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ENERGY HARVESTING USING INTERFACE MODES OF A PHONONIC CRYSTAL ELASTIC ROD SYSTEM

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Abstract *Phononic crystals (PCs) are periodic elastic structures that exhibit interesting wave propagation properties such as band gaps, i.e., frequency bands where elastic waves cannot propagate. The periodicity may be implemented via geometrical and material periodic variations in one, two, or three dimensions. It has recently been shown that coupling two phononic crystals with different geometrical (topological) phase yields interface modes. In the case of one-dimensional elastic structures such as rods, interface modes appear in the second band gap. Interface modes are characterized by a high local amplification of the elastic deformation at the interface of two PCs. In this paper the use of the interface mode to locally amplify elastic energy to improve energy harvesting using a piezoelectric element is investigated. Numerical simulations using spectral finite elements are used. It is shown that a very high amplification of the voltage generated by the piezoelectric element is achieved compared with simply attaching a homogeneous rod to the piezoelectric element. The preliminary results show that interface modes have a good potential for elastic energy harvesting applications.*

Keywords: *Phononic Crystal, Energy Harvesting, Interface mode, Piezoelectric element*

1 INTRODUCTION

Periodic systems exhibit wave propagation properties that have been exploited in applications related to passive vibration and noise control. In more recent years periodic structures have been investigated using the concept of phononic crystals (PCs). PCs are usually analyzed using their frequency band structure, which relate wavenumber of the waves that can propagate in the structural waveguide and frequency. A single unit cell that repeats itself in one or more directions is used to build such diagrams, which reveals the existence of evanescent and propagating wavemodes [Hussein et al., 2014].

Most applications up to date use the evanescent nature of the waves inside a bandgap (frequency band where the wavenumber is imaginary) to reduce vibration and noise. A recent development is the concept of interface modes on periodic acoustic systems Xiao et al. [2015]. Interface modes are related with the geometric (Zak) phase. These modes are characterized by a large local dynamic response and fast spatial attenuation, which may be useful in applications such as energy harvesting.

In previous works [Lima et al., 2017] [Arruda et al., 2017] [Rosa et al., 2017], numerical and experimental analyses were conducted to show that the phononic approach may be used to design periodic systems of acoustic ducts and rods with interface modes. In analogy to the work presented in Xiao et al. [2015], Rosa et al. [2017] proposed a strategy that allows the design of PCs exhibiting interface modes. Broadly speaking, it was shown that the computation of the geometrical phase is not necessary. Otherwise, the band gap closing and reopening is sufficient to observe the topological phase change.

In this work we continue the investigation of interface modes by conducting a numerical investigation of the use of the interface mode by the coupling of two elastic rod PCs with different geometrical phase to enhance the elastic energy harvesting using a piezoelectric rod at the interface. The dynamic stiffness matrices of elastic and piezoelectric rods are obtained using the Spectral Element Method (SEM).

1.1 Spectral Element Method formulation

Using the elementary theory of elastic rods, the elastodynamic local equilibrium equation can be obtained [Ahmida et al., 2003]. The elementary rod theory adopted for thin and long rods supports only axial stresses. Therefore an elastic rod can be considered as a longitudinal waveguide. Considering an external load $q(x,t)$, the resulting displacement $u(x,t)$ in the x direction obeys the following differential equation:

$$\frac{\partial}{\partial x}\left[EA\frac{\partial u}{\partial x}\right] = \rho A\frac{\partial^2 u}{\partial t^2} + \eta A\frac{\partial u}{\partial tq} \tag{1}$$

where EA is the axial stiffness, ρA is the mass per unit length, η is the viscous damping coefficient per unit of volume. Applying the Fourier transform to Eq. (1) yields:

$$u(x,t) = \sum_{\omega} \hat{u}_n(x,\omega)e^{(i\omega t)} \tag{2}$$

Replacing Eq. (2) in Eq. (1):

$$EA\frac{d^2\hat{u}}{dx^2} + (\omega\rho A - i\omega\eta A)\hat{u} = 0 \tag{3}$$

The solution of Eq. (3) is exponential harmonic, resulting in the following representation of the longitudinal displacement of the rod with finite length L:

$$\hat{u}(x) = \mathbf{A}e^{-ikx} + \mathbf{B}e^{-ik(L-x)} \tag{4}$$

where **A** and **B** are constants determined by the boundary conditions and k is the wavenumber:

$$k = \sqrt{\frac{\omega^2\rho A - i\omega\eta A}{EA}} \tag{5}$$

In the undamped case, η is null and the relation becomes linear with frequency, which corresponds to a non dispersive propagation. Expressing Eq. (4) with the time variation yields:

$$u(x,t) = \sum_{\omega}\mathbf{A}e^{-i(kx-\omega t)} + \sum_{\omega}\mathbf{B}e^{i(kx+\omega t)} \tag{6}$$

Equation (6) represents two waves, one is the wave propagating to the right and the other to the left.

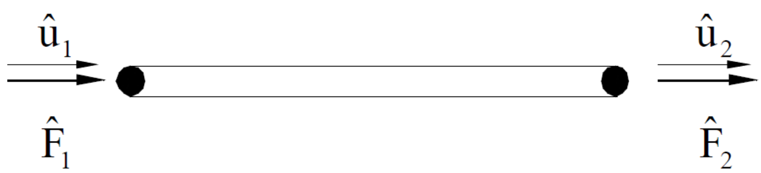

Figure 1: Rod spectral element

The nodal displacements at the extremities of the rod are:

$$\hat{u}(0) \equiv \hat{u_1} = \mathbf{A} + \mathbf{B}e^{-ikL} \qquad \hat{u}(L) \equiv \hat{u_2} = \mathbf{A}e^{-ikL} + \mathbf{B} \tag{7}$$

The displacement at any point along the rod may be computed combining Eq. (4) and Eq. (7):

$$\hat{u}(x) = \hat{g}_1(x)\hat{u_1} + \hat{g}_2(x)\hat{u_2} \tag{8}$$

where the functions $\hat{g}_1$ and $\hat{g}_2$ are the shape functions of the rod:

$$\hat{g}_1(x) = \hat{g}_1(x)\hat{u}_1 + \hat{g}_2(x)\hat{u}_2 \tag{9}$$

where

$$\begin{aligned} \hat{g}_1(x) &= \Big(e^{-ikx} - e^{-ik(2L-x)}\Big)/\Delta \\ \hat{g}_1(x) &= \Big(-e^{-ik(L+x)} - e^{-ik(L-x)}\Big)/\Delta \\ \Delta &= 1 - e^{-i2kL} \end{aligned} \tag{10}$$

These functions play the same role of the shape functions in the Finite Element Method. The nodal loads are related with the nodal displacements:

$$\begin{aligned} F_1 = -F(0) = EA\big[g_1'(0)u_1 + g_2'(0)u_2\big] \\ F_2 = -F(L) = EA\big[g_1'(L)u_1 + g_2'(L)u_2\big] \end{aligned} \tag{11}$$

Writing Eq.(11) in matrix form:

$$\begin{bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{bmatrix} = \frac{EA}{L}\frac{ikL}{(1-e^{-2ikL})}\begin{bmatrix} 1+e^{-2ikL} & -2e^{-ikL} \\ -2e^{-ikL} & 1+e^{-2ikL} \end{bmatrix}\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{bmatrix} = \Big[\hat{K}\Big]\Big\{\hat{u}\Big\} \tag{12}$$

where $[\hat{K}]$ is the dynamic stiffness matrix for the spectral rod element. A global dynamic stiffness matrix can be assembled using the direct stiffness method commonly used in FEM.

1.2 Phononic Crystals

A phononic crystal (PC) is an elastic system with a periodic discontinuity in material and/or geometry. The wave propagation features in such a structure can be obtained from the study of a single periodic unit cell. This can be done using the dispersion diagram, which relates the angular frequency to the wavenumber. Through the manipulation of the pass and stop bands in this diagram, wave control in the system may be achieved.

In consonance with [Xiao et al., 2015], it is possible to obtain a mode that is formed in the border of two distinct PCs having different band gap topological characteristics. This mode is named as interface mode and it amplifies the longitudinal vibration at the interface between the two PCs. The concept of topological phase developed in quantum physics was used to investigate electromagnetic interface modes and recently applied to acoustic waveguides [Xiao et al., 2015] and to elastic rods [Rosa et al., 2017].

In this section piezoelectric rod spectral element with be formulated.

1.3 Formulation of the Piezoelectric Rod Spectral Element

According to [Li and Guo, 2016], the transfer matrix of the piezoelectric element is given by:

$$\begin{bmatrix} \hat{u}(L) \\ \hat{F}(L) \end{bmatrix} = \frac{1}{\delta} \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} \hat{u}(0) \\ \hat{F}(0) \end{bmatrix} \tag{13}$$

where:

$$\begin{aligned} &\delta = -B\left(e^{-ikL} - e^{ikL}\right) + 2\xi \\ &t_{11} = t_{22} = (\xi - B)e^{-ikL} + (\xi + B)e^{ikL} \\ &t_{12} = -\left(e^{-ikL} - e^{ikl}\right) \\ &t_{21} = -\left(\xi^2\right)\left(e^{-ikL} - e^{ikL}\right) + 2\xi B\left(e^{-ikL} + e^{ikl} - 2\right) \end{aligned} \tag{14}$$

and ξ and B are the axial force influence coefficients of the piezoelectric rod, which are non-relative and relative to the electric boundary conditions, respectively. The formulation of these coefficients is given by [Li and Guo, 2016]:

$$\begin{aligned} &\xi = ikA_{PZT}(E_{PZT} + e^2_{PZT}/\alpha_{PZT})A_{PZT} \\ &B = -\frac{E^2_{PZT}C_{PZT}A_{PZT}}{\alpha^2_{PZT}A_{PZT} + \alpha_{PZT}C_{PZT}L_{PZT}} \end{aligned} \tag{15}$$

The constants in the Eq. 15 are given in Table 1.

Table 1: Mechanical and geometrical properties of the PZT

Properties	Nomenlcl	PC1
Young's Modulus (GPa)	E_{PZT}	117
Piezoelectric constant (C/m^2)	e_{PZT}	23.3
Dielectric Constant (nF)	α_{PZT}	13.02
Density (Kg/m^3)	ρ_{PZT}	7500
Length (mm)	L_{PZT}	10
Radius (mm)	r_{PZT}	0.5
Capacitance (pF)	C_{PZT}	1.02

The voltage between the two terminals can be expressed as [Li and Guo, 2016]:

$$V = \frac{e_{PZT} A_{PZT}}{\alpha_{PZT} A_{PZT} + C_{PZT} L_{PZT}} \left[\hat{u_2} - \hat{u_1} \right] \tag{16}$$

where $\hat{u_1}$ and $\hat{u_2}$ are the displacements of the left and right ends of the piezoelectric rod, respectively.

It is possible to transform the transfer matrix into the dynamic stiffness matrix by [Lee, 2009]:

$$\hat{K}(\omega) = \begin{bmatrix} t_{12}^{-1} t_{11} & \left(\frac{t_{12}}{\delta}\right)^{-1} \\ \frac{t_{21} - t_{22} t_{21}^{-1} t_{11}}{\delta} & t_{22} t_{12}^{-1} \end{bmatrix} \tag{17}$$

2 NUMERICAL PROCEDURE

In this section a rod system consisting of two PCs and a piezoelectric element shown in Fig. 2 is investigated. The number of cells used for each PC is eight. Two piezoelectric elements were used to keep the symmetry on the numerical model. The geometry of the periodic cells of PC1 and PC2 are given in Table 2.

Table 2: Geometry of the Phononic Crystals

Dimensions	Nomenlclature	PC1	PC2
radius of the cross-section A (cm)	r_A	36.31	36.31
Length of the section A (cm)	L_A	94.55	68.08
radius of the cross-section B (cm)	r_B	22.69	22.69
Length of the section B (cm)	L_B	34.04	94.55

Both PCs are made of steel and their Young's Modulus is $E = 210GPa$ and mass density is $\rho = 7800kg/m^3$.

The properties of the piezoelectric element are given in Table 1

The structure is assembled as shown in the sketch in Fig.2

The PCs are configured as shown in Fig. 3 with the dimensions given in Table 2.

The total system (condensing the internal degrees of freedom of the PCs) has 5 DOFs, as shown in Fig.4.

3 RESULTS

The displacements at the terminals of the piezoelectric element were obtained using the global dynamic stiffness matrix using the software MatLab.

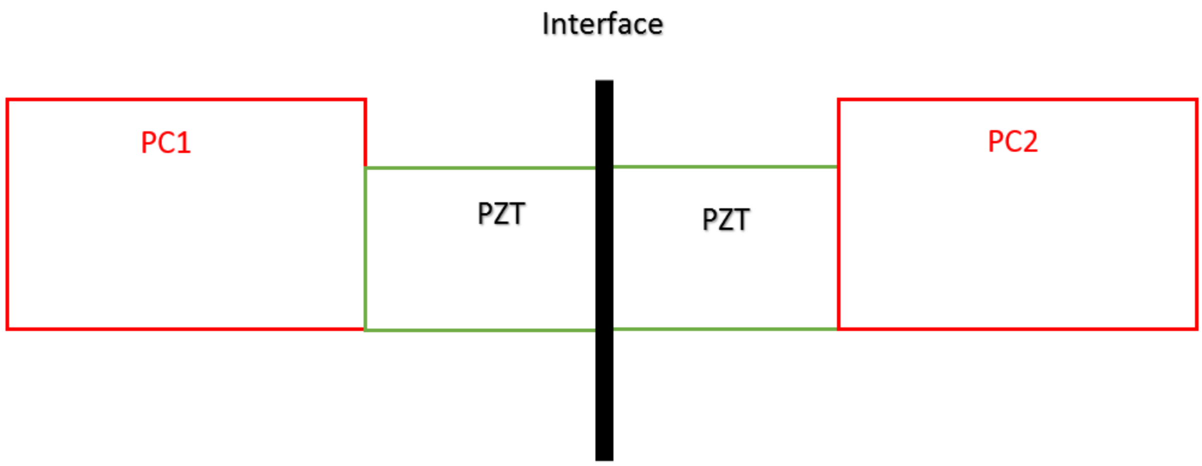

Figure 2: Rod system with two PCs and two piezoelectric rods.

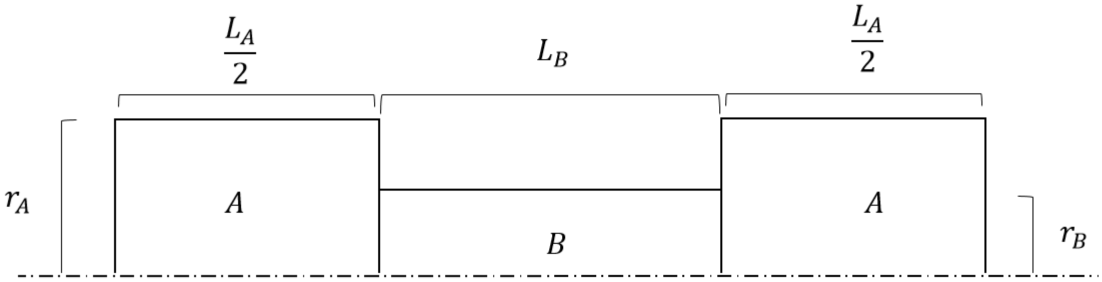

Figure 3: Periodic cell of the phononic crystals

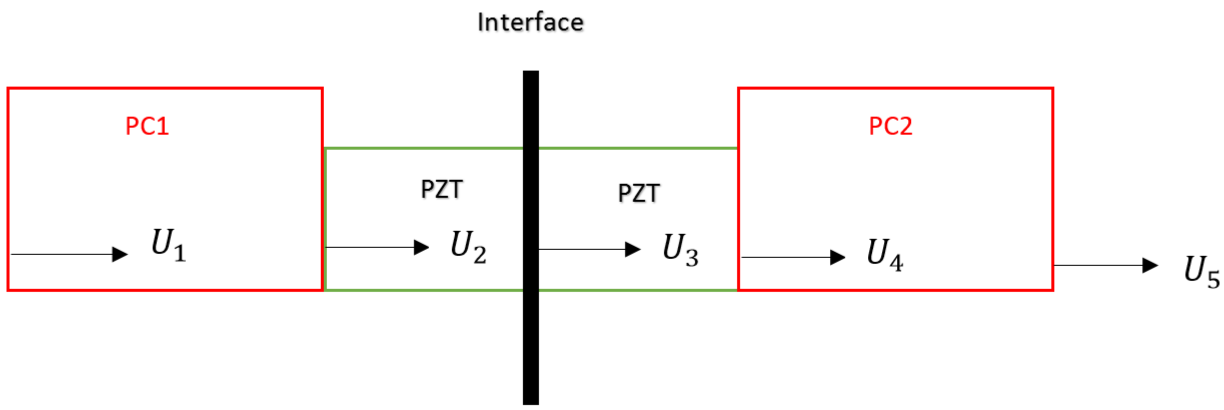

Figure 4: Degrees of freedom of the assembled system after condensation.

The boundary conditions adopted are free-free ends. The sinusoidal unitary force excitation in the range of frequencies of DC to 10KHz is applied at the interface of the two PCs. The Frequency Response Function of the system is obtained by solving a linear system of equations in the frequency domain. The displacements of each degree of freedom and the voltage at the piezoelectric terminals are shown in Fig.5.

Using Eq. (16), it is possible to obtain the voltage at the terminals of the piezoelectric rod shown in Fig.6.

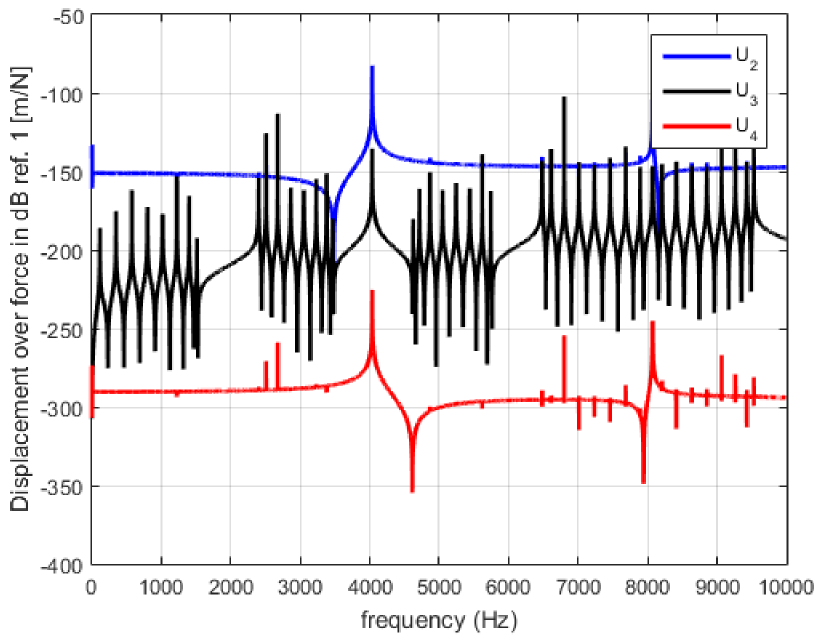

Figure 5: FRFs of the displacements at the piezoelectric element ends.

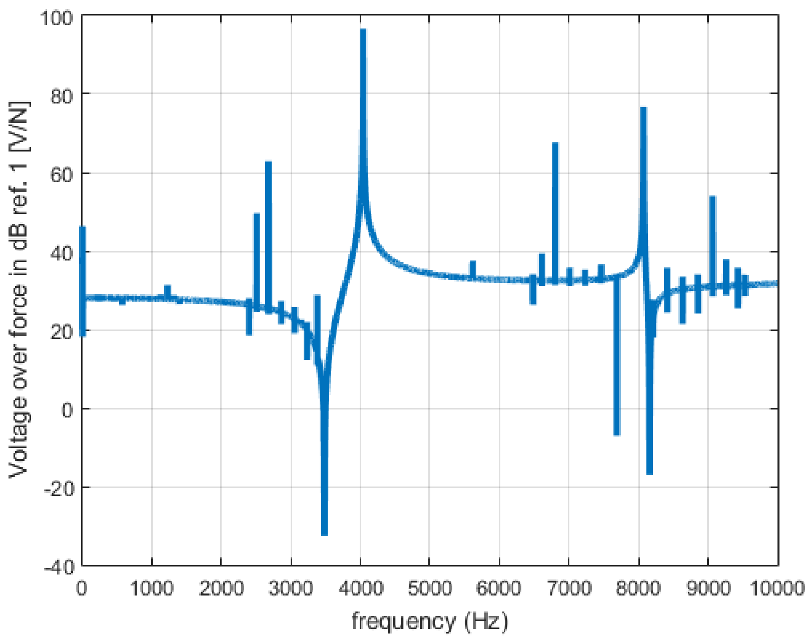

Figure 6: FRF of the voltage generated by the piezoelectric element.

Replacing the PCs by equivalent homogeneous rods with the same mass, the responses in Fig.7 are obtained.

4 CONCLUSION

According to Fig. (6) and Fig. (8), it is possible to see that the use of the interface mode has a strong influence in the FRFs of the displacements at the piezoelectric element ends.

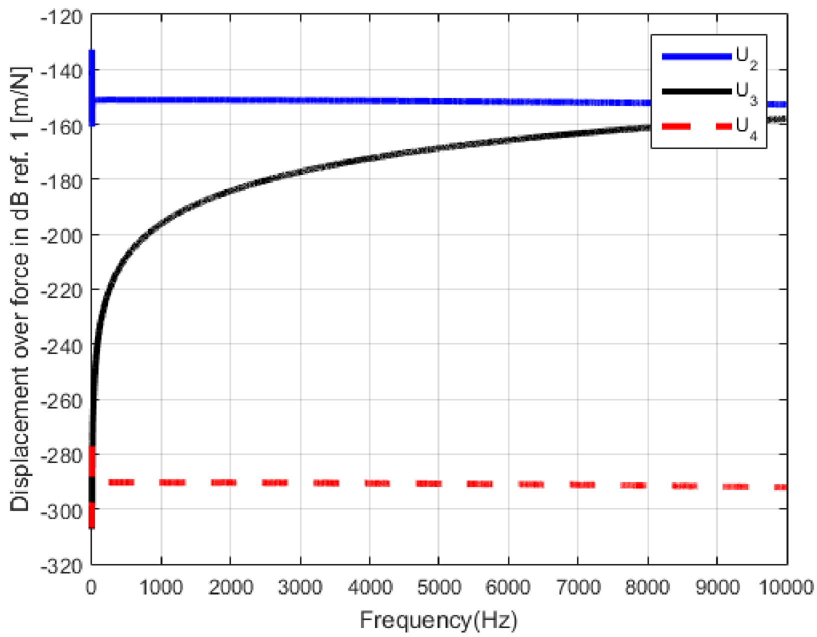

Figure 7: FRFs of the displacements with equivalent homogeneous rods.

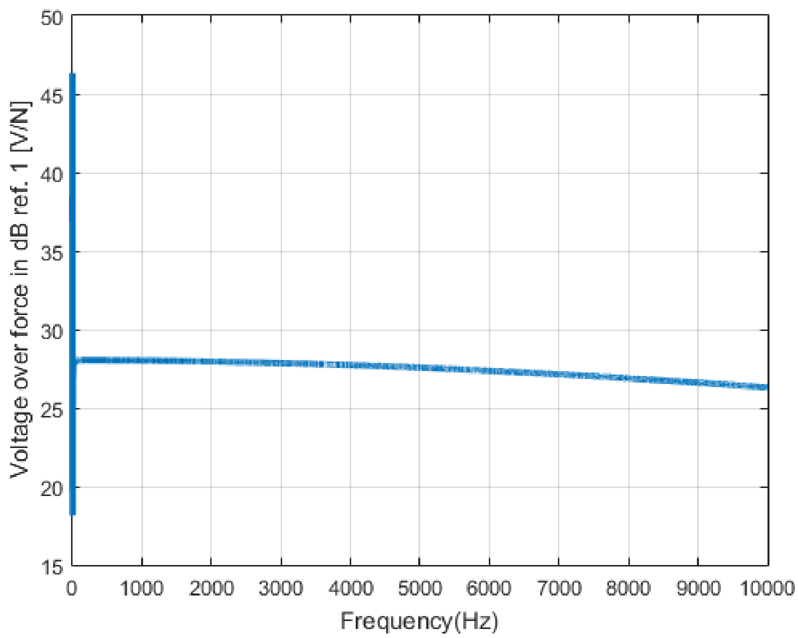

Figure 8: FRF of the voltage with equivalent homogeneous rods.

The interface mode obtained at the interface between two phononic crystals having different topological phase characteristics amplified the displacements and resulting voltage. Taking advantage of this effect, it was possible to obtain, theoretically, a voltage gain of approximately 70dB at the terminals of the piezoelectric crystal when the system was excited at the interface mode frequency of 4035 Hz, when compared to the voltage obtained using equivalent homogeneous rods.

In future work, experiments will be presented to explore the limits of this theoretical prediction. Also, practical issues related to the implementation of this idea will be addressed.

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REFERENCES

K. M. Ahmida, J. R. F. Arruda, and L. Ferreira. A tapered acoustic duct spectral element. *Proc. of the 10th Int. Congress on Sound and Vibration (ICSV), Stockholm, Sweden*, pages 3901–3908, July 2003.

J. R. F. Arruda, M. I. N. Rosa, V. D. Lima, and J. M. C. Santos. Numerical and experimental investigation of interface modes in periodic acoustic waveguides. *International Conference on Structural Engineering Dynamics, ICEDyn 2017 Ericeira, Portugal*, pages 1–10, July 2017.

M. I. Hussein, M. J. Leamy, and M. Ruzzene. Dynamics of phononic materials and structures: Historical origins, recent progress, and future outlook. *Applied Mechanics Reviews*, 66(4): 040802, 2014.

U. Lee. *Spectral element method in structural dynamics*. John Wiley & Sons, 2009.

L. Li and Y. Guo. Analysis of longitudinal waves in rod-type piezoelectric phononic crystals. *Crystals 2016*, 6(4), 2016. 45.

V. D. Lima, J. M. Santos, and J. R. F. Arruda. Passive control of noise propagation in tube systems using bragg scattering. *Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics, DINAME 2017, A. T. Fleury, D. A. Rade, P. R. G. Kurka (Editors), ABCM, São Sebastião, SP, Brazil*, pages 1–5, 5-10 March 2017.

M. I. N. Rosa, J. R. F. Arruda, and M. Ruzzene. Investigating interface modes on periodic acoustic waveguides and elastic rods using spectral elements. *Proceedings of the XVII International Symposium on Dynamic Problems of Mechanics, DINAME 2017, A. T. Fleury, D. A. Rade, P. R. G. Kurka (Editors), ABCM, São Sebastião, SP, Brazil*, pages 1–7, 5-10 March 2017.

M. Xiao, G. Ma, Z. Yang, P. Sheng, Z. Q. Zhang, and C. T. Chan. Geometric phase and band inversion in periodic acoustic systems. *Nature Physics 11*, pages 240–244, february 2015.