FATIGUE ANALYSIS OF VISCOELASTICALLY DAMPED STRUCTURES SUBJECTED TO UNCERTAINTIES

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Abstract. *In the scope of structural dynamics, in most of the cases, engineering systems are evaluated by considering mean values of their physical and geometrical properties. However, since real-word engineering systems are subjected to uncertainties, deterministic approaches may not represent with reasonable accuracy the real dynamic behavior of the structures such as in fatigue failure induced by mechanical vibrations. As a result, it can lead to a catastrophic failure, depending on the application. Thus, to provide a more realistic study of this subject, structural characterization requires the application of statistical methods, considering fluctuation of design variables such as elastic modulus, thickness and temperature. After the presentation of the theoretical foundations, numerical studies are performed with a sandwich thin plate system incorporating viscoelastic damping. The interest is to investigate the influence of the uncertainties and to perform a sensitivity analysis for the three-layer sandwich plate structure treated with passive constrained damping layer under Gaussian random loadings. In this case, the influence of the parametric uncertainties on the stress responses and the fatigue index are performed by means of the well-known Latin Hypercube Sampling (LHS). Numerical results are presented in terms of frequency responses functions (FRFs), stress responses (PSDs) and fatigue indexes estimated by Sines' criterion. Based on the obtained results, it is possible to highlight the importance of considering the effects of uncertainties in the fatigue analysis of more realistic situations.*

Keywords: *Non-parametric uncertainties, viscoelastic damping, fatigue, finite elements method, dynamics.*

1 INTRODUCTION

Viscoelastic materials have been a convenient strategy in surface treatments devices for several engineering systems, due to versatility and mechanical strength. However, its behavior when subjected to mechanical loads is still in characterization, especially when subjected to different excitation frequencies and temperature (de LIMA et al., 2004).

The present work is aimed at evaluating the influence of some parameters to investigate their impact in the responses of the system in study. This kind of research is normally applied in deterministic finite element models, considering properties' mean values on the design of structures subjected to dynamic efforts. However, it may not have a reasonable accuracy into the mechanical behavior characterization. In these cases, it is important to consider uncertainties generated by measurement errors and fluctuation on geometric and mechanical properties. Therefore, by using a finite element method considering non-parametric uncertainties, answers that are better representative of real-world applications are provided.

After a theoretical foundation concerning FEM and sensitivity analysis, numerical applications are presented to demonstrate the influence of various parameters for a three-layer sandwich plate incorporating viscoelastic materials to insure the reliability and their use in real projects.

2 FINITE ELEMENTS FORMULATION FOR FATIGUE ANALYSIS

2.1 Three-Layer Sandwich Plate Finite Element

This section is devoted to present the formulation of a moderately thin three-layer sandwich plate finite element, as shown in Fig. 1. The numerical implementation is introduced herein by the development made by Khatua and Cheung (1973) and applied by de Lima et al. (2010).

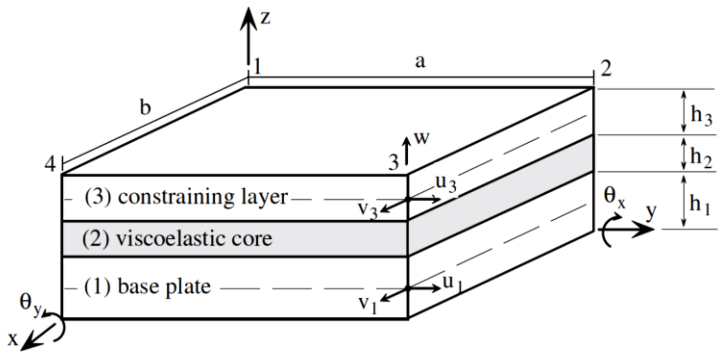


Figure 1. Three-layer sandwich plate (Guedri et al., 2010)

The plate dimensions are $0.30m \times 0.20m$, and it is composed by an elastic base-plate, a viscoelastic core passive damping and an elastic constraining layer. The FEM discretization considered in this model consists in a 14x10 plane rectangular finite elements mesh with four nodes per element and seven degrees of freedom per node, which are five in-plane displacements in directions x, y, z (denoted by u_1, v_1, u_3, v_3, w) and two cross-section rotations (denoted by θ_x, θ_y).

In the development, it is assumed that all materials have homogeneous, isotropic and present linear mechanical behavior. Kirchhoff's theory (Zienkiewicz and Taylor, 2005) was used in the modeling of the elastic base-plate and constraining layer. The viscoelastic core is modeled according Mindlin's theory. In addition, cross-section rotations are assumed to be the same for the elastic layers and transverse displacement is the same for all layers.

The plate's mean mechanical properties are given on Tab. 1 for each layer. In this work, the viscoelastic core is composed of the material 3M ISD112^{TM}.

Table 1. Mechanical properties of the sandwich plate

Layer	**Thickness** (mm)	**Young's Module** (GPa)	**Poisson ratio**	**Mass density** (kg/m³)
Base plate	1.0	69.4	0.29	2700
Viscoelastic core	0.0254	-	0.49	950
Constraining layer	0.5	69.4	0.29	2700

The viscoelastic sandwich system's movement ordinary differential equation is shown on Eq. (1):

$$\left[\mathbf{K}_e + \mathbf{K}_v(\omega,T) - \omega^2\mathbf{M}\right]\mathbf{U}(\omega,T) = \mathbf{F}(\omega) \tag{1}$$

The solution of the eigenproblem related to the homogeneous equation presented in Eq. (1) provides the system's natural frequencies and vibration mode shapes. One may note that this analysis, performed in viscoelastically damped structures, requeires the viscoelastic stiffness matrix, which depends on temperature and excitation frequency. Despite this, initially, the dynamic behavior of viscoelastic structures in the frequency domain is represented assuming poisson ratio as being independent on the temperature and frequency. Thus, longitudinal modulus $E(\omega,T)$ becomes proportional to the shear modulus $G(\omega,T)$ through the relation $G(\omega,T) = E(\omega,T)/2[1+v]$. And from this, there is an inclusion of frequency-dependence of viscoelastic material by the elastic-viscoelastic correspondence principle (Christensen,1982), in which the module is factored-out from the stiffness matrix and then the complex module approach is given by the relation:

$$G(\omega,T) = G'(\omega,T)\left[1 + i\eta_G(\omega,T)\right] \tag{2}$$

where $G'(\omega,T)$, $\eta_G = G''(\omega,T)/G'(\omega,T)$ and $G''(\omega,T)$ are the storage modulus, loss factor and loss modulus, respectively.

The Frequency-Temperature Superposition Principle (Nashif et al., 1985; Cazenove et al., 2012) establishes a relation between the effects of excitation frequency and temperature on the properties of viscoelastic materials. According to this principle, changes in the viscoelastic characteristics at different temperatures can be represented by changes in the actual values of the excitation frequency. Symbolically, this principle can be expressed as follows:

$$G(\omega,T)=G(\omega_r,T_0)=G(\alpha_T\omega,T) \tag{3.a}$$

$$\eta(\omega,T)=\eta(\omega_r,T_0)=\eta(\alpha_T\omega,T) \tag{3.b}$$

where ω_r is the reduced frequency, T_0 represents the reference temperature e α_T is the shift factor.

In general, the strain-displacement relation and displacement relations are given classically by Eqs. (4.a) and (4.b), respectively:

$$\boldsymbol{\varepsilon}(x,y,z,t)=\mathbf{B}(x,y,z)\mathbf{u}_{(e)}(t) \tag{4.a}$$

$$\mathbf{u}(x,y,t)=\mathbf{N}(x,y)\mathbf{u}_{(e)}(t) \tag{4.b}$$

where $\mathbf{N}(x,y)$ is shape function and $\mathbf{u}_{(e)}(t)=\left[u_1^i \;\; v_1^i \;\; u_3^i \;\; v_3^i \;\; w^i \;\; \theta_x^i \;\; \theta_y^i\right]^T$ is the nodal displacement data vector.

Longitudinal displacements of the elastic layers are given by means of a discretization using linear interpolation functions for the displacements in directions *x* and *y* in the middle plane of the elastic layers and using a third-order polynomial function for the transverse displacement:

$$\mathbf{u}_1=\mathbf{u}_1(x,y)=a_1+a_2x+a_3y+a_4xy \tag{5.a}$$

$$\mathbf{v}_1=\mathbf{v}_1(x,y)=a_5+a_6x+a_7y+a_8xy \tag{5.b}$$

$$\mathbf{u}_3=\mathbf{u}_3(x,y)=a_9+a_{10}x+a_{11}y+a_{12}xy \tag{5.c}$$

$$\mathbf{v}_3=\mathbf{v}_3(x,y)=a_{13}+a_{14}x+a_{15}y+a_{16}xy \tag{5.d}$$

$$\mathbf{w}=\mathbf{w}(x,y)=b_1+b_2x+b_3y+b_4x^2+b_5xy+b_6y^2+b_7x^3+b_8x^2y+b_9xy^2+b_{10}y^3+b_{11}x^3y+b_{12}xy^3 \tag{5.e}$$

The solution of Eq. (1) requires the preliminary determination of elementary mass and stiffness matrices for elastics and viscoelastic layers as shown on Eqs. (6).

$$\mathbf{M}^{(e)}=\sum_{k=1}^{3}\rho^{(k)}h^{(k)}\int_{x=0}^{a}\int_{y=0}^{b}\mathbf{N}^T(x,y)\mathbf{N}(x,y)dydx \tag{6.a}$$

$$\mathbf{K}_e^{(e)}=\sum_{k=1}^{3}\int_{z=0}^{h^{(k)}}\int_{x=0}^{a}\int_{y=0}^{b}\mathbf{B}_k^T(x,y,z)\mathbf{C}_k\mathbf{B}_k(x,y,z)dydxdz \tag{6.b}$$

$$\mathbf{K}_v^{(e)}(\omega,T)=\int_{z=0}^{h^{(2)}}\int_{x=0}^{a}\int_{y=0}^{b}\mathbf{B}_2^T(x,y,z)\mathbf{C}_2(\omega,T)\mathbf{B}_2(x,y,z)dydxdz \tag{6.c}$$

where $\mathbf{C}_k$ e $\mathbf{C}_2$ are the matrices of elastic and viscoelastic coefficients, respectively, as shown in Eqs (7).

$$C_k = \begin{bmatrix} \frac{E^{(k)}}{1-\nu^{(k)^2}} & \frac{E^{(k)}\nu^{(k)}}{1-\nu^{(k)^2}} & 0 \\ \frac{E^{(k)}\nu^{(k)}}{1-\nu^{(k)^2}} & \frac{E^{(k)}}{1-\nu^{(k)^2}} & 0 \\ 0 & 0 & G^{(k)} \end{bmatrix} \tag{7.a}$$

$$C_2 = \begin{bmatrix} \frac{E^{(2)}}{1-\nu^{(2)^2}} & \frac{E^{(2)}\nu^{(2)}}{1-\nu^{(2)^2}} & 0 & 0 & 0 \\ \frac{E^{(2)}\nu^{(2)}}{1-\nu^{(2)^2}} & \frac{E^{(2)}}{1-\nu^{(2)^2}} & 0 & 0 & 0 \\ 0 & 0 & G^{(k)} & 0 & 0 \\ 0 & 0 & 0 & G^{(k)} & 0 \\ 0 & 0 & 0 & 0 & G^{(k)} \end{bmatrix} \tag{7.b}$$

Global mass and stiffness matrices can be lately assembled by the system's connectivity, considering the behavior purely elastic and representing the initial strain state. After obtaining these matrices, the modulus is factored-out from the complex stiffness matrix of the viscoelastic layer $K_v(\omega,T) = G(\omega,T)\overline{K}_v$. The frequency response function of a sandwich structure is given by:

$$H(\omega,T) = \left[K_e + G(\omega,T)\overline{K_v} - \omega^2 M\right]^{-1} \tag{8}$$

where $K_e = K_1 + K_3$ and $K_v(\omega,T) = K_2(\omega,T) = G(\omega,T)\overline{K_2}$ correspond to the contributions of global elastic and viscoelastic layers matrices, respectively.

The complex modulus and shift factor of the viscoelastic material 3M ISD112^{TM} are determined by following equations (Drake and Soovere, 1984):

$$G(\omega_r) = 430700 + \frac{1200\times10^6}{1+3.241\times(i\omega_r/154300)^{-0.18} + (i\omega_r/154300)^{-0.6847}} \tag{9.a}$$

$$\log(\alpha_T) = -3758.4\times\left(\frac{1}{T}-\frac{1}{290}\right) + 225.06\times\log\left(\frac{T}{290}\right) + 0.23273\times(T-290) \tag{9.b}$$

The frequency-domain stress response of the viscoelastic system can be obtained considering stress and strain relations (de Lima et al., 2014) for each layer as shown on Eq. (9).

$$\mathbf{s}(x,y,z,t) = \mathbf{C}\boldsymbol{\varepsilon}(x,y,z) = \mathbf{CB}(x,y,z)\mathbf{u}_{(e)}(t) \tag{10}$$

2.2 Sines' Fatigue Criterion

Fatigue is caused by generation and propagation of cracks and is accelerated by effects of environment and frequency of cycling. Dowling (2007) pointed out that this phenomenon's biggest problem is that it occurs generally on a very dangerous, sudden and catastrophic way, without previous notice. Lambert (2007) pointed out that fatigue cracks usually appear on critical points where the stress levels are higher but remarked that working on time domain analyzing stress historic is possible but impracticable due to high time spent on data acquisition and post treatment. Therefore, the viable way is to obtain data from frequency domain, specifically by statistical analysis of the structure's stress spectral power density (stress PSD).

Since most of fatigue criteria require material properties only obtained by experiments, they are highly subjected to uncertainties and measurement errors. Therefore, Weber (1999) studied several fatigue criteria and their formulations, concluding that Sines' criterion is the one that has better accuracy and has a simple formulation, requiring only two fatigue-related properties. Sines (1959) proposed a stress criterion for multiaxial fatigue that consists on the calculation of a factor D_{Sines} that is function of the square root of the stress deviatoric tensor second invariant $\left(\sqrt{J_{2,a}}\right)$. Values greater than one indicate fatigue failure. Thus, the problem becomes estimating $\sqrt{J_{2,a}}$. Sines' fatigue criterion for random loading is formulated as follows:

$$E[D_{Sines}] = \frac{E[\sqrt{J_{2,a}}] + \alpha E\left[p(t)\right]}{t_{\_1}} \tag{11}$$

where *p(t)* is the hydrostatic stress, $\alpha = \frac{3t_{\_1}(R_m + f_{\_1})}{f_{\_1}R_m} - \sqrt{6}$, $t_{\_1}$ is the alternate torsional fatigue resistance, $f_{\_1}$ is the alternate traction fatigue resistance and R_m is the ultimate material's strength.

Li and de Freitas (2002) proposed an estimation method for this variable using the loading path circumscribed in an ellipse. This method is suitable for applications with uniaxial and in-phase loading. For multiaxial and out of phase loads, it is better to use Lambert et al. (2010) approach. They expanded the ellipse method for a five-dimension prismatic hull circumscribing the loading path of the second invariant. The following equation presents its estimation by this method:

$$E\left[\sqrt{J_{2,a}}\right] = \sqrt{E\left[R_1\right]^2 + E\left[R_2\right]^2 + E\left[R_3\right]^2 + E\left[R_4\right]^2 + E\left[R_5\right]^2} \tag{12}$$

Each semi-axe R_i of the prismatic hull is calculated by:

$$E\left[R_i\right] \approx \lambda_{0,i}\left(\sqrt{2\ln N_p\left(\tilde{D}_i\left(t\right)\right)} + \frac{\gamma}{\sqrt{2\ln N_p\left(\tilde{D}_i\left(t\right)\right)}}\right), i = 1,\ldots,5 \tag{13}$$

$$N_p\left(\tilde{D}_i\left(t\right)\right) = \frac{T}{2\pi}\sqrt{\frac{{\lambda_{1,i}}^2}{{\lambda_{2,i}}^2}}, i = 1,\ldots,5 \tag{14}$$

where $\lambda_{n,i}$ is the n[th]-order spectral moment, $\gamma = 0.5772$ is the Euler-Mascheroni constant and $N_p\left(\tilde{D}_i(t)\right)$ is the number of maxima on the gaussian process $\tilde{D}_i(t)$.

2.3 Sensitivity Analysis by the Finite-Difference Approach

The sensitivity analysis is a widely used tool with application in the identification of which parameter affects more the responses when the structure is subjected to fluctuations, model updating and uncertainty propagation (de Lima et al., 2006). Sensitivity can be estimated by finite differences, successively computing *H*, which represents the FRFs response. They are also dependent on p_i^0, that corresponds to the mean value of the design variable and Δp_i is the variation of this parameter. The normalized sensitivity function $\left(S_{FRF}^N(\omega,T,p)\right)$ can be numerically estimated according to the following equations:

$$\frac{\partial H(\omega,T,p_i)}{\partial p_i} = \frac{H(\omega,T,p_i^0+\Delta p_i)}{\Delta pi} - \frac{H(\omega,T,p_i^0)}{\Delta pi} \tag{15.a}$$

$$S_{FRF}^N(\omega,T,p) = \left.\frac{\partial H(\omega,T,p)}{\partial p}\right|_{\omega,T,p_0} \frac{p_0}{H(\omega,T,p_0)} \tag{15.b}$$

3 NUMERICAL APPLICATION

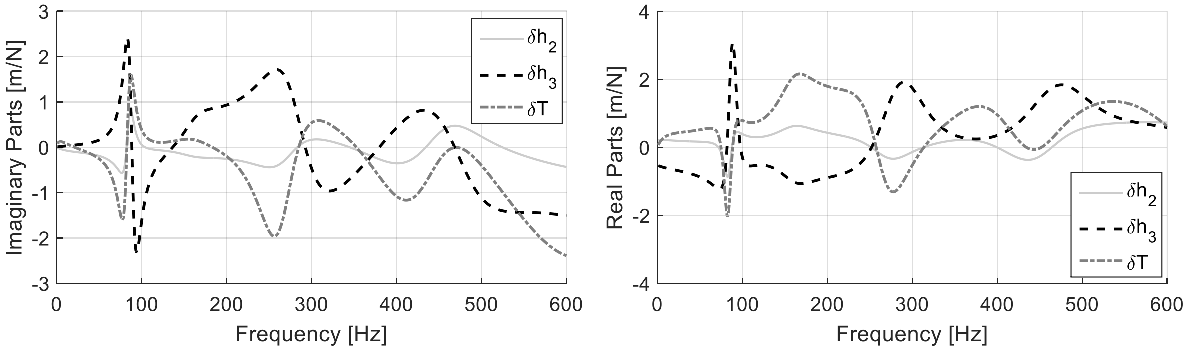
This section outlines the numerical applications and summarises responses obtained in therms of FRFs envelopes, PSDs envelopes and Sines' fatigue coefficient. A sensitivity analysis was also performed to evaluate the influence of the uncertain variables in theese responses of the viscoelastic damped sandwich structure. It was considered in the FE model a clamped-clamped-free-free rectangular plate made of aluminum, fully treated with a layer of 3M ISD112™ between base-plate and constrained-layer. The loading is applied on the central node, and data os aquired under a frequency band of [0-600 Hz].

The model of the three-layer sandwich plate had the uncertain parameter factored out of mass and stiffness matrices allowing the inclusion of fluctuations in its values. In the sequence, Monte Carlo Simulation (MCS) was used as stochastic solver, considering variation in layers thicknesses and temperature. Samples were generated by Latin Hypercube Sampling (LHS). The uncertainty levels, for each case, are shown in table 2.

Table 2. Uncertainty level of random parameters.

	Uncertain parameter	**Nominal value**	**Uncertainty level**
Case 1	h2	0.0254 mm	2%
Case 2	h3	0.5 mm	2%
Case 3	T	25 ºC	2%

In this study, the sensitivities of $H(\omega,T,p_i)$ were estimated considering 2% of variation of constraining and viscoelastic layers thicknesses and nominal temperature. Figure 2 shows the normalized real and imaginary parts of the sensitivity functions by finite differences. It is possible to observe that constraining layer thickness affect the FRF results more than temperature. The viscoelastic layer thickness is the parameter with less influence on results.



(a) **(b)**

Figure 2. Sensitivities of the FRF: (a) Imaginary parts, (b) Real parts

A convergence analysis via root-mean-square deviation (RMSD) was conducted to estimate the optimal number of samples (n_s), by the following equation:

$$RMSD(n_s) = \sqrt{\frac{1}{n_s}\sum_{i=1}^{n_s}\left|G(\omega)-\overline{G}(\omega)\right|^2} \tag{16}$$

From the previous results of sensitivity analysis, the constraining layer thickness presented more influence in FRF responses. So, it was used as the reference for applying RMSD. As seen on Fig. (3) the convergence plot at $n_s = 300$ samples a satisfactory convergence for the method is noted, concluding that this can be set as an optimal numer for LHS sampling.

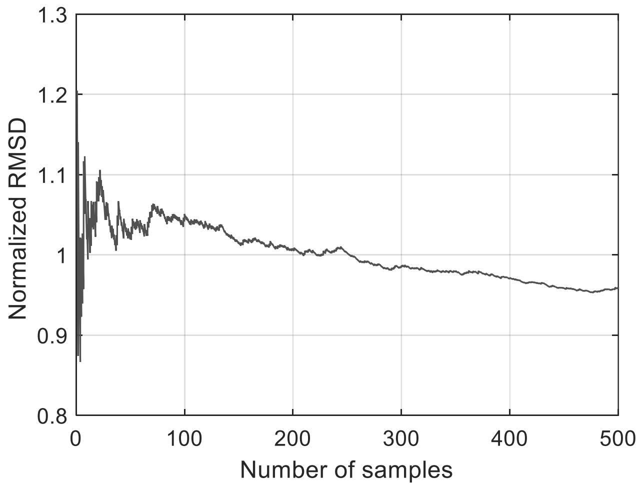
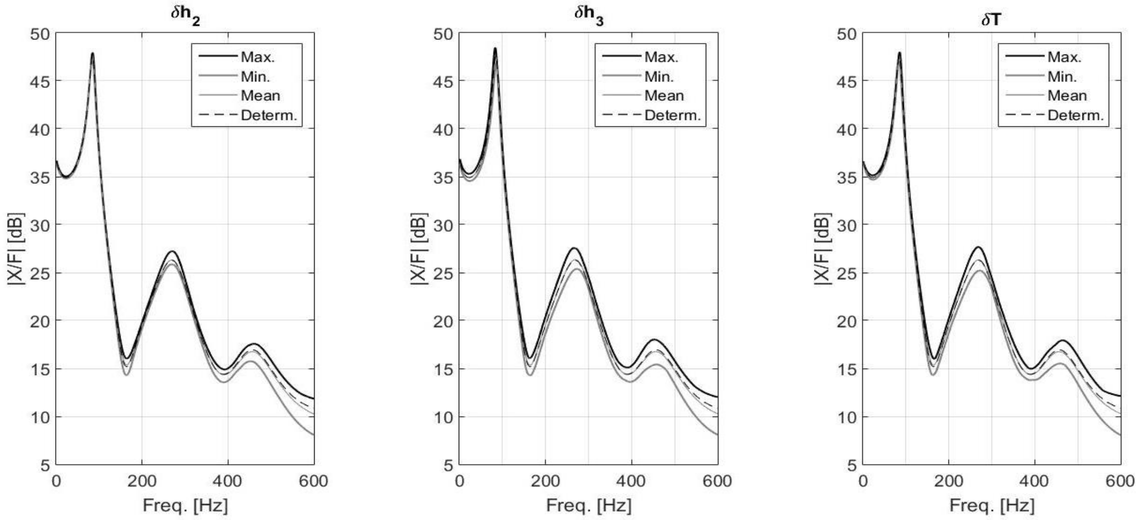


Figure 3. Convergence analysis via RMSD for case 2

The second analysis consists of solving Eq.(8) to obtain FRFs envelopes composed by its maximum, mean and minimum values for each frequency in the range, as shown in Fig.(4). These envelopes represent the relation of the displacement by a unitary impulse loading applied and measured on the central node. The mean model FRF is also plotted together with the envelopes of random results to verify theit consistency. Thus, its curve must fit inside this envelope. It is possible to conclude, that constraining layer thickness and temperature, having larger FRF envelopes, presents greater influence in the responses. Anyway, the recommended technique to identify the most influenctial parameter is the sensitivity analysis, as stated previously.



(a) **(b)** **(c)**

Figure 4. Displacement FRF envelopes for 2% uncertainty level: (a) case 1, (b) case 2, (c) case 3

In the sequence, stress responses in the frequency domain via Power Spectral Densities (PSD) were obtained by applying Eq. (10) for each one of the plate's elements. The loading is a white Gaussian noise with $\phi_f = 85 \times 10^3 Pa^2 / Hz$ applied in the central node. PSDs envelopes are shown on Fig. (5).

Finally, from PSDs responses, Sines' fatigue coefficient expectation for each element of the sandwich plate was calculated considering $t_{\_1} = 92\,MPa$ for two million cycles. For all three cases, the central elements presented the greatest values for $E\left[D_{Sines}\right]$. Table 3 summarizes the maximum values encountered for each case. Figures 6, 7 and 8 present three plate plots for the maximum, mean and minimum expected values of the coefficient for each case, pointing out that the most critical elements are those that have the highest values fatigue index. Fatigue failure is expected in elements presenting Sines' index greater than one. Through these results, one comes to the conclusion that considering uncertainties is of highly importance in the design if dynamics structures, specially when the system is subjected to loadings that may cause the fatigue phenomenon. In all three cases, even with the viscoelastic layer damping, the maximum values were higher than one despite not reaching this level when only mean parameters were considered.

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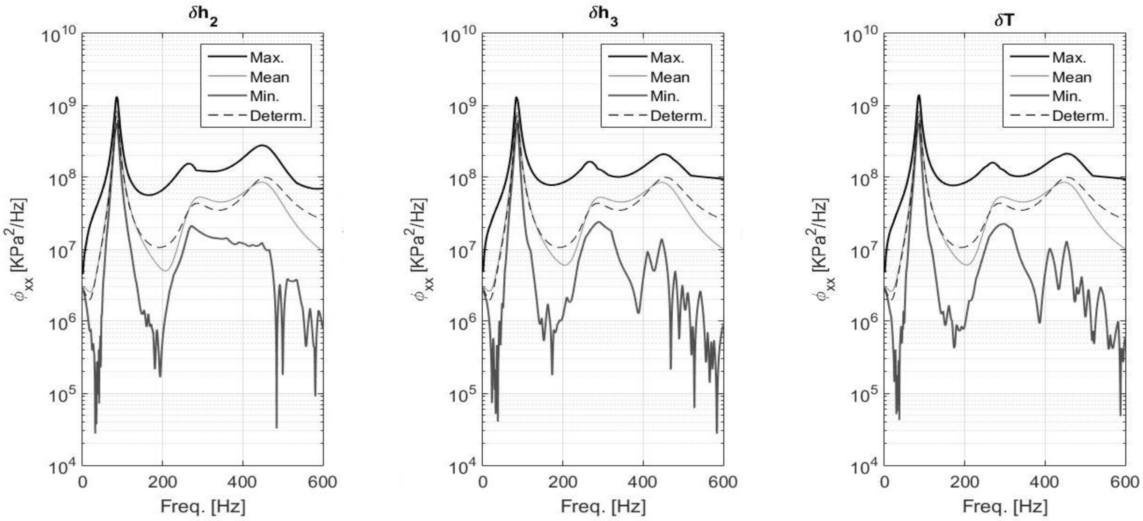
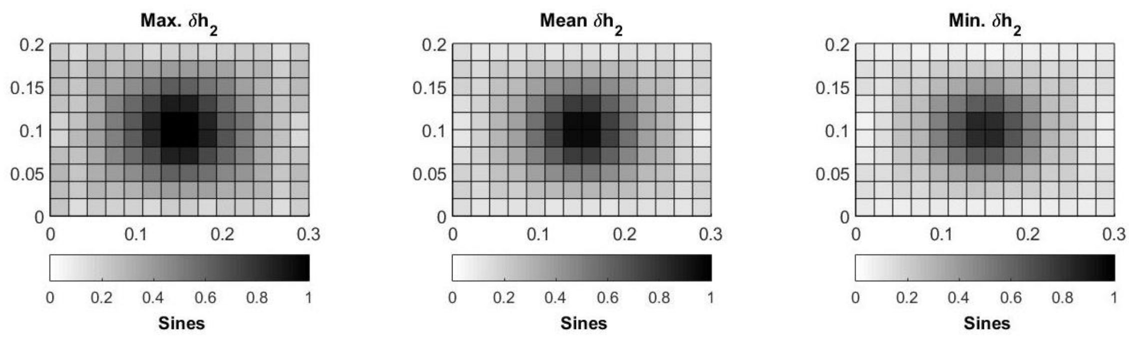


(a) **(b)** **(c)**

Figure 5. Displacement PSD envelopes for 2% uncertainty level: (a) case 1, (b) case 2, (c) case 3

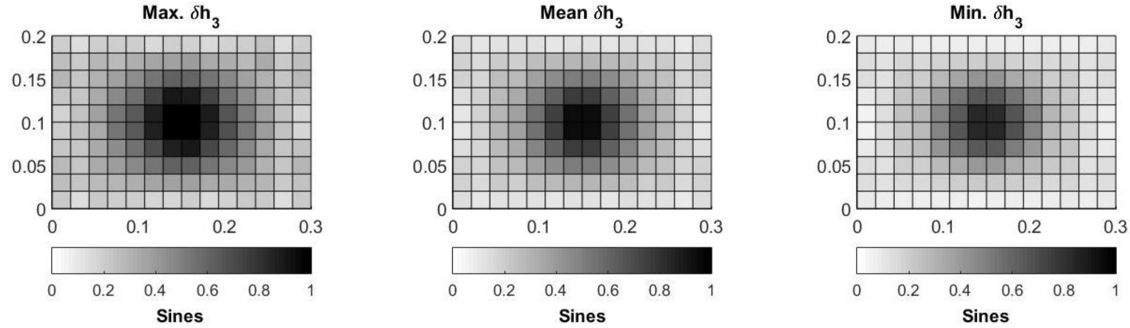
Table 3. Sine's higher coefficient values for each case and set of results.

	Case 1	**Case 2**	**Case 3**
Maximum	1.0765	1.1420	1.1276
Mean	0.9414	0.9428	0.9423
Minimum	0.8341	0.8343	0.8441



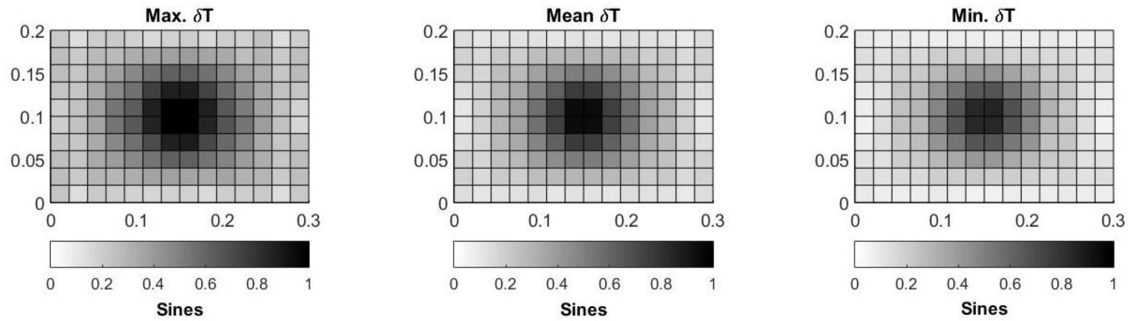
(a) **(b)** **(c)**

Figure 6. Sines coefficient distribution for case 1: (a) maximum, (b) mean, (c) minimum



(a) **(b)** **(c)**

Figure 7. Sines coefficient distribution for case 2: (a) maximum, (b) mean, (c) minimum



(a) **(b)** **(c)**

Figure 8. Sines coefficient distribution for case 3: (a) maximum, (b) mean, (c) minimum

4 CONCLUDING REMARKS

The main objective of this work was to analyze the influence of design parameters in stress response and fatigue damage estimation of viscoelastically-damped structures subjected to uncertainties. Numerical simulations showed that the deterministic modeling of these structures might not characterize with reasonable accuracy the modifications on the dynamics behavior by small changes on their design variables. Another point of discussion was the use of the finite differences to estimate parameter sensitivity. It showed greater influence of constraining layer thickness on the results of the viscoelastic sandwich plate.

Finally, Sines criterion allowed to notice exactly which elements are more susceptible to failure after a certain number of cycles and evidenced how uncertainties act in the variation of plate performance. Linked to this point it was also noted that the consideration of random effects has had great impact on the estimated system life. This reinforces the importance in performing uncertainty analysis during the design of dynamical systems because, as mentioned previously, most materials parameters are obtained from experimental tests that inherently contain measurement errors, fluctuation on geometric and mechanical properties.

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