THERMODYNAMIC OPTIMIZATION OF A PLATE-FIN HEAT EXCHANGER BY OPPOSITIONAL-DIFFERENTIAL EVOLUTION

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Abstract. *In the present study, a single pass cross-flow plate-fin heat exchanger (PFHE) equipped with offset-strip fins was thermodynamically optimized. The more generated entropy in the exchanger, greater is the loss of potential work or heat/mass transfer, therefore, the minimization of the number of entropy generation was the main objective of the study. The method used for the modelling was the effectiveness-NTU. The process fluids are hot ammonia (NH3), referred as fluid A, and cold air, fluid B. They are superheated vapors, with pressure of 101325 Pa and temperatures of 400 K (ammonia) and 200 K (air). The optimization method used was the Oppositional Differential Evolution (ODE), a stochastic evolutionary-based method. The ODE is a variation of the classic Differential Evolution method. The variables subjected to the ODE were the exchanger's length for each fluid, La and Lb, the number of fin layers for A, Na, the external height, H, thickness, t, frequency, n, and the lance length, l, of the fins. The study was carried out in MATLAB®. The heat duty of the exchanger was fixed at 200 W. The achieved results were better than expected, with a number of entropy of 0.0609 and an exchanger effectiveness of 82.78%.*

Keywords: *Oppositional, Entropy, Optimization, Heat Exchanger.*

1. INTRODUCTION

The heat transfer of two fluids is performed by an equipment known as the heat exchanger. There are many applications of such: air conditioning systems, refrigeration cycles, waste heat recovery, industrial scale sterilization, general chemical processing, and others. The fluids flow along the equipment and the hot transfer its energy to the cold through a solid wall. Conduction and convection are of great importance in this equipment and it's essential to work with an overall heat transfer coefficient instead of the particular ones (Lavine *et al.*, 2014; Moran *et al.*, 2013).

According to Çengel & Ghajar (2012), the fluids can flow through the equipment in parallelly (cocurrent), in countercurrent or in crossflow. These different types of flow change various correlations of the equipment, allows different outlet conditions to be reached for the fluids and vary their temperature profiles. There are many ways to classify heat exchangers, as can be seen in Fig. (1) (Rohsenow *et al.*, 1998).

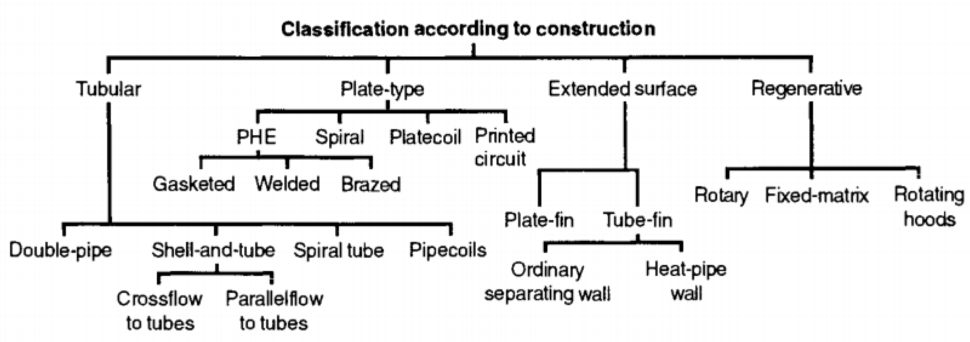

Figure 1 - Construction classification of heat exchangers.

Of the many presented types of exchanger, one of the most common is the shell-and-tube ones, which is composed of a several tubes contained within a shell in which the fluids flow. However, the requirement of high efficiency, flexibility, small and economic heat exchangers led to the development of the Plate Heat Exchanger (PHE). Two types of PHEs stand out: the gaskets and the plate-finned one (Gut, 2003).

The Plate-type Gasketed ones, Fig. (2), are composed of packs of corrugated metal plates installed by gaskets that form flow channels for hot and cold fluids. They flow alternately between the plates, so that each stream of cold fluid is surrounded by two streams of hot fluid, a fact that enhances the heat transfer (Gut, 2003)

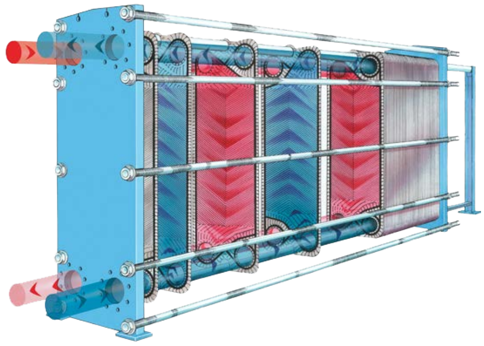

Figure 2 - Gasketed Plate Heat Exchangers.

In accordance do Kuppan (2000) and Rohsenow *et al*. (1998), the Plate-Fin Heat Exchangers (PFHE) are a combination of extended surface and plate exchangers composed of corrugated fins sandwiched between plates, called partition panels. They can be seen in Fig. (3). The fluids flow between the passages formed by the corrugations, and those can be attached to the plates by brazing, soldering, adhesive bonding, welding, or extrusion. The fins serve as a secondary heat exchanger area and grant mechanical support for the internal pressure of the equipment. PFHEs are commonly used for gas-gas operations, and can grant up to 6000 m^2/m^3 of heat transfer area per exchanger volume. Because of their great compactness, they are widely used in cryogenic gas separation processes, electric power plants (gas turbine, nuclear, steam), automobile plants, thermodynamic cycles (refrigeration and heat pump), electronics, air conditioning and aerospace services.

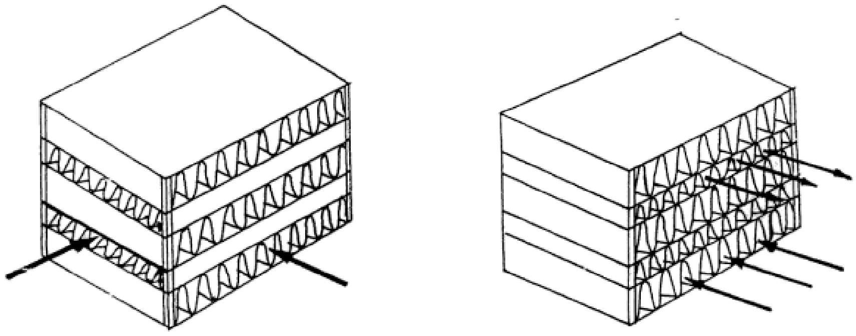

Figure 3 - Plate-fin heat exchangers. Cross-flow (left) and countercurrent (right).

The fin's shape can vary according to the application between wavy, triangular, rectangular, perforated and others. Permissible temperatures range from cryogenic uses to 1073.15 K and up until 14 MPa, depending on the material and fin type. These exchangers are not recommended for use when the fluid is excessively encrusting, since the cleaning of the passage ducts is not simple. In case of encrustation risk, the wavy fins are recommended and the serrated ones must be avoided. It is worth noticing that the combination of high pressure and extreme temperatures should be prevented. These exchangers provide approximately 25 times more thermal exchange area than the shell and tube types (Kuppan, 2000).

With the focus on cost minimization and operation improvement, optimization studies with the PFHEs have emerged, including the Genetic Algorithm (GA), used by Mishra *et al.* (2009), Particle Swarm Optimization (PSO) applied by Rao & Patel (2010) and Tsallis JADE used by Segundo *et al.* (2017). In the present study, the Oppositional Differential Evolution (ODE) proposed by Rahnamayan *et al.* (2008) was used to optimize the number of entropy generation in a Plate-Fin Heat Exchanger using offset strip fins.

The method is based on evolutionary principles where an initial population of possible solutions (solution vectors) are generated and improved by means of mutation, crossover and selection techniques. The ODE is an adaptation of the original Differential Evolution of Storn & Price (1995). In this variant, the opposing vectors are compared to the normal vectors in order to accelerate and possibly improve convergence.

The objective of the present work is to perform the analytic modeling of a cross-flow plate-fin (offset fin) heat exchanger with literature-based equations and its consequential optimization by means of the Oppositional Differential Evolution.

2. METHODOLOGY

2.1 Heat Exchanger Modeling

In the present work, the modeling of a cross-flow plate-fin heat exchanger was fulfilled, followed by its optimization with the Oppositional Differential Evolution. Since only the inlet temperatures of the fluids were known, the effectiveness-NTU method was chosen to calculate the exchanger's parameters. The program used was MATLAB®.

The considerations made for the mathematical modeling were: steady state; same specification fins used for both fluids; ideal gas behavior; constant heat transfer coefficients throughout the exchanger; constant free-flow and thermal exchange area; number of fin layers for fluid B is one more than A's and the fluids' thermophysical properties were taken at the average temperature between inlet and outlet conditions. The properties and the inlet conditions are shown in table 1.

Table 1 - Operation parameters.

Parameters	Ammonia (Fluid A)	Air (Fluid B)
Mass flow [kg/s]	1.5	1.2
Inlet temperature [K]	400	200
Inlet pressure [Pa]	101325	101325
Specific heat [J/kg.K]	2237.871	1006.655
Specific mass [kg/m³]	0.554	1.222
Prandlt number	0.877	0.711
Dynamic Viscosity [N.s/m²]	0.0000128	0.0000176
Specific gas constant [J/kg.K]	489.058	286.986
Exchanger's heat duty [W]	200	

In heat transfer, it is common to express certain parameters as dimensionless numbers. One of extreme importance is the Prandtl number (Pr), which provides a relative measure of the velocity and thermal boundary layer's thickness. Since it depends only on thermophysical properties, it comes as a tabulated value, which can be seen on Table 1. For gases, Pr is normally around 0,7. It is calculated as the ration of kinematic viscosity and thermal diffusivity (Lavine et al., 2014). Furthermore, it can be expressed with the dynamic viscosity, specific heat and thermal conductivity, as shown in Eq. (1).

$$Pr = \frac{\nu}{\alpha} = \frac{\mu . c_p}{k} \tag{1}$$

As the effectiveness-NTU method was applied, the specific heat capacity rates of the fluids were calculated with Eq. (2). The higher the thermal capacity, higher is the energy required to promote a temperature variation in the fluid.

$$C_i = \dot{m}_i . c_{p,i} \tag{2}$$

To calculate the total heat transfer rate, it is either possible to apply an energy balance to the hot and cold fluids, Eq. (3) and Eq. (4) respectively, or use the heat transfer rate equation, Eq. (5), that counts with the use of the overall heat transfer coefficient, U.

$$\dot{q} = C_{hot} . (T_{in,hot} - T_{out,hot}) \tag{3}$$

$$\dot{q} = C_{cold} . (T_{out,cold} - T_{in,cold}) \tag{4}$$

$$\dot{q} = U . A . \Delta T_{ML} \tag{5}$$

There is, though, another parameter of great importance, the effectiveness. It is calculated by the ratio of the actual heat transfer rate and the maximum possible transfer rate, as in Eq. (6).

$$\varepsilon = \frac{\dot{q}}{\dot{q}_{max}} \tag{6}$$

Where the heat transfer reaches its maximum in two situations: the hot fluid is cooled to the cold fluid inlet temperature or the cold fluid is heated up until the hot fluid inlet temperature. Therefore, the greatest temperature difference that could occur in a heat exchanger is the difference of the hot and cold inlet temperatures. Also, knowing that the fluid with the smaller heat capacity suffers greater temperature variations, it is safe to assume that the maximum heat transfer rate shall occur with the fluid of minimum heat capacity value. The maximum rate is calculated by Eq. (7).

$$\dot{q} = \varepsilon . C_{mín} . \left(T_{hot,in} - T_{cold,in}\right) \tag{7}$$

The calculation of the exchanger's effectiveness, ε, counts with specific correlations and charts, which vary with the type of exchanger, flow and number of passes. For a single-pass, cross-flow exchanger with unmixed fluids, the correlation is Eq. (8) and Fig. (2) is the respective chart.

$$\varepsilon = 1 - \left[\exp\left(\frac{1}{C_r}\right) NTU^{0,22} . \{\exp[-C_r . NTU^{0,78}] - 1\}\right] \tag{8}$$

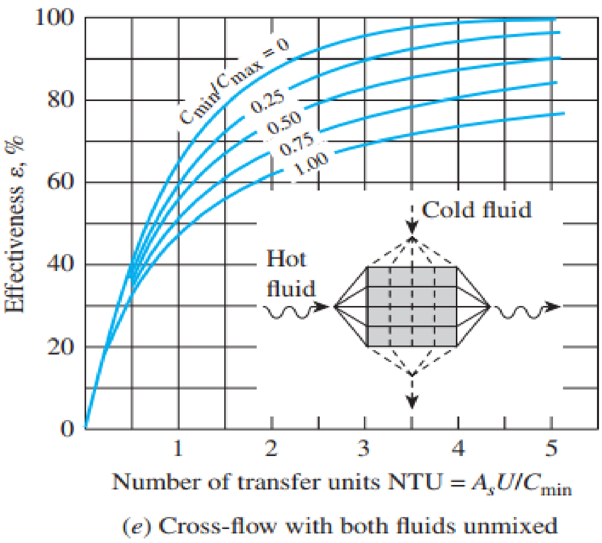

Figure 4 - Effectiveness chart.

The term C_r in Eq. (8) is a dimensionless parameter used in all correlations of effectiveness. It is called the capacitance ratio, which is the ratio of the minimum thermal capacity (the smaller value between two) by the maximum. It is calculated by calculated by Eq. (9).

$$C_r = \frac{C_{mín}}{C_{máx}} \tag{9}$$

From Eq. (8), NTU refers to the dimensionless parameter called number of transfer units. It is a measure of the amount of energy that can be transferred in the process and reflects the size of the heat exchanger. High NTU values represent a high heat transfer rate, as well as large equipment. This is a "thermal size" of the exchanger, which indicates the area of thermal exchange required for a certain service, given an overall transfer coefficient U.

Small NTU values indicate that there are more opportunities for heat transfer if the length of the equipment is increased. High values reflect a large surface area of thermal exchange, which is positive for heat transfer, but can be harmful in economic terms. Çengel and Ghajar (2012) indicate its calculation in the form of Eq. (10).

$$NTU = \frac{U.A}{C_{mín}} \tag{10}$$

The total heat transfer area, A, in the previous equation is the sum of the individual heat transfer area for the fluids, which can be calculated by means of Eq. (11) for fluid A (ammonia) and Eq. (12) for fluid B (air).

$$A_a = L_a.L_b.N_a.[1 + 2n_a.(H_a - t_a)] \tag{11}$$

$$A_b = L_a.L_b.N_b.[1 + 2n_b.(H_b - t_b)] \tag{12}$$

Where L_i (on for each direction of fluid flow) is the exchanger length, as can be seen in Fig. (3). N_a and N_b are the number of fin layers for each fluid, n_i is the fin frequency per meter,

H_i is the fin external height and t_i the fin thickness. The offset-strip fins dimensions can be visualized in Fig. (4).

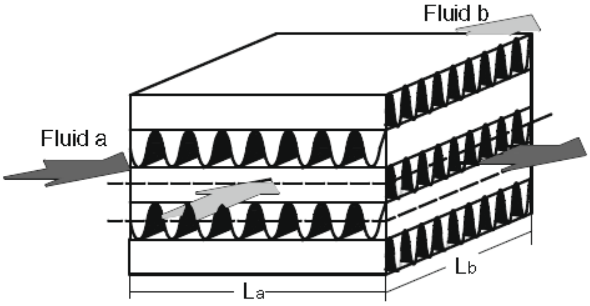

Figure 5 - Length of the Plate-fin Heat Exchanger.

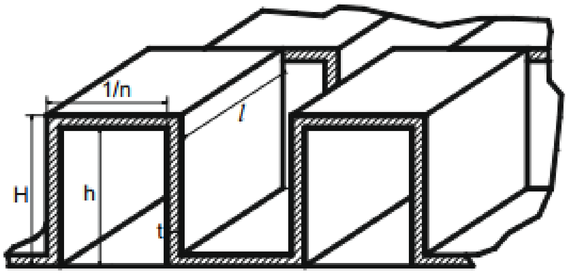

Figure 6 - Offset fin and its dimensions.

As can be observed in Fig. (4), l_i is the length of the fin and h_i is the internal height. Another particular physical parameter required was the fin spacing, s. It can be calculated by the difference of the fin frequency and thickness, as in Eq. (13).

$$s = \frac{1}{(n_i)} - t_i \tag{13}$$

The flow area of the fluids is required for calculations of the exchanger pressure drop and the dimensionless flow parameters, which are closely related with the heat transfer coefficients. The free flow (ff) areas are calculated by Eq. (14) and Eq. (15).

$$A_{ff,a} = (H_a - t_a).(1 - n_a.t_a).L_b.N_a \tag{14}$$

$$A_{ff,b} = (H_b - t_b).(1 - n_b.t_b).L_a.N_b \tag{15}$$

An essential parameter for the application of any heat transfer empirical correlation is the dimensionless number of Reynolds, which classifies the fluid flow as laminar, transition or turbulent. It is calculated by means of Eq. (16).

$$Re_i = \frac{G_i.D_{h,i}}{A_{ff,i}.\mu_i} \tag{16}$$

Where the variable G_i in Eq. (16) is the mass flux (kg/m².s) of the fluid, Eq. (17).

$$G_i = \frac{\dot{m}_i}{A_{ff,i}} \tag{17}$$

The term $D_{h,i}$ refers to the hydraulic diameter. For a circular pipe, its own internal diameter is the hydraulic diameter. For the plate exchanger, however, since the flow is between flat plates, D_h assumes the form of Eq. (18).

$$D_h = \frac{2.(s-t).(H-t)}{[s+(H-t)]+\frac{(H-t).t}{l}} \tag{18}$$

As fluids flow along surfaces they lose pressure, as a result of the friction with said surface. The higher the pressure loss, more is the expense with bombs and blowers to pump the fluids along the equipment, and higher is the operational cost. This parameter is extremely important in the project of a heat exchanger. The total pressure drop can be calculated by Eq. (19). Low velocities for the fluids help to avoid erosion, vibrations and reduce pressures drop (Çengel & Ghajar, 2012 and Lavine *et al.*, 2014).

$$\Delta P_i = \frac{4.f_i.L_i.G_i^2}{2.\rho_i.D_{h,i}} \tag{19}$$

The factor "f" in Eq. (19) is the Moody friction factor, an essential dimensionless parameter in the analysis of fluid flow. It can be obtained from empirical diagrams and correlations. It relates to the Reynolds number and the conditions of the flow surface. Its value increases with increasing surface roughness.

For laminar flow ($Re \ \leq 1500$):

$$f = 8{,}12.(Re_i)^{-0{,}74}.\left(\frac{l_i}{D_{h,i}}\right)^{-0{,}41}.\left[\frac{s_i}{(H_i-t_i)}\right]^{-0{,}02} \tag{20}$$

For turbulent flow ($Re \ > 1500$):

$$f = 1{,}12.(Re_i)^{-0{,}36}.\left(\frac{l_i}{D_{h,i}}\right)^{-0{,}65}.\left(\frac{t_i}{D_{h,i}}\right)^{0{,}17} \tag{21}$$

With the pressure loss of the fluid with Eq. (19), the outlet pressure of the fluid can be determined with Eq. (22).

$$P_{i,out} = P_{i,in} - \Delta P_i \tag{23}$$

Having calculated the exchanger's hydraulic parameters, returning to Eq. (10), the term U, the overall heat transfer coefficient, can be expanded in the form of the convection thermal resistances. As the exchanger is equipped with fins and the objective is to transfer as much heat as possible, the conduction heat resistance was neglected, as its value would be quite small. Also, as mentioned in the previous section, the PFHE's are not to be used with greatly incrusting fluids, making it so that the fouling resistances were also neglected. With those considerations, the overall coefficient results in Eq. (23).

$$U = \left[\frac{1}{(h.A)_a} + \frac{1}{(h.A)_b}\right]^{-1} \tag{24}$$

Besides using the Nusselt number to evaluate the individual convection heat transfer, h_i, it can also be written in terms of the Reynolds Modified Analogy (or the Chilton-Colburn analogy), as described by Lavine *et al.* (2014). The Colburn correlation is represented in Eq. (24), with j being the Chilton-Colburn factor. It represents a relationship between the coefficient of friction (hydrodynamic boundary layer), heat transfer coefficient and mass transfer coefficient. The subscript "H" refers to heat transfer, in order to differentiate the convective mass transfer analogy.

$$j_H = St.Pr^{2/3} \tag{25}$$

Where St is the dimensionless Stanton Number (also referred as the modified Nusselt number), determined by Eq. (25). It is calculated by the ratio of the fluid's convection coefficient by the product of its specific mass, velocity and specific heat.

$$St = \frac{Nu}{Re.Pr} = \frac{h_i}{\rho_i.v_i.c_{p,i}} \tag{26}$$

Substituting Eq. (25) in Eq. (24) and isolating the convective coefficient grants Eq. (26).

$$h_i = j_{H,i}.\frac{\dot{m}_i}{Aff_i}.c_{p,i}.Pr^{-2/3}{}_i \tag{27}$$

The Chilton-Colburn factor can be calculated by means of empirical correlations as a function of the Reynolds number and the physical parameters of the equipment. The used correlations were:

For laminar regime ($Re \ \leq 1500$):

$$j = 0{,}53.(Re_i)^{-0{,}50}.\left(\frac{l_i}{D_{h,i}}\right)^{-0{,}15}.\left[\frac{s_i}{(H_i - t_i)}\right]^{-0{,}14} \tag{28}$$

For turbulent regime ($Re \ > 1500$):

$$j = 0{,}21.(Re_i)^{-0{,}4}.\left(\frac{l_i}{D_{h,i}}\right)^{-0{,}24}.\left(\frac{t_i}{D_{h,i}}\right)^{0{,}02} \tag{29}$$

Taking into account the Colburn factors, convective heat transfer coefficients and the total area of thermal exchange, NTU can ultimately be written in the form of Eq. (29).

$$NTU = \left\{C_{mín}.\left[\frac{1}{(j_a.\dot{m}_a.c_{p,a}.Pr^{-2/3}{}_a)}\frac{A_{ffa}}{A_a} + \frac{1}{(j_b.\dot{m}_b.c_{p,b}.Pr^{-2/3}{}_b)}\frac{A_{ffb}}{A_b}\right]\right\}^{-1} \tag{30}$$

The calculation of the fluid's outlet temperatures is performed by isolating the term $T_{i,out}$ in the effectiveness relation, Eq. (6). The resulting equation for fluid A is Eq. (30) and for fluid B is Eq. (31).

$$T_{a,out} = T_{a,in} - \frac{\varepsilon . C_{mín} . (T_{a,in} - T_{b,in})}{C_a} \tag{31}$$

$$T_{b,out} = T_{b,in} + \frac{\varepsilon . C_{mín} . (T_{a,in} - T_{b,in})}{C_b} \tag{32}$$

The use of the previous equations to determine the fluid's outlet conditions allowed the determination of the entropy variations, the total entropy generation rate and the number of generated entropy. The total variation of entropy for each fluid is calculated as described in Bejan (1977), by Eq. (32) for A end Eq. (33) for B.

$$\Delta S_a = c_{p,a} . ln \frac{T_{a,2}}{T_{a,1}} - R_a . ln \frac{P_{a,2}}{P_{a,1}} \tag{33}$$

$$S_b = c_{p,b} . ln \frac{T_{b,2}}{T_{b,1}} - R_b . ln \frac{P_{b,2}}{P_{b,1}} \tag{34}$$

The total rate of entropy generation in a stream containing multiple fluids depends on the individual generations. The total is the sum of these in the form of Eq. (34).

$$\dot{S} = \dot{m_a} . \Delta S_a + \dot{m_b} . \Delta S_b \tag{35}$$

Finally, then, the number of entropy generation is calculated by the ratio between the total rate and the maximum heat capacity, Eq. (35).

$$N_s = \frac{\dot{S}}{C_{máx}} \tag{36}$$

Substituting equations 30 to 34 in 35 allows to rewrite the number of entropy generation in the format of Eq. (36) like Mishra *et al.* (2009).

$$N_s = \frac{C_a}{C_{máx}} . \left[ln \left\{ 1 - \varepsilon . \frac{C_{mín}}{C_a} . \left(1 - \frac{T_{b,1}}{T_{a,1}} \right) \right\} - \frac{R_a}{c_{p.a}} . ln \left(1 - \frac{\Delta P_a}{P_{a,1}} \right) \right] + \frac{C_b}{C_{máx}} . \left[ln \left\{ 1 + \varepsilon . \frac{C_{mín}}{C_b} . \left(\frac{T_{a,1}}{T_{b,1}} - 1 \right) \right\} - \frac{R_b}{c_{p.b}} . ln \left(1 - \frac{\Delta P_b}{P_{b,1}} \right) \right] \tag{37}$$

Thus, with the application of the equipment's optimized physical dimensions on the mathematical modeling, all of the heat exchanger parameters were calculated and it was possible to evaluate their performance in regard to the heat transfer coefficients, pressure losses and entropy generation.

2.2 Oppositional Differential Evolution Optimization

In this work, the chosen optimization method is called Oppositional Differential Evolution (ODE), which is a variant of the original Differential Evolution (DE) developed by Storn and

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Price (1995). They are very similar, where only a few variations are incremented in the search method of the solution vector of the latter.

The DEs are stochastic search methods that depart from evolutionary principles. An initial population (possible solution vectors) is generated and mutation, crossover and selection techniques are applied in order to reach a desired optimal vector. When information about the solution vector is not known, the population is generated randomly. The advantages that can be observed in ED are presented by Oliveira (2006):

- The search is rarely stuck in a local optimum (since it searches for the best vector in different regions of the search space);
- Effective to optimize discontinuous functions, since no information about the derivative of the function is required;
- Input and output parameters can be manipulated as real numbers (floating points), facilitating computational processing;
- Effective even with small populations;
- Is a good local optimizer, since the different values generated from a population become infinitesimal.

For a population of size equal to N_P, $X_{ri,G}$ (i = 1, 2, 3... Np) is referred to a solution vector of the generation G. The first step in the creation of the new generation is the mutation. It is the formation of a new vector from the sum of a random vector, r_1, with the weighted difference of two random vectors, $r_2$and r_3, in the form of Eq. (37).

$$V_{i,G} = X_{r1,G} + F.(X_{r2,G} - X_{r3,G}) \tag{38}$$

This weighting is performed by means of the mutation factor, F, as can be seen in the previous equation. This value can be any number between 0 and 2.0 (Rahnamayan *et al*., 2008). Storn & Price (1995) suggest the use of 0.8 as its value.

Storn & Price (1995) also propose a technique called dither, in which the mutation factor is made to vary between 0.5 and 1.0 between each iteration. Depending on the objective function, this technique can significantly improve convergence. The dither technique was used in this study's code.

The crossover of vectors is the process of generating new solutions through operations between existing vectors. This technique is used to increase population diversity. A vector V^{q+1}, called the donor, shares its characteristics with those of the target vector, X^q, in order to generate the trial vector, U^{q+1}. This vector, therefore, inherits data from both, and the degree of similarity with the generating vectors depends on the crossover parameter, C_r.

When C_r is equal to 0, all inherited properties come from X^q. When C_r is equal to unity the properties all come from V^{q+1}. The value of the Cr factor, therefore, varies between 0 and 1.0 (Oliveira, 2006 and Storn & Price, 1995).

Selection is the last step of ED. It can be visualized in Figure 5.

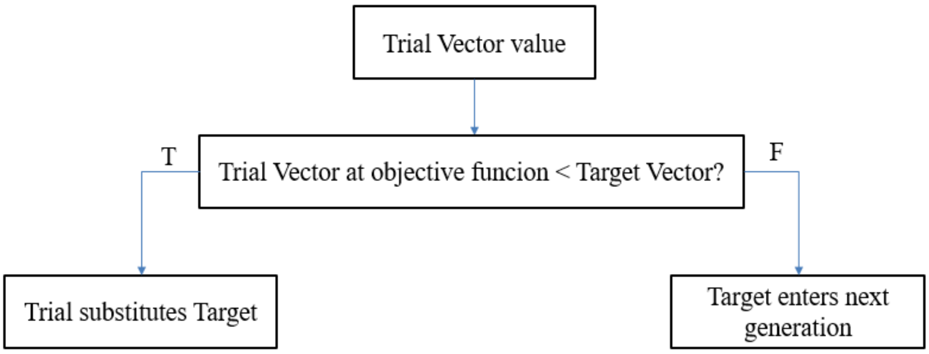

Figure 7 - Scheme of DE's selection method.

In this stage, it is decided which vector will be a member of the next generation, G+1. As how it is defined, the selection process is the comparison of the cost of the target and trial vectors. The one with the lower cost (greater suitability to the solution) advances (Rahnamayan *et al*., 2008). This could be translated at the application of these vectors in the objective function and verifying which one suits the desired value the most. In these case, the vector who would promote the lower value for the number of generated entropy, N_s, shall advance.

Evolutionary optimization methods start from initial assumptions of the solutions (creation of the first population) and are improved to an optimum that fits previously described conditions (for example, upper and lower limits for the optimized variables). When no information about the solution is known, the first vector is created from total random assumptions. As this is a random choice, computational time may be higher for different rounds of optimization.

One way to reduce this computational time (which is translated as the distance from the initial solution vector) is using the so-called opposing vectors. The theory of probability confirms that in 50% of the times the opposition vector is closer to the optimal solution than the initial vector. Therefore, it is perceived that checking the opposite is favorable to convergence. All solutions, not only the first, have their opposite computed in order to accelerate convergence and grant better results. That is the difference in the Oppositional Differential Evolution. All other parameters and strategies are in accordance with classical ED.

For a vector $P_{i,j}$ within the interval [a_j,b_j], the opposite vector, $OP_{i,j}$, is defined by Eq. (38).

$$OP_{i,j} = a_j + b_j - P_{i,j} \tag{39}$$

Based on a certain jumping rate, J_r, the operation presented in Eq. (38) is dynamically performed throughout the search space after the normal generation has already been created. By comparing the normal and opposite vectors, the best one (least distance of the solution) is chosen for the current generation. To assure that the calculated opposite does not remain outside the valid search space for the given variable (for example, a negative fin measure) or accidentally jump outside the shrunken search space, the opposite vector is calculated from the maximum and minimum values of the current population, in the form of Eq. (39).

$$OP_{i,j} = MIN_j^p + MAX_j^p - P_{i,j} \tag{40}$$

The jumping rate is another control parameter in the ODE (along with F, C_r and N_P) that influence in the program's performance. Its values range from 0 to 0.6. As indicated by Rahnamayan *et al*. (2008), values of J_r higher than 0.6 proportionate premature convergence, while smaller values are more suited to high-dimensional problems. For the present work, the value of 0.3 was used.

The seven variables submitted to the optimization program were the exchanger's physical dimensions. Their significance and constraints are presented in Table 2. There were 30 runs, 5000 iterations per run and the population size, N_P, was 40, as in accordance to Storn and Price (1995), whose empirical practices have shown that increasing the N_P value beyond 40 does not provide substantial improvements to convergence. The crossover factor, C_r, was 0.9 and the mutation factor varied with an exponential function dither, $F = {}^{1}\Big/\left[1 + e^{\left(-2*\frac{iter}{1000}\right)}\right]$. ED's Strategy 7 (binominal) was used.

Table 2 - Variables' constraints.

Variable	Constraint	Unit
Exchanger's outside length for fluid A, L_a	$0.1 \leq L_a \leq 1.0$	m
Outside length for fluid B, L_b	$0.1 \leq L_b \leq 1.0$	m
Fin external height, H	$0.002 \leq H \leq 0.01$	m
Fin frequency, n	$100 \leq n \leq 1000$	Fin/m
Fin thickness, t	$0.0001 \leq t \leq 0.0002$	m
Fin length, l	$0.001 \leq l \leq 0.01$	m
Number of fin layers, N_a	$1 \leq N_a \leq 10$	m

The optimization achieved results were applied in the mathematical modeling in order to calculate the transfer coefficients and output exchanger variables and grant the equipment's thermal and hydraulic analysis.

3. RESULTS AND DISCUSSION

The results obtained by the Oppositional Differential Evolution for the exchanger's physical parameters can be verified in Table 3.

Table 3 - Optimized dimensions.

Variable	Unit	Value
L_a	m	0.787
L_b	m	1.0
H	m	0.01
n	Fin/m	221.592

t	m	0.0001
l	m	0.01
N_a	-	10.00

Analyzing Table 3 and according to Rohsenow *et al.* (1998) it is possible to observe that the obtained dimensions were within the standard dimensions for finned plate changers. The frequency of the fins is generally around 120 to 700 fins/m, the common thicknesses are 0.00005 to 0.0025 m and the heights range from 0.002 to 0.025 m. Among the 30 rounds performed by the optimization method, it was at the sixteenth that the minimum of the objective function was reached more quickly, with such occurring in the iteration 3400 of 5000.

Inserting the previous values for the exchanger's physical dimensions in the mathematical modelling generated the results for the heat transfer and flow areas, heat transfer coefficients, pressure drops and the dimensionless parameters. They are displayed in Table 4.

Table 4 - Equipment's heat and flow parameters.

Variable	Unit	ODE Value
$\dot{q}$	(W)	199.990
h_A (W/m^{2}K)	(W/m^{2}K)	106.501
h_B (W/m^{2}K)	(W/m^{2}K)	185.127
G_A (kg/m^{2}s)	(kg/m^{2}s)	15.496
G_B (kg/m^{2}s)	(kg/m^{2}s)	14.319
ΔP_A (N/m^2)	(N/m^2)	1872.141
ΔP_B (N/m^2)	(N/m^2)	1065.006
C_{max}	(W/K)	3356.806
$\dot{S}$ (W/K)	(W/K)	204.485
N_S	-	0.06092
A (total heat transfer area)	m^2	89.0243
$A_{ff,a}$	m^2	0.0968
$A_{ff,b}$	m^2	0.0838
U	(W/m^{2}K)	185.137
NTU	-	2.479

ε	-	0.8278
Re_A	-	7196.4
Re_B	-	4821.8
$T_{a,out}$	K	340.4225
$T_{b,out}$	K	365.5565

As can be seen in Table 4 , the number of entropy generated was considerably low, despite the high generation rate. This is a consequence of the high heat capacity of the ammonia, a fact that is also reflected in the small variation of temperature suffered by this fluid in the exchanger, which enters the equipment with 400 K and exits with 340.4 K. The air, on the other hand, possesses a low value of heat capacity, thus suffering a major change in its condition. This is expected and it should be noted that its outlet temperature is higher than that of ammonia. This occurrence agrees with the literature, since only the countercurrent and cross flow exchangers the cold fluid can exit the exchanger at a higher temperature than the hot.

As a finned heat exchanger, it is supposed to have a high value for the heat transfer surface area. That can be observed as the resulting equipment has 89 m² available for energy transfer, even though the optimized dimensions do not appear to be so grand. Although laminar flows usually occur in the compact exchangers (Kuppan, 2000), the flow of both fluids was turbulent. That can be justified by the low values of the flow areas and high mass flow rates (resulting in high mass flows). These same factors promoted a large increase in the individual coefficients of heat transfer by convection, as can be verified by Eq. (26). The resulting overall coefficient was also of high value.

The number of transfer units was 2.479, indicating possible opportunities for heat transfer. According to Çengel & Ghajar (2012), NTU values up to 5.0 are interesting as it becomes possible to promote considerable variations in the temperature of the flowing fluid. However, as explained earlier, this parameter evaluates the equipment only thermally. The economic factor directly interferes with the design of heat exchangers and therefore, not always high values of NTU are feasible in a real equipment.

Another variable that is closely related to the operating cost of the equipment is the pressure drop. The value obtained for both was low, smaller than expected. The cost of pumping fluids through pumps and blowers in the exchanger is a major factor in the economical design of the equipment. Higher pressure losses mean higher costs as fluids need to have pressure to successfully flow throughout the equipment. The exchanger’s effectiveness was that of 82.78%, meaning that out of all the possible energy to transfer, 82% of that was achieved.

Opposite Differential Evolution proved to be an efficient method for the optimization of heat exchangers, operating simply and rapidly. The 30 runs took approximately 5 minutes to complete. It was possible to observe during the study that the ED has guaranteed convergence, presenting very satisfactory results.

4. CONCLUSIONS

In the present study, the design and thermodynamic optimization of a heat exchanger of finned plates (offset type) of cross flow with hot ammonia and cold air were carried out. The seven major physical dimensions of the exchanger and fins were constrained and optimized by Oppositional Differential Evolution. Their resulting values were applied in the mathematical modeling of the exchanger in order to calculate the equipment's heat transfer parameters. The total heat transfer rate was set at 200 W and the code was used in Matlab®.

The ODE method proved itself simple and efficient, with low computational time and guaranteed convergence. It has provided satisfactory results, with high transfer coefficients, low pressure drops and a low number of generated entropy. As expected, the ammonia underwent low temperature variation, as a result of its high thermal capacity, while the cold air varied by more than 150 K, leaving the equipment at a higher temperature than the hot fluid. The effectiveness was 82.78%, allowing to conclude that the resulting equipment has a good thermal, hydraulic and thermodynamic performance.

Optimizations aimed at the reduction of entropy generation are very interesting since they improve the thermal performance of the equipment. Less generation of entropy represents a greater opportunity for the transfer of energy. As mentioned throughout this paper, attention should be paid to the economic bias, which is a project limiter. Small gains in thermal efficiency can mean a great increase in initial and operation costs, so a compromise between the two must be achieved when projecting heat exchangers.

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