CILAMCE 2017
Florianópolis - Santa Catarina - Brazil

TOPOLOGY OPTIMIZATION WITH STRESS CONSTRAINTS CONSIDERING UNCERTAINTY IN APPLIED LOADS

Gustavo Assis da Silva

André Teófilo Beck

gustavoassisdasilva@gmail.com

atbeck@sc.usp.br

São Carlos School of Engineering, University of São Paulo

Department of Structural Engineering, 13.566-590, São Carlos, SP, Brazil

Abstract. *This paper proposes a formulation for solving continuum Reliability-Based Topology Optimization (RBTO) problems, of volume minimization subject to local stress constraints, considering uncertainty in applied loads. The local nature of stress constraints makes this problem very challenging to solve, even in a deterministic setting, since there is one constraint for every finite element in the mesh. The consideration of uncertainties in applied loading makes the problem even more challenging to solve, because stochastic problems are naturally more non-linear than deterministic ones, and additional effort is needed for uncertainty quantification and propagation. The solution proposed in this paper avoids aggregation techniques, and imposes stress constraints by means of the augmented Lagrangian method. The paper also discusses some issues related to the density approach, namely: the singularity phenomenon and artificial stress concentration on jagged boundaries. Probabilistic stress constraints are handled via Performance Measure Approach (PMA), where a first-order approximation is considered for probability of failure. In order to alleviate the computational effort associated with solution of inverse reliability analyses at each step of outer optimization problem the principle of superposition is used. The proposed methodology leads to crisp black-and-white topologies, at a reasonable number of iterations. Through numerical examples it is verified that solutions of the reliability-based problems are different from solutions of deterministic problems, and that it is generally not possible to establish equivalences between these solutions.*

Keywords: *Reliability-based topology optimization, Stress constraints, Uncertainty in loads*

1 INTRODUCTION

Topology optimization is an important tool often used during the design of high-performance structures (Bendsøe and Sigmund, 2003), that consists in obtaining the structure with the best layout given a design domain, boundary conditions, a performance measure and design constraints. There are several approaches in the literature that have been successfully used to perform topology optimization, for instance: density, topological derivative, level set, phase field and others (Sigmund and Maute, 2013); where each approach has its own advantages and disadvantages.

In this paper, the classical density approach is applied to solve the particular problem of volume minimization subjected to local stress constraints under uncertainty in applied loads. This is a very challenging problem to solve, since the traditional difficulties associated with local stress constraints, like the local nature of the stress failure criterion (Pereira et al., 2004) and the singularity phenomenon (Duysinx and Bendsøe, 1998), are added to the difficulties associated with uncertainty quantification.

Continuum topology optimization problems with stress constraints have been studied since the work of Duysinx and Bendsøe (1998), and since then several works were developed to handle this particular problem. Recent papers employ the concepts of density (Le et al., 2010; Luo et al., 2013; Holmberg et al., 2013), topological derivative (Amstutz and Novotny, 2010; Amstutz et al., 2012), level set (Guo et al., 2011; Emmendoerfer and Fancello, 2014, 2016) and phase field (Jeong et al., 2014; Lee et al., 2016). Although several authors have been developing new formulations for handling the deterministic stress-based problem, there is a gap in the literature regarding the stochastic stress-based formulations.

For instance, to the author's knowledge, there is only one paper in the literature addressing the volume minimization problem with stress constraints under uncertainties through a reliability-based framework (Luo et al., 2014). In Luo et al. (2014), a formulation based on density approach is presented for solving continuum RBTO problems of volume minimization under uncertainty in applied loads and material properties: a PMA-based approach (Tu et al., 1999) is proposed for handling the probability requirements associated with local stress computations; local stresses are relaxed with ε-relaxed proposed in Duysinx and Sigmund (1998) associated with target reliability index relaxation proposed in Luo et al. (2014); and stress constraints are grouped with aggregation technique proposed in Luo et al. (2013) for avoiding solution of the topology optimization problem with the whole set of local stress constraints.

In this paper, an alternative is adopted to solve the RBTO problem with local stress constraints. Instead of using an aggregation technique, as presented in Luo et al. (2014), we propose the solution of the topology optimization problem with the whole set of local stress constraints with the augmented Lagrangian method (Birgin and Martínez, 2014). Probability requirements are ensured with a first-order PMA approach and all stress constraints of the optimization problem are included into the augmented Lagrangian function, such that resulting optimization subproblems are subjected to bounding constraints only, being solved with algorithm proposed in da Silva et al. (2017). Density filtering with Heaviside step function (Guest et al., 2004) is applied to avoid checkerboard like areas and to guarantee a minimum length scale in the optimal topology. Reliability-based solutions are compared with traditional solutions of the deterministic problem and post-processed with Monte Carlo Simulation (MCS) for validating proposed approach.

2 DETERMINISTIC TOPOLOGY OPTIMIZATION

In this work, the equilibrium configuration of the structural problem is obtained with a displacement-based finite element method under the hypotheses of linear elasticity subjected to static loads (Bathe, 1996), where the Solid Isotropic Material with Penalization (SIMP) approach is considered for penalizing stiffness (Bendsøe and Sigmund, 1999). Considering the von Mises yield criterion as material failure constraint, one can write the deterministic topology optimization problem in discrete form as

$$\begin{array}{lll} \underset{\boldsymbol{\rho}}{\text{Min.}} & V(\overline{\boldsymbol{\rho}}) = \sum_{e=1}^{N_e} V_e f_v\left(\overline{\rho}_e\right) & \\ \text{s. t.} & \frac{\sigma_{eq}^{(k)}(\overline{\boldsymbol{\rho}})}{\sigma_{adm}} - 1 \leqslant 0 & k = 1, 2, ..., N_k \\ & \mathbf{K}(\overline{\boldsymbol{\rho}})\mathbf{U}(\overline{\boldsymbol{\rho}}) = \mathbf{F} & \\ & 0 \leqslant \rho_e \leqslant 1 & e = 1, 2, ..., N_e \end{array}, \tag{1}$$

where $\boldsymbol{\rho} \in \mathbb{R}^{N_e}$ are the design variables of the optimization problem, $V(\overline{\boldsymbol{\rho}})$ is the objective function which depends on the physical pseudo densities $\overline{\boldsymbol{\rho}} \in \mathbb{R}^{N_e}$, N_e is the number of finite elements in the finite element mesh, V_e is the volume of element e, $\overline{\rho}_e$ is the physical pseudo density of element e, $\sigma_{eq}^{(k)}(\overline{\boldsymbol{\rho}})$ is the von Mises equivalent stress at point k, σ_{adm} is the admissible stress equal to σ_y/λ_f, σ_y is the yield stress, λ_f is the safety factor, N_k is the number of points where the von Mises equivalent stress is computed, $\mathbf{K}(\overline{\boldsymbol{\rho}})$ is the global stiffness matrix, $\mathbf{U}(\overline{\boldsymbol{\rho}})$ is the global displacement vector and $\mathbf{F}$ is the global load vector. The local stiffness matrix of element e is computed based on SIMP approach as $\mathbf{k}_e(\overline{\rho}_e) = \left(\overline{\rho}_e^p + \rho_{min}\right) \mathbf{k}_e^b$, as performed in Guest et al. (2011); Tootkaboni et al. (2012), where $\rho_{min} = 1 \times 10^{-9}$ is adopted to ensure a well-conditioned system of linear equations, $p > 1$ is a penalization factor and $\mathbf{k}_e^b$ is the stiffness matrix obtained directly from finite element analysis. In this work, function $f_v\left(\overline{\rho}_e\right)$ is chosen equal to $f_v\left(\overline{\rho}_e\right) = 1 - \mathrm{e}^{-\delta_v \overline{\rho}_e} + \overline{\rho}_e \mathrm{e}^{-\delta_v}$ (the same function proposed in Guest et al. (2004) for filtering with Heaviside step approximation). If $\delta_v = 0$, we have $f_v\left(\overline{\rho}_e\right) = \overline{\rho}_e$ and, as a consequence, $V(\overline{\boldsymbol{\rho}}) = \sum_{e=1}^{N_e} V_e \overline{\rho}_e$, which corresponds to the structural volume. For $\delta_v > 0$ we penalize pseudo densities in order to make intermediate material uneconomical to the optimizer, with the drawback of increasing non-convexity of optimization problem.

The von Mises equivalent stress at any point k can be computed as (Duysinx and Bendsøe, 1998)

$$\sigma_{eq}^{(k)}\left(\overline{\boldsymbol{\rho}}\right) = \sqrt{\boldsymbol{\sigma}_k^T\left(\overline{\boldsymbol{\rho}}\right) \mathbf{M} \boldsymbol{\sigma}_k\left(\overline{\boldsymbol{\rho}}\right)}, \tag{2}$$

where, for plane problems

$$\mathbf{M} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \tag{3}$$

and $\boldsymbol{\sigma}_k\left(\overline{\boldsymbol{\rho}}\right)$ is the stress vector, computed as $\boldsymbol{\sigma}_k\left(\overline{\boldsymbol{\rho}}\right) = \overline{\mathbf{C}}_k(\overline{\rho}_k)\mathbf{B}_k\mathbf{u}_k(\overline{\boldsymbol{\rho}})$, which depends on: the constitutive matrix of base material of the element which contains point k, $\mathbf{C}_k^b$, such that,

$\overline{\mathbf{C}}_k(\overline{\rho}_k) = f_\sigma(\overline{\rho}_k)\,\mathbf{C}_k^b$; the strain-displacement transformation matrix evaluated at point k, $\mathbf{B}_k$; and the local displacement vector of the element which contains point k, $\mathbf{u}_k(\overline{\boldsymbol{\rho}})$.

The singularity phenomenon is avoided by choosing $f_\sigma^\delta(\overline{\rho}_k) = 1 - \mathrm{e}^{-\delta_\sigma \overline{\rho}_k} + \overline{\rho}_k \mathrm{e}^{-\delta_\sigma} + \rho_{min}$, already adopted in volume penalization, with $\delta_\sigma > 0$, where the small value $\rho_{min} = 1 \times 10^{-9}$ was added in order to avoid numerical instabilities in inverse reliability analysis, as presented in recent submitted work of the authors (da Silva and Beck, 2017).

In this paper, physical pseudo densities are computed through density filtering with Heaviside step function (Guest et al., 2004), as presented in Sigmund (2007). In addition to reducing the amount of material with intermediate pseudo densities, the Heaviside projection technique avoids checkerboard-like areas and imposes a minimum length scale in the optimal solution (Guest et al., 2004).

In order to solve the deterministic optimization problem, Equation (1), an augmented Lagrangian method is used, as presented in da Silva et al. (2017).

3 RELIABILITY-BASED TOPOLOGY OPTIMIZATION

In this paper, a probabilistic RBTO formulation is used to handle uncertainty in applied loads. Considering that von Mises equivalent stress at each point of stress computation depends on uncertain parameters which can be represented through random variables, the constraints of the original problem, Equation (1), can be simply changed into probabilistic constraints, as follows

$$\begin{array}{lll} \underset{\boldsymbol{\rho}}{\text{Min.}} & V(\overline{\boldsymbol{\rho}}) = \sum_{e=1}^{N_e} V_e f_v(\overline{\rho}_e) & \\ \text{s. t.} & P\left(\frac{\sigma_{eq}^{(k)}(\overline{\boldsymbol{\rho}},\mathbf{Z})}{\sigma_y} - 1 \geqslant 0\right) \leqslant P_{adm} & k = 1,2,...,N_k \\ & \mathbf{K}(\overline{\boldsymbol{\rho}})\mathbf{U}(\overline{\boldsymbol{\rho}},\mathbf{Z}) = \mathbf{F}(\mathbf{Z}) & \\ & 0 \leqslant \rho_e \leqslant 1 & e = 1,2,...,N_e \end{array}, \tag{4}$$

where $\mathbf{Z} \in \mathbb{R}^N$ is a vector containing all random loads of the problem, $P(\cdot)$ is the probability of event $\cdot$, i.e., in this case, the probability that a realization of $\mathbf{Z}$ may cause a von Mises equivalent stress higher than yield stress at point k (probability of failure associated with k-stress constraint), and P_{adm} is the admissible failure probability. It should be noted that the same hypotheses of linear elasticity and limitations associated with singularity phenomenon and artificial stress concentration on jagged boundaries are also present in the reliability-based formulation.

In this work, problem presented in Eq. (4) is not solved directly. In order to avoid computation of probability of failure and its sensitivities with respect to design variables, a first-order alternative often used in the literature is employed: PMA formulation (Tu et al., 1999).

The PMA formulation consists in a double-loop strategy (Tu et al., 1999). At each update of design variables $\boldsymbol{\rho}$ (one step of outer problem) a first-order inverse reliability analysis must be performed, in standard normal space $\mathbb{Y}$, for each stress constraint (inner problems), in order to achieve N_k Minimal Performance Points (MiPPs). The MiPPs are coordinates of random space leading to smallest value of the performance function (stress constraints in Eq. (4)), which are found over the reliability-index hyper-sphere. RBTO problem of Eq. (4) is then rewritten as a double-loop problem, where outer problem is defined as

$$\begin{array}{lll}\underset{\boldsymbol{\rho}}{\text{Min.}} & V(\overline{\boldsymbol{\rho}}) = \sum_{e=1}^{N_e} V_e f_v\left(\overline{\rho}_e\right) & \\ \text{s. t.} & \frac{\sigma_{eq}^{(k)}\left(\overline{\boldsymbol{\rho}}, \left(\mathbf{Y}^{(k)}\right)^*\right)}{\sigma_y} - 1 \leqslant 0 & k = 1, 2, ..., N_k \\ & \mathbf{K}(\overline{\boldsymbol{\rho}})\mathbf{U}(\overline{\boldsymbol{\rho}}, \mathbf{Y}) = \mathbf{F}(\mathbf{Y}) & \\ & 0 \leqslant \rho_e \leqslant 1 & e = 1, 2, ..., N_e\end{array}, \tag{5}$$

where $\mathbf{Y} \in \mathbb{R}^N$ is a vector containing all random variables of the problem in standard normal space and $\left(\mathbf{Y}^{(k)}\right)^*$ is the MiPP associated with k-th stress constraint. One can verify that the outer problem, Equation (5), is very similar to the deterministic problem, Equation (1). The difference is in each stress constraint, which must be evaluated at correspondent MiPP (obtained after solving inner problems).

The objective functions of the inner problems are associated with the constraints of the outer problem and are defined as $-\frac{\sigma_{eq}^{(k)}(\overline{\boldsymbol{\rho}}, \mathbf{Y})}{\sigma_y} + 1$. Since additive and multiplicative (positive) constants do not change optimal solution, one can rewrite objective functions as $-\sigma_{eq}^{(k)}\left(\overline{\boldsymbol{\rho}}, \mathbf{Y}\right)$. Thus, the inner problems can be written as

$$\begin{array}{ll}\underset{\mathbf{Y}}{\text{Min.}} & -\sigma_{eq}^{(k)}\left(\overline{\boldsymbol{\rho}}, \mathbf{Y}\right) \\ \text{s. t.} & \|\mathbf{Y}\| = \beta_T\end{array}, \tag{6}$$

where $\|\mathbf{Y}\|$ is the Euclidean norm of $\mathbf{Y}$. The solution $\left(\mathbf{Y}^{(k)}\right)^*$ of k-th inner problem (k-th MiPP) depends on the target reliability index β_T defined for each constraint k. The target reliability index is related to admissible failure probability through the standard Gaussian cumulative distribution function Φ, such that $\beta_T \cong -\Phi^{-1}\left(P_{adm}\right)$, which consists in an approximate relation consistent with FORM based approaches (Lopez and Beck, 2012).

Since the inner problems are solved in $\mathbb{Y}$, a transformation must be performed from original space $\mathbb{Z}$, which can be non Gaussian, to the standard normal space $\mathbb{Y}$. There are some techniques used to perform this transformation, mapping from $\mathbf{Z}$ to $\mathbf{Y}$, but they are beyond the scope of this work. The reader may consult Ditlevsen and Madsen (2007) for details. In this work, all random variables are Gaussian, such that Hasofer and Lind transformation is sufficient (Lopez and Beck, 2012).

In order to solve the outer optimization problem, Equation (5), an augmented Lagrangian method is used, as presented in da Silva et al. (2017). The difference between solution of deterministic and reliability-based problems is in von Mises equivalent stress computation. As presented in this section, von Mises stresses are evaluated in MiPPs in the reliability-based problems, by solving inner problems, Equation (6). In this work, inner problems are solved at each step of the outer problem and for each point of stress computation with the Hybrid Mean Value (HMV) algorithm (Youn et al., 2003).

For details regarding sensitivity analysis of both inner and outer problems and solution algorithm the reader may consult da Silva and Beck (2017).

4 NUMERICAL RESULTS

In this section, a set of topology optimization problems with rectangular design domain, Figure 1, is solved to demonstrate the accuracy and efficiency of the methodology proposed herein. There are two applied loads in this problem: one vertical, with known intensity of 10N; and one horizontal, whose intensity is defined as a zero-mean Gaussian random variable with standard deviation equal to 1N. Applied loads are distributed along 0.2m in order to avoid stress concentration. This problem was similarly solved in da Silva et al. (2017), where uncertainty in direction of applied load was considered instead. Solutions are post-processed with MCS, in order to demonstrate accuracy of proposed approach.

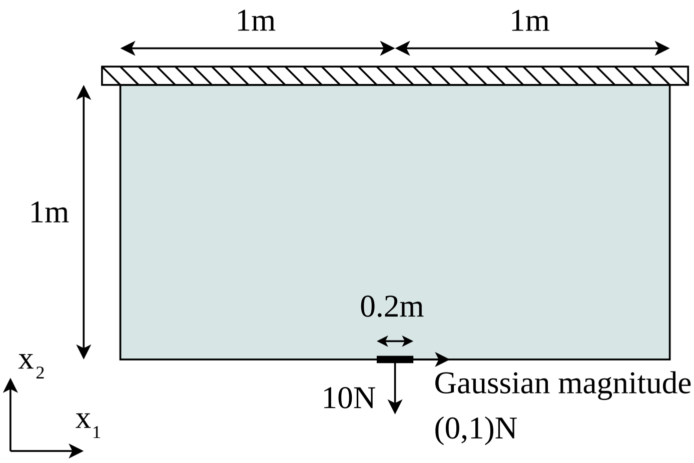

Figure 1: Rectangular domain considered in topology optimization with geometrical dimensions and boundary conditions.

Input data associated with numerical model: parameter $p = 3$ for stiffness parameterization (SIMP); parameter $\delta_\sigma = 3$ for stress interpolation; parameter $\delta_v = 5$ for volume penalization; parameter β, associated with density filtering with Heaviside step function (Guest et al., 2004), starts in 0 and ends in 100, considering continuation approach proposed in da Silva et al. (2017). Initial value of design variables is $\boldsymbol{\rho}^{(1)} = \mathbf{1}$ for all problems. Structural problems are discretized with structured mesh of 80000 four-node bilinear isoparametric finite elements and stress computation is performed at the center of each finite element, since this is the superconvergent point for stress computations in this element (Barlow, 1976). The problem is solved considering four values of target reliability index: $\beta_T = 0, 1, 2$ and 3. For comparison purposes, the problem is also solved without the random horizontal load, through the traditional deterministic framework, considering a safety factor of $\lambda_f = 1.6$. In order to verify accuracy of the FORM handling of probabilistic constraints, reliability index associated with each stress constraint of each optimal structure is obtained with MCS considering 1×10^6 realizations.

Material properties and minimum length scale: Young's modulus of 1MPa, thickness of 1mm, Poisson's ratio of 0.3, yield stress of 100kPa and a filter radius of $R = 0.04$m. Hypotheses of plane stress is considered.

Topologies, von Mises equivalent stresses and post-processed reliability indices are illustrated in gray scale images, Figure 2. In topology plots: white represents no material ($\overline{\rho} = 0$) and black represents full material ($\overline{\rho} = 1$). In von Mises equivalent stress plots: white represents next to zero stress and black represents maximum stress (next to yield or admissible stress in all cases presented). In reliability index plots: white represents maximum reliability index and black represents minimum reliability index.

Figure 2: Color scale used for plotting graphics.

Figure 3, first and second columns, shows optimal topologies with respective von Mises equivalent stresses, evaluated at MiPPs. Figure 3, third column, shows post-processed reliability indices. Table 1 shows objective function ratio, structural volume ratio, maximal von Mises equivalent stress of each optimal structure and minimum reliability index obtained through post-processing with MCS.

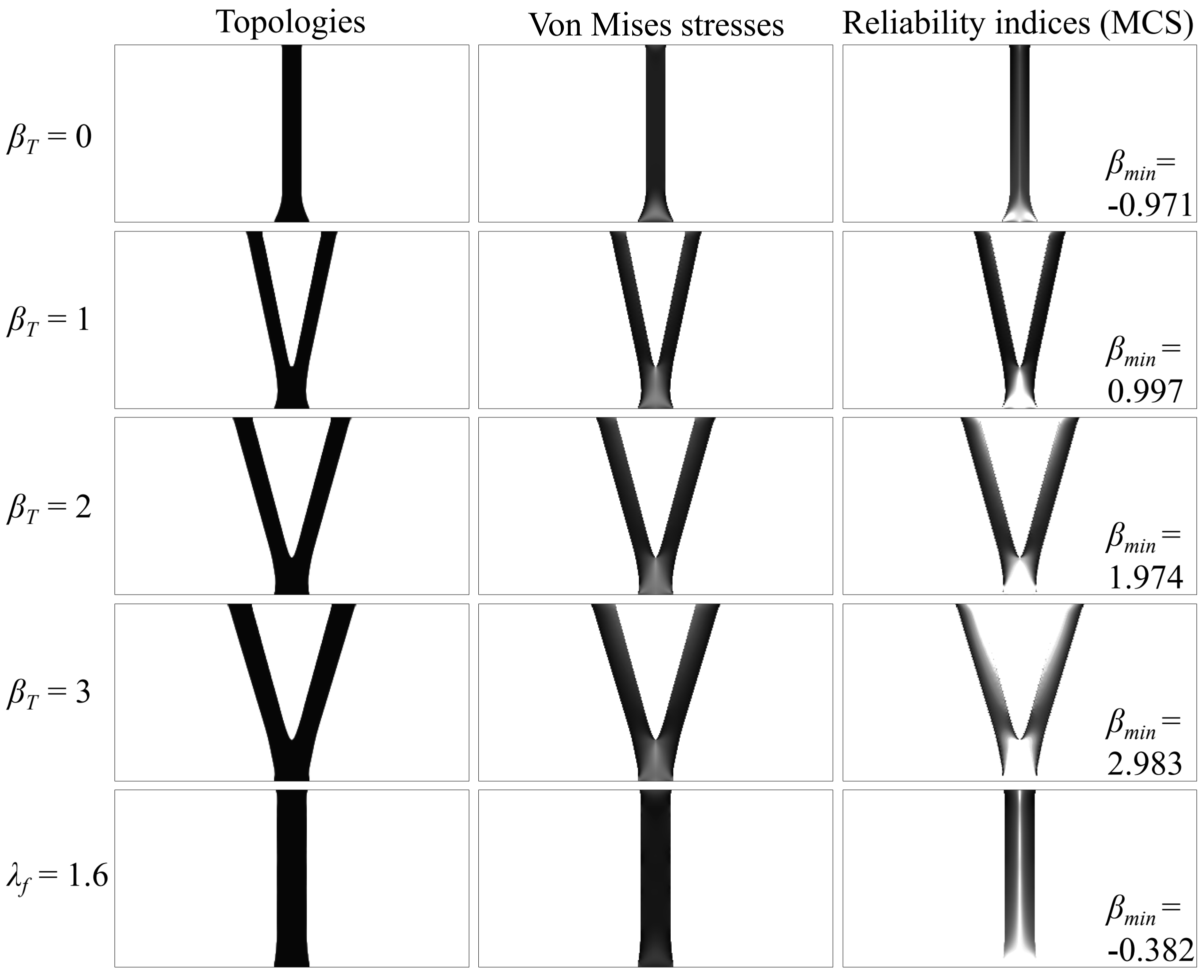

Figure 3: Optimal topologies (first column); von Mises equivalent stresses evaluated at MiPPs (second column); and reliability indices computed with MCS with 1×10^6 realizations (third column); considering solution of stochastic problems with $\beta_T = 0$, 1, 2 and 3; and considering solution of deterministic problem with $\lambda_f = 1.6$.

The most striking result is that optimal topologies are largely dependent on target reliability indices used as constraint in the RBTO approach. For $\beta_T > 0$ different topologies are obtained, with two members instead of only one member as the solution for $\beta_T = 0$. Also, analyzing Table 1, one can verify differences among structural volumes of optimal structures: the greater the value of β_T, the greater the structural volume of optimal structure. The same conclusions can be

Table 1: Optimal objective function ratios $V(\overline{\boldsymbol{\rho}})/V_{domain}$, structural volume ratios, maximal von Mises equivalent stresses and minimal reliability indices (MCS) of topologies in Fig. 3.

Target reliability	Obj. function ratio (%)	Struc. volume ratio (%)	Max. v. M. (kPa)	Min. post-processed reliability MCS
$\beta_T = 0$	5.807	5.771	100.24	$\beta_{min} = -0.971$
$\beta_T = 1$	8.745	8.547	100.10	$\beta_{min} = 0.997$
$\beta_T = 2$	10.48	10.27	100.47	$\beta_{min} = 1.974$
$\beta_T = 3$	12.26	12.05	100.18	$\beta_{min} = 2.983$

made when objective function is analyzed. Importantly, since crisp black and white topologies are obtained, small differences are verified between original objective function (penalized volume) and structural volume. This confirms results obtained in Luo et al. (2014), where a similar problem is solved with a different approach. This also confirms the case in Beck and Gomes (2012): to solve an actual design problem, optimal reliability constraints should be obtained from a risk optimization approach where consequences of failure are quantified and included in the objective function.

The result obtained for $\beta_T = 0$, of a single bar aligned with the vertical load of known intensity, and which agrees with Luo et al. (2014), reveals a drawback of the FORM approach adopted herein. Since MiPPs coincide with origin of standard normal space, we have horizontal load of zero intensity in the PMA constraint. This makes the RBTO solution identical to a deterministic solution neglecting load uncertainty (Figure 3). However, it is evident that any deviation from zero intensity, for the random horizontal load, leads to higher stresses in the single bar solution. Hence, the problem becomes highly non-linear, and the FORM solution is no longer appropriate. This is confirmed by MCS results in Fig. 3, first line, third column, where it is observed that reliability indices are not respected, especially in those regions of the design domain dominated by bending stresses arising from the random horizontal load. Anyway, this drawback is irrelevant for practical purposes, as actual structures are expected to have reliability indices much larger than zero.

Small differences between target reliability index β_T and minimum reliability index obtained through the use of MCS can be verified for $\beta_T \geqslant 1$, as shown in Table 1. When $\beta_T = 0$, large difference is observed, since $\beta_{min} = -0.971$ is obtained. It should be emphasized that in some points of design domain failures were not observed with this number of simulations, so that maximum reliability index, in Fig. 3, third column, was limited in $\beta_{max} = -\Phi^{-1}\left(1 \times 10^{-6}\right) \cong 4.75$, since 1×10^{6} realizations were used for computing probability of failure.

As a final comparison, solution of reliability-based problem considering $\sigma_y = 100$kPa and $\beta_T = 1$ is compared to solution of deterministic problem. In order to make a "fair" comparison, deterministic problem is solved considering a yield stress of $\sigma_y = 100$kPa and safety factor of $\lambda_f = 1.6$, resulting in admissible stress of $\sigma_{adm} = 62.5$kPa. Resulting structure has same topology as solution of reliability-based problem obtained considering $\sigma_y = 100$kPa and $\beta_T = 0$, but greater structural volume ratio (8.582%). Reliability indices are also post-processed for this optimal topology by MCS and considering the uncertainty of the RBTO problem, as presented in Fig. 3, fifth line, third column.

The structural volume ratio of deterministic solution for $\sigma_{adm} = 62.5$kPa (8.582%) is nearly the same as the structural volume ratio of RBTO solution with $\beta_T = 1$ (8.547%). Hence, these solutions are compared in terms of reliability indices. We observe that the minimum reliability index for deterministic solution is $\beta_{min} = -0.382$, whereas $\beta_{min} = 0.997$ for RBTO solution with $\beta_T = 1$. Hence, it is not possible to establish equivalences between the deterministic and the reliability-based formulations, as noted in Beck and Gomes (2012).

Although this paper focuses on RBTO problems, it is also important to comment about a relevant aspect regarding both deterministic and reliability-based topology optimization with stress constraints based on density approach: artificial stress concentration on jagged boundaries.

Considering reliability-based solutions obtained for $\beta_T \geqslant 1$, some comparisons can be drawn with recent work regarding stress-based topology optimization taking artificial stress concentration on jagged boundaries into account (Svärd, 2015). Since we obtain crisp black and white topologies, artificial stress concentration is obtained on jagged boundaries, in agreement with Svärd (2015). As demonstrated in Svärd (2015), optimal topologies obtained without alleviating effects of jagged boundaries on stress computation tend to overestimate optimal structural volume. This fact does not change conclusions drawn in this subsection, since we would obtain even lighter results if some technique, as the Interior Value Extrapolation technique (Svärd, 2015), was applied for alleviating jagged boundary effects. Jagged boundary effects are also verified in graphics regarding post-processed reliability indices, Figure 3, third column, since their computations are directly related to von Mises equivalent stresses, Figure 3, second column. Post-processed reliability indices are minima in jagged boundaries, where von Mises equivalent stresses are maxima. This can be clearly seen for the case of $\beta_T = 3$ in Fig. 3.

5 CONCLUSIONS

In this paper, a probabilistic RBTO framework is proposed to solve continuum topology optimization problems of volume minimization with local stress constraints under uncertainty in applied loads. In order to formulate the problem, a first-order PMA approach was considered, where one inverse reliability analysis was performed per point of stress computation at each iteration of outer optimization problem, with the HMV algorithm. The topology optimization problems were solved with an augmented Lagrangian method, where the whole set of local stress constraints was considered.

Comparisons among solutions of reliability-based problems were performed. Allowable probability of failure was indirectly imposed through the use of target reliability index. Solutions were post-processed with MCS, where agreement between target reliability indices and minimum reliability indices were observed for large values of target reliability indices. Results shown that there are no equivalent deterministic solutions to RBTO problems, as deterministic solutions lead to different topologies of compromised safety.

ACKNOWLEDGEMENTS

The authors kindly acknowledge financial support of this research project by the agencies CNPq (National Council for Research and Development), grant number 303749/2012-1, and

FAPESP (São Paulo Research Foundation), grant number 2015/25199-0.

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