Computation of geometric properties in solids written in IGES files

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Abstract. *This work presents the implementation of a numerical code for calculating the geometrical properties of solids modeled in CAD (Computer Aided Design) programs and exported in IGES (Initial Graphics Exchange Specification) format. Initially, various types of representation of curves and surfaces are shown finishing with NURBS (Non Uniform Rational Basis Spline) surfaces. Then it is shown how the surface area and volume were calculated on trimmed NURBS surface using the radial integration method. Finally, numerical results obtained by the program were verified through analytical results and, for more complex solids, the surface area and volume values given by SOLIDWORKS software.*

Keywords: *NURBS, Surface area, Volume, Trimmed surfaces, Radial integration method*

1 INTRODUCTION

Numerical simulations requires a large amount of time for sending data from CAD (Computer Aided Design) generated models to solvers, as seen in Bazilevs (2010), Beer (2015) and Cottrell (2009). Some reasons for that are the translation from CAD files to a geometry compatible with the analyzing software and the mesh generation.

According to Beer (2015), one option for making the numerical simulation more efficient is to adopt the CAD generated geometry within the analytical software, avoiding, this way, one step of the process (the mesh generation).

As CAD softwares represent only the surface of a solid, the Boundary Elements Method (BEM) is an outstanding technique to use without the need of creating a mesh grid, owing to the fact that it needs only the solid surface to be applied.

This work presents the implementation of a numerical code for calculating the geometrical properties of solids generated in CAD programs and exported as IGES (Initial Graphics Exchange Specification) files. Domain integrals are transformed into boundary integrals by using the radial integration method and Gauss theorem. As a result, no discretization is necessary provided that the NURBS (Non Uniform Rational Basis Spline) entities from IGES file are used directly in the numerical integration, as in Beer (2015). In future works, the same procedure will be used in a boundary element implementation. So, the mesh generation, that takes a lot of time of the engineer in a boundary element analysis, will be completely eliminated from the process of analysis.

The code was developed based on the isogeometric boundary element method, which is the same used for the geometric representation. In the isogeometric formulation, the polynomials form functions of the traditional BEM are substituted by NURBS, i.e., the mathematical basis used by most part of CAD softwares to represent solids. Because of that, the mesh generation process, one of the parts that most consumes time from the engineer, is eliminated.

2 GEOMETRIC PROPERTIES CALCULATION

2.1 Radial Integration Method (RIM)

The Radial Integration Method (RIM) transforms a domain integral into a boundary one. Consider an arbitrary surface Ω and boundary Γ. The integral of a function $f(x, y)$ over the area is represented by:

$$I = \iint_{\Omega} f(x, y)dxdy \tag{1}$$

In polar coordinates:

$$I = \iint_{\Omega} f(x(\rho, \theta), y(\rho, \theta))\rho d\rho d\theta \tag{2}$$

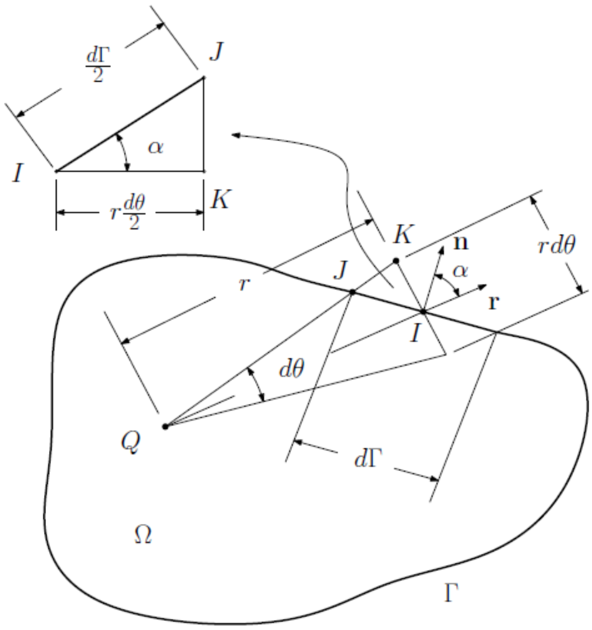

Figure 1: ***Domain*** Ω ***and its geometrical relations.***

or

$$I = \int_\theta \int_0^r f(x(\rho,\theta), y(\rho,\theta))\rho d\rho d\theta \tag{3}$$

where r stands for the value of ρ in a boundary point.

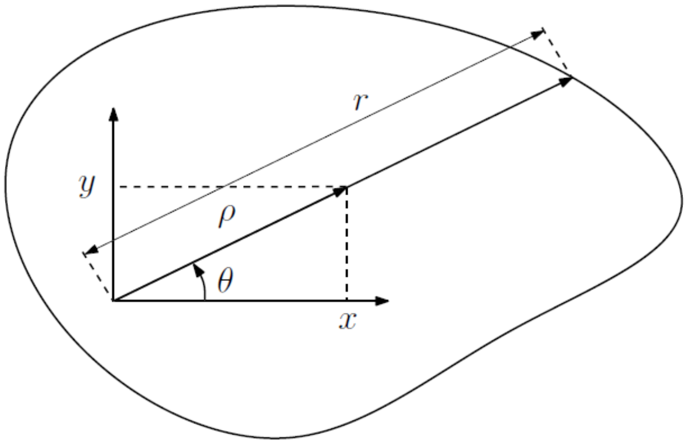

Figure 2: ***Radial Integration Method Variables.***

Equation (3) may also be represented as follows:

$$I = \int_\theta F(\rho,\theta) d\theta \tag{4}$$

where F is:

$$F(\rho,\theta) = \int_0^r f(x(\rho,\theta), y(\rho,\theta))\rho d\rho \tag{5}$$

Considering an infinitesimal angle $d\theta$, the relation between the arc length $rd\theta$ and the infinitesimal boundary length $d\Gamma$ is given by:

$$\cos\alpha = \frac{r\frac{d\theta}{2}}{\frac{d\Gamma}{2}} \tag{6}$$

Isolating the term $d\theta$:

$$d\theta = \frac{\cos\alpha}{r} d\Gamma \tag{7}$$

where α is the angle between the unit vectors r and n.

Using the properties of the inner product of both unit vectors n and r:

$$d\theta = \frac{\mathbf{n}\cdot\mathbf{r}}{r} d\Gamma \tag{8}$$

Substituting equation (8) on equation (4), the domain integral may be written as a boundary integral:

$$I = \int_\Gamma F\ \frac{\mathbf{n}\cdot\mathbf{r}}{r} d\Gamma = \int_s F\ \frac{\mathbf{n}\cdot\mathbf{r}}{r}\ \frac{d\Gamma}{ds}\ ds \tag{9}$$

$$d\Gamma\ = \sqrt{du^2 + dv^2}\ \ => \ \frac{d\Gamma}{ds}\ = \sqrt{\left(\frac{du}{ds}\right)^2 + \left(\frac{dv}{ds}\right)^2} \tag{10}$$

$$u\ = \sum_{i=0}^{n} (u_{i,p}(s) w_i u_i) \tag{11}$$

$$v\ = \sum_{i=0}^{n} (v_{i,p}(s) w_i v_i) \tag{12}$$

where $u_{i,p}(s)$ and $v_{i,p}(s)$ are the i-th basis functions with degree p.

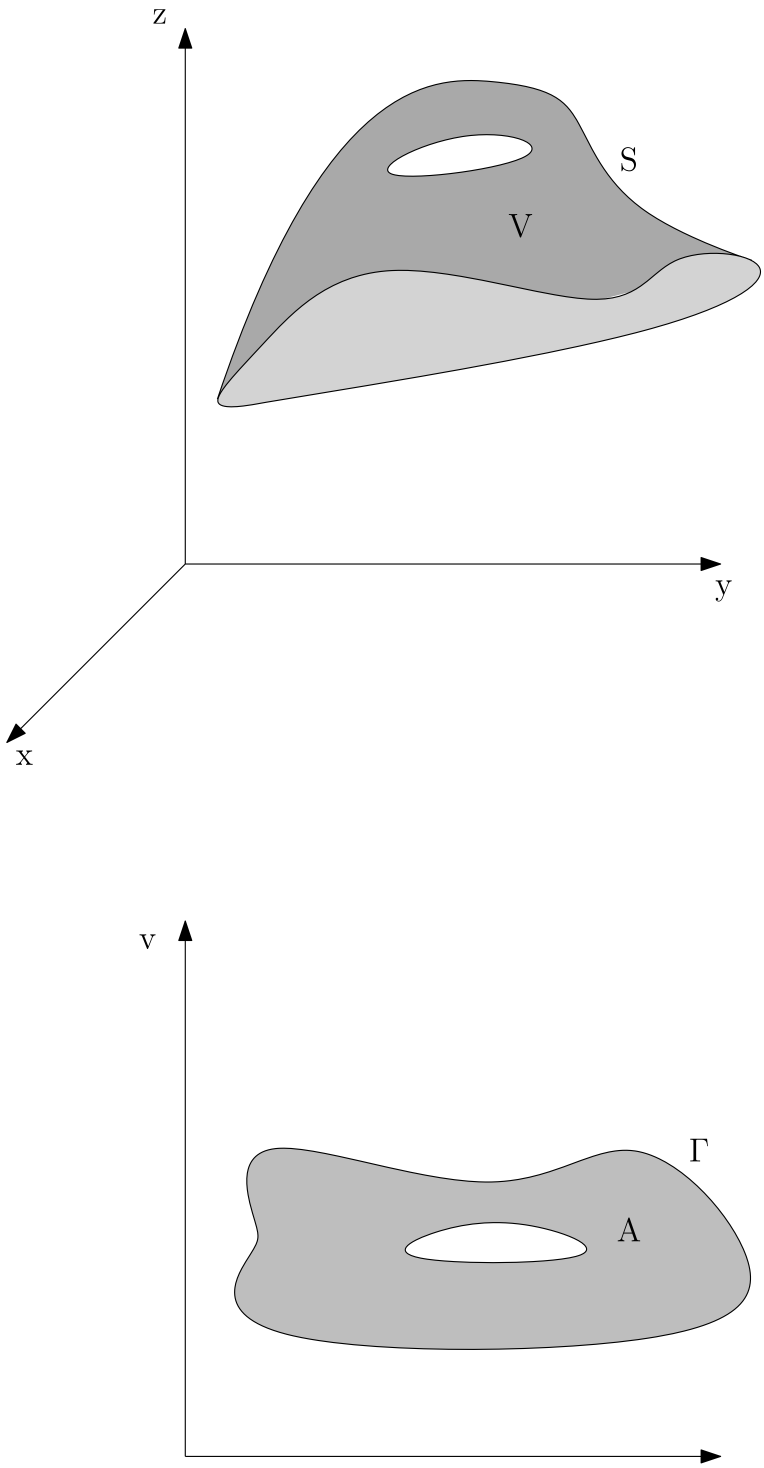

Figure 3: ***From u and v coordinates to x,y and z.***

2.2 Calculation of the surface area of three-dimensional solids

Consider a three-dimensional figure with an arbitrary surface S. Its surface area is given by:

$$A_S = \iint_S dS \tag{13}$$

With a variables change from x,y and z to u and v, as shown in Figure 4, the equation of surface area can be written as follows:

$$A_S = \iint_S dS = \iint_A J(u,v)dudv \tag{14}$$

where the function $J(u,v)$ is the transformation's jacobian given by:

$$J(u,v) = \sqrt{g_1^2 + g_2^2 + g_3^2} \tag{15}$$

and g_1, g_2 e g_3 are:

$$g1 = \frac{\partial y}{\partial u}\frac{\partial z}{\partial v} - \frac{\partial y}{\partial v}\frac{\partial z}{\partial u} \tag{16}$$

$$g2 = \frac{\partial x}{\partial v}\frac{\partial z}{\partial u} - \frac{\partial x}{\partial u}\frac{\partial z}{\partial v} \tag{17}$$

$$g3 = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u} \tag{18}$$

Using (19), equation (13) may be represented as:

$$A_S = \int_\Gamma F\frac{\mathbf{n}\cdot\mathbf{r}}{r}d\Gamma \tag{19}$$

where F is:

$$F = \int_0^r J\rho d\rho \tag{20}$$

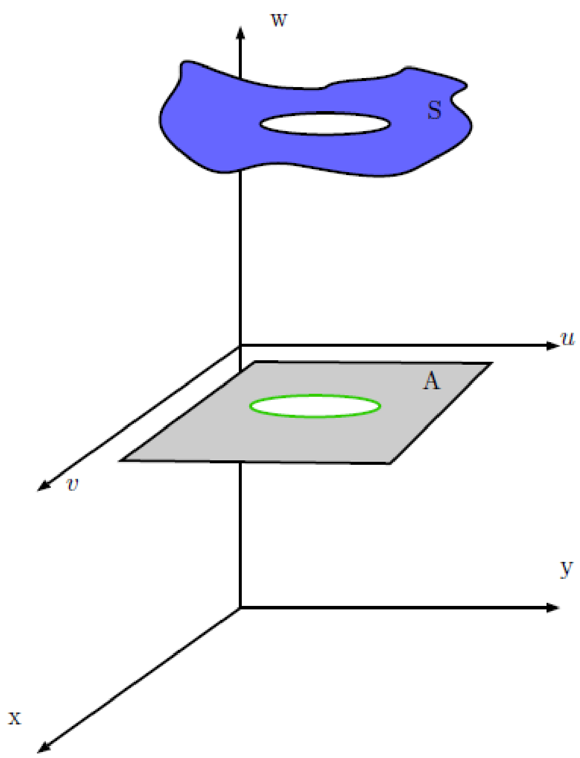

Figure 4: ***Transformation from 3D to 2D.***

and γ is the boundary of the area A. For the calculation of F it will be used the Gaussian quadrature. It is often solved over $[-1, 1]$, so a variable change is needed for a change of interval.

$$F = \int_0^r J\rho d\rho = \int_{-1}^1 J\rho \frac{d\rho}{d\xi} d\xi \tag{21}$$

To know the value of $\frac{d\rho}{d\xi}$, a linear interpolation shall be done:

$$\rho = a\xi + b \tag{22}$$

$$\begin{cases} \rho(\xi = -1) = 0 \\ \rho(\xi = 1) = r \end{cases} \rightarrow \begin{cases} 0 = a(-1) + b \\ r = a(1) + b \end{cases} \tag{23}$$

Solving the system:

$$a = b = \frac{r}{2} \tag{24}$$

which means:

$$\rho = \frac{r}{2}(\xi + 1) \tag{25}$$

$$\frac{d\rho}{d\xi} = \frac{r}{2} \tag{26}$$

Solving F using the Gaussian quadrature leads to:

$$F = \int_0^r J\rho d\rho = \int_{-1}^1 J\rho \frac{d\rho}{d\xi} d\xi = \sum_{k=1}^{p} J_k \rho_k \frac{r}{2} w_k \tag{27}$$

where p is the number of Gauss' evaluation points.

2.3 Calculation of the volume of 3D figures

The volume of an arbitrary three-dimensional figure is given by:

$$V = \iiint_V dV \tag{28}$$

To transform the volume integral into a surface one, the Gauss' theorem is used, which states that a volume integral of a function's **R** divergence equals to a surface integral **R**. It means that:

$$\iiint_V \nabla \cdot \mathbf{R}\, dV = \iint_S \mathbf{R} \cdot dS = \iint_S (\mathbf{R} \cdot \mathbf{n})\, dS \tag{29}$$

where **n** is given by:

$$\mathbf{n} = n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k} \tag{30}$$

For the volume integral theorem to equals the volume, the divergence of the function **R** must be 1. Thus, the function **R** becomes:

$$\mathbf{R} = \frac{1}{3}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \tag{31}$$

So, equation (28) may be written as:

$$V = \iiint_V dV = \iint_S (\mathbf{R} \cdot \mathbf{n})\, dS = \iint_S \frac{1}{3}(xn_x + yn_y + zn_z)dS \tag{32}$$

Now that the volume may be solved by an surface integral, the same steps used in the calculation of the surface area are followed. First, transform the variables x, y and z into u and v:

$$V == \iint_S \frac{1}{3}(xn_x + yn_y + zn_z)dS = \iint_A \frac{1}{3}(x(u,v) + y(u,v) + z(u,v))J(u,v)dudv \tag{33}$$

where $J(u,v)$ is the transformation's Jacobian (15).

Then the RIM is used to reach the equation:

$$V = \int_\Gamma F \frac{\mathbf{n} \cdot \mathbf{r}}{r} d\Gamma \tag{34}$$

In this case, F is given by:

$$F = \int_0^r \frac{1}{3}(xn_x + yn_y + zn_z)J\rho d\rho \tag{35}$$

For the calculation of F, the Gaussian quadrature is used.

$$\begin{gathered} F = \int_0^r \frac{1}{3}(xn_x + yn_y + zn_z)J\rho d\rho = \int_{-1}^1 \frac{1}{3}(xn_x + yn_y + zn_z)J\rho \frac{d\rho}{d\xi} d\xi = \\ \sum_{k=1}^p \frac{1}{3}(x_k n_x + y_k n_y + z_k n_z)J_k \rho_k \frac{r}{2} w_k \end{gathered} \tag{36}$$

where p is the number of evaluation points and w_k is the Gauss weight.

3 SOFTWARE'S DESCRIPTION

With the necessary equation for obtaining both the surface area and the volume, it was possible the implementation of the software *PropGeoIGES*, coded in MATLAB, which is capable of computing surface areas and volume of solids. Those solids are modelated using a CAD like SolidWorks or FreeCAD and exported in IGES files.

The software *PropGeoIGES* uses the IGES toolbox (available at `https://www.mathworks.com/matlabcentral/fileexchange/13253-iges-toolbox`) to extract the parameter data of an IGES file and to plot the solid. It has the following steps:

- Parameter data extraction from the IGES file.
- Plotting of the solid.
- Surface area and volume calculation.
- Presentation of the results.

The following flowchart given in Figure 5 shows the set of implemented functions. Those functions which constitutes the software will be described next:

iges2matlab: Extracts IGES parameter data from a CAD-file into Matlab.

plotIGES: Plots the IGES CAD-model.

calc_F : Makes the calculation of the F integral, in equation (27), for each integration point of an element. This function receive, as input, the value of r (radius), as well as Gauss' points and weights and outputs the integral F solved for surface area and volume.

calcula_PropGeo: Makes the calculation of each isogeometric elements integral and sums them, resulting in the surface area and volume of the solid.

nrbDerivativesIGES: Returns first and second derivative representation of NURBS.

PropGeo: Receive, as input, the NURBS surface data, its derivatives, the number of surfaces that the solid will be divided and the values of surface area and volume for each surface, solved by the function calcula_PropGeo. Makes the sum of each surface, resulting in the area and volume of the solid.

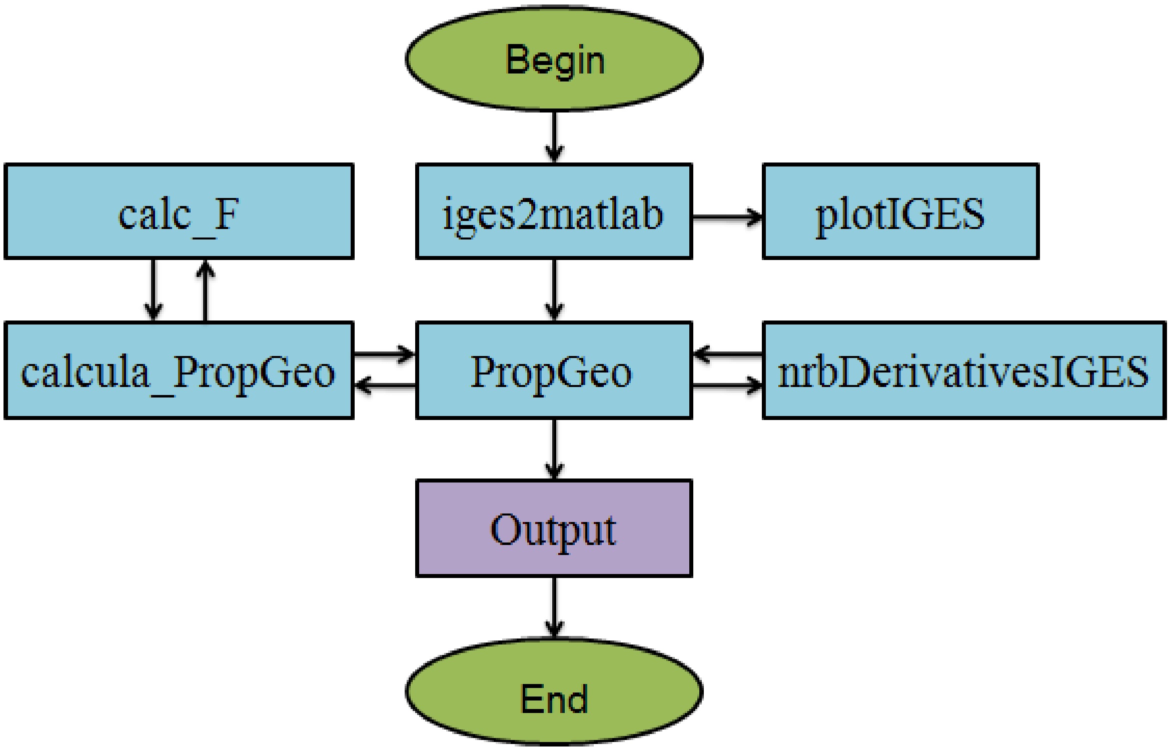

Figure 5: ***Software's flowchart (PropGeoIGES).***

4 RESULTS

The results obtained with the software based on the RIM are now shown. As discussed before, it was coded in Matlab and the solid was modeled in SolidWorks and exported as an IGES file.

First, simple solids - which both the surface area and volume may be obtained analitically - are shown; then more complex ones. Besides the surface area and volume values obtained by the software, those obtained analitically are also described with the corresponding error. For the more complex surfaces, the value obtained by software calculation is compared to values given by the SolidWorks CAD.

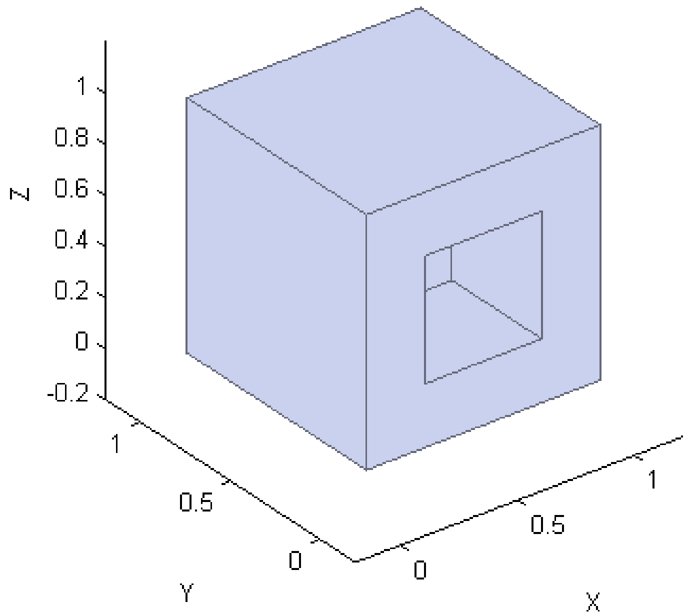

Figure 6: ***Cube with hole.***

Table 1: Properties of the cube with hole

Property	Value
Surface area obtained numerically	7
Surface area obtained analitically	7
Absolute error	1,7497 10^{-13}
Percentage error	2,4996 10^{-12}

IGES entity types used in this work are:

102: Groups other curves to form a composite. Can use Ordered List, Point, Connected Point, and Parameterized Curve entities.
110: Defines a line using an end point and a start point.
124: Transforms entities by matrix multiplication and vector addition to give a translation.
126: Composes analytic curves. Form: 0=Determined by data 1=Line 2=Circular arc 3=Elliptical arc 4=Parabolic arc 5=Hyperbolic arc.
128: This is a surface entity defined by multiple surfaces. The form number describes the general type: 0=determined from data, 1=Plane, 2=Right circular cylinder, 3=Cone, 4=Sphere, 5=Torus, 6=Surface of revolution, 7=Tabulated cylinder, 8=Ruled surface, 9=General quadratic surface.
142: Associates a curve and a surface, gives how a curve lies on the specified surface.
144: Describes a surface trimmed by a boundary consisting of boundary Curves.

Their definitions can be seen in $https://wiki.eclipse.org/IGES\_file\_Specification$.

Table 2: Cube with hole - Entities in ParameterData

Entity type	Number of entities	Entity name
128	11	B-NURBS Surface
126	48	B-NURBS Curve
110	48	Line
102	24	Composite Curve
142	12	Curve on a Parametric Surface
144	11	Trimmed Surface
124	0	Transformation Matrix

Table 3: Properties of the plate with hole

Property	Value
Surface area obtained numerically	41,7756
Surface area obtained analitically	41,8584
Absolute error	0,0828
Percentage error	0,1978

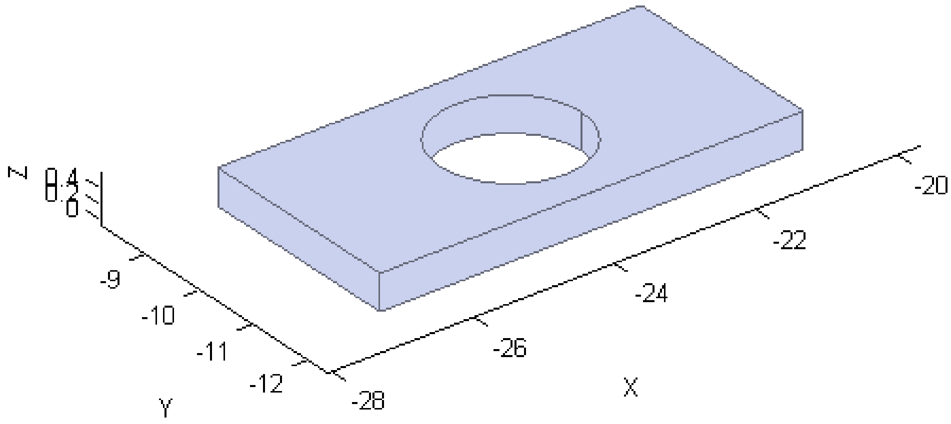

Figure 7: ***Plate with hole*.**

Table 4: Plate with hole - Entities in ParameterData

Entity type	Number of entities	Entity name
128	8	B-NURBS Surface
126	44	B-NURBS Curve
110	40	Line
102	22	Composite Curve
142	10	Curve on a Parametric Surface
144	8	Trimmed Surface
124	2	Transformation Matrix

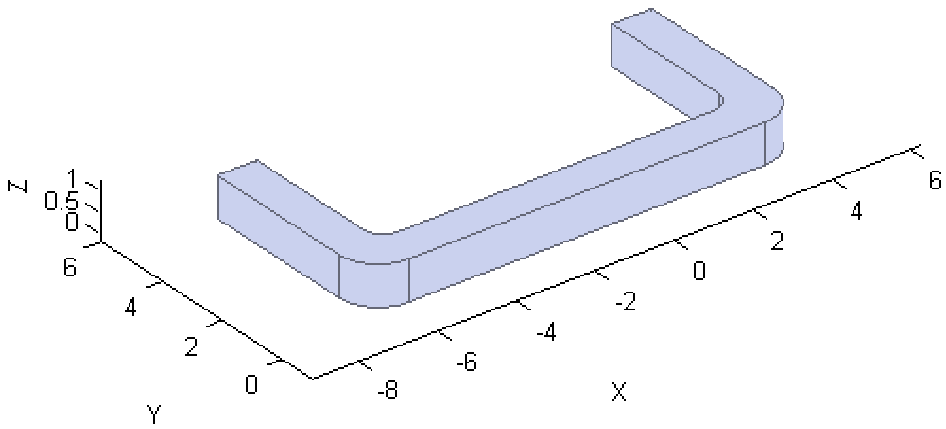

Figure 8: ***U shape*.**

Table 5: U shape properties

Property	Value
Surface area obtained numerically	76,0298
Surface area given by SolidWorks	76,07
Absolute error	0,0402
Percentage error	0,0528

Table 6: U shape - Entities in ParameterData

Entity type	Number of entities	Entity name
128	14	B-NURBS Surface
126	88	B-NURBS Curve
110	80	Line
102	32	Composite Curve
142	14	Curve on a Parametric Surface
144	18	Trimmed Surface
124	8	Transformation Matrix

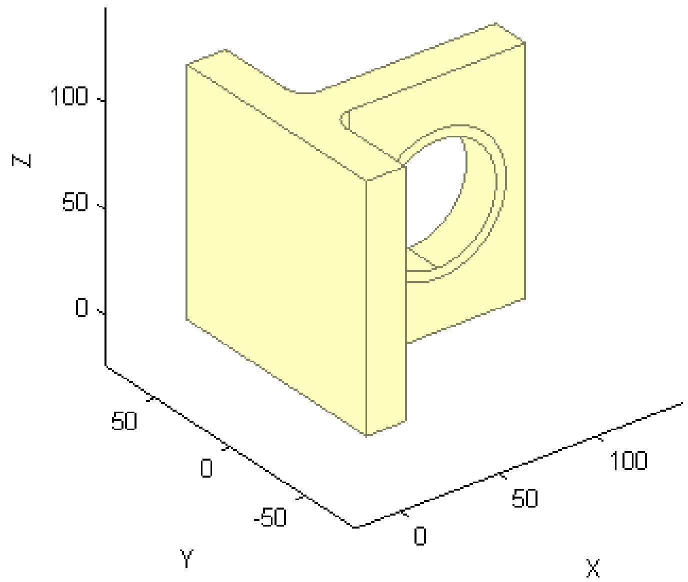

Figure 9: ***Bracket*.**

Table 7: Properties of the bracket

Property	Value
Surface area obtained numerically	63427,0946
Surface area given by SolidWorks	63570,80
Absolute error	143,0754
Percentage error	0,2231

Table 8: Bracket - Entities in ParameterData

Entity type	Number of entities	Entity name
128	16	B-NURBS Surface
126	104	B-NURBS Curve
110	80	Line
102	44	Composite Curve
142	20	Curve on a Parametric Surface
144	16	Trimmed Surface
124	18	Transformation Matrix

Table 9: Properties of the gear

Property	Value
Surface area obtained numerically	165950,5517
Surface area given by SolidWorks	166145,70
Absolute error	196,1483
Percentaqge error	0,1181

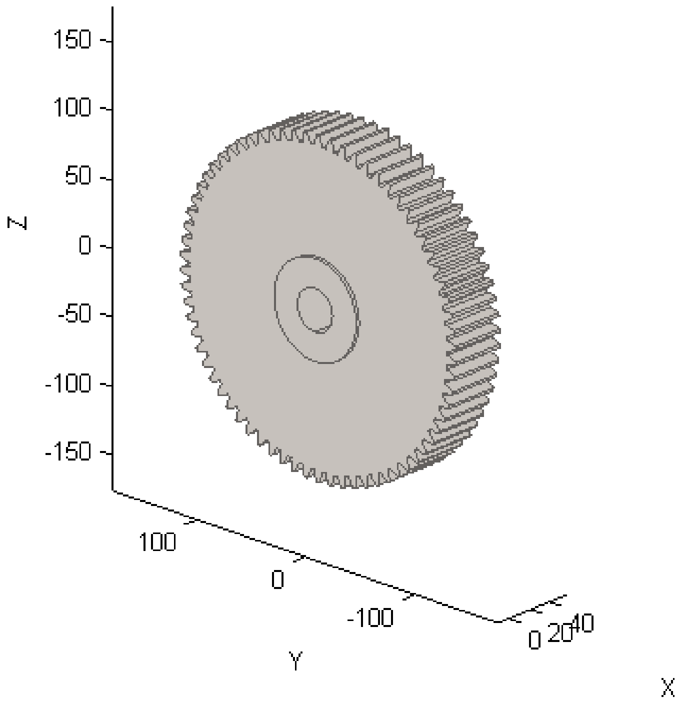

Figure 10: ***Gear*.**

Table 10: Gear - Entities in ParameterData

Entity type	Number of entities	Entity name
128	295	B-NURBS Surface
126	2910	B-NURBS Curve
110	2328	Line
102	889	Composite Curve
142	299	Curve on a Parametric Surface
144	295	Trimmed Surface
124	1164	Transformation Matrix

5 CONCLUSION

For this study, a software was coded in Matlab for the calculation of the surface area using the RIM applied to solids modelled in CAD and exported as IGES files. The numerically obtained results were close to the analitically obtained or given by the SolidWorks CAD. All errors were up to 1%.

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