TRI-ROTOR TILTED UAV CONTROL AND STABILITY INVESTIGATIONS BASED ON THE NONLINEAR DYNAMIC INVERSION TECHNIQUE

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Abstract. *Unmanned Aerial Vehicles(UAV's) have been used in a variety of scenarios such as vigilance, mapping and load transportation. The tri-rotor configuration requires one less motor which can lead to a reduction in weight, volume and energy consumption. The challenge is to find a control strategy such that the tri-rotor can hover at a stable position. According to the literature, the number of studies that have been carried out considering this configuration are small. The use of a tri-rotor UAV must deal with the problem identified using Locally Asymptotically Stabilization(LAS). a control law using only the rotor velocities cannot have a hover stabilization for position and attitude simultaneously via continuous state feedback. So, this paper presents the use of a tilt technology, adding one input for each rotor with a tilt angle (Tilt Rotor UAV – TRUAV)). The control will be done using Nonlinear Dynamic Inversion (NDI), with this methodology a nonlinear control law is created to produce a wanted set of dynamics for the selected controlled variables (CVs). The overall proposed flight controller performance is tested via numerical simulations on the mathematical model of the UAV. The present work was only possible thanks to the FAPEMIG, CAPES and CNPQ support.*

Keywords: *UAV, NDI, Stability, Non-linear*

1 INTRODUCTION

There is an increasing interest on autonomous Unmanned Aerial Vehicles(UAV) due to the growth of their military and civil application, the improvement of their capabilities requires contributions from different disciplines including aeronautics, electronics, signal processing, automatic control, computer science, etc.

There are multiples designs for UAV's presented in the literature, but two major classes can be highlighted: Fixed wings and Rotary Wings. As discussed in (Liu, et al., 2017) fixed-wing aerial vehicles are equipped with a set of nonmoving wings and driven forward by propellers or turbine jet engines. Therefore, they can cover greater distances during flight and have higher cruising speed with low consumption. However, takeoff demands more space and complex maneuvers, in addition to the impossibility to hover at a fixed position. Rotary wings aerial vehicles, such as helicopters can hover at one spot but have lower speed and higher consumption considering cruising conditions.

So, as an effort to combine the advantages of these two designs, it was developed small flying machines capable of performing hover as well as forward flight using tilt technology. This strategy consists of an extra degree of freedom on the rotors so they can turn and change the propulsion direction as showed in (Papachristos, et al., 2014). Another functionality for this technology is to reduce the rotor efforts, decreasing the battery consumption and improving the flight performance, which is not so explored in the literature.

Rotary wings UAV's can be sub classified by the number of rotors, such as: quadrotor, hexa-rotor, tri-rotor, etc. Considering that rotors are the components which drain the most significant parcel from the batteries, trying to reduce their number is justifiable due to the weight, energy and control complexity reduction. In the case of the tri-rotors configuration, the challenge is to find a control strategy that is able to make it perform as a classical helicopter, having a hovering stable position. According to literature, few studies have been carried out for such configuration, as presented in (Salazar-Cruz, et al., 2009).

The control strategy proposed in this paper is a modified version of that one realized by (Damato, et al., 2015). Due to different rotors configuration, the dynamic inversion is different but it is considered the same control law for the forces. NDI (Nonlinear Dynamic Inversion) is a control technique which ignore inherent dynamics and force the dynamics from a reference mode to the system as discussed in (Caverly, et al., 2016). The best benefits of NDI are the ability to compute a closed form of the problem, linearize the system equations and separate the dynamic inversion from the model dynamics. However, NDI exhibits strong sensitivity to modelling errors and external disturbances.

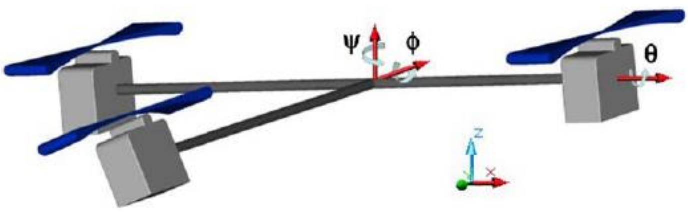

Figure 1 Tri-rotor schema. Adapted from (Salazar-Cruz, et al., 2009).

Considering the tri-rotor architecture showed in the Fig. 1, with its respective Euler angles (ϕ, θ, ψ), attitude and position cannot be defined simultaneously. So, the additional degree of freedom with the tilt using a nonlinear control is explained and analyzed using Lyapunov function, helping solving this problem and reduce the control efforts.

The following section will describe the model equations and control application, then the simulation results will be discussed. The final section presents possible studies for future research.

2 PROBLEM ANALYSIS

In this section, the nonlinear state equations for the tri rotor tilting UAV using Euler angles is derived, followed by the stability problem and the control strategy description.

2.1 UAV Dynamics

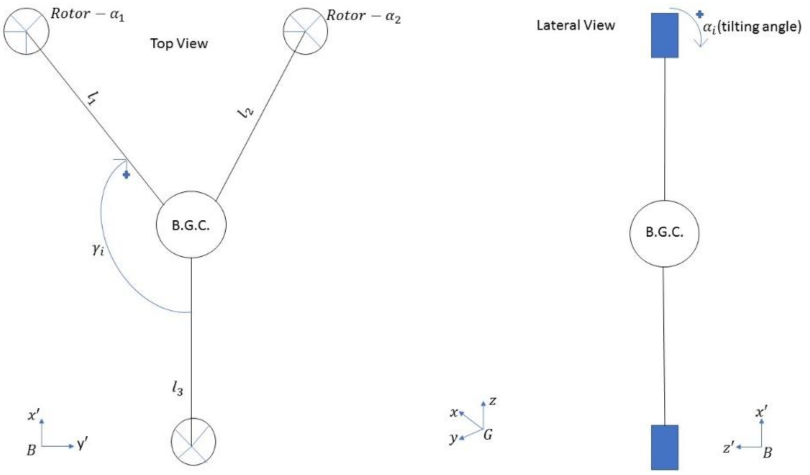

Figure 2. UAV Schema

Figure (2) shows a Y tri-rotor configuration with two frontal tilt angles (α_1, α_2) and each arm positioned in a direction γ_i with a length of l_i at the body frame (B).

The mathematical model for the UAV architecture showed in Fig (2) is based on Newton equations and uses Euler angles to represent attitude. For the tilt angle, an adaptation from (Damato, et al., 2015) is proposed.

First, the force vector applied to the model will be stablished considering the transformation matrix (R_{BE}) from the local to the global system based on the aircraft attitude $[\phi(Roll), \theta(Pitch)\ and\ \psi(Yaw)]$. Eq. (1) presents this transformation, as defined in (Tang, et al., 2017).

$$R_{BE} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \tag{1}$$

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And to transform the coordinate system of the rotor, where the tilt is applied to the local system B, the rotation presented in Eq. (2) must be applied.

$$R^i_{BM} = \begin{bmatrix} \cos(\alpha_i) & 0 & \sin(\alpha_i) \\ 0 & 1 & 0 \\ -\sin(\alpha_i) & 0 & \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} \cos(\gamma_i) & \sin(\gamma_i) & 0 \\ -\sin(\gamma_i) & \cos(\gamma_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

where, $\alpha_i = 0$ for the non-tilting rotor.

The gravitational force,F_g, is defined in Eq. 3, where m and g stands for the UAV mass and the acceleration due to the gravity force, respectively, as showed by (Flores-Colunga & Lozano-Leal, 2014):

$$F_g = R_{BE} \cdot \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = \begin{bmatrix} sin(\theta)mg \\ -sin(\phi)cos(\theta)mg \\ -cos(\phi)cos(\theta)mg \end{bmatrix} \tag{3}$$

The combined thrust of all rotor, T, presented in the local system B, is given by Eq. (4) and (5), where k_f denotes the propeller force constant and Ω_i the velocity of the rotor i:

$$T = R^1{}_{BM} \cdot \begin{bmatrix} 0 \\ 0 \\ k_f \Omega_1{}^2 \end{bmatrix} + R^2{}_{BM} \cdot \begin{bmatrix} 0 \\ 0 \\ k_f \Omega_1{}^2 \end{bmatrix} + R^3{}_{BM} \cdot \begin{bmatrix} 0 \\ 0 \\ k_f \Omega_1{}^2 \end{bmatrix} \tag{4}$$

$$T = \begin{bmatrix} \cos(\gamma_1)\sin(\alpha_1)k_f\Omega_1^2 + \cos(\gamma_2)sin(\alpha_2)k_f\Omega_2^2 \\ -\sin(\gamma_1)\sin(\alpha_1)k_f\Omega_1^2 - \sin(\gamma_2)\sin(\alpha_2)k_f\Omega_2^2 \\ \cos(\alpha_1)k_f\Omega_1^2 + cos(\alpha_2)k_f\Omega_2^2 + k_f\Omega_3^2 \end{bmatrix} \tag{5}$$

So, disregarding any drag present on the UAV, Eq. 6 presents the total force vector (F) applied to the aircraft.

$$F = F_g + T = \begin{bmatrix} sin(\theta)mg + \sin(\alpha_1)k_f\Omega_1^2 + sin(\alpha_2)k_f\Omega_2^2 \\ -sin(\phi)cos(\theta)mg \\ -cos(\phi)cos(\theta)mg + \cos(\alpha_1)k_f\Omega_1^2 + cos(\alpha_2)k_f\Omega_2^2 + k_f\Omega_3^2 \end{bmatrix} \tag{6}$$

The total moment applied to the model is obtained using the inertial matrix presented in Eq. (7).

$$J = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{7}$$

where, I_{xx}, I_{yy} and I_{zz} are the inertia at the directions x,y and z, respectively.

The moment due to the thrust, τ_{thrust}, and the drag of the helices, τ_{air}, is defined in equations 8, 9, 10 and 11, where b denotes the torque constant.

$$\tau_i = \begin{bmatrix} l_i \cos(\gamma_i) \\ l_i \sin(\gamma_i) \\ z_{cg} \end{bmatrix} R^i{}_{BM} \begin{bmatrix} 0 \\ 0 \\ k_f \Omega_i{}^2 \end{bmatrix} = \begin{bmatrix} l_i \sin(\gamma_i)\cos(\alpha_i) k_f \Omega_i^2 \\ -l_i \cos(gamma_i) cos(\alpha_i) k_f \Omega_i^2 \\ -l_i sin(\gamma_i) sin(\alpha_i) k_f \Omega_i^2 \end{bmatrix} \tag{8}$$

$$\tau_{thrust} = \begin{bmatrix} l_i \sin(\gamma_1)\cos(\alpha_1) k_f \Omega_1^2 + l_2 \sin(\gamma_2)\cos(\alpha_2) k_f \Omega_2^2 + l_3 \sin(\gamma_3) k_f \Omega_3^2 \\ -l_1 \cos(\gamma_1) cos(\alpha_1) k_f \Omega_1^2 - l_2 \cos(\gamma_2) cos(\alpha_2) k_f \Omega_2^2 - l_3 \cos(\gamma_3) k_f \Omega_3^2 \\ -l_1 sin(\gamma_1) sin(\alpha_1) k_f \Omega_1^2 - l_2 sin(\gamma_2) sin(\alpha_2) k_f \Omega_2^2 \end{bmatrix} \tag{9}$$

$$\tau_{air} = R^1{}_{BM} \cdot \begin{bmatrix} 0 \\ 0 \\ b\Omega_1^2 \end{bmatrix} + R^2{}_{BM} \cdot \begin{bmatrix} 0 \\ 0 \\ b\Omega_1^2 \end{bmatrix} + R^3{}_{BM} \cdot \begin{bmatrix} 0 \\ 0 \\ -b\Omega_1^2 \end{bmatrix} \tag{10}$$

$$\tau_{air} = \begin{bmatrix} \cos(\gamma_1)\sin(\alpha_1)\,\mathrm{b}\,\Omega_1^2 + \cos(\gamma_2) sin(\alpha_2)\,\mathrm{b}\,\Omega_2^2 \\ -\sin(\gamma_1)\sin(\alpha_1)\,\mathrm{b}\,\Omega_1^2 - \sin(\gamma_2)\sin(\alpha_2)\,\mathrm{b}\,\Omega_2^2 \\ \cos(\alpha_1)\,\mathrm{b}\,\Omega_1^2 + cos(\alpha_2)\,\mathrm{b}\,\Omega_2^2 - b\Omega_3^2 \end{bmatrix} \tag{11}$$

The total moment, M, with the components in each direction defined as L_x,M_y and N_z, is defined in Eq. (12):

$$M = \begin{bmatrix} L_x \\ M_y \\ N_z \end{bmatrix} = \tau_{thrust} + \tau_{air} \tag{12}$$

From the forces and moments in the local coordinate system, the motion equations can be obtained from the work of (Salazar-Cruz, et al., 2009):

$$m \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = - \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \times m \begin{bmatrix} u \\ v \\ w \end{bmatrix} + F \tag{13}$$

$$J \begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix} = - \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \times J \begin{bmatrix} P \\ Q \\ R \end{bmatrix} + M \tag{14}$$

where, (u, v, w) are the translational velocities and (P, Q, R) the rotational velocities in (x, y, z) respectively in the body frame, B, and they can be transformed to the global system using Eq. (15) and (16).

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\theta)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{bmatrix} \begin{bmatrix} \dot{P} \\ \dot{Q} \\ \dot{R} \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = R_{BE}{}^{-1} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} \tag{16}$$

With Eq. (15) and (16) constructed, a nonlinear control can be used on the plant using the parameters showed in Table 1 obtained from (Conroy, et al., 2014) and the dimensional parameters were obtained based on an initial project of a tri-rotor UAV that has been developed by the authors.

Table 1. Model Parameters.

Parameter	Value	Unity
m	1.2	kg
k_f	7.9e-6	Ns^2
l_1, l_2, l_3	0.2,0.2,0.15	m
$\gamma_1, \gamma_2, \gamma_3$	135, -135,0	º
I_{xx}, I_{yy}, I_{zz}	5.08e-3,5.34e-3,5.09e-3	kgm^2
b	9.46e-7	Nms^2

2.2 Attitude and stability

The idea is to determine if hovering stabilization is possible for a tri-rotor aircraft configuration using the control strategy adopted. Controlling all degrees of freedom from the model, as proven using the results presented by (Ishikawa & Sampei, 1998) and (Kataoka, et al., 2011), does not guarantee a locally asymptotically stabilizable(LAS) system, which means that it is not possible to get a hover stabilization with attitude and position.

Reproducing some conclusions from (Kataoka, et al., 2011), the following condition must be stablished: "*A system is not feedback stabilizable if the number of inputs is less than the dimension of the equilibria submanifold*" where the following theorem can be applied to the tri-rotor schema : **It is impossible to locally asymptotically stabilize the UAV model with only tri-rotor input by state feedback control**.

The equations of motion for the three-rotor design at the global coordinate system can be put in a general form using Eq. (17).

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} f(\psi)*f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2) \\ f(\psi)*f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2) \\ f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2) \\ f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2) \\ f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2) \\ f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2) \end{bmatrix} \tag{17}$$

The equilibrium manifold is found by setting the equation (17) to zero. So, observing the acceleration in x and y, the Yaw angle appears independent of the others parameters and can have any value canceling the function $f(\phi,\theta,\Omega_1,\Omega_2,\Omega_3,\alpha_1,\alpha_2)$, forcing the equilibrium manifold to be $N^b = \{x, y, z, \psi\}$. So, it is not possible to have a locally asymptotically stabilization with continuous control feedback with the input $u = \{\Omega_1, \Omega_2, \Omega_3\}$.

Thus, using the tilting angles as a new input addition (α_1, α_2) allows for the system to be LAS for the manifold N^b, considering the conclusion from (Kataoka, et al., 2011).

2.3 Non-Linear Dynamics Inversion (NDI)

Dynamic inversion is an easy way to implement control considering the idea, directly invert the efforts into the inputs using a nonlinear control law to produce the wanted set of dynamics for the selected controlled variables (CVs). After the inversion, a closed loop system is designed to make the CVs exhibit the wanted responses while satisfying the usual disturbance response requirement for the overall system. The main problem happens when the aircraft dynamics have high non linearities, which makes the inversion process very difficult.

However, one of the advantages pointed out by (Zhang & Morton, 1998), is that the NDI "…can offer greater generality for re-use across different airframes, greater flexibility for handling changing models as an airframe evolves during its design cycle, and greater power to address nonstandard flight regimes such as super maneuvers…".

NDI is a non-linear control technique that does not have a high computational cost. After the inversion is done, the aircraft can be controlled with a simplicity which depends on the control law. Besides it can give an insight of the main variables influencing the model behavior.

The use of NDI requires a diffeomorphism for the transformation from the control efforts to the CV guaranteeing the unicity of the solution. So, to achieve the dynamic inversion, the control law will, at first, be linearized around the equilibrium point on ϕ,θ, α_1 and α_2, with the inputs defined by (Damato, et al., 2015).

The equilibrium forces can be written as defined in equation (18):

$$\begin{aligned} &\cos(\psi)\ddot{x} + \sin(\psi)\ddot{y} = \sin(\theta)\cdot\cos(\phi)\cdot f_z + \sin(\phi)\cdot\sin(\theta) f_y + \cos(\theta) f_x \\ &\sin(\psi)\ddot{x} - \left(\cos(\psi)\ddot{y}\right) = -\left(\cos(\phi) f_y\right) + \sin(\phi) f_z \\ &\ddot{z} = \cos(\theta)\left(\cos(\phi) f_z + \sin(\phi) f_y\right) - \left(\sin(\theta) f_x\right) - (gm) \end{aligned} \tag{18}$$

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Considering the translational dynamics, the following control law will be applied, where k_u,k_v, k_w,k_x,k_y and k_z are the control parameters, and x_c,y_c and z_c the desired displacement:

$$\begin{aligned}
\ddot{x} &= -\left(k_u\dot{x}\right)-\left(k_u\cdot k_x\left(x-\left(x_c\right)\right)\right)\\
\ddot{y} &= -\left(k_y\dot{y}\right)-\left(k_v\cdot k_y\left(y-\left(y_c\right)\right)\right)\\
\ddot{z} &= -\left(k_z\dot{z}\right)-\left(k_w\cdot k_z\left(z-\left(z_c\right)\right)\right)
\end{aligned} \tag{19}$$

Linearizing Eq. (18) and not considering the input effort on $\theta(\theta_c = 0)$, the Eq. (20) is obtained.

$$\begin{aligned}
&\cos\left(\psi\right)\ddot{x}+\sin\left(\psi\right)\ddot{y} = f_x\\
&\sin\left(\psi\right)\ddot{x}-\left(\cos\left(\psi\right)\ddot{y}\right)=\phi_c f_z\\
&\ddot{z} = f_z-\left(gm\right)
\end{aligned} \tag{20}$$

Solving the Eq. (20) on f_x, f_z, ϕ_c and obtaining the control efforts for the momentum, Eq. (21) is derived, where k_p,k_q, k_r,k_ϕ,k_θ and k_ψ are the control parameters.

$$\begin{aligned}
L_x &= \sin\left(\theta\right)I_{xx}k_pk_\psi(\psi-\psi_c)+I_{xx}k_pk_\phi(\phi_c-\phi)-\left(QRI_{yy}\right)+QRI_{zz}-\left(k_pPI_{xx}\right)\\
M_y &= \cos\left(\theta\right)\sin\left(\phi\right)I_{yy}k_\psi k_q(\psi_c-\psi)-\cos\left(\phi\right)I_{yy}k_qk_\theta(\theta_c-\theta)-k_qQI_{yy}+PRI_{xx}-PRI_{zz}\\
N_z &= \cos\left(\phi\right)\cos\left(\theta\right)I_{zz}k_\psi k_r(\psi_c-\psi)+\sin\left(\phi\right)I_{zz}k_rk_\theta(\theta_c-\theta)-PQI_{xx}+PQI_{yy}-k_rRI_{zz}
\end{aligned} \tag{21}$$

The simplified inversion process proposed by (Damato, et al., 2015) does not suit the system due to different arms configuration and tilt rotation. So, for the present model another inversion is proposed without simplifications.

The Y configuration for the tri-rotor UAV creates a coupling effect on the tilt at the directions x and y, so the NDI must consider the force docomposition on both directions when α_i is non-zero.

Therefore, combining the efforts in Eq. (6) and (12) it is possible to obtain the CV $(\Omega_1, \Omega_2, \Omega_3, \alpha_1, \alpha_2)$ as showed in Eq. (22), (23) and (24):

$$\begin{aligned}
\Omega_1^2 = &-\frac{1}{2}\frac{\left(\sqrt{2}k_f{}^2bl_1^2+2k_f{}^2bl_3l_1+2\sqrt{2}b^3\right)\sqrt{2}f_x}{k_f{}^2l_1\left(\sqrt{2}k_f{}^2l_1^2+2k_f{}^2l_1l_3+2\sqrt{2}b^2\right)}+1/2\frac{\sqrt{2}bN_z}{\sqrt{2}k_f{}^2l_1^2+2k_f{}^2l_1l_3+2\sqrt{2}b^2}-\\
&-1/2\frac{\left(-\sqrt{2}k_f{}^3l_1^2-2k_f{}^3l_1l_3-2\sqrt{2}b^2k_f\right)\sqrt{2}L_x}{k_f{}^2l_1\left(\sqrt{2}k_f{}^2l_1^2+2k_f{}^2l_1l_3+2\sqrt{2}b^2\right)}-1/2\frac{\left(-\sqrt{2}k_f{}^3l_1^2l_3-b^2k_fl_1\right)\sqrt{2}f_z}{k_f{}^2l_1\left(\sqrt{2}k_f{}^2l_1^2+2k_f{}^2l_1l_3+2\sqrt{2}b^2\right)}+\\
&+\frac{k_fl_1M_y}{\sqrt{2}k_f{}^2l_1^2+2k_f{}^2l_1l_3+2\sqrt{2}b^2}
\end{aligned} \tag{22}$$

$$\Omega_2^2 = -1/2\frac{\left(\sqrt{2}k_f{}^2 bl_1^2 + 2k_f{}^2 bl_3 l_1 + 2\sqrt{2}b^3\right)\sqrt{2}f_x}{k_f{}^2 l_1\left(\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2\right)} +$$
$$+1/2\frac{\sqrt{2}bN_z}{\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2} - 1/2\frac{\left(-\sqrt{2}k_f{}^3 l_1^2 - 2k_f{}^3 l_1 l_3 - 2\sqrt{2}b^2 k_f\right)\sqrt{2}L_x}{k_f{}^2 l_1\left(\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2\right)} - \tag{23}$$
$$-1/2\frac{\left(-\sqrt{2}k_f{}^3 l_1^2 l_3 - b^2 k_f l_1\right)\sqrt{2}f_z}{k_f{}^2 l_1\left(\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2\right)} + \frac{k_f l_1 M_y}{\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2}$$

$$\Omega_3^2 = \frac{\left(\sqrt{2}k_f{}^2 l_1^2 + \sqrt{2}b^2\right)f_z}{k_f\left(\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2\right)} - 2\frac{k_f l_1 M_y}{\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2} - \tag{24}$$
$$\frac{\sqrt{2}bN_z}{\sqrt{2}k_f{}^2 l_1^2 + 2k_f{}^2 l_1 l_3 + 2\sqrt{2}b^2}$$

The same solution process can be done for the tilt angles equation but, due to their size and complexity, they cannot be shown here.

2.4 Global Stability

One of the first stability theory proposed on the literature for non-linear dynamic system is the Lyapunov criteria. An interesting view for stability analysis, considering the NDI control, is presented in (MORTON, et al., 1996), where the use of a Lyapunov function is established to determine if the system is globally stable or not.

The Lyapunov function ($V(X,t)$) which depends on the state variables(X), for multirotor system given as Eq. (25) and the time, t, must satisfy the conditions defined in Eq. (26) for a global stable system (Khalil, 2014):

$$X = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \psi \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ P \\ Q \\ R \end{bmatrix} \tag{25}$$

$$V(0,t_0)=0$$
$$V(\mathrm{X},\mathrm{t})>0\forall t,\mathrm{X}\neq 0 \tag{26}$$
$$\frac{dV}{dt}\leq 0\forall t$$

Considering the definitions above, possible candidates for V are the vector norm or the system energy. Thus, the goal in this article is analyze the Lyapunov function to set a group of control parameters that guarantee the stability. To achieve this objective the Lyapunov function is considered as the kinetic energy of the system, which can be divided into translational ($V_{x,y,z}$) and rotational ($V_{\phi,\theta,\psi}$) functional, and then derived in time to apply the condition in Eq. (26), obtaining Eq. (27):

$$\frac{dV_{x,y,z}}{dt}=-\dot{x}k_u k_x(x-x_c)-\dot{y}k_v k_y(y-y_c)-\dot{z}k_w k_z(z-z_c)-\dot{x}^2k_u-\dot{y}^2k_y-\dot{z}^2k_z\leq 0$$
$$\begin{aligned}\frac{dV_{\phi,\theta,\psi}}{dt}=&-\left(\psi\cos(\theta)\sin(\phi)Qk_\psi k_q\right)-\left(\psi\cos(\theta)\cos(\phi)Rk_\psi k_r\right)+\\&\cos(\theta)\cdot\sin(\phi)Qk_\psi k_q\psi_c+\cos(\theta)\cos(\phi)Rk_\psi k_r\psi_c+\psi\sin(\theta)Pk_p k_\psi-\\&\left(\sin(\theta)Pk_p k_\psi\psi_c\right)+\sin(\phi)\theta Rk_r k_\theta-\left(\theta\cos(\phi)Qk_q k_\theta\right)+\\&\phi_c Pk_p k_\phi-\left(\phi Pk_p k_\phi\right)-\left((P)^2k_p\right)-\left((Q)^2k_q\right)-\left((R)^2k_r\right)\leq 0\end{aligned} \tag{27}$$

3 RESULTS AND DISCUSSION

The inequalities on Eq. 27 are valid only if the negative terms in each inequality become the most significant ones. To achieve that, the control parameters at the position (k_x, k_y, k_z) must be small enough so the quadratic velocities, which are negatives fractions on each inequality, become more relevant. However, if those parameters are lowered to much, the output of the controller becomes underactuated and the system will present a poor response. Therefore, a set of parameters having a good balance is important to the system response.

Considering the settling time as the time spent by the system to achieve 90% of the step input and analyzing the model with different control parameters, a trial/error process followed by a few heuristics process has produced a set of pseudo-optimum parameters (not optimized), which are presented in Table 2.

Table 2. Settling times considering multiple control parameter sets.

CP	$k_x k_u, k_y k_v, k_z k_w$	k_u, k_v, k_w	$k_\phi k_p, k_\theta k_q,$	k_p, k_q, k_r	Settling Time[s]
Exp. 1	0.1	1	1.3	15	22.7
Exp. 2	0.1	0.5	1.3	15	25
Exp. 3	0.2	1	1.3	15	NC

Exp. 4	0.1	1	3	15	22.2
Exp. 5	0.1	1	3	5	NC

The control response is proportional in respect to the equilibrium manifold and their derivative, so the system will have a slow convergence implying in a low maximum velocity.

The parameters applied on the Euler angles, except for the Yaw (ψ), will not be a concern because the terms that can be positive on Eq. (27) multiply the Pitch and Roll angles. Only ψ, included on the equilibrium manifold, will have significant interference with our stability.

Analyzing the results for different parameters, is possible to observe that the dynamics of the model allows a maximum value for the velocities components on the Lyapunov function ($|-\dot{x}k_u k_x(x-x_c)|$) of 0.11, consequently, the maximum velocity that can be obtained with this control law is approximately $0.05\left[\frac{m}{s}\right]$, which is very slow considering the usual applications.

The mathematical simulations are presented below, considering a unit step on all directions and $\frac{\pi}{6}$ on ψ.

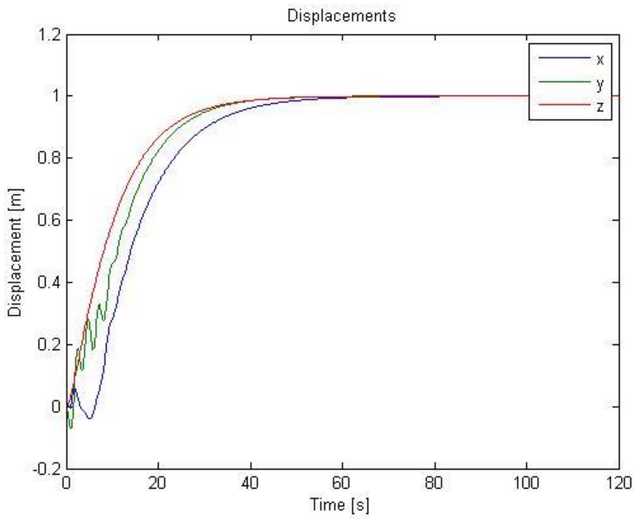
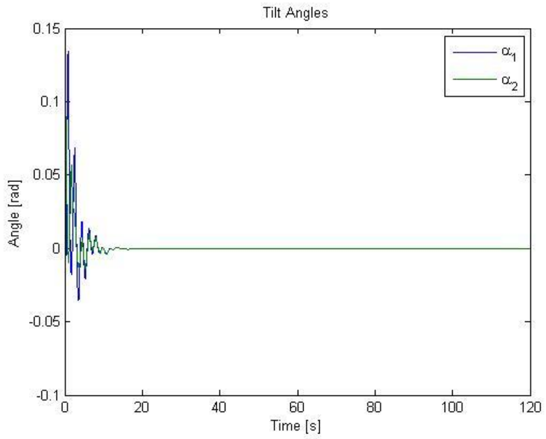
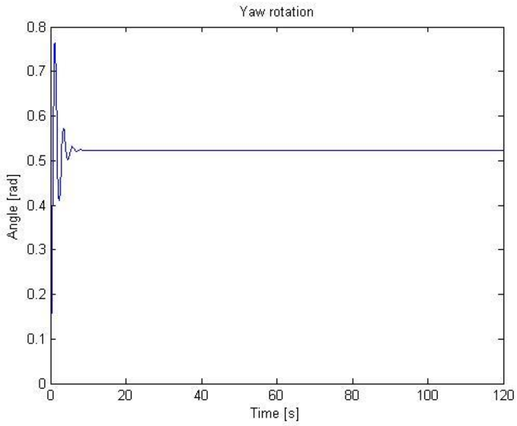


Figura 3. Displacement

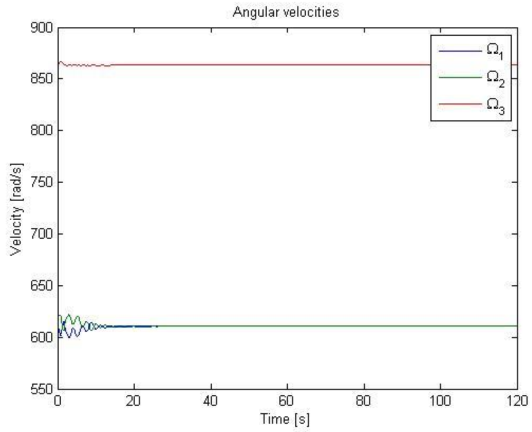
Figure 3 shows that the *x* direction has a slower convergence. This happens because this direction activates the tilt instead of changing the attitude, creating an oscillation at the y direction which need to be stabilized. The residual force caused by the tilt activation in the y direction is balanced changing the attitude, creating a delay in the x direction, that can be observed in Fig. 3, of approximately 7 seconds. The *z* direction does not have coupling effects, because the tilt angles are small, so the force variation will not appear on the results.



(a)



(b)



(c)

Figura 4. Atitude and Control efforts
a. Tilt Angles
b. Yaw Rotaion
c. Angular Velocities

Considering the Yaw displacement, it can be seen in the Fig. 4.b that it will have a faster stabilization due to the decoupling considering others degrees of freedom, it can achieve its stable point without interfering other states.

The angular velocities have an oscillation at the beginning due to the tilt activation and to set the attitude so the UAV can dislocate in y as showed in Fig. 4.c. Figure 4.a also shows that the tilt angles try to stay equal, lowering their interference at the y direction.

4 CONCLUSION

In this paper developed the dynamic model of a tri-rotor aircraft system and applied a control law based on dynamic inversion to find a set of parameters that guarantee the global stability. Considering the energy of the system as the Lyapunov function, a not yet optimized set of control parameter has been identified in order to guarantee global stability.

It was possible to verify that the NDI could be a very easy control to calculate after the inversion was obtained. However, modifications at the control law used must be done so the system become more effective considering the time response. In addition, as pointed out by (Damato, et al., 2015), the NDI control should not operate alone.

Hence, artificial intelligence, such as neural networks acting as a "teacher" with the purpose to reduce the model and disturbance sensitive, is an option and can help the construction of an adaptive system model.

Since the NDI does not handle well variations on the parameters, some of the them were obtained experimentally and may have significant error. The direct consequence is that the control parameters must be modified depending on the input magnitude.

Future research works must focus on how the control parameters can be optimized with a low computational cost. The optimization must be done in real time and by the main avionics controller in order to minimize costs and to improve performance.

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