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Evaluation of a Cantilever Beam Energy Harvester Subjected to Impacts

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Abstract. *In many applications the use of energy harvesters are suitable for generating electrical output from vibration input. However, in some practical situations the harvesters are subjected to high amplitudes of oscillations which may induce shocks in them, shortening their* *life service. In this way, bump stops can be introduced to limit the energy harvesters amplitude of vibration to prevent mechanical failure from stresses developed from shocks. In this paper, a cantilever beam energy harvester model including a bump stop is presented. The contact of the beam with the stop is modeled using the Hertz contact model, which is a nonlinear impact model. The responses obtained from the system are compared and in agreement with experimental results found in the literature.*

Keywords: *Energy harvester, vibro-impact, Hertz contact model, bump stop, nonlinear model.*

1 INTRODUCTION

The increase in the number of portable and wireless devices has raised the attention towards the energy harvesting field. Since some current devices have to carry some sort of power source which in turn can diminish their operational lifetime, as well as increase the total weight of the equipment needed, the use of a power energy harvester can be a good alternative. The concept of power harvesting works towards developing self-powered devices that do not require replaceable power supplies (Anton et al., 2007). Regarding the vibrating energy harvesting, piezoelectric transducers are more often used, which can be monomorph, bimorph, stack or membrane (Shenck et al., 2001), (Feenstra et al., 2008), (Gao, et al., 2007), (Wang et al., 2006), (Mancosu et al., 2008), (Keck, 2007) and (Roundy, 2003).

In many practical situations the cantilever beam configuration is used as the vibration energy harvester due to its simplicity and ease of fabrication (Mak et al., 2011). For the power generated by the harvester to be optimum, the natural frequency of the beam has to match to that of the excitation source, which can be achieved by placing a mass at the free end of the beam. For such reason, the energy harvesters might be subjected to high amplitudes of oscillations which in turn can induce impacts and high mechanical stresses on them. The most practical overcome is the reduction of the harvester operation lifetime due to reduced fatigue life. In that sense, bump stops are sometimes added to the system to prevent high amplitude oscillation and reducing the mechanical stresses acting on the beam. Also, Tsai and Wu (1996), Lo (1980) and Fathi et al. (1994) treat beam models impacting bump stops, and others vibro-impact models can be seen in Navarro et al. (2014), Moraes et al. (2013) and De Souza et. al (2008).

In this work, a single-degree of freedom model of a cantilever energy harvester beam impacting a bump stop is presented. The impact is modeled using the Hertz contact theory. For the evaluation of the model, the responses are compared to results taken from the literature. In addition, the influence of the bump stop in the power and voltage output are studied for the proposed system.

2 VIBRO-IMPACT MODEL

The system studied in this work consist in a cantilever beam with a mass made of copper at its free end and a steel bump stop, as depicted in Fig. 1. The whole system is mounted in a single structure which is excited by an external acceleration. As shown in the figure, the beam considered is in a bimorph configuration, where the two piezoelectric layers are separated by a substrate which is commonly made of copper. For the sake of simplicity, the beam will considered to be made entirely of piezoelectric material, i.e., the stiffness of the substrate will not be considered in the model.

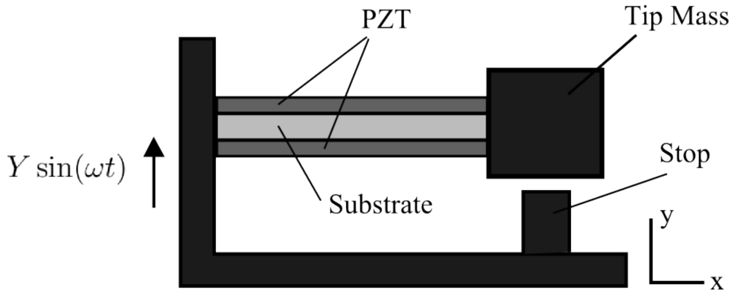

Figure 1: Cantilever Energy Harvester Studied

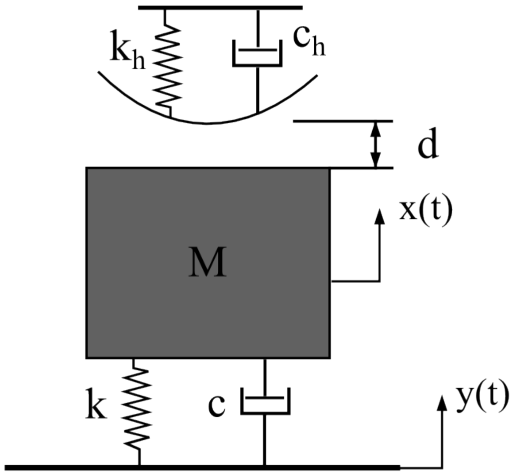

Figure 2: Single-degree of freedom model

Figure 2 presents the single-domain of freedom model used to represent the system, where its equation of motion is given by the following system of equations,

$$\begin{aligned} m\ddot{x}(t) + c\dot{x}(t) + kx(t) &= mYsin(\omega t); \quad \text{for } x < d \\ m\ddot{x}(t) + c\dot{x}(t) + kx(t) + F_c(x,\dot{x}) &= mYsin(\omega t); \quad \text{for } x \geq d \end{aligned} \tag{1}$$

where m is the mass of the system, t the time, x the displacement, c the system damping coefficient, k the system stiffness, F_c the contact force, $y(t) = Y/\omega^2 \sin\omega t$ the excitation function, Y the excitation amplitude, ω the excitation frequency, d is the gap distance and the dots represents time derivatives. The contact force F_c according to the damped Hertz contact model is given as follows (Půsst et al., 2003),

$$F_c(x,\dot{x}) = k_h x(t)^{3/2}(1 + c_h\dot{x}(t)) \tag{2}$$

where the Hertz stiffness and damping coefficient, k_h and c_h, depend on the colliding bodies mechanical properties and their relative velocities after and before the impact, respectively. By using Eq. (2), the equation of motion, Equation (1), can be written as,

$$\ddot{x}(t) + c\dot{x}(t) + kx(t) + H(x-d)k_h x(t)^{3/2}(1 + c_h\dot{x}(t)) = mYsin(\omega t) \tag{3}$$

where $H(x-d)$ is the Heaviside function. The Hertz stiffness can be obtained for two isotropic spheres of radii R_1 and R_2 by the following (Goldsmith, 1978),

$$k_h = \frac{4}{3\pi(h_1 + h_2)}\left[\frac{R_1R_2}{R_1 + R_2}\right] \tag{4}$$

being,

$$h_i = \frac{1+\nu_i}{\pi E_i}; \quad i = 1,2 \tag{5}$$

where ν is the Poisson ratio and E the modulus of elasticity of the bodies. Since the collision will be of two non-spherical bodies, an equivalent sphere radius has to be estimated, which can be done using (Muthukumar et al.,2006),

$$R_i = \left(\frac{3m_i}{4\pi\rho_i}\right)^{1/3} \quad i = 1,2 \tag{6}$$

As for the Hertz damping coefficient c_h, a model was proposed by Nikravesh et al. (1990), which has the form,

$$c_h = \frac{3k_h(1-e^2)}{4v_1} \tag{7}$$

where e is the coefficient of restitution and v_1 the velocity before impact.

3 ENERGY HARVESTER MODEL

To obtain the electromechanical model, the Kirchhoff's loop law is applied for the electrical circuit of the system. The coupled differential equations are given by (Dutoit et al., 2005),

$$\begin{aligned} m\ddot{x}(t) + c\dot{x}(t) + kx(t) + H(x-d)k_h x(t)^{3/2}(1 + c_h\dot{x}(t)) - kd_{33}V(t) = mY\sin(\omega t) \\ R_{eq}C_p\dot{V}(t) + V(t) + mR_{eq}d_{33}\omega_n^2\dot{x}(t) = 0 \end{aligned} \tag{8}$$

where d_{33} is the piezoelectric constant, R_{eq} the system electric resistance, C_p the system capacitance, V the voltage and ω_n the system natural frequency. It is worth noting that this single-degree of freedom model is only valid when the harvester have a linear dependence between the electrical damping and the system velocity, but it's still useful in the study of the power output, as discussed by Dutoit et al. (2005). In addition, a parameter that will be calculated in the simulations is the power dissipated in the electrical circuit, which is given by,

$$P = \frac{V^2}{R_{eq}} \tag{9}$$

4 RESULTS AND DISCUSSIONS

The responses of the model presented in the previous section will be compared with the one proposed by Mak et al. (2011), which is an infinite degree of freedom model. For such, the parameters used in the simulation were taken from that work and are listed in Table 1. It's worth noting that in the model presented by Mak et al. (2011), the stop was placed at some distance from the tip mass, thus the impact bodies are the cantilever beam and the stop. In the S-DOF model presented here, however, the impacting bodies are the tip mass and the stop, being the former made of copper. This fact alters the Hertz impact parameters, k_h and c_h, as they are function of the impacting bodies mechanical properties.

Table 1: Parameters used in the simulations

Parameter	Value
Natural Frequency (f_n)[Hz]	150
Frequency of Excitation (f)[Hz]	140
Excitation Amplitude (Y)[m/s^2]	1
Gap distance (d)[μm]	1
System's mass (m)[g]	2.5
Piezoelectric Constant (d_{33})	190×10^{-12}
Electrical Resistance (R_{eq})[kΩ]	10
Damping Ration (ζ)	0.01

In order to obtain the responses of the system, first the Hertz stiffness and damping coefficient, k_h and c_h, were calculated, using Eqs. (4) and (7), respectively, giving the following results,

$$k_h = 9.03 \times 10^8 \text{ N/m}^{3/2} \quad \text{and} \quad c_h = 1.07 \times 10^{11} \text{ (m/s)}^{-1}$$

The mechanical properties of the copper and steel needed to obtain k_h were taken from Boyer and Gall (1985). As for the calculation of c_h, the coefficient of restitution (e) was taken as 0.85 and the velocity before the impact as 1.76×10^{-3} m/s. These values were obtained through the simulations.

Figure 3 show the results obtained for the S-DOF model, which can be compared with the ones from Mak et al. (2011). In the displacement graph, Figure 3a, the stop is located at the negative part of the y label, thus under this point the mass is in contact with the stop. In the model presented by Mak et al. (2011), the five first modes of vibration were considered for the beam and the stop, the latter which was modeled as a longitudinal vibrating rod, and it's seen from the displacement of the beam model that higher modes of vibration influence the overall response of the system. The displacement obtained for the S-DOF model, Figure 3a, showed a two period oscillation, and a very similar behavior compared with the beam model. The same can be said to the output voltage graph, Figure 3b, where the S-DOF responses showed similar characteristics as the beam results. The maximum voltage, however, where different comparing both models, as the maximum value for the S-DOF was 0.08 V while the beam model presented a value of 0.05 V.

The power dissipated in the electrical system obtained for the S-DOF model is shown in Fig. 3c. As the power is function of the voltage squared, the differences seen in the voltage amplitude were increased in the expected power dissipated, and in numbers the maximum power of the S-DOF and the beam model were, 64 and 24 μW, respectively. Despite of this, the behavior of the power dissipated of both models were still close related. The contact force obtained for the S-DOF is presented in Fig. 3d. As discussed in the first paragraph of this section, different materials are impacting in both models. For this reason, the average contact force were also different from both models, as one can note comparing Fig. 3d with the results from Mak et al. (2011). In addition, the contact force of the beam model presented by Mak et al. (2011) is seen to vary its amplitude because of the stop longitudinal vibration. As in the S-DOF model no mass is considered for the stop, the force stays almost the same during the impacts.

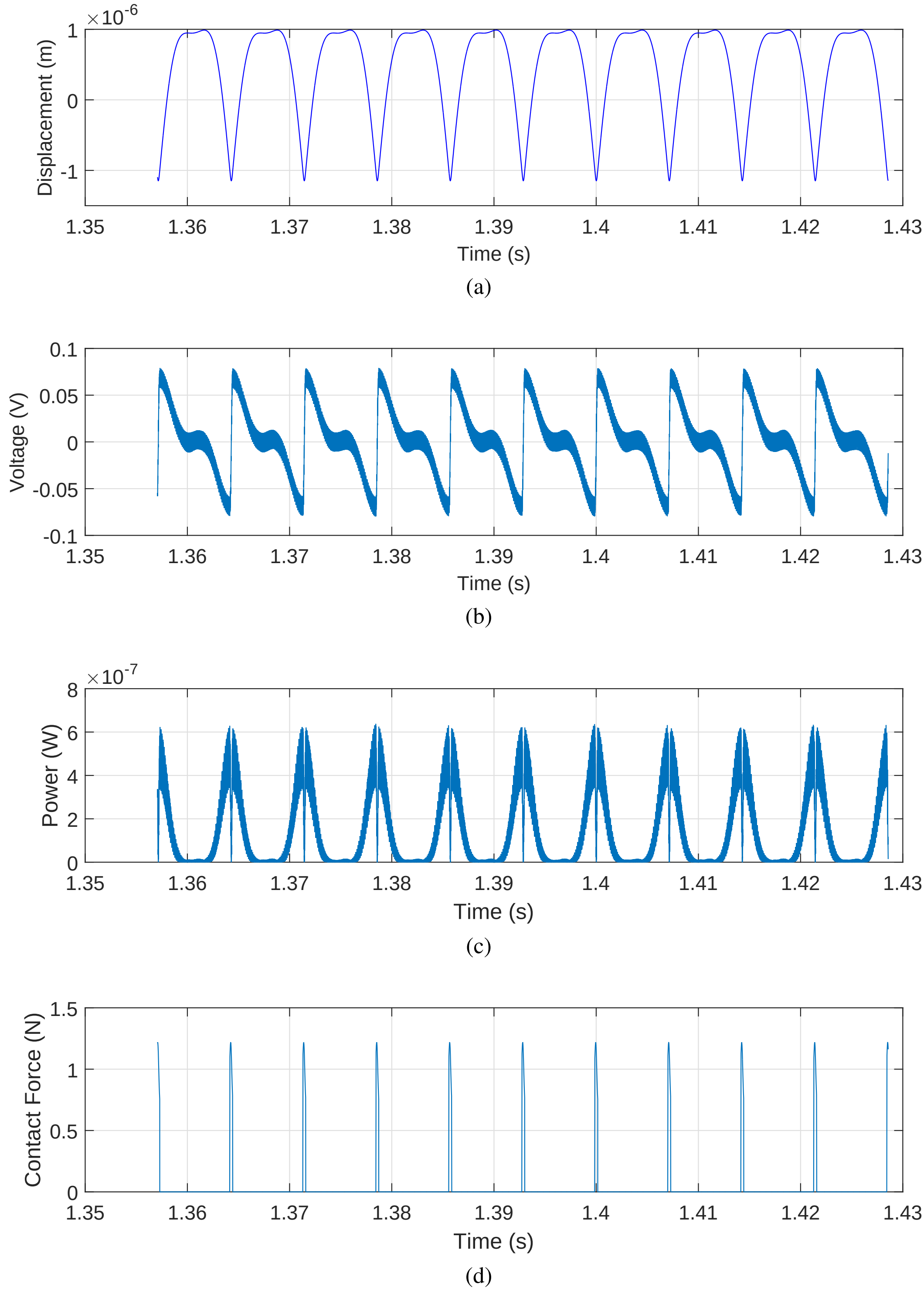

Figure 3: Results obtained for the S-DOF model: (a) Displacement, (b) Voltage, (c) Dissipated Power and (d) Contact Force.

5 CONCLUSION

In this paper, a single-degree of freedom model was presented with the purpose of modeling the vibro-impact phenomenon of a cantilever beam energy harvester impacting a bump stop. The evaluation of the model proposed was done by comparing its responses with an infinite degree of freedom beam model from the literature, which in turn was evaluated by an experimental procedure. The model presented in this work has the advantage of being simpler than an continuous beam model, and the results obtained were very close to that more complex model. In addition, the S-DOF model can be improved by considering the vibration of the stop, as it was done by Mak et al. (2011).

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