac{\ln(1-d)}{1-d} \tag{13}$$

Which $R(d,c)$ it's the resistance of the frame and it depends on the varieties of damage and corrosion, $R_0(\mathrm{c})$ is the initial resistance $q(c)$ is the increase of resistance by the action of the reinforcement. The parameters R_0 e q can be determinates by the critical moment, both are determinate by Eq. 13 and the Fig. 5:

$$m^2 = \frac{6EI(1-d)^2}{L}R_0 + \frac{6qEI}{L}(1-d)ln(1-d) \tag{14}$$

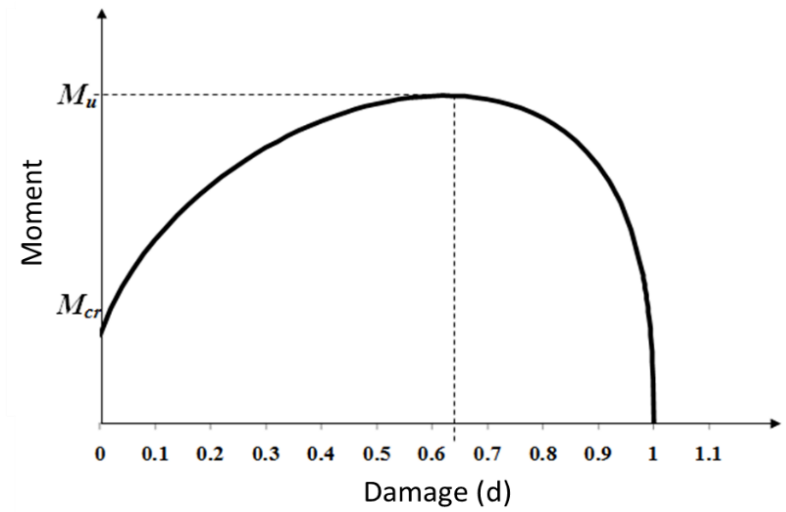

Figure 5. Moment as a function of damage (Flórez-López, Marante and Picón, 2015)

When $m = M_{cr}$, which corresponds to the critical moment, then the damage is zero and the initial resistance is defined as:

$$R_0 = \frac{M_{cr}^2 L}{6EI} \tag{15}$$

The parameter q it is calculate with ultimate damage that is obtained by derivative of the Eq. 13 equal to zero. The criterion of Griffith is defined as:

$$y = \frac{F^0 m^2}{2(1-d)^2} \tag{16}$$

Where F^0 is the flexibility matrix with zero damage. Then, as long as the value obtained through Griffth criterion is less than the value of the resistance function, there won't be any damage to the frame. However, when there is an equality, the increment of the damage variable and consequently the corrosion, so:

$$\begin{cases} y < R(d,c) \Rightarrow \dot{d} = 0 \\ \dot{d} > 0 \Rightarrow y = R(d,c) \end{cases} \tag{17}$$

2.2 Thermodynamics

The principle of virtual power

This principle states that the virtual power of external forces ($\hat{P}_{ext}$) is equal to the sum of the virtual power of inertia's force ($\hat{P}_{ine}$) and the virtual power of deformations ($\hat{P}_{def}$).

$$\hat{P}_{ext} = \hat{P}_{ine} + \hat{P}_{def} \tag{18}$$

Where the virtual power of a force system is a continuous linear function of the virtual velocity scalar value $\left(\dot{\hat{U}}\right)$, equal to the work (τ) performed per time unit (Δt) of the phenomenon (Lemaitre and Chaboche, 1994).

$$\hat{P} = \frac{\tau}{\Delta t} \tag{19}$$

The first principle

The variation of the total energy of the system occurs if there is a mechanical work and/or heat transfer, so the first principle of thermodynamics represents the energy conservation of the system, establishing the balance between the mechanical power and the heat transfer rate with the heat rate of total energy's change, that is to say the mechanical work rate (P_{ext}) (external force power) plus the heat rate (Q) is equal to the rate of kinematic energy $(\dot{E})$ plus the variation's rate of the internal energy $(\dot{K})$ (Proença, 2000).

$$\dot{E} + \dot{K} = P_{ext} + Q \tag{20}$$

Whereas the first thermodynamics' principle and the virtual power's principle, considering $P_{ext} = \{\dot{U}\}\{F\}$ and $\dot{K} = P_{ine}$, the following equation is obtained:

$$\Rightarrow Q = \sum_{b=1}^{m} Q_b \qquad \Rightarrow \dot{E} = \sum_{b=1}^{m} \dot{E}_b \quad \therefore \ \dot{E}_b = P_{def}^b + Q_b \tag{21}$$

The second principle

The second principle increases two variables, entropy and temperature (T_b), assuming it is possible to represent the temperature by a scalar field of positive values defined for each instant time, where the entropy ($\dot{S}_b$) corresponds to the energy variation associated with a change in temperature. Thus, the principle determines that entropy rate is always greater than or equal to the heat rate (Q_b) divided by the absolute temperature (Lemaitre and Chaboche, 1994).

$$\dot{S}_b - \frac{Q_b}{T_b} \geq 0 \tag{22}$$

This principle can be used both for reversible processes as well as irreversible processes, different from the first principle, which doesn't restrict the direction of conversion, it only requires a balance between energies. Thus, in the equation above, equality corresponds to reversible processes and inequality for irreversible processes (Proença, 2000).

In thermodynamics, thermodynamic potentials are used, in which they present the continuity property, that is, the potential is a continuous function of all the variables of the space, being thus differentiable in a relation to them (Campos et al, 2015).

The free energies of Helmholtz and Gibbs can be used as thermodynamic potentials, assuming that Helmholtz free energy is a function of generalized strains, temperature and a set of state variables and also considering that Gibbs free energy is a function of generalized stresses, temperature and the same set of state variables (Campos et al, 2015).

So, the Clausius-Duhem inequality is obtained through the free energy of Helmholtz ($\dot{\psi}_b$), where this inequality is defined as (Lemaitre and Chaboche, 1994).

$$P_{def}^{b} - \dot{\psi}_b - S_b \dot{T}_b \geq 0 \tag{23}$$

And the variation of Gibbs' free energy (ΔG) indicates the reaction tendency of in a metal with its environment. In this way, the more negative the value of ΔG the more the tendency of reaction occurrence, however, it can't be said that corrosion rate will also be high (Revie and Uhlig, 2008).

Thermodynamics of frames

The virtual power of external forces corresponds to a function of the virtual velocities $\left(\dot{\hat{U}}\right)$ for the external forces, $\hat{P}_{ext} = \left\{\dot{\hat{U}}\right\}^T \{F\}$, where {F} is the matrix of equivalent nodal forces. And the virtual power of the strain corresponds to the sum of the virtual powers defined for each frame element, in which the power of each element is given by the product of the generalizes deformation rate $\left\{\dot{\hat{\Phi}}\right\}$ for generalizes stresses' matrix $\{M\}_b$:

$$\hat{P}_{def} = \sum_{b=1}^{m} \hat{P}_{def}^{b} \Rightarrow \hat{P}_{def}^{b} = \left\{\dot{\hat{\Phi}}\right\}_b^T \{M\}_b \tag{24}$$

Which, m is the number of the frames' elements. And, at last, the virtual power of the inertia's is defined as:

$$\hat{P}_{ine} = \left\{\dot{\hat{U}}\right\}^T [Mass]\{\ddot{U}\} \tag{25}$$

Where, $[Mass]$ is the mass matrix of the structure and $\{\ddot{U}\}$ is the acceleration matrix. This way, we obtain the dynamic equilibrium equation of the frame:

$$\{F\} = \sum_{b=1}^{m} [B]^T \{M\} + [Mass]\{\ddot{U}\} \tag{26}$$

When $[Mass]\{\ddot{U}\} \cong 0$, there's the static equilibrium. Combining these two principles of thermodynamics, one of them obtains the fundamental inequality:

$$T_b \dot{S}_b + \{\dot{\Phi}\}^T \{M\} - \dot{E}_b \geq 0 \tag{27}$$

And whereas Helmholtz's free energy can be written as $\psi_b = E_b - T_b S_b$, we can rewrite the fundamental inequality as a function of Gibbs' free energy (G_b):

$$G_b = -E_b + T_b S_b + \{\Phi\}^T \{\mathrm{M}\} \tag{28}$$

Therefore, in deriving the equations, there is the following expression:

$$\left\{\frac{\partial \Psi_b}{\partial \Phi}\right\} = \{\mathrm{M}\} \quad -\frac{\partial \Psi_b}{\partial T_b} = S_b \quad -\frac{\partial \Psi_b}{\partial V_\alpha} \dot{V}_\alpha \geq 0 \tag{29}$$

This inequality must be verified to be a thermodynamically possible process. Consequently, obtains the state laws for a frame element. The variable A_α is the thermodynamic force associated with state variables (V_α). In that inequality indicates that energy dissipation during an inelastic process must be positive.

$$\underbrace{\boxed{\left(\frac{\partial G_b}{\partial \dot{M}} \{\dot{M}\} - \{\dot{\Phi}\}\{\dot{M}\}\right)}}_{\text{Elasticity}} + \underbrace{\boxed{\left(\frac{\partial G_b}{\partial \dot{T}_b} \{\dot{T}_b\} - S_b \dot{T}_b\right)}}_{\text{Heat transfer}} + \underbrace{\boxed{\left(\frac{\partial G_b}{\partial V_\alpha} \{V_\alpha\}\right)}}_{\text{Energy dissipation}} \geq 0 \tag{30}$$

3 EXAMPLE

3.1 Reinforcement concrete beam in flexure

For the implementation of this model, it was used a simple example of reinforcement concrete beam (Fig. 6) in flexure. The section is composed by two compression bars of 14 mm and by four tension bars of 20 mm with 500 MPa of yield stress. The concrete has 28-day strength of 30 MPa and a section of 0,25x0,50 m with 4 mm of cover. The length (L) of each beam's elements is equal to 4 m.

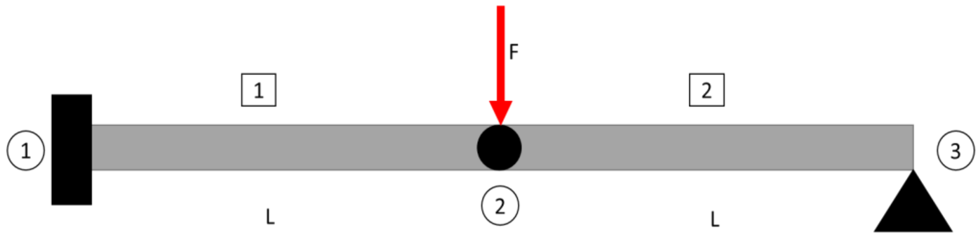

Figure 6. Example's design

First, calculates the initial proprieties (Table 1) of the section based in NBR 6118.

Table 1. Section's proprieties

Proprieties	kNm
Critical moment	45,3
Plastic moment	216,2
Ultimate moment	226,8

This way, it is applied a force in the middle of the span until a damage of 0.4, i.e. a force of 150,0 kN and a displacement of 0,020 m. After that, the force is maintained continuous until the critical time (tcr = 20 years), this time was choice based in a commonly critical time that starts the depassivation (Coelho, 2017).

So, the corrosion rate is increased until the service live of the structure, it was assumed that the service life is 100 years. Notice, that the only change in the initial data in time is in the effective area of steel, and the calculous is repeat with new proprieties for each time interval.

3.2 Results

Based in the concepts presented before, in the first step we have an increasing of force (Fig. 7), that obtains a damage of 0,41 in the fixed support and 0,34 in the mid-span.

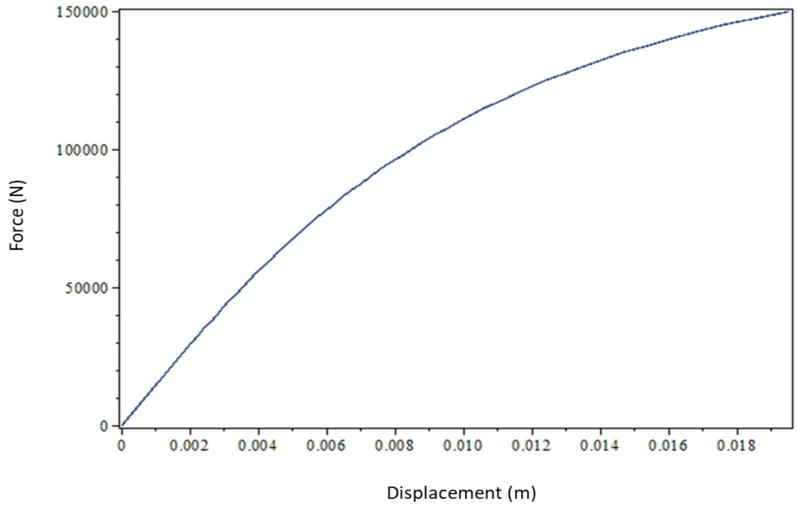

Figure 7. Displacement's evolution with force

The damage's evolution in time increases quickly when the force is increased, i.e., before the critical time, how shows in the Fig. 8:

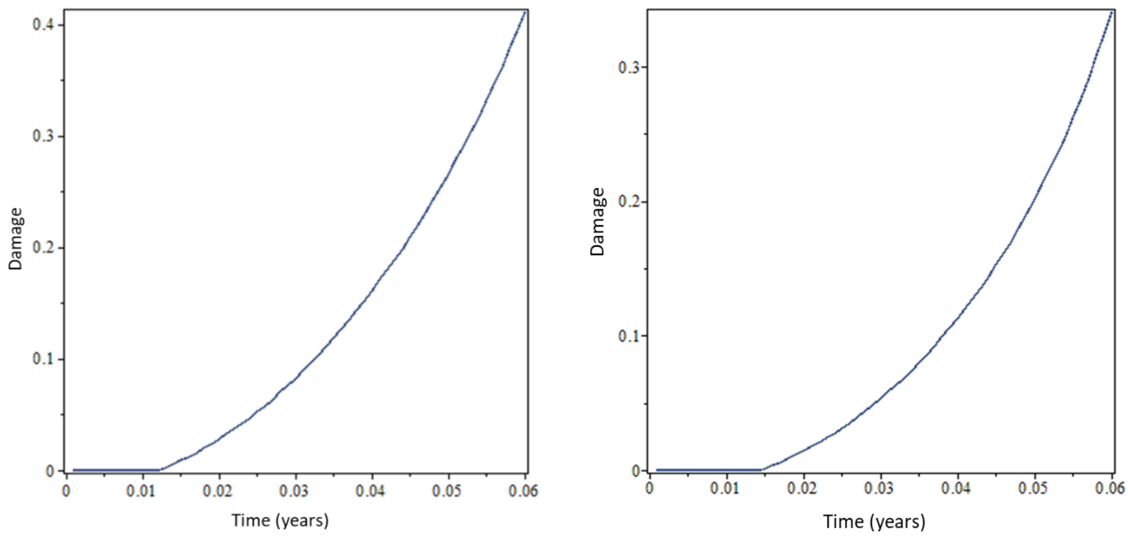

Figure 8. Damage's evolution in time (t=0…tcr), fixed support (left) and in the mid-span (right)

The second step maintained the force constant in time with no corrosion's rate. After the critical time the corrosion's rate is increased and the analysis is made to each year. This way, the displacement rises and we have the yield of steel, first, in the fixed support and after where the force is applied. The variable of corrosion (c), reaches 0,34 and the damage variable reaches 0,53.

After the critical time can be observed a reduction of the velocity of damage, because there is no increase in the force in this step. The corrosion's rate is increase until the structure reaches the service life. The Fig. 9 shows the damage's evolution in time in the fixed support and the Fig. 10 in the mid-span.

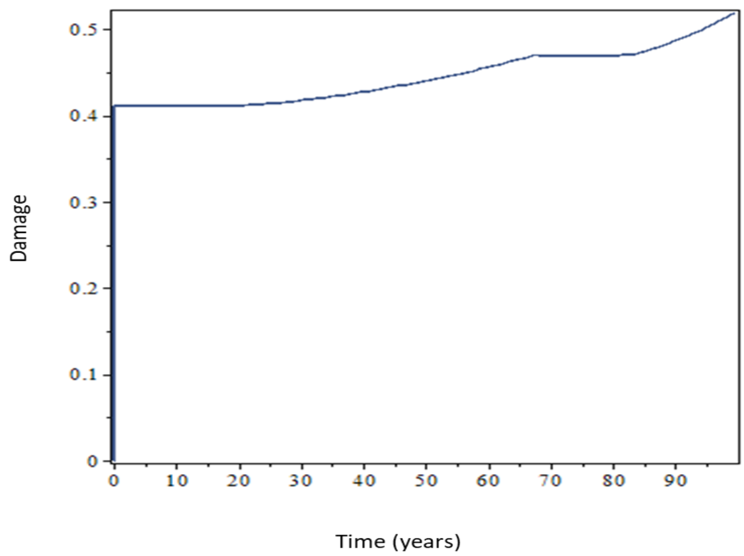

Figure 9. Damage's evolution in time (t=0...100), in the fixed support

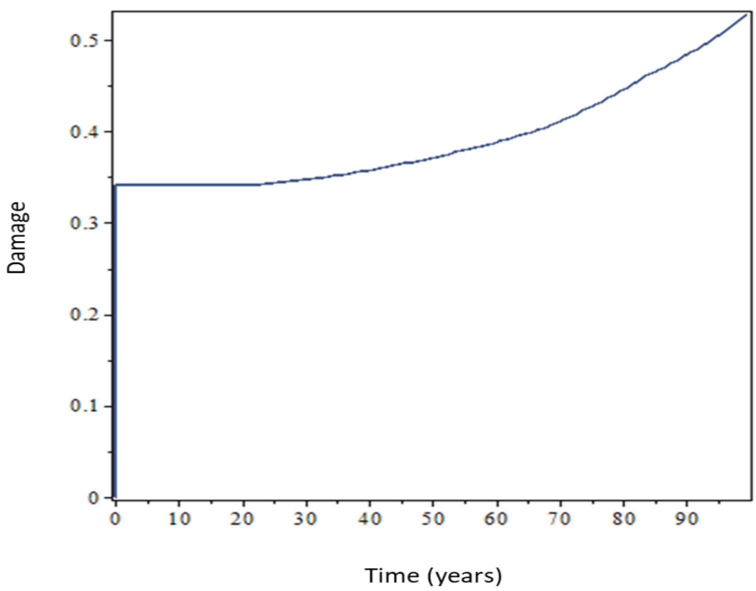

Figure 10. Damage's evolution in time (t=0...100), in the mid-span

The graphic of displacement in time (Fig. 11) shows a large displacement after the year 83, in this moment the steel begins to yield in the mid-span. This represents the collapse of the structure with displacements which vary of 0,02 m in the year 83 until 0,16 m with 100 years.

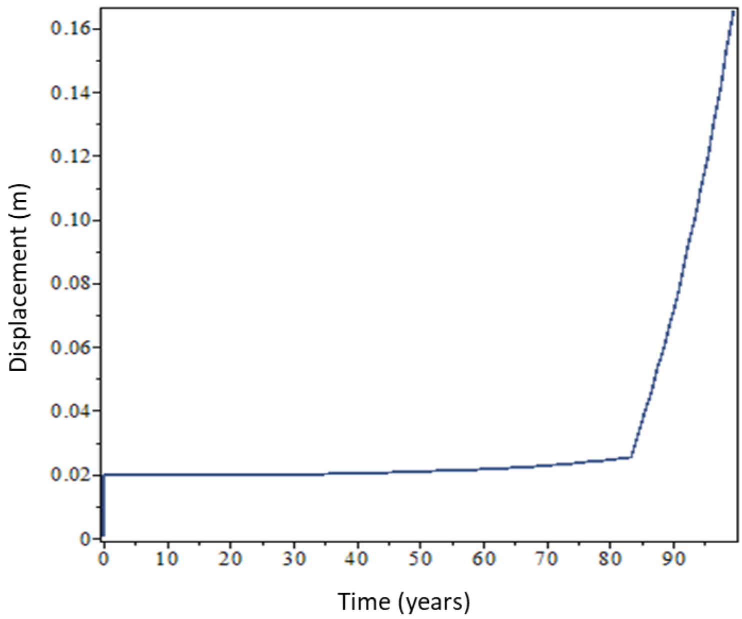

Figure 11. Displacement's evolution in time (t=0...100), in the mid-span

4 CONCLUSIONS

In this paper a formulation was presented that allows to incorporate a literature's law of corrosion in the structural calculation. This model can be used for any corrosion law existing in the literature. In the example presented it is possible to consider the influence of corrosion in the mechanic behavior during all the service live of the structure.

In the future it is expected to include through of the thermodynamics the crack influence in the corrosion propagation, to evaluate mechanic behavior closest to reality of structures subjected to corrosion.

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