ANALYTICAL EVALUATION OF WATER SUPPLY TANKS AS DAMPERS

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Abstract. *The urban agglomeration of the cities has consequently led to the construction of tall and slender buildings. These structures are subjected to horizontal dynamic forces, resulting in higher displacements. Among the devices used to avoid the undesirable effects caused by these displacements, the Tuned Liquid Dampers (TLDs) stand out by its low cost of operation. In Brazil, the water supply tanks are conceived on the top of buildings. So, the present article handles the potential use of these tanks as TLD. In order to verify its potential damping effect, the water tank is modeled associating models proposed by Housner (1963) and Yu et al. (1999). For that, two different models were created to represent analytically the tank at the top of the building and the differences among such models were verified. In addition to that, the dynamic analysis of a building is made analytically. Different heights of water in the tank were tested and it was concluded that the reservoir dampens the horizontal displacements and that the larger the water height, the lower is the maximum displacement. Therefore, it can be concluded from the analysis performed that the water tank can work like damper.*

Keywords: *TLD, Rectangular tank, Analytical analysis*

INTRODUCTION

Nowadays, the construction of tall and slender buildings has led to the need for performing dynamic analysis due to horizontal forces acting on these structures, such as the wind and earthquakes. The range of effects induced by these forces is varied, and they can cause from small displacements until the complete collapse of the structure.

Damping devices have been developed as a way of mitigating the undesirable effects caused by horizontal forces acting in buildings. Among these devices, it is highlighted the Tuned Liquid Dampers (TLDs), which are tanks filled with water. They use the sloshing energy of the water to reduce the dynamic response of the system when it is subjected to excitation (Mondal et al., 2014). In this case, the TLD natural frequency is tuned to the natural frequency of the structure. They are a good option to reduce oscillations because of their low cost of operation and maintenance (Malekghasemi et al., 2015). Such dampers have been proposed in the 1980's applicating for ground structures. Besides it was suggested to use the existing water tanks in buildings by simply configuring internal partitions into multiple dampers, increasing energy dissipation and damping (Kareem et al., 1999; Livaoglu et al., 2011).

In Brazil, there are water supply tanks placed at the top of the buildings because there is an unpredictability associated with the public water supply network (Carvalho, 2014). These water tanks meet basic requirements to be considered as TLDs: they are located at the top of the buildings and they are containers with rigid walls filled with water.

When a reservoir partially filled with liquid oscillates, the horizontal movement associated with its fluid is of two types. There is a part of the liquid that moves a sloshing motion, while the rest part moves rigidly with the tank walls (Dutta et al., 2004). Besides, an interaction between the fluid and the reservoir walls takes place. There are many mathematical models in literature to represent the interaction between water and the reservoir. The most well-known models are developed by Housner (1963) and Yu et al. (1999). Housner (1963) considered the fluid-structure interaction with only linear stiffness springs for elevated water tanks. Yu et al. (1999) represented TLDs where the fluid-structure interaction was simulated through stiffness springs and dampers nonlinear.

The model proposed in this paper consists in associating the two models mentioned above. The liquid is divided into two masses, as proposed by Housner (1963) while the fluid-structure interaction will be taken into account through stiffness springs and dampers nonlinear calculated according to Yu et al. (1999). Hence, the main focus of this work is to evaluate if the water supply tanks located at the top of Brazilian buildings can work as TLDs. To do so, it is made a comparison between Yu's two-degree of freedom model and the model proposed in this paper. Results are obtained using Newmark's method to solve the nonlinear dynamic equilibrium equations whose algorithm was created in the free software Octave. Results show that water supply tanks can be considered as TLDs and the higher is the volume of water inside the tank the lower is the horizontal displacement after the structure stabilization.

1 WATER MODELS

First, it is important to differentiate shallow water tanks from deep water tanks. Some authors may model water tanks differently depending on this condition, while Yu et al. (1999) only consider the shallow water theory. According to these authors, the shallow water tank presents the ratio of tank length over the water height (h/L) ranging from 0.04 to 0.5 (Malekghasemi et al., 2015). Housner (1963) considers shallow tanks those which ratio h/L is less than 1.5 (Livaoglu, 2008). Otherwise, the tank is considered to be a deep water one. Brazilians water supply reservoirs have this ratio below the value of 1.5; hence they can be considered shallow tanks. It is worth mentioning that the models presented here are valid only to rectangular reservoirs, and the equations are specific for each shape of the tank.

1.1 Housner's Model

Motivated by the Chilean earthquakes and the behavior of elevated water tanks during the earthquakes, Housner (1963) studied the dynamic behavior of these reservoirs taking into consideration the relative movement between the water and the tank. When horizontal forces act in this structure, they induce on the free surface movement of the water, and the amplitude of this movement is dependent on the intensity of the external action.

According to the author, a filled tank may be considered as an element of only one mass only, since the water does not have a free surface to oscillate. Therefore, the water and the reservoir have the same movement in this condition. Otherwise, when there is a free oscillating surface, the tank can be considered as an element with two different masses. The first one is called impulsive mass (M_0), and it is rigidly attached to the tank; the second one is called convective mass (M_1), and it represents the part of the water which oscillates inside the container. In Housner's model, M_1 is connected to the tank through stiffness springs $k_1/2$ (Fig. 1b). Figure 1a shows the equivalent dynamic system for an elevated water tank which M_0' consists of the sum of M_0 with the empty tank mass.

For rectangular tanks with length $2L$ and water height h, the masses and stiffness associated with the system can be determined by the following equations, reviewed by Epstein (1976 cited by Livaoglu, 2008):

$$M_0 = 0.577\,\frac{h}{L}\tanh\left(1.732\,\frac{L}{h}\right)M\,, \tag{1}$$

$$h_0 = \frac{3}{8}h\,, \tag{2}$$

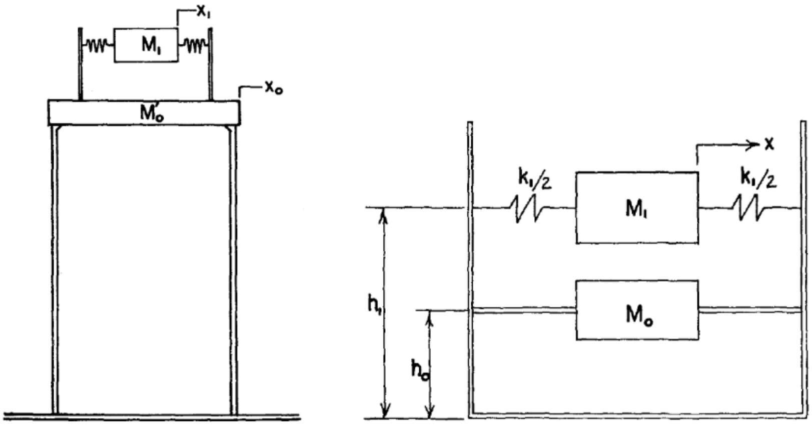
$$M_1 = 0.527\,\frac{L}{h}\tanh\left(1.581\frac{h}{L}\right)M\,, \tag{3}$$

$$k_1 = 1.581\,\frac{g}{L}\tanh\left(1.581\frac{h}{L}\right)M_1\,, \tag{4}$$

$$h_1 = \left(1 - \frac{\cosh\left(1.581\frac{h}{L}\right) - 1}{1.581\frac{h}{L}\sinh\left(1.581\frac{h}{L}\right)} \right) h . \tag{5}$$

Where M is the total water mass of the tank, h_0 is the height of the impulsive mass, g is the gravity acceleration and h_1 is the height of the convective mass. The natural frequency of the reservoir (f_1) considers only the convective mass, and it can be calculated as the frequency of a simple harmonic motion, given by the following equation:

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}} . \tag{6}$$



(a) Equivalent system **(b) Tank model**

Figure 1. Housner's model for an elevated water tank (Housner, 1963)

Many authors have used Housner's model to assess water tanks. Bauer (1964) and Chen and Barber (1976) analyzed tanks sitting on the ground considering more than one convective mass. However, Livaoğlu and Doğangün (2006) concluded that the effect of adding multiple convective masses could be neglected in elevated tanks, even when the natural frequency of the structure was close to the natural frequency of the water.

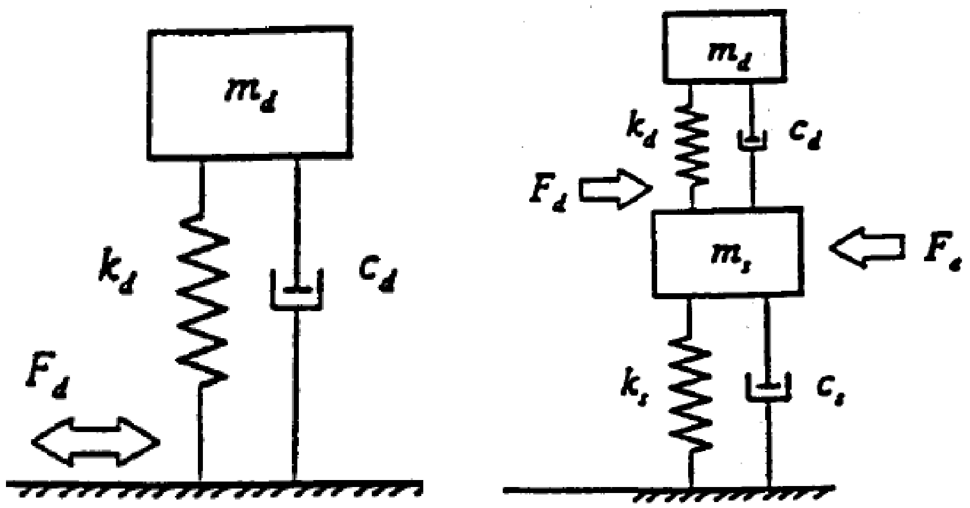
Other studies focus only on evaluating the fluid-structure and soil-structure interactions of reservoirs subjected to earthquakes (Dutta et al., 2004; Livaoğlu and Doğangün, 2006). The fluid-structure interaction is made through the Housner's model while the soil-structure interaction is accomplished through a translating spring and a torsion spring.

Vidal (2015) evaluated if an existing water supply tank located at the top of a building could mitigate horizontal displacements induced by the wind. A finite element analysis was performed using Housner's model and a model of purely impulsive mass to represent the water tank. When comparing the results, the author concluded that Housner's model had a smaller effect on damping horizontal displacements than the model of purely impulsive mass. This may have happened because Housner's model was not developed for TLDs but for water tanks. The presence of an empty tank at the building top increased horizontal displacements and reduced the natural frequency of vibration of the structure approaching the excitation frequency. This phenomenon is known as resonance.

The water supply tank from Vidal's model was placed with eccentricity on the building and this might have affected displacements. The tank material is reinforced concrete that has a large specific weight, what it might have increased the displacements due to the large mass. It is important to mention that the building studied by Vidal (2015) is already built, and its water supply tank was not dimensioned to mitigate vibrations.

1.2 Yu's Model

While Housner (1963) developed equations for water tanks, Yu et al. (1999) created a model representing TLDs with nonlinear stiffness and damping, also known as Non-linear Stiffness and Damping (NSD) model (Figure 2a). The parameters associated to the nonlinearities of the model were derived from extensive shaking table tests, in such a way that the energy dissipated by the TLD was the same dissipated by the NSD. Besides, the model considers all the mass of water inside the tank as convective mass (m_d). In this case, the energy is dissipated through wave breaking and the water viscous action (Kareem et al., 1999).



(a) NSD **(b) 2DOFS (Building + NSD)**

Figure 2. Yu's model (Yu et al., 1999)

When a damping ratio is introduced, the water viscous behavior can be better described; hence, the system can be better characterized. The fundamental frequency of vibration of the water (f_d) can be determined through surface waves analysis, which depends on the length of the tank (L), the height of the water (h) and the acceleration of gravity (g) (Lamb, 1932 cited by Yu et al., 1999). This frequency is calculated by the following equation:

$$f_d = \frac{1}{2\pi}\sqrt{\frac{\pi g}{L}\tanh\left(\frac{\pi h}{L}\right)}. \tag{7}$$

It can be obtained larger values of damping ratio when this frequency (Eq. 7) is equal to the natural frequency of the structure since the liquid would resonate with the structure. The stiffness (k_d) and damping ratio (c_d) associated with the water movement can be determined with the values from Eq. (7) and the convective mass. These parameters are given by the following equation:

$$
\begin{aligned}
k_d &= 1.075\left(\frac{A}{L}\right)^{0.007} m_d\left(2\pi f_d\right)^2 \Rightarrow \left(\frac{A}{L}\right) \leq 0.003 \\
k_d &= 2.52\left(\frac{A}{L}\right)^{0.25} m_d\left(2\pi f_d\right)^2 \Rightarrow \left(\frac{A}{L}\right) > 0.003
\end{aligned}
, \qquad (8)
$$

$$
c_d = 1.04\sqrt{k_d m_d}\left(\frac{A}{L}\right)^{0.35}. \qquad (9)
$$

The A parameter is the oscillation amplitude of the reservoir and characterizing the nonlinear stiffness and damping. In Eq. (8), the first expression is referred to the weak wave breaking region, while the second one is the strong wave breaking region. According to Malekghasemi et al. (2015), Yu's model provides more realistic results (displacements and accelerations) than other models for TLDs available in the literature.

Dattatray et al. (2014) studied reservoirs at the top of buildings with different heights of water as TLDs. The authors have used the finite element software SAP 2000 v14 to simulate such structures subjected to three known earthquakes. For each earthquake, three different heights of water were adopted: empty tank, 2/3 of its volume filled or completely filled. They concluded that the reduction in the horizontal displacements is more effective when the ratio between the mass of water and mass of structure is close to 3%.

1.3 Proposed model

The purpose of this paper is to model a water supply tank (Fig. 3) associating the model proposed by Housner (1963) to the model proposed by Yu et al. (1999). From the first author, it is obtained both impulsive and convective masses (Eq. 1 and Eq. 3); from the second one, the nonlinear stiffness and damping resulting from the movement of the liquid (Eq. 8 and Eq. 9).

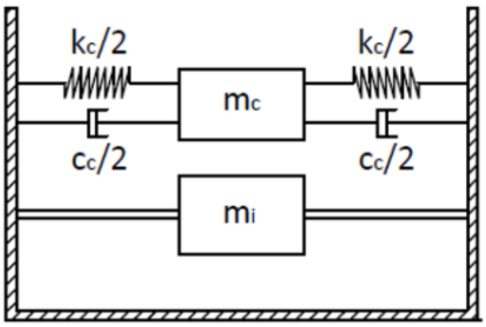


Figure 3. Model proposed for water supply tanks

2 MODELS ANALYZED

TLDs are placed right over the roof slab and their maximum displacement is the same as the floor below. However, the Brazilians water supply tanks are built on short columns over the roof slab to avoid percolation problems. TLDs are modeled as two-degree of freedom system (2DOFS – Fig. 4), while Brazilians reservoirs are modeled as three-degree of freedom systems (3DOFS – Fig. 5).

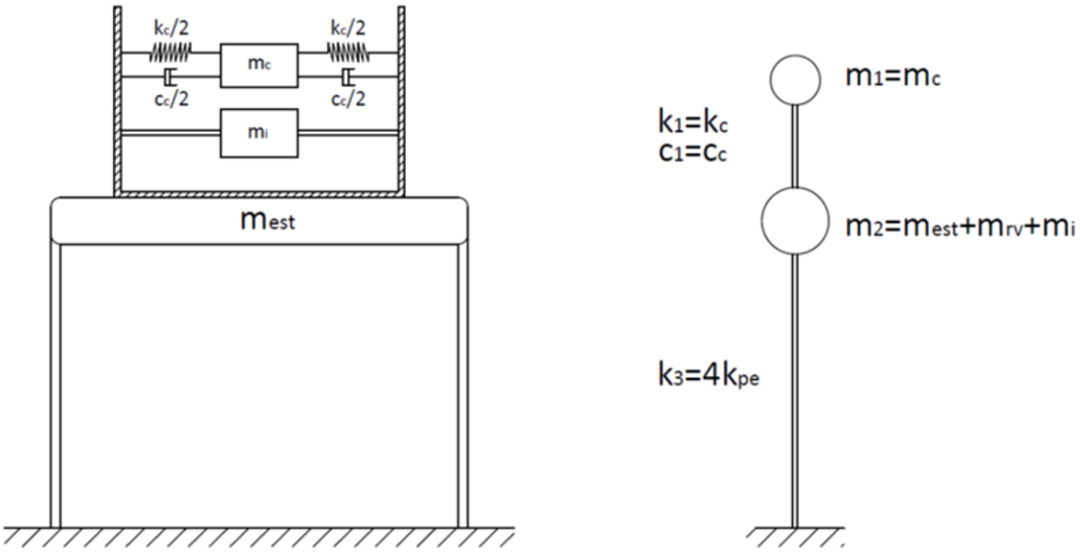


Figure 4. 2DOFS

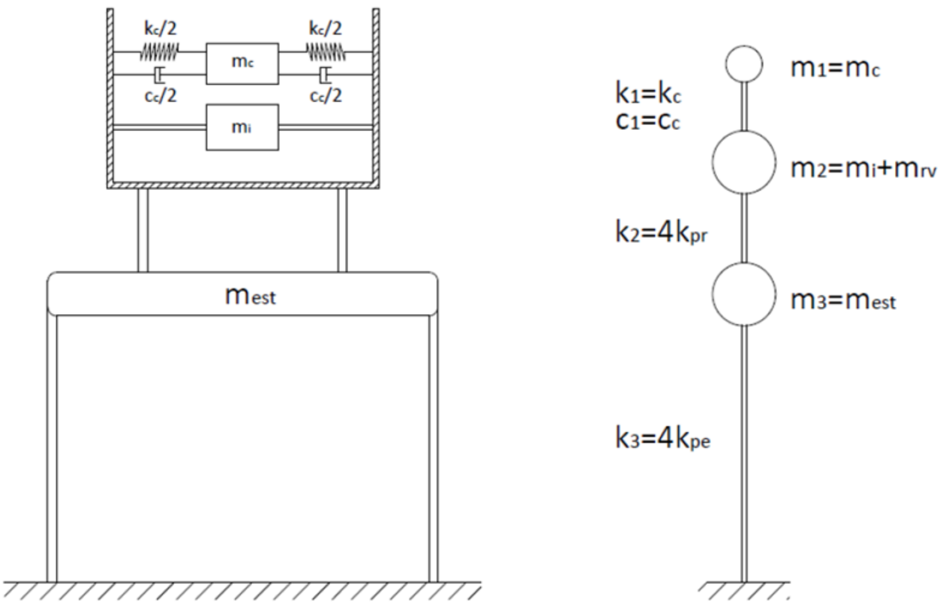


Figure 5. 3DOFS

Therefore, in order to evaluate the potential damping effect of a water tank located at the top of a building, three different frames are analyzed. The first is a 2DOFS (equivalent to Yu et al.'s model – Fig. 4), the second is a 3DOFS (Fig. 5) and the third one is a four-story building. These models are excited by seismic motion. The movement equation is written in matrix form as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1(A) & -c_1(A) \\ -c_1(A) & c_1(A) \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1(A) & -k_1(A) \\ -k_1(A) & k_1(A)+k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ k_2 x_{sismo}(t) \end{Bmatrix}. \tag{10}$$

Where m_1 is the convective mass, m_2 is the sum of the impulsive mass and structure mass, k_2 is the equivalent stiffness for the columns of the frame and x_{sismo} is a time-dependent function that describes the soil movement. k_1 and c_1 are calculated by Eq. (8) and Eq. (9), respectively. $\ddot{x}$, $\dot{x}$, and x are the accelerations, velocities, and displacements of the respective masses.

The equation for the 3DOF system is written in matrix form as:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c_1(A) & -c_1(A) & 0 \\ -c_1(A) & c_1(A) & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1(A) & -k_1(A) & 0 \\ -k_1(A) & k_1(A)+k_2 & -k_2 \\ 0 & -k_2 & k_2+k_3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ k_3 x_{sismo}(t) \end{Bmatrix} \tag{11}$$

m_2 represents the sum of the impulsive mass and the mass of the reservoir, m_3 is the mass of the structure only, k_2 is the equivalent stiffness of the columns supporting the water tank and k_3 is the equivalent stiffness of the frame columns. $\ddot{x}$, $\dot{x}$, and x represent the accelerations, velocities, and displacements of the masses corresponding to the subscripts 1, 2 and 3.

Equations (10) and (11) may be rewritten as:

$$[M]\{\ddot{X}\}+[C]\{\dot{X}\}+[K]\{X\}=\{F\}, \tag{12}$$

where $[M]$, $[C]$ and $[K]$ are mass, damping and stiffness matrices, respectively; $\{\ddot{X}\}$, $\{\dot{X}\}$, and $\{X\}$ are the accelerations, velocities, and displacements vectors, respectively; $\{F\}$ is the vector of external forces applied as a function of time.

2.1 Dynamic load

The seismic action is a dynamic load that induces the base displacement of the structure. It is represented by the non-zero vector on the right-hand side of Eq. (10) and Eq. (11). It is considered a time-dependent external force which is dependent on the soil movement (x_{sismo}). It is expressed by the following equation:

$$F_{ext}(t)=k_i x_{sismo}(t). \tag{13}$$

The subscript i (Eq. 13) represents the degree of freedom associated with the column fixed at the soil. The soil movement (x_{sismo}) is considered as a sine function, as suggested by Yu et al. (1999). This movement is described by:

$$x_{sismo}(t)=X_s\sin(\omega_{sismo}t). \tag{14}$$

Where X_s is the amplitude of the seismic movement, and ω_{sismo} is the circular excitation frequency (rad/s).

3 NEWMARK'S METHOD

Equation (12) consists of the nonlinear second order differential equations, which solution is obtained through Newmark's method of numerical integration. The displacement (x) and velocity ($\dot{x}$) at time t + Δt are given by:

$$x(t+\Delta t) = x(t) + \dot{x}(t)\Delta t + \ddot{x}(t)\frac{\Delta t^2}{2} + \dddot{x}(t)\frac{\Delta t^3}{6}, \tag{15}$$

$$\dot{x}(t+\Delta t) = \dot{x}(t) + \ddot{x}(t)\Delta t + \dddot{x}(t)\frac{\Delta t^2}{2}. \tag{16}$$

By the finite difference method, the following approximation is obtained:

$$\dddot{x}(t) = \frac{\ddot{x}(t+\Delta t) - \ddot{x}(t)}{\Delta t}. \tag{17}$$

Substituting Eq. (17) in equations (15) and (16), it is obtained:

$$x(t+\Delta t) = x(t) + \dot{x}(t)\Delta t + \left(\frac{1}{2} - \beta\right)\Delta t^2 \ddot{x}(t) + \beta\Delta t^2 \ddot{x}(t+\Delta t), \tag{18}$$

$$\dot{x}(t+\Delta t) = \dot{x}(t) + (1-\alpha)\Delta t\ddot{x}(t) + \alpha\Delta t\ddot{x}(t+\Delta t). \tag{19}$$

α and β are constants that define the acceleration variation within each time interval Δt and determine the stability and accuracy of the method. Considering an average rate of change constant, these parameters assume values of 1⁄2 and 1⁄6, respectively. The dynamic equilibrium equation (Eq. 12) at time t + Δt is given by:

$$[M]\{\ddot{X}\}_{t+\Delta t} + [C]_{t+\Delta t}\{\dot{X}\}_{t+\Delta t} + [K]_{t+\Delta t}\{X\}_{t+\Delta t} = \{F\}_{t+\Delta t}. \tag{20}$$

Equations (18), (19) and (20) define the Newmark's method. Substituting the first two equations on the latest, the acceleration ($\ddot{x}$) at time t + Δt can be determined since the conditions of displacement, velocity, acceleration, and force at time t are known. From the acceleration at time t + Δt, the velocity and acceleration at this time are calculated with Eq. (18) and Eq. (19).

The initial conditions $\{X\}_{t=0}$, $\{\dot{X}\}_{t=0}$, and $\{F\}_{t=0}$ are needed to begin the algorithm of the method. With these values determined, the initial acceleration can be calculated by:

$$\{\ddot{X}\}_{t=0} = [M]^{-1}\left(\{F\}_{t=0} - [C]_{t=0}\{\dot{X}\}_{t=0} + [K]_{t=0}\{X\}_{t=0}\right). \tag{21}$$

With the initial acceleration, a constant time interval Δt must be adopted throughout the interactive process. The effective stiffness matrix [K_e] is then determined at time t, and it is composed by stiffness, damping and mass matrices. Besides, two additional matrices [A_1] and [A_2] are used to assist the calculus. These equations are:

$$[K_e]_t = [K]_t + \frac{\alpha}{\beta\Delta t}[C]_t + \frac{1}{\beta\Delta t}[M], \tag{22}$$

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$$[A_1]_t = \frac{1}{\beta \Delta t}[M] + \frac{\alpha}{\beta}[C]_t, \tag{23}$$

$$[A_2]_t = \frac{1}{2\beta}[M] + \Delta t\left(\frac{\alpha}{2\beta} - 1\right)[C]_t. \tag{24}$$

The changes in displacements, velocities, and accelerations are determined at each time interval Δt in the interactive process. This is due to the change in the external forces. Assuming the acceleration is known from the previous step, the interaction *i* corresponding to the instant *t* is given by:

1. Calculating $\Delta\{F\}_t$:

$$\Delta\{F\}_t = (\{F\}_t - \{F\}_{t-\Delta t}) + [A_1]_t\{\dot{X}\}_{t-\Delta t} + [A_2]_t\{\ddot{X}\}_{t-\Delta t}. \tag{25}$$

2. Calculating the change in displacement $\Delta\{X\}_t$, velocity $\Delta\{\dot{X}\}_t$, and acceleration $\Delta\{\ddot{X}\}_t$:

$$\Delta\{X\}_t = [K_e]_t^{-1}\Delta\{F\}_t, \tag{26}$$

$$\Delta\{\dot{X}\}_t = \frac{\alpha}{\beta \Delta t}\Delta\{X\}_t - \frac{\alpha}{\beta}\{\dot{X}\}_{t-\Delta t} + \left(1 - \frac{\alpha}{2\beta}\right)\{\ddot{X}\}_{t-\Delta t}, \tag{27}$$

$$\Delta\{\ddot{X}\}_t = \frac{1}{\beta \Delta t^2}\Delta\{X\}_t - \frac{1}{\beta \Delta t}\{\dot{X}\}_{t-\Delta t} - \frac{1}{2\beta}\{\ddot{X}\}_{t-\Delta t}. \tag{28}$$

3. Updating the variables $\{X\}_t$, $\{\dot{X}\}_t$, and $\{\ddot{X}\}_t$:

$$\{X\}_t = \{X\}_{t-\Delta t} + \Delta\{X\}_t, \tag{29}$$

$$\{\dot{X}\}_t = \{\dot{X}\}_{t-\Delta t} + \Delta\{\dot{X}\}_t, \tag{30}$$

$$\{\ddot{X}\}_t = \{\ddot{X}\}_{t-\Delta t} + \Delta\{\ddot{X}\}_t. \tag{31}$$

The algorithm described above was implemented in the mathematical software Octave. Figure (6) provides the illustration of the process. Stiffness and damping parameters depend on the displacement amplitude (*A* at Fig. 6) and the reservoir mass.

4 RESULTS AND DISCUSSION

The external force was applied to the models with different excitation frequencies while keeping the amplitude constant, simulating the action of seismic motion. The damping ratio to the structure material was disregarded in the analyses to allow better evaluating the effect of the damping from the water into the tank.

4.1 S2GL vs. S3GL

Table 1 displays the geometric and mechanical properties adopted for both 2DOFS and 3DOFS frames. They were defined in an arbitrary way and do not correspond to a specific building. In order to compare both systems, the 3DOFS is analyzed considering both amplitudes of displacement at the reservoir and at the top of the structure. This amplitude is used to obtain the liquid parameters through Eq. (8) and Eq. (9).

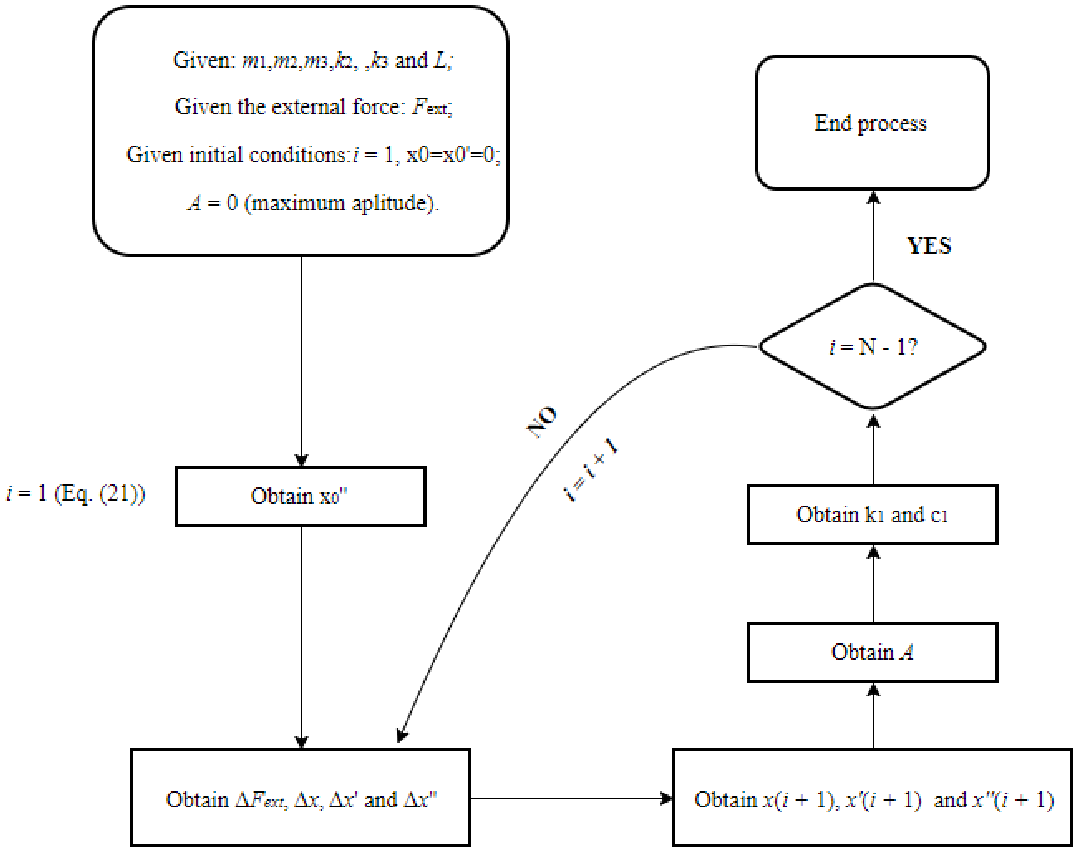

Figure 6. Scheme of algorithm implemented on Octave

The amplitudes obtained separately in the 3DOFS are then compared to the displacements from 2DOFS, which corresponds to the displacement of the structure mass added to the empty tank mass and the convective mass of water. The excitation frequency of the force is 0.08 Hz. Time history of displacements from both systems is shown in Fig. 7.

Table 1. Mechanical and geometrical properties of both 2DOFS and 3DOFS frames

Mass in each story	100,000 kg
Equivalent stiffness of each story	$8{\cdot}10^8$ kN/m
Equivalent stiffness of columns supporting the tank	10^5 kN/m
Displacement amplitude of soil	0.01 m
	Length of the tank L = 2.5 m
Geometrical properties of the tank	Width of the tank b = 6 m
	Height of the tank h = 1 m

Height of water in the tank $h_a = 0.5$ m

It is observed that curves from both systems have similar behavior (Fig. 7). However, the 2DOFS exhibits larger displacements (0.61 cm) at the beginning of the analysis when compared to the 3DOFS (0.59 cm). This difference is approximately 3.28%. After movement stabilization, both cases of 3DOFS show very close values of displacements (0.16 cm), while 2DOFS displayed again larger displacements (0.20), a difference of 20%.

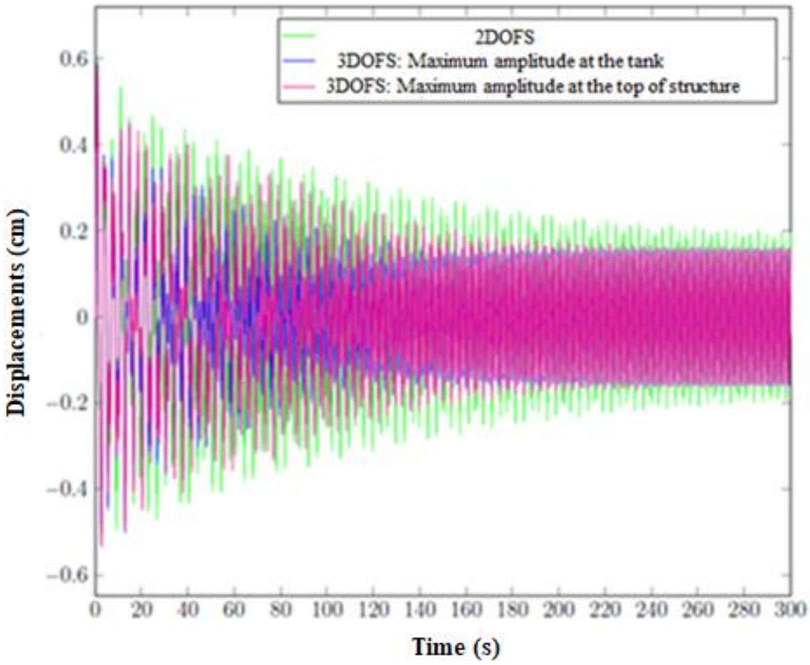

Figure 7. 2DOFS and 3DOFS displacements

4.2 Relationship between the excitation frequency and displacement

With the purpose of evaluating maximum displacements at the beginning motion as well as after its stabilization for the 3DOFS, an analysis was performed varying the excitation frequency of the external force (1 to 3 rad/s). The results are displayed in Fig. 8, where the ratio of frequencies is the excitation frequency divided by the natural frequency of the frame (2 rad/s).

It is observed that the displacement reaches its maximum value when the ratio of frequencies is 0.9. The shape of the graph obtained was as expected for a damped forced system. Therefore, the water tank is working as a damping system.

4.3 Evaluation of the effect caused by the water tank as a damper

In order to evaluate the use of water supply tanks as damping devices, it is modeled the structure (Fig. 9) used by Miranda (2010), which analyzed its seismic vulnerability. The water reservoir of this building is dimensioned to meet the code number 006/2008 from the Fire Department of Ceará.

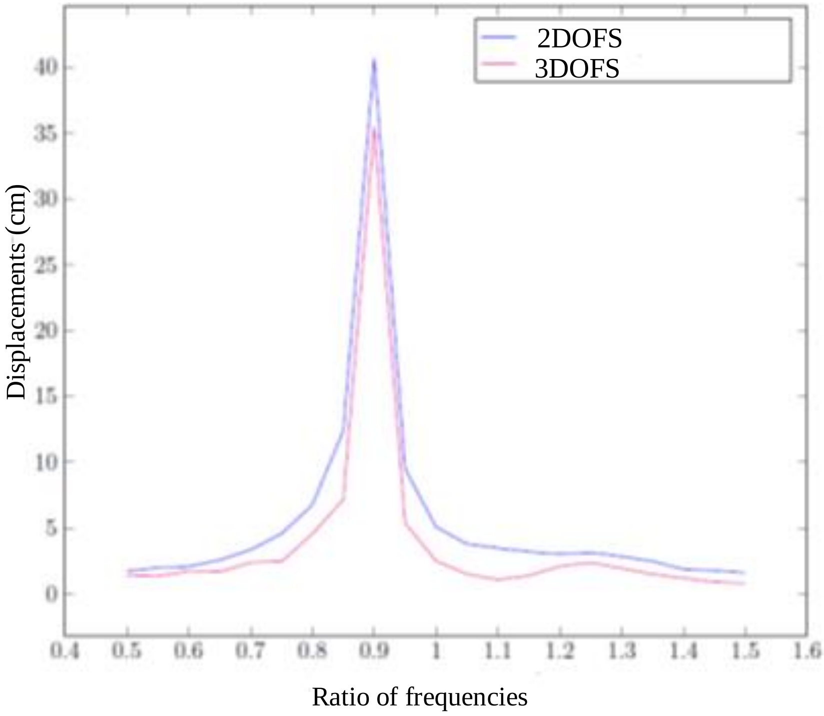

Figure 8. Relationship between excitation frequencies and displacements

(a) Building modeling (Miranda, 2010) **(b) 6DOFS**

Figure 9. Perspective of the building and the analytical model

The model consists of a four-story building (Fig. 9a). The height of each story is 2.8 meters and the span between all columns is 4.0 m (Fig. 10). All beams have the same cross-section (15 x 40) cm², and the cross-section for columns is (20 x 35) cm².

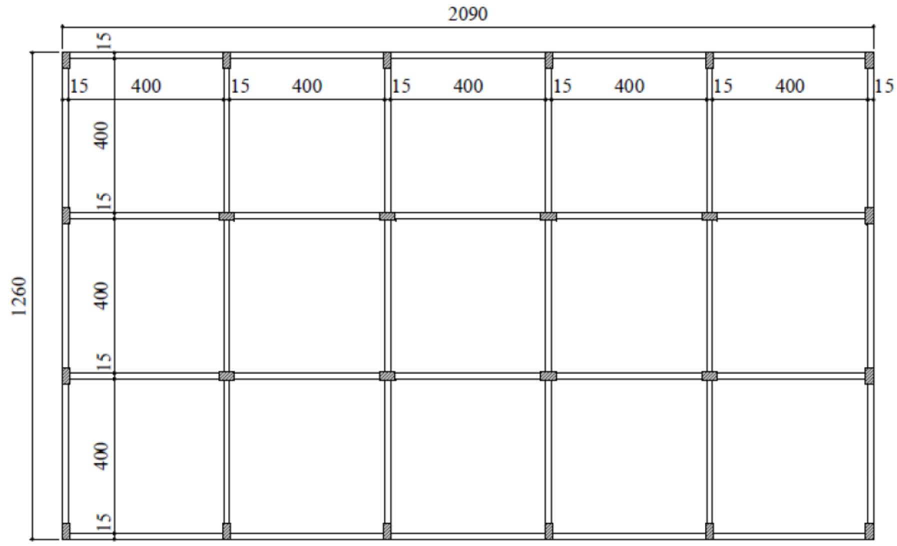

Figure 10. Structural plan for each story

While dimensioning the water supply tank, it was assumed that each floor would accommodate 5 occupants and that their water consumption per day is 175 liters per person. An additional volume of water of 4.5 m^3 is required for fire protection. Thus, the dimensions of the tank adopted are the following: width equal to 2.0 m, length of 4.0 m and height of 1.5 m. The water height will reach the maximum value of 1.0 m and the corresponding volume is 8.0 m^3.

Different masses are assigned to each floor, and its value was equal to those proposed by Miranda (2010). The columns are considered fixed on the base and its equivalent stiffness is calculated in such a way to obtain the model of six degrees of freedom (6DOFS) shown in Fig. 9b. The properties of this structure are shown in Table 2.

Table 2. Properties of the model

Story	Mass (kg)	Equivalent stiffness (kN/m)
1	74,962	$1.2915 \cdot 10^8$
2	56,281	$1.2915 \cdot 10^8$
3	37,599	$1.2915 \cdot 10^8$
Reservoir	8,817.5	$4.0000 \cdot 10^5$

The natural frequencies of vibration of the building are calculated through the algorithm and their values are shown in Table 3. An external force is applied with the vibration frequency equal to 1.0074 Hz, which is equivalent to the natural frequency of the structure. Three different heights of water in the tank are considered during the analysis of forced vibration: 100 cm, 75 cm, and 56 cm. The height of 56 cm represents the volume required for fire protection.

Table 3. Natural frequencies of vibration of the structure

Mode of vibration	Frequency of vibration (Hz)
1st	1.0074
2nd	1.1116
3rd	2.6703
4th	3.9710
5th	5.5766

An analysis with the empty tank is also performed. This means that the convective and impulsive masses are both equal to zero, and only the tank mass is considered in the model, reducing the number of degrees of freedom of the system to five. It is observed that the structure is in resonance with the external force; thus, displaying large displacements (Fig. 11), as expected. It should be noticed that, after certain amplitudes of displacement, the structure exhibits nonlinear behavior.

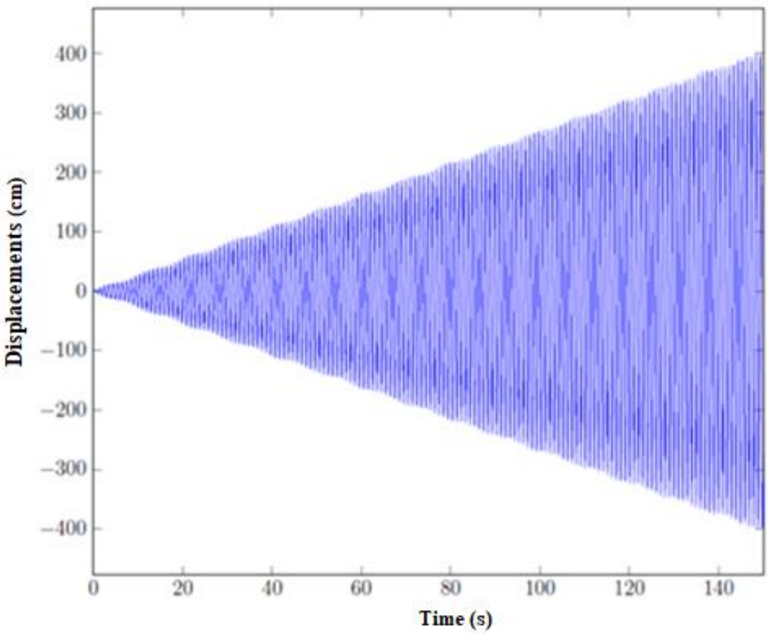

Figure 11. Resonating structure with empty tank (*f* = 1,0074 Hz)

Evaluating the three different heights of water (Fig. 12), it is observed water height equal to 1.0 m presents smaller displacements after the movement stabilization, followed by the tanks containing water heights of 0.75 m and 0.56 m. Therefore, even the reservoir containing the water volume corresponding to the fire protection has contributed slowly to reduce displacements. Thus, the higher the water level in the reservoir, the greater the damping of the structure (Fig. 13).

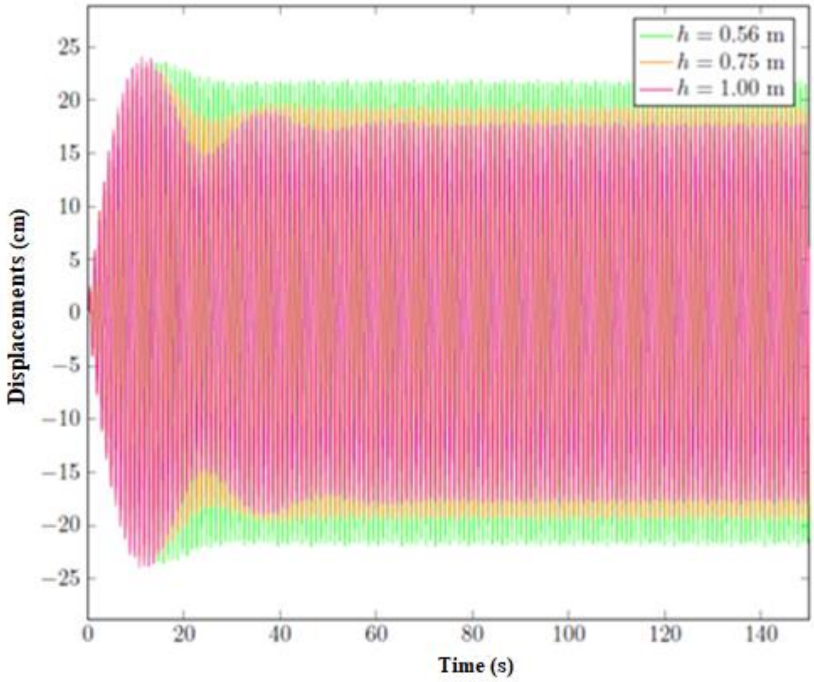

Figure 12. Structure with tank completely filled (*f* = 1,0074 Hz)

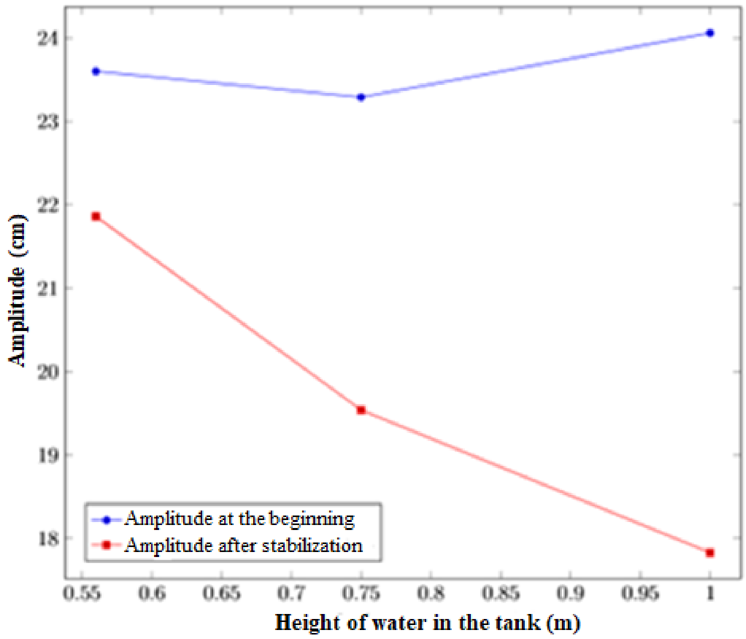

Figure 13. Amplitude as a function of height of water in the tank

Plotting the damping ratio ($\xi = c_d/(2m_d\omega_d)$) versus time (Fig. 14), it can be observed that this parameter converges to constant values being 50%, 52%, and 55% for the water heights equal to 1 m, 0.75 m and 0.56 m, respectively. In this case, a larger value of damping ratio does not imply smaller maximum displacement after movement stabilization but imply quicker stabilization. The tank containing 1 m of water height showed a larger damping ratio at the beginning of the movement when compared to the others. This fact can explain its lower displacement after motion stabilization.

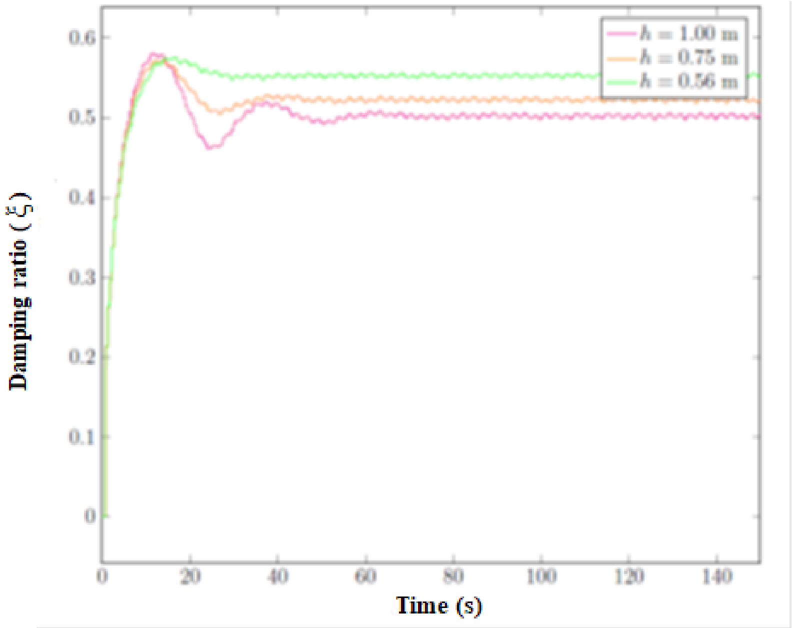

Figure 14. Damping ratio vs time of structure (*f* = 1,0074 Hz)

5 CONCLUSION

The main focus of this paper is to evaluate the potential damping effect caused by water supply tanks in buildings. Two analytical models were compared and a structure was analyzed. It should be emphasized that the damping associated with the structure was not considered during simulations.

It is concluded that the 2DOFS, which represents the tank placed right above the roof slab, shows similar behavior to the 3DOFS, supported by short columns. Therefore, it can be concluded that the two models are equivalents. However, 3DOFS produces lower displacements after motion stabilization is reached. For the four-stories building analyzed, the water supply tank contributes to damp the lateral displacements, reducing them as the water height increases.

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