EVALUATION OF SAFETY PUNCHING IN SLABS FLATS WITH SHEAR REINFORCEMENT BY RELIABILITY ANALYSIS

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Abstract. *The objective of this work is to evaluate the safety index of punching in slabs flats with shear reinforcement by reliability analysis using the FORM (First Order Reliability Method). The random variables considered in the problem will be generated randomly considering each type of probability distribution involved in the problem and parameters established in the literature. Variations of critical perimeter, rate of slab shear reinforcement, coefficient of resistance of concrete to compression, coefficient of resistance of steel to traction and permanent and accidental loads were performed in order to analyze the level of safety for the oscillations performed. The analysis was done for the formulation NBR 6118:2014 to study the behavior of the variables.*

Keywords: *Slabs Flats, Punching, Shear Reinforcement, Structural Reliability, FORM.*

INTRODUCTION

The structure of a building must be guaranteed for safety, functionality and durability throughout its life, while maintaining its design characteristics at a reasonable cost of execution and maintenance.

In the execution of the structural projects the designer coexists with the existence of uncertainties. Often the existing data are incomplete or insufficient and certainly contain variability. The consequence of these uncertainties is the existence of a probability of failure of the structure (SAGRILO, 2004). The risk of failure of the structure must be within acceptable safety margins. The structural reliability analysis aims to evaluate the probability that it does not exceed these limits. In this work the reliability analysis will be used as a tool for the evaluation of puncture safety in slabs.

The design of a structure involves a set of essential and prior steps to the constructive process. According to Pantoja (2012), the ultimate goal of this whole process is that the structure performs satisfactorily all the functions for which it was designed, with maximum economy, efficiency and durability.

Flat slabs, also called flat slabs, are horizontal laminated structures in reinforced or prestressed concrete that rest rigidly and directly on pillars, and there are no beams for transferring stresses. The use of plain slab structures has become customary for many years. This is fundamentally due to the greater simplicity of the construction system, time savings, execution and costs, as well as the flexibility of using the constructed spaces (ALBUQUERQUE, 2010).

In recent years the use of the flat slab system has become quite common, especially in commercial buildings, due to the absence of beams this system allows a great flexibility in the layout of the building, as well as in the distribution of electrical and hydraulic installations. It is possible to mention several other advantages in the use of this type of slab, as simplification in the execution of the forms and less waste, reduction of the total height of the building and greater ease in the launching, densification and deformation of the concrete. However, this system also has certain disadvantages, among which the possibility of a puncture rupture in the pillar slab attachment region can be highlighted by a smaller load than would be the ruin by flexion (PANTOJA, 2016).

Albuquerque (2010) cites as one of the main disadvantages of flat slabs the punching of slabs by pillars. According to Trautwein (2006) the punching is characterized by the action of a concentrated force on an area of a flat structural element. This force will cause high shear stresses in its surroundings, which can cause the element to be ruined. In smooth slabs, the pillar introduces this concentrated force, and the ruin occurs in the slab-pillar connection. Puncture rupture can occur abruptly and without notice. The objective of this work is to evaluate the puncture safety indices in shear reinforced plain slabs by reliability analysis by the FORM (First Order Reliability Method).

1 THEORETICAL REFERENCE

In this topic it explains some subjects that gave the basis for the development of this work.

1.1 Structural Security

The concept of structural safety is linked to the capacity of a given structure to withstand the various actions imposed on it during its useful life, also satisfying the functional conditions to which its construction was destined. It is possible to observe a close relationship between the reliability and safety of a structure (PANTOJA, 2012).

According to Lima (2013) failure in the structure can generate total or partial collapse, ultimate limit state, or interruption of its normal use, service limit state. Since the second can be reversible, in this case, failure will only be characterized when the passage to the undesirable state occurs at an inopportune time, for a very prolonged period, an excessive number of times or any combination thereof.

The question of structural security is based on boundary states, which represent the various interfaces between a permissible or unacceptable performance of the structure (BECK, 2010).

Rules that limit the values of actions, displacements and deformations present in a structure represent the level of security associated with it. Limits of this nature exist as a result of the evolution of knowledge in engineering and are constantly modified. The degree of uncertainty present in the variables can cause unforeseen that the deterministic hypotheses even conveniently used are not able to detect (PANTOJA, 2012).

1.2 Reliability Analysis

Statistically we can define reliability as an event complementary to the possibility of failure, in the form of Eq. 1.

$$C = 1 - P_f \tag{1}$$

Where P_f is the probability of failure of the structure during its useful life, that is, the probability of a collapse situation occurring in that period in which reliability is applied. In the case of civil construction, probability values of failure in the order of 10^{-7} to 10^{-3} are used.

The reliability allows to obtain the various failure modes of the structure by means of the calculation of the probability of failure (P_f), in this way it is essential for a decision on structural security and economic viability.

The reliability compares the effect of the load S with the resistance R of a structural element, where R and S are random variables and can be described by probability distributions f_R (.) and f_S (.) respectively (NOGUEIRA, 2005).

According to Beck (2008), a structural problem involving only random variables is classified as a problem independent of time, according to equation 2, where the random variables are R and S, and the vector of random variables that characterizes the problem is called of X.

$$g(X) = g(X_R, X_S) = g(X_1, X_2, \ldots, X_n) = 0 \tag{2}$$

Therefore, the fault domain D_f is defined as the set of all values that X can assume and that lead to failure of the structure. And the survival or insurance domain DS is the set of other values that X can assume, Eq. 3 and 4.

$Df = \{x|g(x) \leq 0\}$ (3)

$DS = \{x|g(x) > 0\}$ (4)

According to Beck (2008), the probability of failure of a structural component or system is given by Eq. 5.

$$P_f = \int_{D_f} f_X(x)dx \quad (5)$$

Where, $f_X(x)$ is the joint probability density function.

The boundary between the behavioral passages of the reversible and irreversible states, represented by equations of the boundary states, is given by $G(x) = 0$, where x are the project variables.

These state functions $G(x)$ are usually defined (LOPES, 2007):

$G(x) = R(x) - S(x)$ (6)

Where,

x is the vector of random variables;

$R(x)$ is the resistance of the element;

$S(x)$ is the request imposed on the element.

Thus, there are two domains, as indicated in Fig. 1: fault domain when $G(x) \leq 0$, which indicates that the structure is not secure; And survival domain when $G(x) > 0$, indicating that the structure is secure (BECK, 2008).

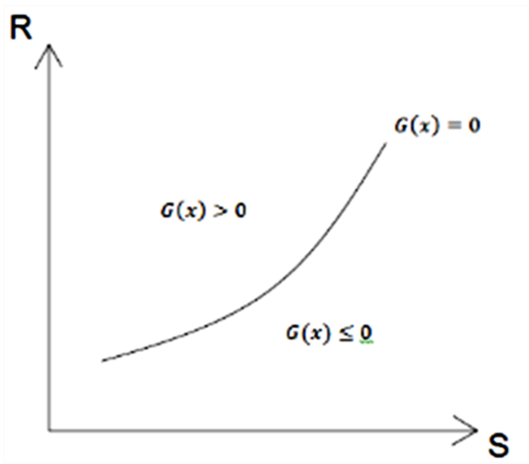

Figure 1: Domains of failure and not failure. Source: LIMA (2013).

In a formulation considering reliability, a structural risk acceptance criterion corresponds to the minimum reliability level defined as the target reliability index. Thus the safety level of a structure is expressed in terms of a minimum reliability index or an acceptable failure probability. Reliability index reference values for ultimate limit states were proposed by JCSS (2001) and are reproduced in Table 1 (PANTOJA, 2012).

Table 1. Reference values for the reliability index β and associated failure probabilities related to ultimate limit states.

Relative cost of security considerations	Light Failure Consequences	Moderate Failure Consequences	Serious Failure Consequences
Big	$\beta = 3{,}1(P_f \approx 10^{-3})$	$\beta = 3{,}3(P_f \approx 10^{-4})$	$\beta = 3{,}7(P_f \approx 10^{-4})$
Normal	$\beta = 3{,}7(P_f \approx 10^{-4})$	$\beta = 4{,}2(P_f \approx 10^{-5})$	$\beta = 4{,}4(P_f \approx 10^{-6})$
Little	$\beta = 4{,}2(P_f \approx 10^{-5})$	$\beta = 4{,}4(P_f \approx 10^{-6})$	$\beta = 4{,}7(P_f \approx 10^{-6})$

1.3 First Order Reliability Method (FORM)

The FORM is constituted by different methodologies to determine the reliability index and the probability of failure. All the methods that constitute it are characterized by working with the first degree of derivation of the performance function and all are restricted to using the first and second statistical moment (mean and standard deviation).

In this article, will be discretized the technique known as FOSM-HL-RF, developed by Rackwitz and Fiessler who adapted the FOSM-HL method, which was developed by Hasofer and Lind. The FOSM-HL has as principle that the design point is not necessarily on the average as the original FOSM method estimated. Thus, the reliability index represents the shortest distance perpendicular to the surface formed by the limit state function ($G(X)$=0) in relation to the mean in the normalized space and consequently the design points can be around the mean of the analyzed variables or not. Fig. 2 graphically depicts what has been said above.

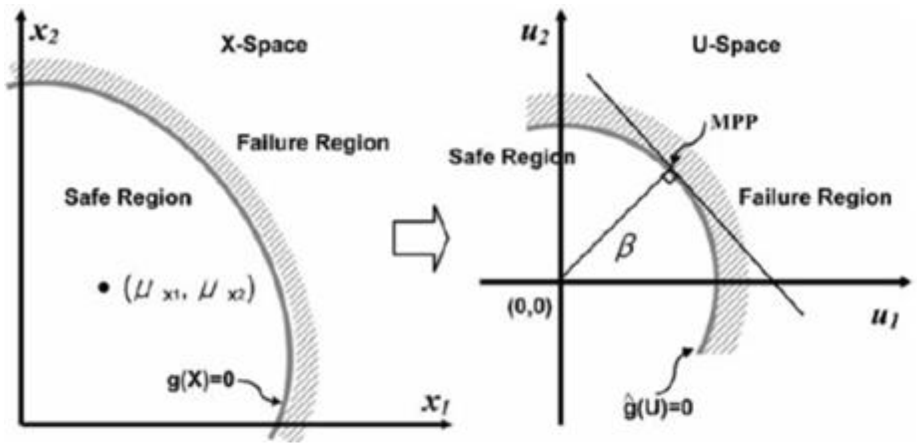

Figure 2 - Mapping of the space X for the normalized space U and geometric illustration of the reliability index. Source: Choi et al (2007).

However, FOSM-HL did not assess the influence of non-normal distributions and correlations. Thus, FOSM-HL-RF, also just called FORM, aims to address these deficiencies.

The step-by-step of how the algorithm works to calculate the FORM is demonstrated in Choi et al (2007).

1.4 Punching and NBR 6118 (ABNT, 2014)

Punching is the ultimate limit state by shear in the surroundings of concentrated forces. This phenomenon can be characterized as a rupture mode that occurs by shearing in elements submitted to the load or reaction applied transversely, generating high shear stresses in the region near the point of application, which can cause abrupt rupture without previous notice of the element. The analysis of the puncture is different from that performed for the ultimate limit state by shearing force, being of fundamental importance in the case of slabs that lean directly on pillars (ARAÚJO, 2010).

In the smooth slabs and mushroom, the typical region of the rupture is in the slab-pillar connection, due to the absence of beams. In this way, in practice, it is tried to minimize the tensions acting in the region where the greatest shear stresses occur with the use of capitals, thickening of the slab as a whole or in the region of force application, increasing the resistant capacity of the slab through the use high-strength concrete and the use of puncture reinforcement (BARBOSA, 2012).

The calculation model of NBR 6118 (ABNT, 2014) corresponds to the verification of shear on two or more critical surfaces defined in the surroundings of concentrated forces. In the first critical area (contour C), the pillar or the concentrated load, the diagonal compression stress of the concrete must be verified indirectly through the shear stress.

At the second critical surface (contour C') 2d of the pillar or concentrated load, the puncture attachment capability associated with diagonal tensile strength must be checked. This verification is also done by a shear stress in the contour C'. If necessary, the connection must be reinforced by transverse reinforcement.

The third critical surface (contour C'') should only be checked when transverse reinforcement is required.

When it comes to an internal pillar case with symmetric loading the shear stress of request must be given by Eq. 7:

$$\tau_{Sd} = \frac{P_{Sd}}{u.d} \tag{7}$$

Where:

d: useful height of the slab along the critical contour C', external to the contour C of the area of application of the force and distant 2d in the plane of the slab. It should be calculated as the average of the sum of the useful heights in the two orthogonal directions (dx and dy);

u: perimeter of critical contour C';

P_{Sd}: force or concentrated reaction of calculation.

The punching force PSd can be reduced from the distributed force applied to the opposite face of the slab, within the contour considered in the verification, C or C', as shown on Fig. 3:

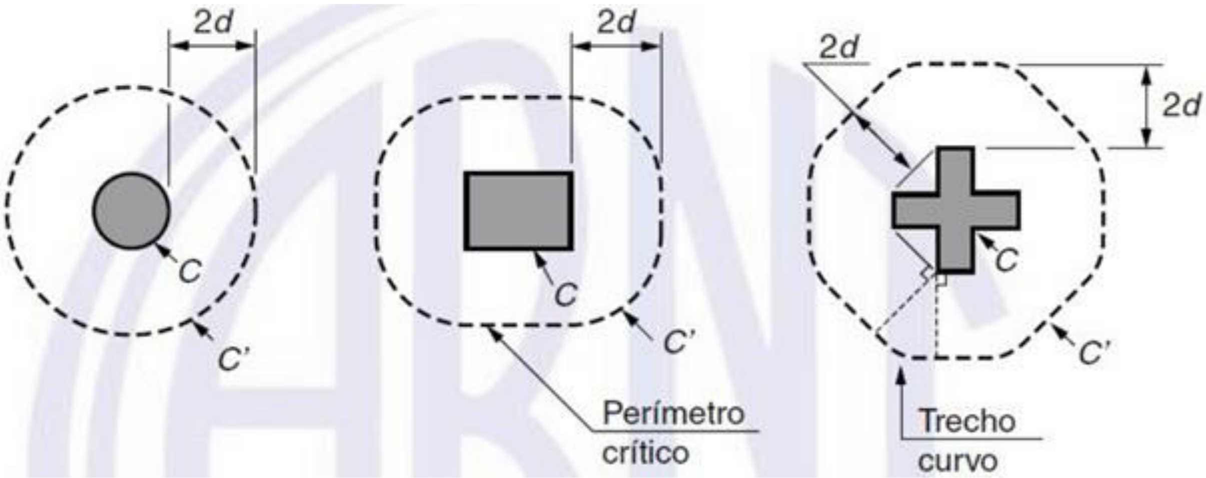

Figure 3 - Definition of critical perimeter.

A check of the tensile stress of diagonal compression of the concrete on the critical surface C (Eq. 8), ie, close to the abutment, is started.

$$\tau_{Sd} \leq \tau_{rd2} = 0{,}27\alpha_v f_{cd} \quad (8)$$

Where:

$\alpha_v = (1 - f_{ck}/250)$, to f_{ck} in megapascal.

The tensile strength at contour C' should be calculated for the case of slab without shear reinforcement according to Eq. 9:

$$\tau_{Sd} < \tau_{rd1} = 0{,}13\left(1 + \sqrt{\frac{20}{d}}\right)(100\rho f_{ck})^{1/3} \quad (9)$$

It being:

ρ: area of steel divided by concrete area.

The tensile strength at the contour C' should be calculated for the case of shear with the shear reinforcement according to Eq. 10:

$$\tau_{Sd} < \tau_{rd1} = 0{,}1\left(1 + \sqrt{\frac{20}{d}}\right)(100\rho f_{ck})^{1/3} + 1{,}5\frac{d}{S_r}\frac{A_{sw} f_{ywd} \sin\alpha}{u.d} \quad (10)$$

Where:

S_r: radial spacing between punching armature lines (must be less than 0,75d);

A_{sw}: area of puncture reinforcement;

α: slope angle between the puncture reinforcement and the plane of the slab;

u: critical perimeter (mm);

f_{ywd}: Punching armature calculation resistance (MPa).

When it is necessary to use transverse reinforcement, it must be extended in contours parallel to C' until, in a distant contour 2d of the last reinforcement contour, it is no longer necessary, that is, $\tau_{Sd} \leq \tau_{rd}1$.

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2 METHODOLOGY

The failure modes of punching verification on smooth slabs are studied. Failure functions are defined based on the checks defined in standard NBR 6118 (ABNT, 2014) as demonstrated in the previous chapter.

The smooth slab is punching by a full load applied to the abutment on which it is supported (Fig. 4). The breakage of the part can occur on the face C, located next to the pillar, or on the face C' located at a distance 'x' from the colunm, which varies from formulation to formulation.

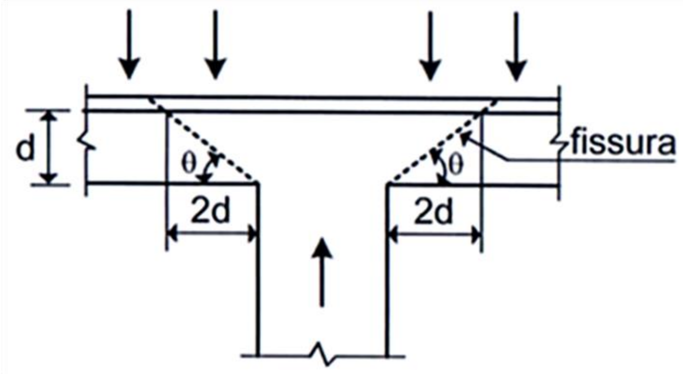

Figure 4 - Ruin by punching in smooth slab.

For structural reliability analysis, an example is taken from the book Reinforcement Concrete Course (ARAÚJO, 2010) vol. 4, page 235. In the example it is requested to verify the punching resistance of the abutments of the plain slab indicated in Fig. 5.

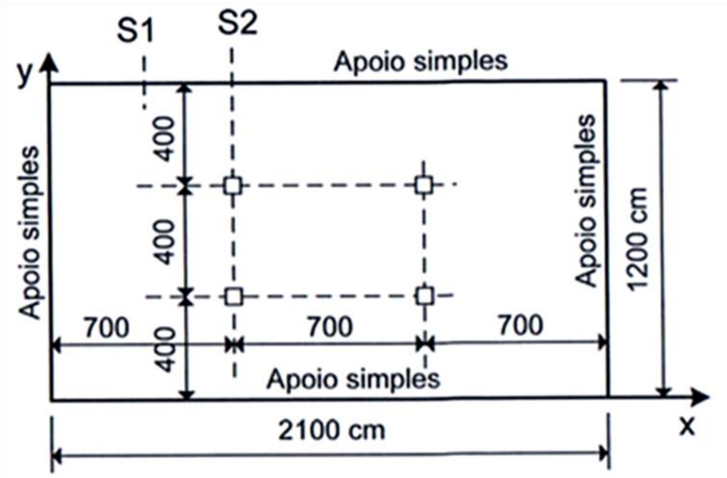

Figure 5 - Plain slab with center column.

For verification, are given:

Pillar section: $c1 = c2 = 40\ cm$

Concrete: $fck = 30\ MPa$

Slab thickness: h=24 cm;d=20 cm

Slab punching force: Fk=300 kN

Steel: fyd=300 MPa

In the example, the average compressive strength (fcm), mean tensile strength (ftm), critical perimeter (dm), shear reinforcement (Asw) and accidental (Pq) and permanent (Pg) loads are considered as random variables. The parameters that will vary and the distributions that will represent them are determined according to the JCSS (Joint Committee on Structural Safety). Table 2 shows the values adopted for the random variables of the problem during the reliability analysis.

Table 2 - Probabilistic models of the random variables for slab with shear reinforcement.

Random Variable	Medium value	Standard deviation	Coefficient of variation (%)	Type of Distribution
fcm (MPa)	30	5,1	17	Lognormal
d (m)	0,2	0,015	7,5	Normal
Pg (MN)	0,21	0,0105	5	Normal
Pq (MN)	0,14	0,035	25	Gumbel
ftm (MPa)	300	15	5	Lognormal
Asw (cm^2)	-	-	-	Deterministic

The variations were performed for shear reinforcement, assuming the following values: 20 cm^2/m, 30 cm^2/m, 40 cm^2/m and 50 cm^2/m.

For slabs with shear reinforcement failure mode 1 indicates the crushing of concrete close to the abutment, failure mode 2 is the crushing of the concrete at the critical perimeter, failure mode 3 is the flow of steel at the critical perimeter and Failure mode 4 is the crushing of the concrete outside the critical perimeter, as shown on Fig. 6.

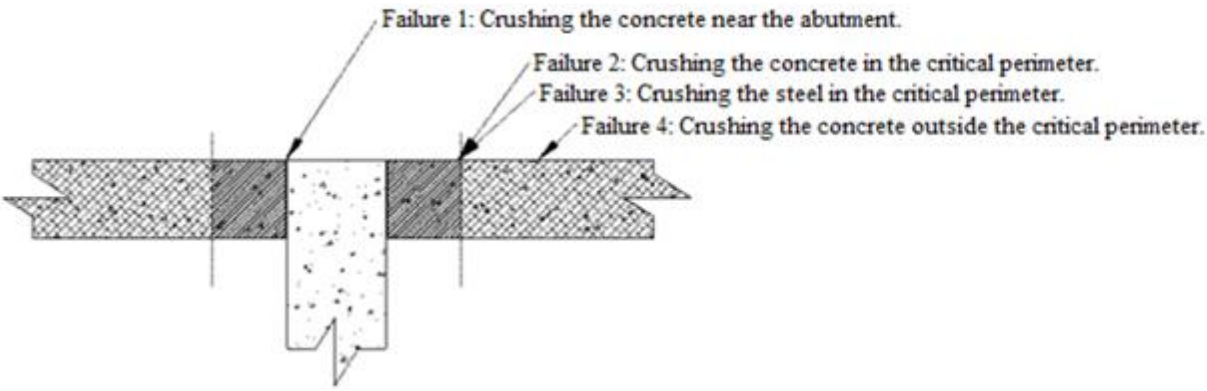

Figure 6 - Failure modes of the slab with shear reinforcement.

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3 RESULTS ANALYSIS

For the slabs with shear reinforcement the variations were performed for the following steel areas (A_{sw}): 20 cm²/m, 30 cm²/m, 40 cm²/m and 50 cm²/m. As in the slabs without shear reinforcement, a percentage variation of the permanent and accidental loads was first made, so that the sum was always 100%. The safety level was obtained via the reliability index (β) for the formulation of NBR 6118 (ABNT, 2014).

Fig. 7 shows the variation of the reliability index as a function of the increase of the accidental load. As the accidental load increases, there is a drop in the reliability indexes for all values of A_{sw} adopted.

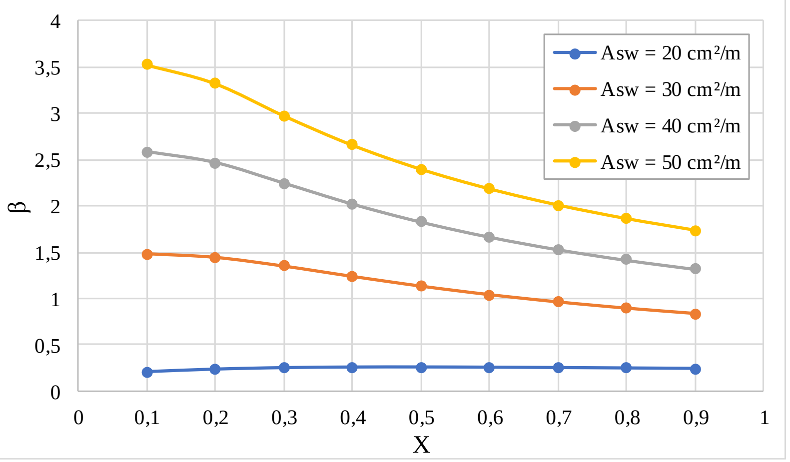

Figure 7 - Variability of the reliability index as a function of the increase of the accidental load.

The best results are seen for the lower proportion of accidental load built into the total load load in the structure, it can be concluded that there is an increase in structural confidence as steel reinforcement increases.

In the Fig. 8 shows the variation of the average compressive strength of the concrete. The increased resistance of the concrete has an immediate response in the confidence of the structure, the increase in the β index exists, but it is according to what was cited on Fig. 7.

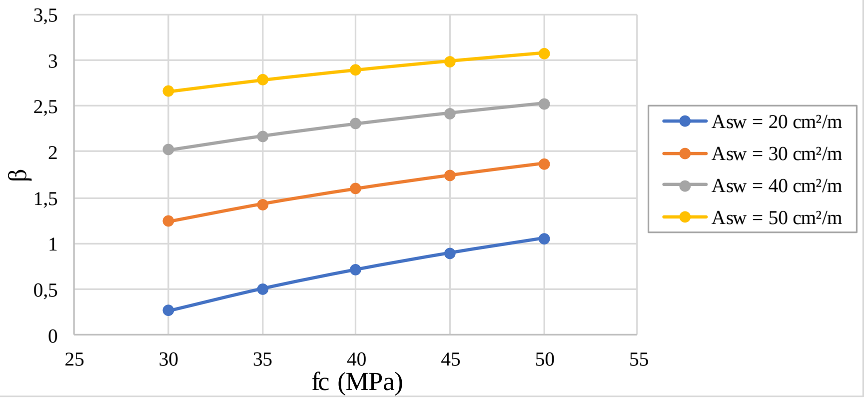

Figure 8 - Variation of the average compressive strength of the concrete.

In the Fig. 9 shows the variation of the tensile strength of the steel to change the area of shear reinforcement. This simulation was very important because it could be seen that the behavior is directly related to the strength of the steel of the armature and reflects in an expressive way to the results. When the resistance is 200 and 250 MPa, for A_{sw} of 20 cm²/m, we obtained values of negative β, indicating that we do not have confidence for such a situation because the structure failed there.

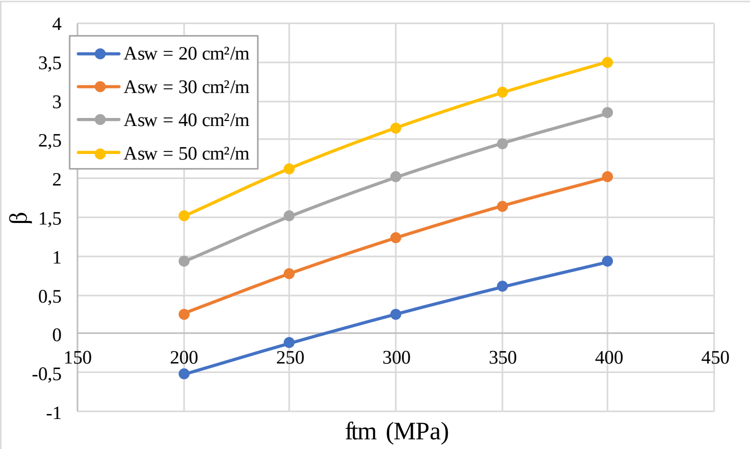

Figure 9 - Variation of the average tensile strength of the steel.

The critical perimeter is a parameter that greatly influences the slab's reliability to puncture. In the Fig. 10 shows the critical perimeter variation illustrates this fact, we have that with d equal to 15 cm, the structure loses confidence (β) in all cases, except when A_{sw} exceeds 50 cm²/m. For our example in which we only increase the useful height up to 35 cm we had a value of β very close to 6 (considered high > 4,7).

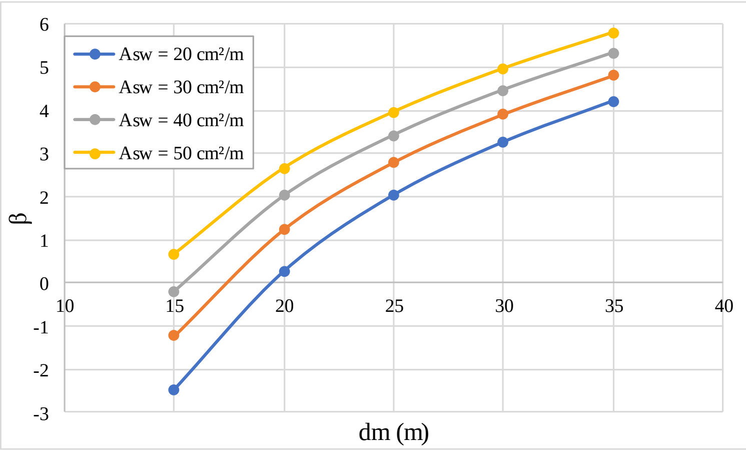

Figure 10 - Variation of the critical perimeter.

Four failure modes were mapped through NBR 6118 (ABNT, 2014) as described in the previous chapter. The Fig. 11 below shows the variation of the shear armature for each failure mode. However, except for Mode 3, there was no change in behavior because of the domain parameters of these modes are not connected to A_{sw}, in this way the reliability was made and a value of β was generated for the specific formulation of each one.

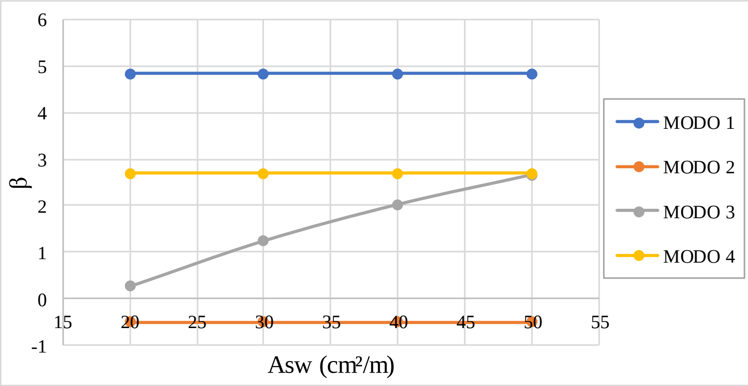

Figure 11 - Shear armature variation for each failure mode.

For Mode 3 it can be concluded that there was an exponential increase and that for armature from 50 cm²/m it is possible to obtain acceptable β.

4 FINALS CONSIDERATIONS

The study analyzed and compared how the different normative models of punching verification in slabs, via reliability analysis, reacted to the variation of the following parameters: variation of the permanent and accidental loads, the average compressive strength of the concrete, the average tensile strength The critical perimeter, the reinforcement area of the slab, and finally, for the slabs with shear reinforcement, the area of shear reinforcement. This study was done with the purpose of identifying which of the formulations would present a lower likelihood of failure, also showing which variables have the greatest influence on the safety of the structure according to each method and which of the formulations presents the highest reliability index for each of the variations.

The application of reliability analysis in the verification of the slab resistance to puncture provides parameters for a more accurate evaluation of its behavior, which can subsidize projects safer and more economical.

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