ANGLE CONTROL OF A SIMPLE STAGE INVERTED PENDULUM: COMPARISON BETWEEN PID, PI-D AND FUZZY

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Abstract. *There are several studies based on inverted pendulum control, because of its simplification of a nonlinear system, which is extremely unstable and multivariable. Its importance is given by the applications of this type of system, such as rockets launching, robots, altitude control of satellites, among others. The purpose of this work is to simulate different angle controls of a single-stage inverted pendulum using MATLAB® software in combination with SIMULINK, these computational tools present great performance in development and simulation of control strategies. After modeled the problem, three control strategies were proposed: a parameterization of the PID controller via root locus, a controller with modified structure PI-D and, finally, a Mamdani fuzzy controller. The simulation consisted of two stages: in the first, different amplitudes disturbances were applied and, in the second, a noise was added. The fuzzy controller presented greater overshoot in both cases, but obtained lower oscillation and faster stabilization for smaller amplitude disturbances and disturbances with noise. In the first stage, the PI-D presented lower stabilization time for major disturbances.*

Keywords: *Inverted pendulum simulation, Matlab®, Fuzzy control, Root locus.*

INTRODUCTION

An inverted pendulum is a classic nonlinear multivariable and extremely unstable control problem, and it is also a simplification of critical applications with great importance in real world, such as rockets stabilization, design of robot, human walking, among others.

The inverted pendulum control and dynamics are theme of several studies often recently, as in Pal and Chakrabarty (2014), who proposed a fuzzy control acting on scaling factor fuzzy, i.e. the weights of the parameters are updated online according to the trend of the process, that is determined by the system error change. These controller results were compared with a predefined weights fuzzy controller and a fuzzy PD controller. Based on the pendulum angle and angular velocity, system drives the output voltage. The system with fuzzy scale update became the best response, getting less overshoot and shorter time to stabilize the system with less error.

Meenakshi and Manimozhi (2016) proposed to compare root locus, LRQ and ANFIS on behalf of an inverted pendulum control. All proposed systems stabilized the system, though ANIFIS stabilized faster and with less overshoot. Ghosh et al (2011) did a robust PID inverted pendulum control by pole allocation obtained through LRQ (Linear Quadratic Regulator). The authors concluded that even when the LRQ design fails, PID remains responding satisfactorily.

The article of Luo Hong-yu and Fang Jian (2014) presents an angular position fuzzy control of an inverted pendulum, which is compound by the fuzzyfication knowledge base, fuzzy reasoning and clarity, whose entries are angle error and error variation. The controller is a Mandani fuzzy with Max-Min algorithm and the output with center of gravity method. The fuzzy controller had better results than the PID: there was no overshoot and the system was stabilized at a time four times smaller.

Shehu *et al* (2015) controlled an inverted pendulum with LRQ, double PID and pole allocation. In the simulation in SIMULINK the signal control disturb was a noise. They concluded that all controllers had a good performance, highlighting the LRQ design, that behaved better to disturbances.

The PID control is widely used in several applications; however, in some cases its configuration may not be so simple. On the other hand, the fuzzy control bases on human experience, in which the decision to practical problems takes base on linguistics (Jang *et al*, 1997).

This paper proposes to compare the stability control of inverted pendulum angle from three simulations in SIMULINK (MATLAB®): PID control with parameters found by trial and error root locus method; PID structure modification to PI-D; and a type Mamdani fuzzy control.

The rest of this paper is organized as follows. The system modelling is presented in Section 1. The three controllers are proposed in Section 2. In Section 3, the results with the disturb response of each system is given. Section 4 draws some conclusions.

1 SYSTEM MODELING

The inverted pendulum with car model parameters (Fig. 1) studied in this article comes from Ghosh *et al* (2011), where $\theta \leq 0.1$ rad is the angle between the pendulum and its vertical position, x is the car position reference, $M = 2.4$ kg is the car mass, $m = 0.23$ kg is the pendulum mass, $L = 0.4$ m is the pendulum length, $J = 0.099$ kg is the pendulum moment of inertia, $b = 0.055$ Ns/m is the car friction coefficient, $g = 9.81$ m/s² is the acceleration of gravity, $F = \pm$ 24N is the force applied on the car.

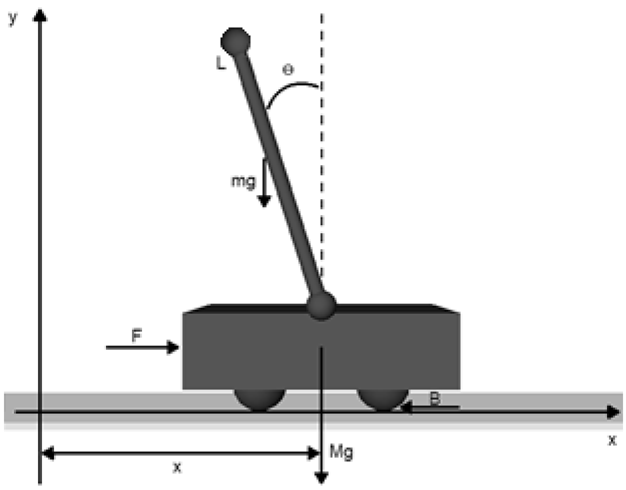


Figure 1. Inverted pendulum model

The inverted pendulum has two degrees of freedom: one linear, corresponding to the X axis car movement, as in Eq. (1), and one non-linear, regarding the pendulum rotation in XY plane, in Eq. (2).

$$\ddot{x} = \frac{(J + mL^2)\left(F - b\dot{x} - mL\dot{\theta}^2 sen\theta\right) + mL^2 g sen\theta cos\theta}{mL^2(M + mcos^2\theta) + J(M + m)} \tag{1}$$

$$\ddot{\theta} = \frac{mL[(F - B\dot{x})cos\theta - mL\dot{\theta}^2 cos\theta sen\theta + (m + M)g sen\theta]}{mL^2(M + mcos^2\theta) + J(M + m)} \tag{2}$$

The angle θ is very small for vertical balance point, hence Eq. (1) and Eq. (2) can be linearized, where $sen\theta \approx \theta$ and $cos\theta \approx 1$, as in Eq. (3) and Eq. (4), respectively.

$$\ddot{x} = \frac{-b(mL^2 + J)\dot{x} + m^2L^2 g\theta + (J + ML^2)F}{mML^2 + J(M + m)} \tag{3}$$

$$\ddot{\theta} = \frac{-mbL\dot{x} + mgL(M + m)\theta + MLF}{mML^2 + J(M + m)} \tag{4}$$

In the state space the Eq. (3) and (4) can be respectively represented by Eq. (5) and Eq. (6), where x, $\dot{x}$, θ and $\dot{\theta}$ are the states and y is the output vector.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-b(mL^2 + J)}{mML^2 + J(M + m)} & \frac{m^2L^2 g}{mML^2 + J(M + m)} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-bmL}{mML^2 + J(M + m)} & \frac{mgL^2(M + m)}{mML^2 + J(M + m)} & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{mL^2 + J}{mML^2 + J(M + m)} \\ 0 \\ \frac{mL}{mML^2 + J(M + m)} \end{bmatrix} \cdot F \tag{5}$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \tag{6}$$

Substituting values of system variables and discounting the force of friction b for being much smaller than the other parameters, it results in the transfer function in Eq. (7), since the system is closed loop controlled (Ghosh *et al*, 2011). As the intrinsic feature of an inverted pendulum is instability, Eq. (7) confirms this condition by presenting two roots ($s = 2{,}609$ and $s = -2{,}609$), where one of them is on the right semi plane (Fig. 2a).

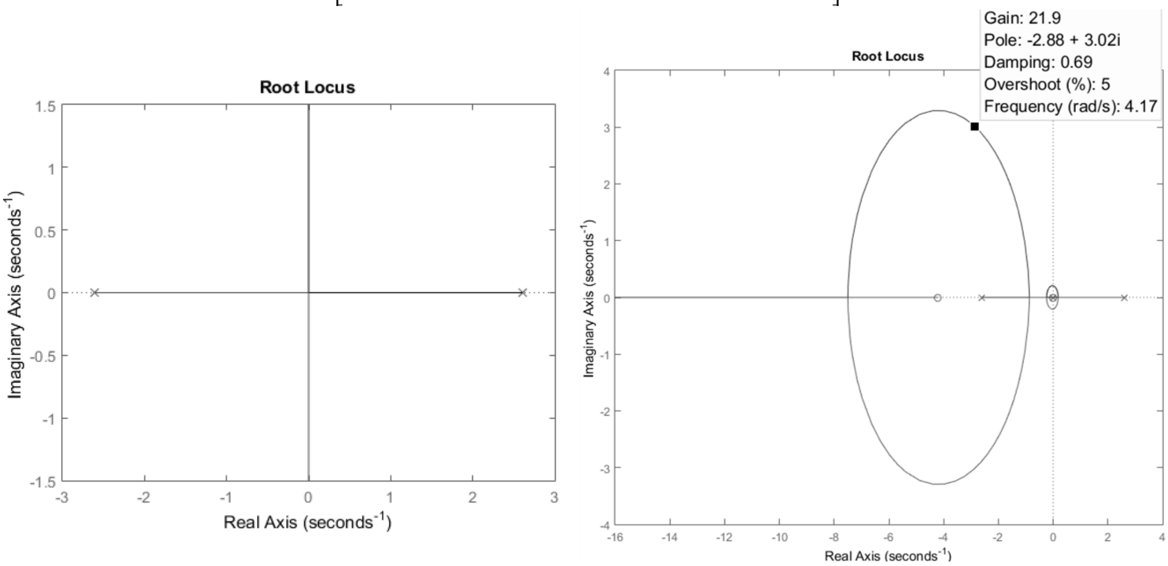
$$\frac{\theta(s)}{F(s)} = \frac{(mL)s^2}{s^2[(J(m+M) + MmL^2)s^2 - mgL(M+m)]} = \frac{0{,}2638s^2}{s^2(s^2 - 6{,}807)} \approx \frac{0{,}2638}{s^2 - 6{,}807} \tag{7}$$

2 CONTROLLERS

2.1 Root locus parametrization

The first method presented in this article is root locus, which consists of allocating closed loop poles in any position from the feedback of states, since the system is completely controllable (Ogata, 2010). The controllability matrix *C* in Eq. (8) has rank 4, which means that the system is completely controllable. From system root locus (Fig. 2a), Table 1 controllers were proposed by trial and error to bring the system to the left semi plane, in order to make him stable as the gain increases.

$$C = \begin{bmatrix} B & A \cdot B & A^2 \cdot B & A^3 \cdot B \end{bmatrix} = \begin{bmatrix} 0 & 0.3895 & -2.7578 & 19.5907 \\ 0.3895 & -2.7578 & 19.5907 & -138.7242 \\ 0 & 0.2638 & -0.0057 & 1.8361 \\ 0.2638 & -0.0057 & 1.8361 & -0.3228 \end{bmatrix}, \; |C| = -3.03 \neq 0 \tag{8}$$



(a) **(b)**

Figure 2. Root locus of (a) original system and (b) PID system

The overshoot Mp and settling time Ts calculations are in Eq. (9) and Eq. (10) respectively, where ξ is the damping and ω_n is the frequency in rad/s (Ogata, 2010, p. 159).

$$Mp = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \tag{9}$$

$$Ts = \frac{4}{\xi\omega_n} \tag{10}$$

Table 1. Proposed controllers by trial and error

Controllers	PID 1	PID 2	PID 3	PID 4
Transfer function	$\frac{(s+2.3)(s+1.9)}{s}$	$\frac{(s+2)(s+0.5)}{s}$	$\frac{(s+1.5)(s+1)}{s}$	$\frac{(s+4.2)(s+0.01)}{s}$
Gain for 5% overshoot	28.6	23.6	32.7	21.9

The controllers' impulse response is shown in Fig. 3. The PID 4, whose root locus is in Fig. 2b, had the best result with settling time in 1.39 s. Therefore it was chosen for the system control, with Kd=21.9, Kp=92.2 and Ki=0.9198.

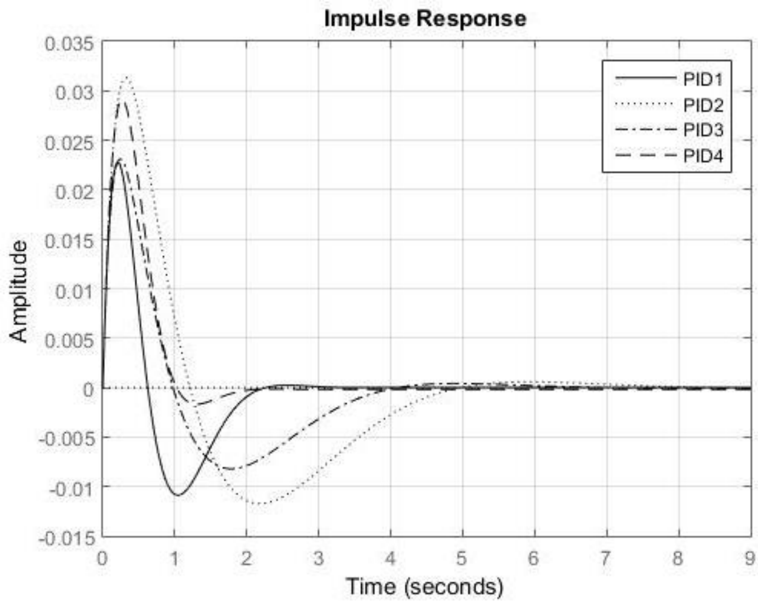


Figure 3. Impulse response of proposed controllers

Figure 4 shows the PID control simulation in SIMULINK, where disturbance is modifiable to other types of sign.

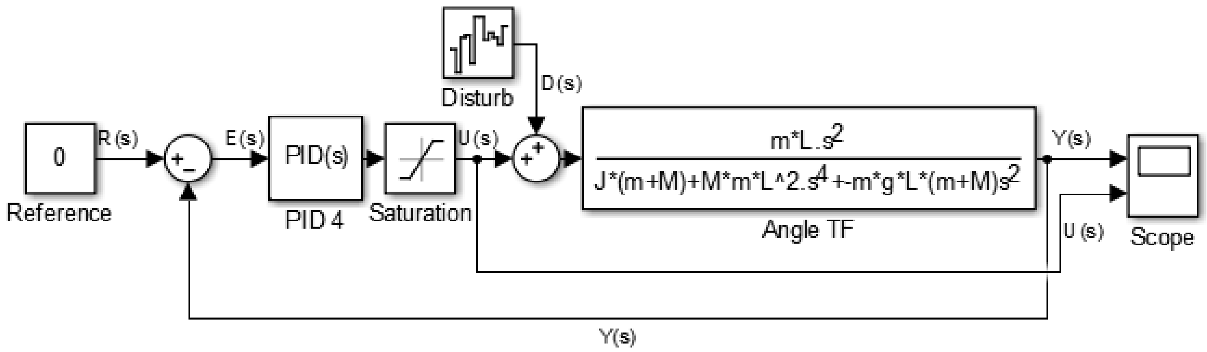


Figure 4. PID control blocks in SIMULINK

2.2 PID structure modification to PI-D

The derivative action have only acting on the feedback signal in PI-D structure, while proportional and integral actions act in the reference signal. This modification delays the transitional action, avoiding that a peak occurs in the reference value, as the Eq. (11) shows the manipulated signal U(s) (Ogata, 2010, p. 542-543). The modified structure PI-D control simulation in SIMULINK is in Fig. 5, where disturbance is modifiable to other types of sign.

$$U(s) = K_p \left(1 + \frac{1}{T_i\ s}\right) R(s) - K_p \left(1 + \frac{1}{T_i s} T_d s\right) B(s) \tag{11}$$

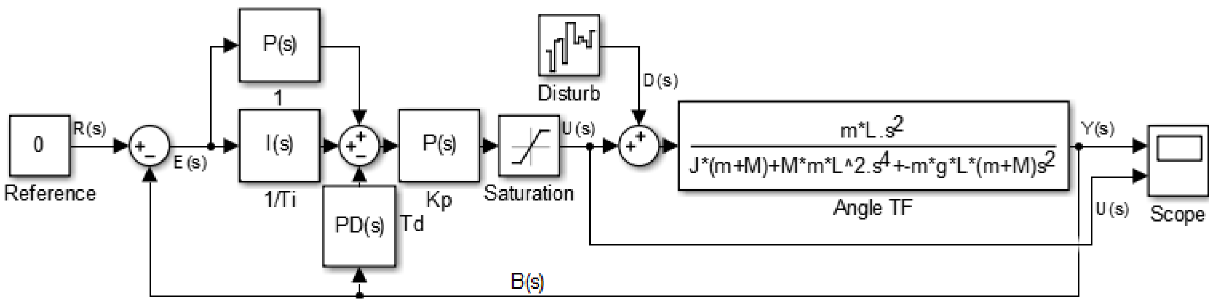


Figure 5. PI-D control blocks in SIMULINK

2.3 Fuzzy controller

The Fuzzy Logic Toolbox in MATLAB® was used to model a Mamdani fuzzy control, since this is an intuitive and widely accepted inference method. The Mamdani fuzzy controller with Min-Max algorithm proposed has the angle error and angular velocity as inputs and the applied force on inverted pendulum car as output (Fig. 6). The maximum angular variation considered feasible to control this system is an angle of ± 0.8 rad, corresponding to ± 45º. The angular velocity set is ± 1.5 rad/s and the output saturation is ± 24N, according to the problem modeling.

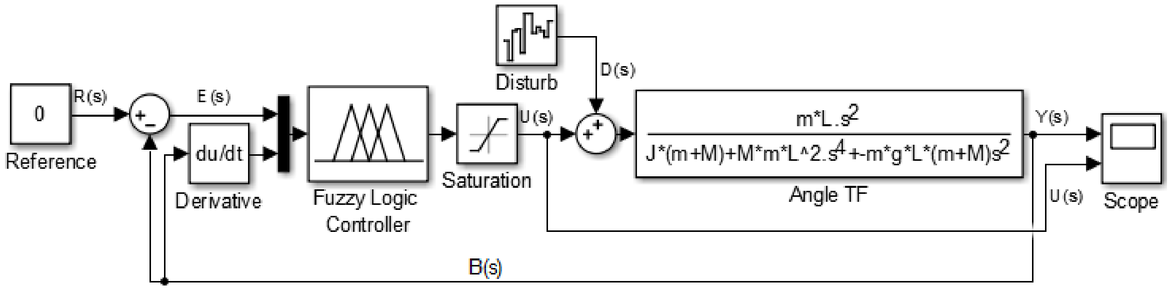
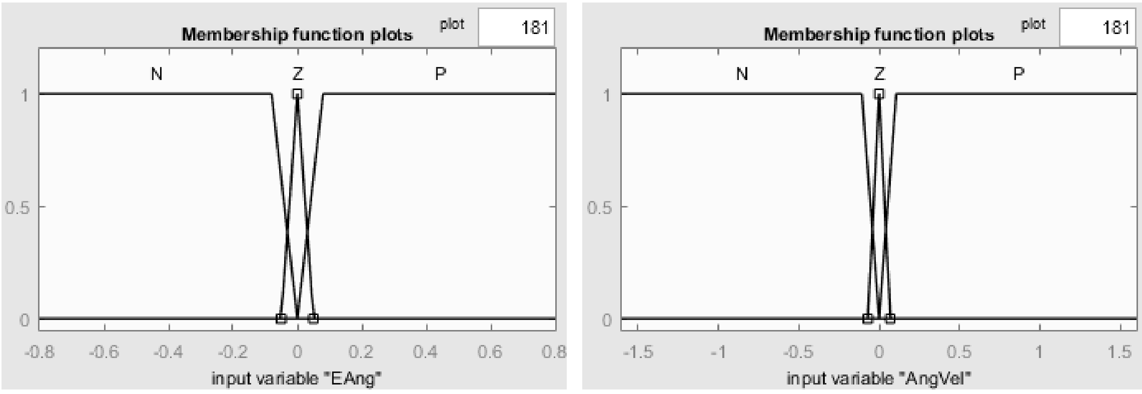


Figure 6. Fuzzy control blocks in SIMULINK

The fuzzy controller inputs and outputs use two triangular and one trapezoidal membership functions, as shown in Fig. 7a for angle error *EAng* and in Fig. 7b for angular velocity *AngVel*. The membership functions of force *F* output are presented in Fig. 8 and the defuzzification method used was the center of gravity. These chosen parameters have presented more satisfactory response than the other membership functions and defuzzification methods tested.



(a) **(b)**

Figure 7. Membership functions of inputs (a) EAng and (b) AngVel

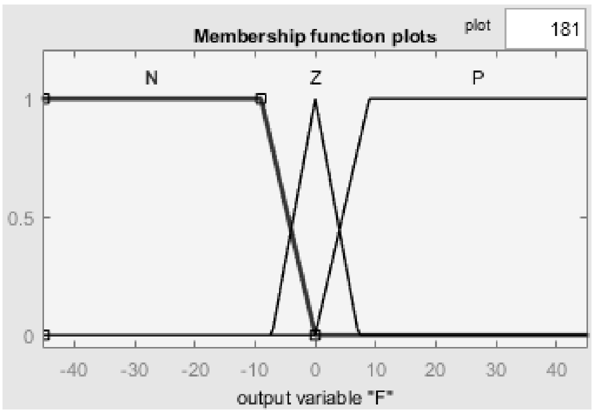
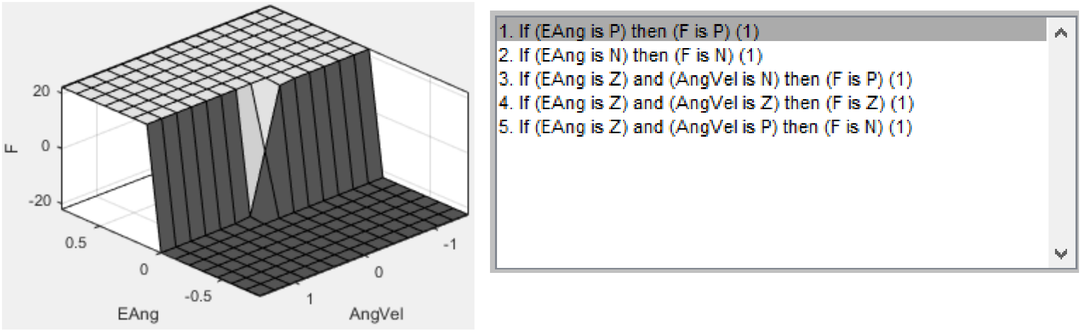


Figure 8. Membership functions of output F

The rules surface created from IF-THEN rules of fuzzy controller (Fig. 9) is a main component of fuzzy inference system (FIS), which controls the output *F* as a function of inputs based in human knowledge (Jang et al., 1997, p. 32-33). The proposal was to use the least number of rules for a satisfactory response to control the inverted pendulum angle, thus resulting in 5 rules that take angle to stabilize at zero position.



(a) **(b)**

Figure 9. (a) Rules surface and (b) fuzzy controller rules

3 RESULTS

To analyze and compare the proposed controllers, simulation had two stages. The first moment (Fig. 10) compound by system response analysis to disturbances of different amplitudes, where the first disturb had 75N in 0s, the second had -35N in 10s and the third had 60N in 20s. In a second stage (Fig. 11), random noises were added to the disturbance signal with amplitude of 6N, simulating an open location subject to wind.

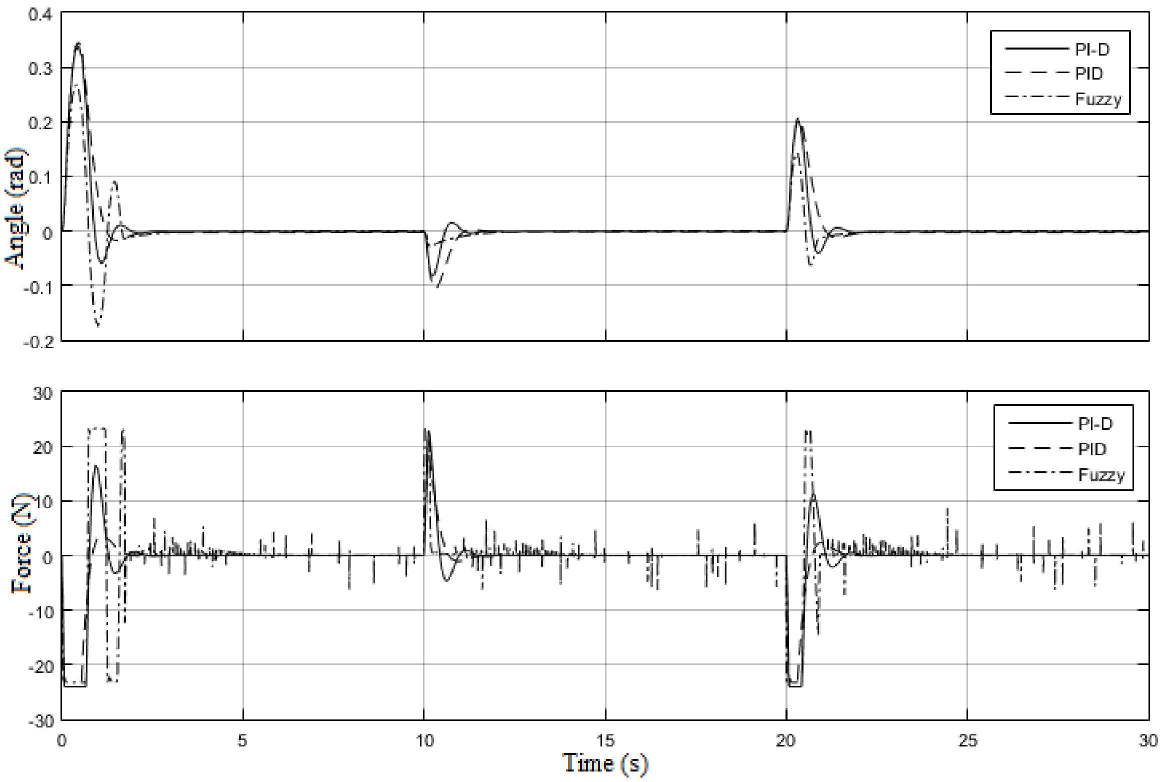


Figure 10. Disturbs system response (angle) and controllers signal (force) by time

In Figure 10, for first major disturbance, the fuzzy controller had the highest oscillation and overshoot, followed by PI-D. PID converged more quickly, with less overshoot and no oscillation. For slightest disturbance, fuzzy control presented no overshoot and stabilized more quickly. PID response was slower, but the overshoot was smaller and without oscillation. PI-D had shorter rise, however greater overshoot. For the last disturbance, fuzzy had highest overshoot and less rise time, however without oscillation and spent less time to stabilize. The PI-D, unlike the PID, oscillated and had greater settling time.

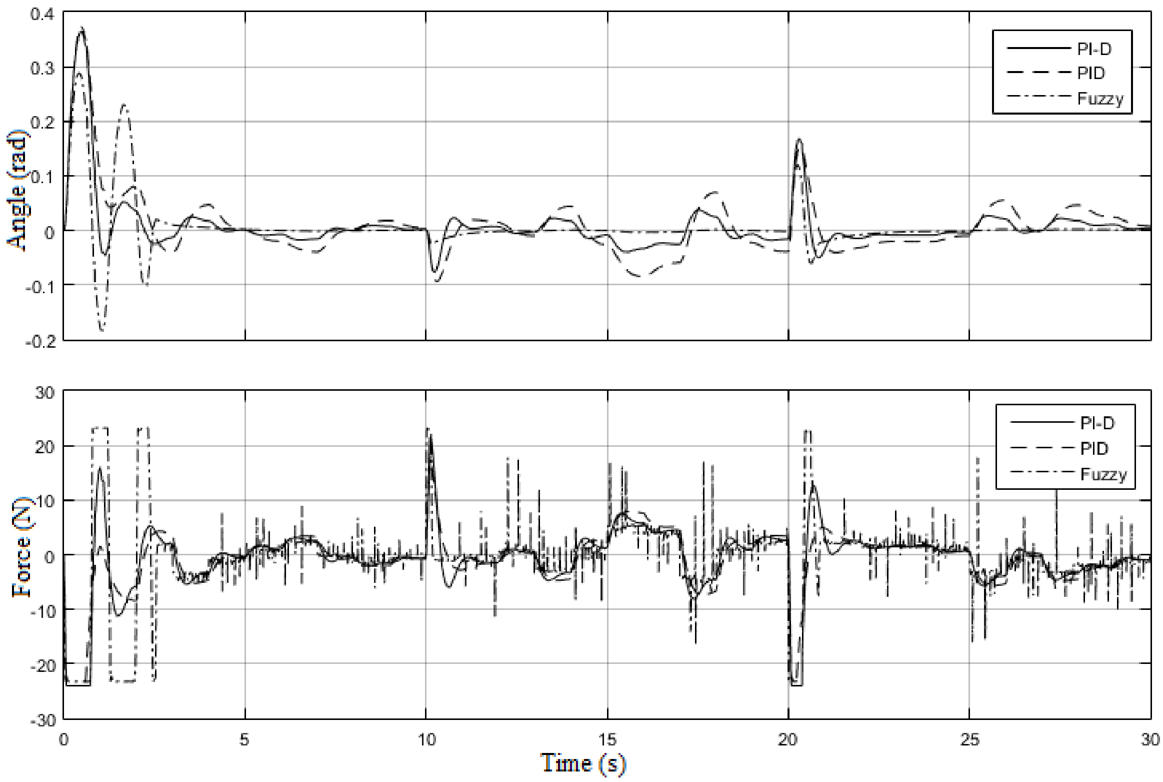


Figure 11. Disturbs with noise system response (angle) and controllers signal (force) by time

Adding noise changed the behavior of the system (Fig. 11). For the greatest disturbance, fuzzy had higher oscillation, but stabilized before, followed by PI-D and PID. For the slightest disturbance, fuzzy stabilized first without any overshoot. PI-D had shorter rise time and stabilized before PID. For the last disturbance, fuzzy stabilized first, followed by PI-D and PID. Fuzzy remained the angle close to zero in response to noises after system stabilized, but PID and PI-D presented greater oscillations. Table 2 contains the comparison of all controllers.

Table 2. Comparison between Fuzzy, PI-D and PID controllers simulation

1st Stage	**Disturb 1**			**Disturb 2**			**Disturb 3**		
	Fuzzy	PI-D	PID	Fuzzy	PI-D	PID	Fuzzy	PI-D	PID
Settling time(s)	1.70	1.40	1.39	0.80	0.90	1.00	0.9	1.18	1.00
Overshoot (%)	64	14	5	-	22	1	40	21	10
2nd Stage	**Disturb 1 + Noise**			**Disturb 2 + Noise**			**Disturb 3 + Noise**		
	Fuzzy	PI-D	PID	Fuzzy	PI-D	PID	Fuzzy	PI-D	PID
Settling time(s)	3.00	4.20	4.45	0.75	1.10	2.60	1.50	2.40	4.55
Overshoot (%)	64	14	11	-	26	22	50	29	30

4 CONCLUSION

The root locus method to parameterize PID and PI-D controller was able to stabilize the system, as well as the Mamdani Type-Fuzzy Inference controller with 5 rules. In the first stage, without noise, PID had lower settling time for major disturbances. Fuzzy controller demonstrated to be less robust than PID and PI-D, in other words, when there were several system parameters modifications, such as disturbance amplitude and noise, fuzzy suffered greater change in control. Fuzzy controller presented greater overshoot in all cases, but had shorter oscillation and lower settling time for smaller amplitude disturbances and disturbances with noise. Fuzzy results were satisfactory, leading to the convergence of the system in all cases, even with a simplified modeling, which includes just three membership functions in each variable and five rules. This controller can be a great option for many situations, especially when the system is too complex or mathematical modelling is unknown, allowing defining control rules from linguistic variables.

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