Building plane and solid modeling using NURBS

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Abstract. *This work presents a reimplementation of the programs ProgGeoIso, which calculates the geometric properties of solid and flat figures applying the Radial Integration Method (RIM) in conjunction with the NURBS. The reimplementation aims to make the program accessible to an individual with basic knowledge in programming, without the need for prior knowledge of NURBS. The extrude command was also developed from the program PropGeoIso3D, which is capable of extruding solids from user defined curves. First, a brief historical introduction is made indicating the relevance of the study of isogeometric modeling. The curves and Bzier surfaces, B-splines, and finally the NURBS are presented. Following, the steps for the implementation of the algorithm based on the radial integration method are described. The programs were developed using Matlab which were tested, whenever possible, on problems whose analytical solutions were known. Otherwise, results from a commercial CAD program were used for comparisons. Numerical results show good agreement to the analytical ones.*

Keywords: *CAD, Isogeometric Formulation, NURBS, Radial integration method, geometric properties.*

1 INTRODUCTION

During the early 20th century, several numerical methods were improved to solve differential equations, such as the Ritz methods [3] Trefftz [4] and the finite difference method [1]. However, all the methods mentioned took a long time in their solutions due to the large number of calculations required. From the 1960s, with the development of computers, the use of the finite difference method (FDM), as well as the finite element method (FEM) and the boundary element method (BEM) became very advantageous. The emergence of computers also led to the creation of Computer Aided Design (CAD) which originated from the work of two French automotive engineers, Pierre Bzier and Paul de Faget de Casteljau [2].

With the development of CAD and numerical methods, there were also the development of non-uniform rational B-Splines (NURBS), which are still the dominant technology in the area of computational design because they can accurately describe several curves.

It is known that the design and physical analysis of engineering projects feed back, but nowadays time is wasted trying to transmit information generated in CAD (Computer Assisted Design) to CAE (Computer Assisted Engineering, that is, the analysis or computational simulation of the problem).

In order to have a cost dimension of this transmission, translation of CAD files into geometry compatible with analysis, mesh generation and input data for a large-scale finite element method analysis consumes about 80% of the total analysis time [9]. In addition, the mesh is not an exact representation of the geometry of the problem which generates errors in the analysis.

One way, then, to improve this process is to adopt the CAD-generated geometry itself, considered to be exact, coupled with the methods of analysis. In this case, as indicated by [6], the choice of MEC as a method of analysis is quite natural, since both use only surface representations. With this combination, there is no longer any need for mesh generation, accounting for about 20% of the total analysis time.

In this work, flat and solid models constructed using non uniform rational B-splines (NURBS) curves and surfaces are generated through a program written in modified MATLAB called PropGeoiso. Taking advantage of this mathematical structure, based on NURBS, the geometric properties of flat and solid models are calculated.

The growing use of NURBS has led to the development of many programs in the area of computational drawing, among them PropGeoiso, developed by [5], which can be used for analysis in conjunction with third party programs.

The purpose of this work is the improvement of PropGeoiso 2D and 3D programs in MATLAB created by [5], with the goal of making its use simpler, without the user being required to have a prior knowledge of NURBS and so that in the future it may be used for other types of analysis as in isogeometric boundary element analysys. In addition, new commands were developed to increase the range of application for the code. As an example, the extrude command was created so that any 2D section can now be extruded in order to build a solid.

2 Solid representation

This chapter describes different ways of representing solids using parametric equations:

2.1 Nurbs curves

A mathematical representation of a NURBS curve is given by [12]:

$$C(u) = \frac{\sum_{i=0}^{n} N_{i,p}(u) w_i P_i}{\sum_{i=0}^{n} N_{i,p}(u) w_i} \qquad a \leq u \leq b \tag{1}$$

for a given knot vector:

$$U = \{\underbrace{a, ..., a}_{p+1}, u_{p+1}, ..., u_{m-p-1}, \underbrace{b, ..., b}_{p+1}\} \tag{2}$$

control points P and weights w. $N_{i,p}$ are basis functions. The order zero basis function is given by:

$$N_{i,0}(u) = 1 \tag{3}$$

if $u_i \leq u \leq u_{i+1}$

$$N_{i,0}(u) = 0 \tag{4}$$

in other cases. Higher order basis functions are calculated using the following iterative equation:

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_i + 1} N_{i+1,p-1}(u) \tag{5}$$

in which, n is the degree of the curve.

An elegant way of representing the points of the rational curves is through homogeneous coordinates. These coordinates allow the easy application of the geometric transformations and the simple representation of the rational curve of dimension n in a curve of $n+1$ control points.

In the case of a rational curve in 3 dimensions, we have that each control point is represented in Euclidean space by $P = (x, y, z)$ and in the new coordinates is represented by $P^w = (wx, wy, Wz, w) = (X, Y, Z, W)$ with $w \neq 0$. These values would be related by the mapping H, given by:

$$P = H\{P^w\} = H\{(X, Y, Z, W)\} = \left(\frac{X}{W}, \frac{Y}{W}, \frac{Z}{W}\right) \qquad W \neq 0 \tag{6}$$

The representation of a NURBS curve in homogeneous coordinates is given by:

$$C(u)^w = \sum_{i=0}^{n} N_{i,p}(u) P_i^w \tag{7}$$

2.2 NURBS surfaces

NURBS surfaces naturally arise through the concepts presented above. They are represented by the following equation:

$$S(u, v) = \frac{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j} P_{i,j}}{\sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) w_{i,j}} \qquad 0 \leq u, v \leq 1 \tag{8}$$

and knot vectors are given by:

$$U = \{\underbrace{0, ..., 0}_{p+1}, u_{p+1}, ..., u_{r-p-1}, \underbrace{1, ..., 1}_{p+1}\} \tag{9}$$

$$V = \{\underbrace{0, ..., 0}_{q+1}, v_{q+1}, ..., v_{s-q-1}, \underbrace{1, ..., 1}_{q+1}\} \tag{10}$$

where U have $r+1$ knots and V, $s+1$.

Like any other rational function, these surfaces can be represented by homogeneous coordinates:

$$S^w(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)P^w_{i,j} \tag{11}$$

where $P^w_{i,j} = (w_{i,j}x_{i,j}, w_{i,j}y_{i,j}, w_{i,j}z_{i,j}, w_{i,j})$ and $S(u,v) = H(S^w(u,v))$.

Another representation of this surface uses the rational base function:

$$R_{i,j}(u,v) = \frac{N_{i,p}(u)N_{j,q}(v)w_{i,j}}{\sum\limits_{k=0}^{n}\sum\limits_{l=0}^{m} N_{k,p}(u)N_{l,q}(v)w_{k,l}} \tag{12}$$

such that:

$$S(u,v) = \sum_{i=0}^{n}\sum_{j=0}^{m} R_{i,j}(u,v)P_{i,j} \tag{13}$$

2.3 Trimmed NURBS surfaces

Complex shapes, especially involving joining of surfaces, are represented in CAD programs using trimmed surfaces. A trimmed surface consists of a NURBS surface and a set of trimming curves. A trimming curve shows the mapping of a 1D parametric space to a 2D parametric space as indicated in the figure 1.

In order extend the formulation for more general solids, the trimmed surfaces will be considered in this work. The use in isogeometric analysis of these types of structure presented by [10], [11], [7], and [13].

These curves must have the following characteristics: order greater than 1, significant regions on the left.

A trimmer curve can be represented by the following equation:

$$C(t) = \sum_{i=1}^{n} R^C_{i,p}(t)Q_i \tag{14}$$

A trimmed surface can be represented by the following equation:

$$S(u,v) = \sum_{i=1}^{n}\sum_{j=1}^{m} R^S_{i,j}(u,v)P_{i,j} \tag{15}$$

In which Q and P are the control points and R is the base function indicated by the equation 12.

3 Computation of solid geometric properties

3.1 Computation of the area of geometric properties

Be a three-dimensional surface figure S. We want to calculate the area of this figure, starting with the assumption that it can be found by calculating a surface integral over S.

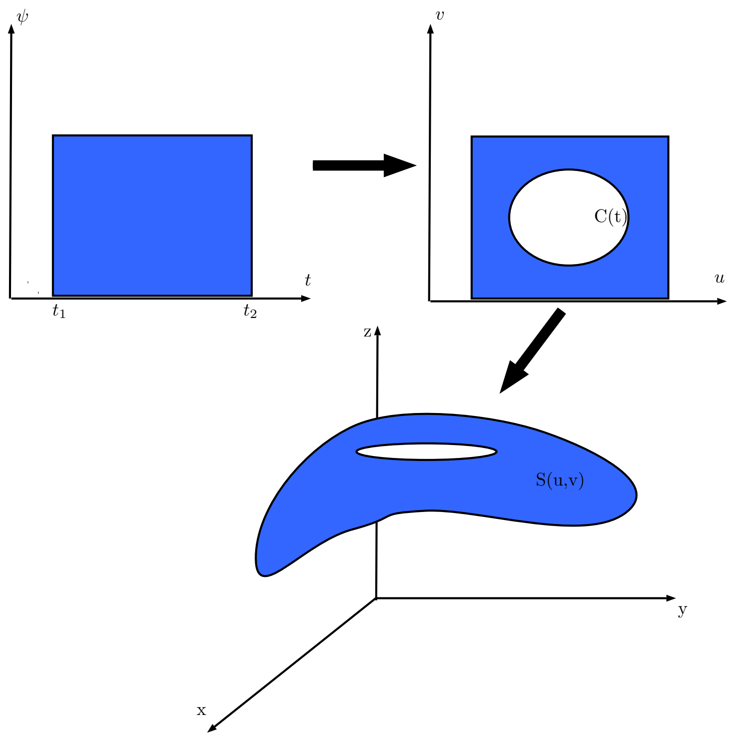

Figure 1: Mapping a Trimmed Surface

The area is then given by:

$$I = \oint_S dS. \tag{16}$$

A strategy for calculating the integral of the area along the surface S is the division of the surface S into a sum of small pieces $S1, S2, ..., Sn$, That is, $S = \sum_{i=1} nS_i$, where n is the number of pieces in which the surface is divided. Since these pieces may have any shape, each piece S_i will be approximated by a known shape. For simplicity, this form is Almost always given by elementary areas (triangle, square, etc.). In this work, this surface will be represented by NURBS.

In this way, each piece $S_1, S_2, ..., S_n$ is approximate by known forms $\Gamma_1, \Gamma_2, ..., \Gamma_n$ called 3D boundary elements.

Hence, we have that:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Gamma_i = S \tag{17}$$

where, Γ= surface written by NURBS.

3.2 Numerical integration

Gauss integration will be used to performe the integration of the solid properties in this work. Thus, we observe the real element, described by NURBS, in the figure 2, and the integration element in the figure 3.

In these figures, the coordinates for the functions x, y and z are given by the equation 8.

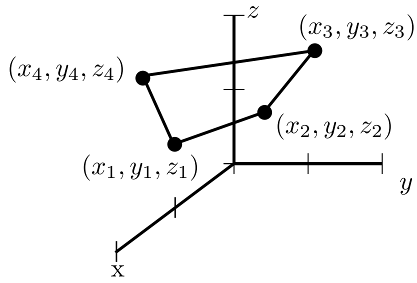

Figure 2: Real element

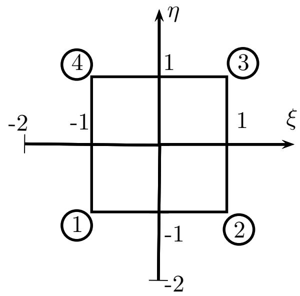

Figure 3: Integration element

With this data, it is necessary to perform a variable change in order to allow the integration of the element area.

Thus, integrating the quadrilateral element, we obtain the following equation:

$$I_i = \int_{-1}^{1}\int_{-1}^{1} J d\xi d\eta \tag{18}$$

The jacobian J is given by:

$$J = \sqrt{g_1^2 + g_2^2 + g_3^2} \tag{19}$$

where g_1, g_2 e g_3 are given by:

$$g_1 = \frac{dy}{d\xi}\frac{dz}{d\eta} - \frac{dy}{d\eta}\frac{dz}{d\xi} \tag{20}$$

$$g_2 = \frac{dz}{d\xi}\frac{dx}{d\eta} - \frac{dz}{d\eta}\frac{dx}{d\xi} \tag{21}$$

$$g_3 = \frac{dx}{d\xi}\frac{dy}{d\eta} - \frac{dx}{d\eta}\frac{dy}{d\xi} \tag{22}$$

Since the real surface will be presented by NURBS, that is a function $S(u, v)$, we have to make more changes of variables so that the value of the area can be found. Considering $x(u, v)$, $y(u, v)$ and $z(u, v)$, equations 20, 21 and 22 will be modified as follows:

$$g_1 = (\frac{dy}{du}\frac{du}{d\xi})(\frac{dz}{dv}\frac{dv}{d\eta}) - (\frac{dy}{dv}\frac{dv}{d\eta})(\frac{dz}{du}\frac{du}{d\xi}) \tag{23}$$

$$g_2 = (\frac{dz}{du}\frac{du}{d\xi})(\frac{dx}{dv}\frac{dv}{d\eta}) - (\frac{dz}{dv}\frac{dv}{d\eta})(\frac{dx}{du}\frac{du}{d\xi}) \tag{24}$$

$$g_3 = (\frac{dx}{du}\frac{du}{d\xi})(\frac{dy}{dv}\frac{dv}{d\eta}) - (\frac{dx}{dv}\frac{dx}{dv})(\frac{dy}{du}\frac{du}{d\xi}) \tag{25}$$

These relations were obtained considering that u is a linear function of ξ and v is a linear function of η. Consequently, we have:

$$u = a\xi + b \tag{26}$$

$$v = c\eta + d \tag{27}$$

Considering:

$$u(\xi = -1) = u_k \tag{28}$$

$$u(\xi = 1) = u_{k+1} \tag{29}$$

$$v(\eta = -1) = v_t \tag{30}$$

$$v(\eta = 1) = v_{t+1} \tag{31}$$

we finally obtain:

$$u = \frac{\xi(u_{k+1} - u_k) + (u_{k+1} + u_k)}{2} \tag{32}$$

$$\frac{du}{d\xi} = \frac{(u_{k+1} - u_k)}{2} \tag{33}$$

$$v = \frac{\eta(v_{t+1} - v_t) + (v_{t+1} + v_t)}{2} \tag{34}$$

$$\frac{dv}{d\eta} = \frac{(v_{t+1} - v_t)}{2} \tag{35}$$

Thus, we know the results of the derivatives $\frac{du}{d\xi}$ and $\frac{dv}{d\eta}$. By applying these values in 23, 24 and 25, it is possible to get the jacobian for any quadrilateral contour element. Then, we simply apply the Gaussian quadrature in the 18 equation to obtain the area of the triangular element.

$$I_i = \int_{-1}^{1}\int_{-1}^{1} J d\xi d\eta = \sum_{k=1}^{p}\sum_{t=1}^{m} J(u_k, v_t) w_k w_t \tag{36}$$

By adding the area of the elements, it is possible to obtain the surface area as follows:

$$A = \sum_{i=1}^{n}\left[\sum_{k=1}^{p}\sum_{t=1}^{m} J(u_k, v_t) w_k w_t\right] \tag{37}$$

Computation of volume integrals using 3D Radial Integration Method (RIM)

The 3D radial integration method (RIM) used in this work follows the procedure presented by [8]. Consider a three-dimensional domain Ω delimited by the surface S, whose Cartesian coordinate system is represented by (x, y, z), and the spherical coordinate system is represented by (ρ, θ, ϕ) with the origin of the two systems in Cartesian coordinates $(0, 0, 0)$.

The relation between the coordinates will be represented by:

$$x = \rho \sin\theta \cos\phi \tag{38}$$

$$y = \rho \sin\theta \sin\phi \tag{39}$$

$$z = \rho \cos\theta \tag{40}$$

An infinitesimal segment of the domain, $d\Omega$, is represented by:

$$d\Omega = \rho^2 \sin\theta d\rho d\theta d\phi \tag{41}$$

Consequently, the integral of a function $f(x, y, z)$ in this domain will be:

$$\int_\Omega f(x, y, z) d\Omega = \int_0^{2\pi} \int_0^{\pi} F \sin\theta d\theta d\phi \tag{42}$$

where

$$F = \int_0^{r(\theta,\phi)} f(\rho, \theta, \phi)\rho^2 d\rho \tag{43}$$

The relation between the spherical surface element $d\ Gamma$ and the real element of the surface is given by:

$$d\Gamma = \rho^2 \sin\theta d\theta d\phi = dS \cos\psi = dS\vec{n}.\vec{r} \tag{44}$$

This relation refers to the figure 4.

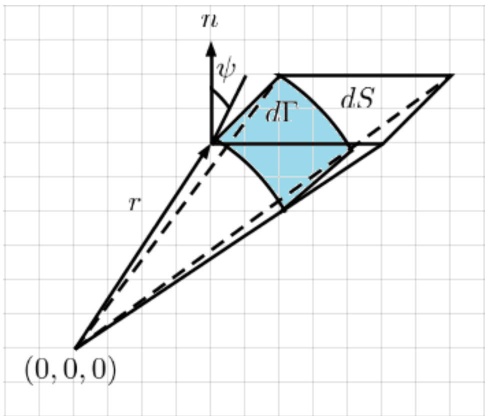

Figure 4: Relation between $d\Gamma$ e dS

We conclude that the integral of the function is given by:

$$\int_\Omega f(x, y, z) d\Omega = \int_0^{2\pi} \int_0^{\pi} \int_0^{r(\theta,\phi)} f(\rho, \theta, \phi)\rho^2 d\rho \sin\theta d\theta d\phi = \int_S F \frac{\vec{n}.\vec{r}}{r^2} dS \tag{45}$$

In order to solve this equation, it is sufficient to apply Gauss's quadrature.

Following this idea, the F function will be given by:

$$F = \int_0^{r(\theta,\phi)} f(\rho, \theta, \phi)\rho^2 d\rho = \int_{-1}^{1} \rho^2 \frac{d\rho}{d\xi} d\xi = \sum_{k=1}^{p} \frac{r}{2} \rho_k^2 w_k \tag{46}$$

Therefore, by using the equation 36 in the equation 45, we obtain:

$$\int_S F \frac{\vec{n}.\vec{r}}{r^2} dS = \int_{-1}^{1} \int_{-1}^{1} F \frac{\vec{n}.\vec{r}}{r^2} J d\xi d\eta = \sum_{k=1}^{p} \sum_{t=1}^{m} F \frac{\vec{n}.\vec{r}}{r^2} J(u_k, v_t) w_k w_t \tag{47}$$

Calculation of properties using trimmed surfaces

To represent more complex figures, with holes, for example, it is appropriate, as indicated in the previous section, to use trimmed surfaces. The calculation of its properties, then, becomes slightly different from that presented until then.

In this case, the simpler surfaces that make up the figure are drawn using NURBS and only the surfaces with cuts or holes receive a different treatment.

To understand this process, imagine a S hole domain A domain.

The area of this surface, for example, is calculated by following the equation 47, but a problem arises using this method: how to represent the holes? Using the concept of trimmed surfaces, a NURBS curve is drawn in the domain, which will exclude nodes on the right, as shown in Figure 5.

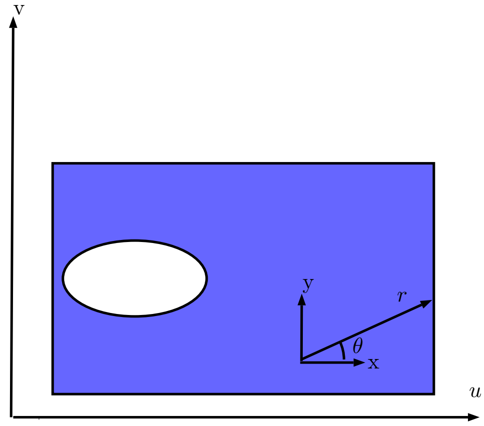

Figure 5: Domain scan using RIM

The equation, then, of an integral I of the function $f(u, v)$, which represents a geometric property, sweeps the domain with the RIM. So:

$$I = \int_{\Gamma} f d\Gamma = \int_{-1}^{1} F_1 \frac{nr}{r} \frac{d\Gamma}{du} \frac{du}{d\xi} d\xi = \sum_{k=1}^{p} F_1 \frac{nr}{r} \frac{d\Gamma}{du} \frac{du}{d\xi} w_1 \tag{48}$$

The terms of this equation were given previously except F_1 which is given by:

$$F_1 = \sum_{k=1}^{p} f_k \rho_k \frac{dr}{d\xi} w_2 \tag{49}$$

In this work, the function f will be the value obtained from one of the two 3D geometric properties of interest (volume or centroid):

Thus, we have:

$$F_1 = \sum_{k=1}^{p} F_2 \frac{nr}{r^2} \rho J(u_k, v_k) \frac{dr}{d\xi} w_2 \tag{50}$$

In this equation, the jacobian $J(u_k, v_k)$ for the transformation of domain A into spherical coordinates and F_2 is given by:

$$F_2 = \sum_{k=1}^{p} g_k \rho_k^2 \frac{dr}{d\xi} w_3 \tag{51}$$

where g is the function for each geometric property (for example, for volume, $g = 1$).

If more than one curve is needed in the domain, the resulting equation is simply:

$$I = \sum_{i=1}^{n} \left[\sum_{i=1}^{n} F_1 \frac{nr}{r} \frac{d\Gamma}{du} \frac{du}{d\xi} w_1 \right] \tag{52}$$

With these solutions, it is enough to add (or subtract) the geometric properties obtained for the trimmed curves with the result obtained for the rest of the surfaces, thus obtaining the numerical value for the solid.

4 NUMERICAL SIMULATION AND RESULTS

4.1 Introduction

In this section, some results obtained with the programs developed with the radial integration method applied to NURBS will be presented. Implementations were done in $MATLAB$ language.

Simple forms will be approached whose value of their geometric properties can be calculated analytically.

For all objects, 6 Gauss points were used in the integrations, the order of NURBS was 3, except for the rectangles whose order was 2, and the integration limits of each element were u_i and u_{i+1}, that is the space between the nodes.

C profile beam

From the NURBS, a C profile beam (Figure 6) was constructed by the extrusion. The beam is 10 units in length and the extrusion surface (transversal section) is defined by points described in the tables 1 and 2.

Table 1: ***Extrusion surface input parameters - Extrusion surface point coordinates.***

Point numbers	x	y
P_1	0,5000	1,0000
P_2	2,0000	1,0000
P_3	2,5000	1,5000
P_4	2,5000	2,5000
P_5	2,0000	3,0000
P_6	-2,0000	3,0000
P_7	-2,5000	2,5000
P_8	-2,5000	-2,5000
P_9	-2,0000	-3,0000
P_{10}	2,0000	-3,0000
P_{11}	2,5000	-2,5000
P_{12}	2,5000	-1,5000
P_{13}	2,00000	-1,0000
P_{14}	0,5000	-1,0000
P_{15}	0,5000	-1,5000
P_{16}	1,8000	-1,5000
P_{17}	2,0000	-1,7000
P_{18}	2,0000	-2,3000
P_{19}	1,8000	-2,5000
P_{20}	-1,7000	-2,5000
P_{21}	-2,0000	-2,2000
P_{22}	-2,0000	2,2000
P_{23}	-1,7000	2,5000
P_{24}	1,8000	2,5000
P_{25}	2,0000	2,3000
P_{26}	2,7000	1,7000
P_{27}	1,8000	1,5000
P_{28}	0,5000	1,5000

Properties calculated by the program $SolidWorks^{\circledR}$ 2016 were used as references for the comparisons.

The geometry generated by the program is shown in figure 6.

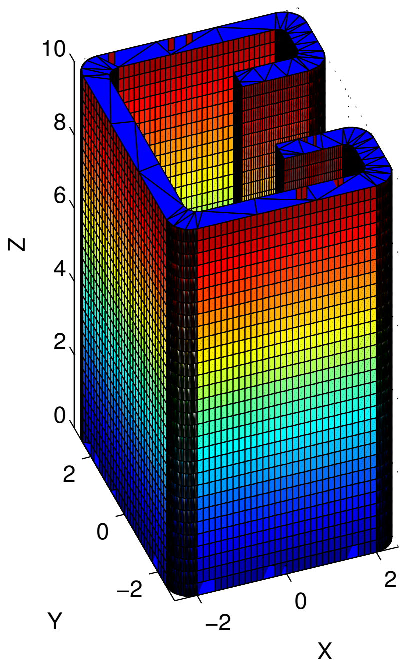

Figure 6: C beam generated from a extruded section

Table 2: ***Extrusion surface input parameters - conectivity.***

Segments	Point numbers	r
1	P_1 e P_2	0,0000
2	P_2 e P_3	0,5000
3	P_3 e P_4	0,0000
4	P_4 e P_5	0,5000
5	P_5 e P_6	0,0000
6	P_6 e P_7	0,5000
7	P_7 e P_8	0,0000
8	P_8 e P_9	0,5000
9	P_9 e P_10	0,0000
10	P_{10} e P_{11}	0,5000
11	P_{11} e P_{12}	0,0000
12	P_{12} e P_{13}	0,5000
13	P_{13} e P_{14}	0,0000
14	P_{14} e P_{15}	0,0000
15	P_{15} e P_{16}	0,0000
16	P_{16} e P_{17}	-0,2000
17	P_{17} e P_{18}	0,0000
18	P_{18} e P_{19}	-0,2000
19	P_{19} e P_{20}	0,0000
20	P_{20} e P_{21}	-0,3000
21	P_{21} e P_{22}	0,0000
22	P_{22} e P_{23}	-0,3000
23	P_{23} e P_{24}	0,0000
24	P_{24} e P_{25}	-0,2000
25	P_{25} e P_{26}	0,0000
26	P_{26} e P_{27}	-0,2000
27	P_{27} e P_{28}	0,0000
28	P_{28} e P_1	0,0000

Numerical results obtained by the program are presented in table 3.

Table 3: ***Results of PropGeoiso3D_extrude for the C beam.***

Property	Value of the property.
Volume ($SolidWorks^{®}$ 2016)	102,5106
Volume (this work):	102,5106
Percent error of the volume	6,3656.10^{-6}
Area $SolidWorks^{®}$ 2016	431,6172
Area (this work)	431,6172
Percent error of the area	7,9663.10^{-7}
Centroid ($SolidWorks^{®}$ 2016)	(-0,062315 , 0,0000 , 5,0000)
Centroid (this work)	(-0.062315 , 1,213.10^{-16} , 5,0000)
Absolute error of the centroid	(6,2721.10^{-9} , 1,213.10^{-16} , 4,2067.10^{-7})

Wheel

Just as an indication of the robustness of the program with solids obtained by rotation of a section, a wheel is analysed in this section. The flywheel was represented by NURBS surfaces formed from a revolved section (Figure 7) around the $y = -5$ axis.

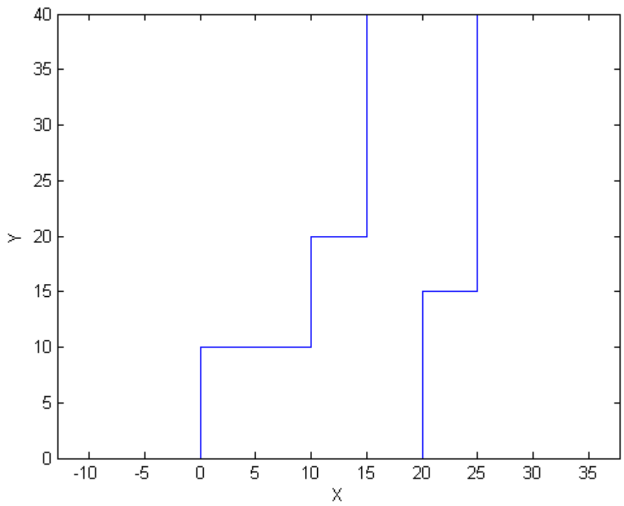

Figure 7: Wheel section

The resulting geometric form is shown in the figure 8.

The results obtained by the program are presented in the table 4.

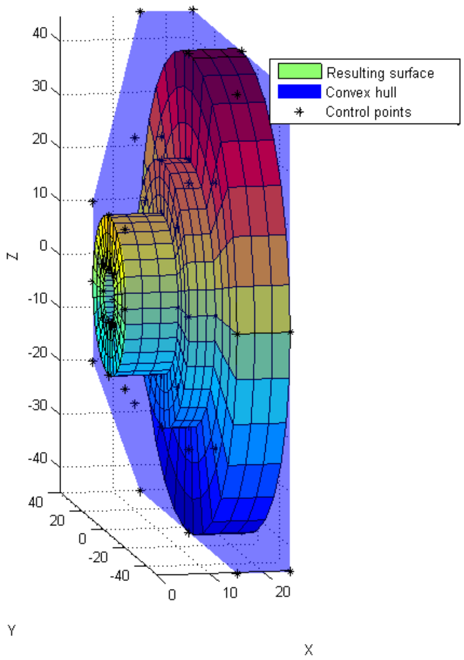

Figure 8: Wheel generated from a revolved section

Table 4: ***Results for the wheel***

Properties	Value
Volume ($SolidWorks^{®}$) 2016	72649.3301
Volume (this work):	72649.3301
Percent error of the volume	$2.4036\ 10^{-12}$
Area ($SolidWorks^{®}$)2016	18378.317
Area (this work)	18378.317
Percent error of the area	$2.7713\ 10^{-13}$
Centroid ($SolidWorks^{®}$) 2016	(0.0000 , -5.0000 , 0.0000)
Centroid (this work)	((17.5270 , -5.0000 , -2.8985 10^{-15})
Absolute error of the centroid	(2.973 10^{-9} , 1.7764 10^{-15} , 2.8985 10^{-15})

5 CONCLUSIONS

In this article, we developed two programs in $MATLAB$ language capable of making geometric analyzes of three-dimensional figures using RIM applied to NURBS. The described geometries were initially presented in a CAD. Then, the values for the area, volume and centroid were obtained numerically and analytically, as well as their respective errors (absolute and relative).

Two solid geometries were analysed. Numerical results were very close to analytical values even with complex shapes. It is noteworthy that good results were obtained using only 6 Gauss points in the integrations. If the number of integration points were larger, the error would decrease until it became constant, which would indicate that it is only due to the truncation of the floating-point variables of $MATLAB$ and no longer due to the numerical method.

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