



FINITE ELEMENT MODELING OF HETEROGENEOUS BEAMS CONSIDERING THE VISCOELASTICITY OF THE CEMENT PASTE AND THE ELASTICITY OF AGGREGATES

Fernando Martins Ribeiro Filho

Amanda Araújo da Silva

fernandofilho337@gmail.com

amanda.as96@gmail.com

Campus Universitário Darcy Ribeiro, Faculdade de Tecnologia, Departamento de Engenharia Civil e Ambiental - ENC, Universidade de Brasília, CEP 70910-900, Brasília-DF, Brasil.

Fabiano Campos Macedo

Francisco Evangelista Júnior

fabianocampos87@hotmail.com

fejr.unb@gmail.com

Programa de Pós-graduação em Estruturas e Construção Civil, Prédio SG-12, 1º andar, Campus Darcy Ribeiro, Universidade de Brasília, CEP 70910-900, Brasília - DF, Brasil.

Abstract. *The present work analyzed the time dependent behavior of a three point bending beam using the finite element approach. The initial data was the creep compliance of a cement paste (J_p). Then, a creep test was simulated numerically in which the specimen was composed of a matrix of cement paste in which elastic aggregates were inserted following a particle-size distribution to obtain the creep compliance of the mixture (J_{hom}). The beam was divided in two zones: a homogenous zone (with compliance J_{hom}) towards the supports, and a heterogeneous central zone composed of a matrix of cement paste in which aggregates were inserted following a particle-size distribution. Different meshes and different aggregates positioning were analyzed, trying to show their influence on the results. The results showed that with the insertion of aggregates in the cement paste, the beam's displacements, both the initial and the time-dependent ones, significantly decreased. It was also possible to conclude that the distribution of the aggregates inside the viscoelastic matrix is relevant.*

Keywords: *Viscoelasticity, Finite Elements, Creep Compliance.*

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1 INTRODUCTION

One of the phenomena presented by cementitious materials, especially at early ages, is the creep flow, which is characterized by an increase in strain even when the material is under constant stress. However, even though creep is a fairly known subject, there are not many models that can explain it in a satisfactory level when it comes to concrete. This lack of a good model is mainly due to the fact that concrete is not a homogeneous material, but a mix of cement paste (which presents viscoelastic behavior) and aggregates (which present elastic behavior) (Neville, 2012). This way, it is complicated to formulate a model that considers both behaviors simultaneously.

Many studies have been performed trying to better understand the heterogeneous behavior of concrete. Granger (1995) and Alexander (1996) concluded that the primary source of creep in concrete is the paste content. Lavergne *et al.* (2015) performed a numerically investigation on the effect of size distribution and shape of elastic inclusions on the long-term creep of concrete.

In order to try to analyze the concrete creep, this work applied the Finite Element Method to simulate the deflections on a simply supported concrete beam subjected to a point central load. Through the Finite Element Method, it was possible to consider both the viscoelasticity of the cement paste and the elastic characteristics of the aggregates.

2 BACKGROUND

Viscoelastic materials are those that present both the characteristics of linear-elastic solids, which follow Hooke's law, and of Newtonian fluids (Christensen, 1982). These materials are described by two properties: the relaxation modulus, $E(t)$, and the creep compliance, $J(t)$. The constitutive equations for viscoelastic material are shown in Eq. (1) for the stress formulation (σ) and Eq. (2) for the strain formulation (ε).

$$\sigma(t) = \int_{-\infty}^t E(t-\tau) \frac{\partial \varepsilon}{\partial \tau} d\tau \quad (1)$$

$$\varepsilon(t) = \int_{-\infty}^t J(t-\tau) \frac{\partial \sigma}{\partial \tau} d\tau \quad (2)$$

Where t is the time in a chosen referential and τ is the integration variable.

These two properties $E(t)$ and $J(t)$ relate to each other through the Laplace transform as shown in Eq. (3), where s is the Laplace variable.

$$E(s)J(s) = \frac{1}{s^2} \quad (3)$$

There are some mathematical formulations that model the creep compliance materials, most of which are based in two basic models: the Maxwell material and the Kelvin material (Christensen, 1982). By generalizing these two basic models, many different models were obtained, one of which is the Prony Series model, which is presented in Eq. (4) for the compliance function.

$$J(t) = \left[J_0 + \sum_{i=1}^n J_i \left(1 - e^{-\frac{t}{\tau_i}} \right) \right] \quad (4)$$

Where J_0 , J_i and τ_i are parameters fitted from creep experiments.

3 METHODOLOGY

In order to obtain the compliance of concrete (J_{hom}), which would be necessary to analyze the heterogeneous beams, a creep test was simulated on the software Abaqus. The specimen was prismatic and made of a matrix of viscoelastic cement paste in which elastic aggregates were inserted following a certain particle-size distribution (Fig. 1a). For the simulation, the specimen was modeled with plane stress elements. The base of the specimen was restricted from moving in the y direction ($\delta_y=0$) and uniform stress of $\sigma=1\text{MPa}$ was applied at the top of the specimen. Two materials were used for the analyses: the cement paste and aggregates. The cement paste data was its creep compliance, J_p , which was proposed by the authors of this paper based in the experimental data presented by the following authors: Alexander (1996), Atrushi (2003) and Kalintzis and Kuperman (2005). As for the aggregate, its Young's modulus was considered $E=80\text{GPa}$. The Poisson's ratio for both materials was considered 0.17.

Once the simulation was over, the graphs of displacement vs. time for every node at the top of the specimen were obtained. Afterwards, for each time step, an average of the displacements of all the nodes was calculated in order to obtain a single graph of displacements vs. time. Dividing the displacement by the initial height of the specimen and then dividing the result by the applied stress, J_{hom} was obtained. Figure 1b shows both compliances, J_p and J_{hom} .

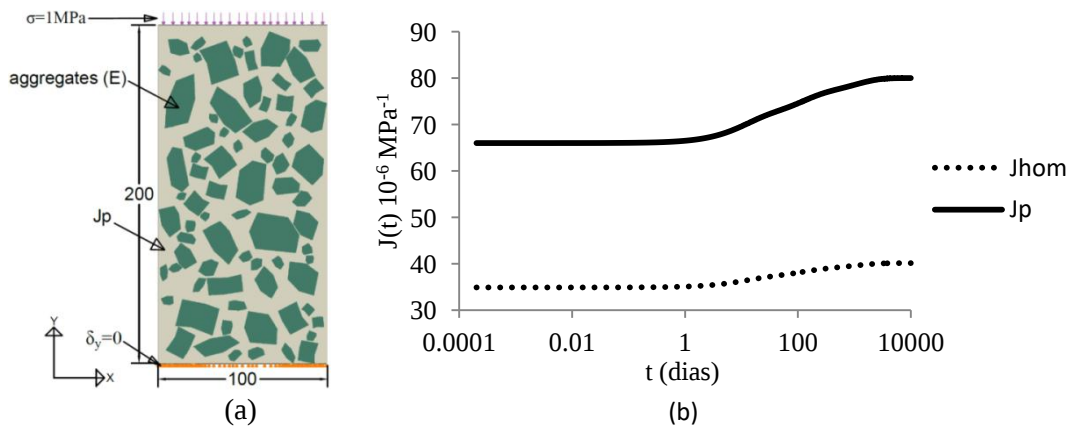


Figure 1. Uniaxial configuration (a) geometry, dimensions in mm (b) creep compliances used for analyses, J_{hom} and J_p

A simulation of three point bend (TPB) was then conducted subjecting a beam to a point central load that represented the beam's self-weight. Initially, the beam was considered homogenous with a creep compliance equal to J_p (Fig 2a). As the result, a graph of displacement of the center of the beam vs. time was obtained. The procedure was then repeated, but now the beam's creep compliance was J_{hom} (Fig 2a). Subsequently the beam was divided in two zones: a homogeneous zone (with compliance J_{hom}) towards the supports and a heterogeneous central zone composed of a matrix of cement paste in which elastic aggregates

were inserted following a certain particle-size distribution. Two meshes were analyzed trying to show their influences of the results. The results obtained from the simulation of the heterogeneous beams were the same graphs explained previously. It is also important to notice that even though all heterogeneous zone had the same particle-size distribution, the positioning of the aggregates inside the mix was variable. The three arrangements of aggregates tested are illustrated in Figs. 2b, 2c and 2d.

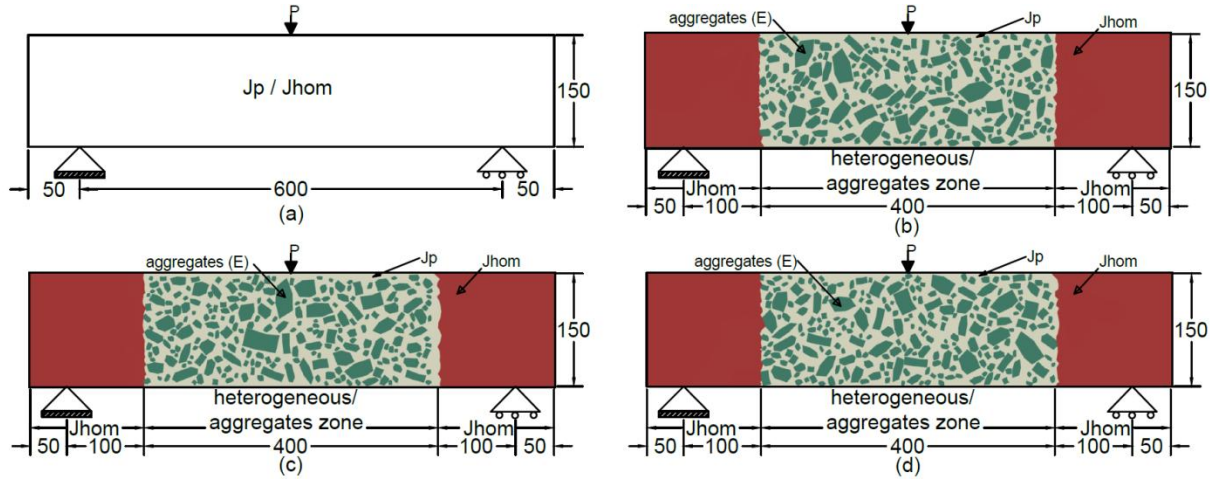


Figure 2. Beams configurations for the TPB (a) homogeneous beam (b) heterogeneous beam 1 (heter1) (c) heterogeneous beam 2 (heter 2) (d) heterogeneous beam 3 (heter 3); all dimensions are in mm

4 RESULTS

Figure 3 shows the results obtained from the simulation of the beams. In the results, $\delta_{\max}$ represents the maximum deflection of the beam. Meshes 2 and 2b are similar and with the same density, but slightly different due to the positioning of the aggregates.

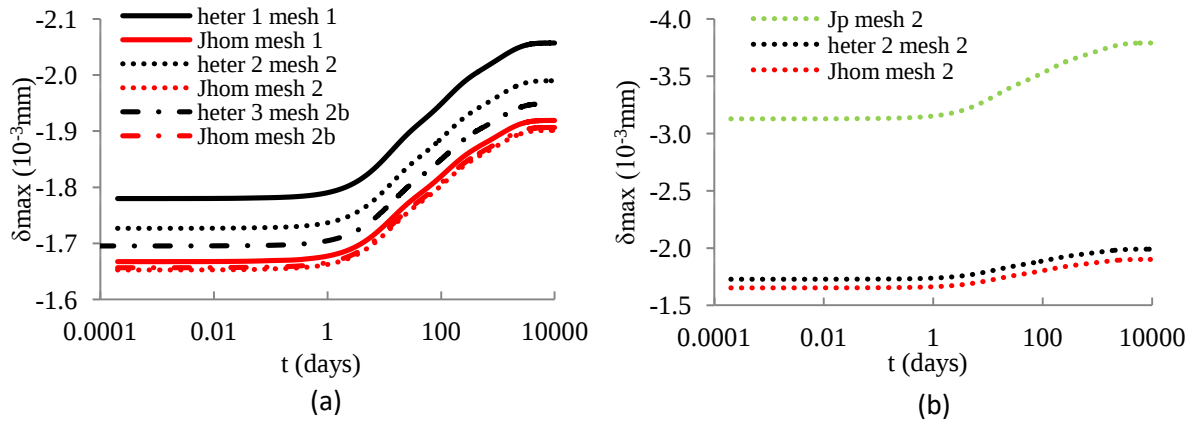


Figure 3. Maximum deflection for the beam (a) $\delta_{\max}$ vs. t for a heterogeneous beams with varied aggregate positions and homogeneous beams (J_{hom}) (b) $\delta_{\max}$ vs. t for a fixed mesh and different compliances

Figure 3a shows that the mesh influence is neglectable for homogeneous beams, since the $\delta_{\max}$ values for the beams with creep compliance J_{hom} were almost the same regardless of the chosen mesh. It was also possible to show that the change in the positioning of the aggregates inside the matrix of cements paste result in a considerable change of the rigidity of the beam, even for the same particle-size distribution. For instance, the results for the beam "heter 1

mesh 1" were around 5.6% higher than the ones for the beam "heter 3 mesh 2b". However, in order to be certain of how much the positioning of the aggregates can influence the creep compliance of the final material, it would be ideal to perform a greater number of simulations with aggregates in different positioning.

Observing Fig. 3b, it is clear how the insertion of aggregates in the matrix of cement paste causes a decrease in the values of $\delta_{\max}$. The results for both the beam with creep compliance J_{hom} and the heterogeneous one decreased about 47% in relation to the values obtained for the homogeneous beam with creep compliance J_p . Also, it is possible to conclude that the quality of the regression of J_{hom} is satisfactory due to the proximity of the displacements presented by homogeneous beams with creep compliance J_{hom} and the ones presented by heterogeneous beams for the relation aggregate zone/span equal to 2/3.

5 CONCLUSIONS

From the results obtained, it was possible to conclude that, by adding aggregates to the cement paste, the beam deflections decrease, both the initial ones and on time. Also, it was possible to observe that the quality of the J_{hom} regression was satisfactory. Finally, it was showed that the distribution of the aggregates on the heterogeneous zone is relevant for the cases studied and that the effect of the mesh is neglectable.

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