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CONSTRUCTAL DESIGN OF TWO T-SHAPED FINS COOLING A CYLINDRICAL SOLID CONSIDERING SEVERAL THERMAL CONDUCTIVITIES

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Abstract. *This paper considers a numerical study about geometrical optimization of two T-shaped fins cooling a cylindrical solid body as function of the thermal conductivity of the two fins. It relies on Constructal Design associated to the Exhaustive Search Method to discover the best geometries. The objective is to minimize the maximal excess of temperature between the solid cylindrical body and the ambient. Internal heat generation is distributed uniformly* *throughout the solid body. The two fins are bathed by a steady stream with constant temperature and heat transfer coefficient. The outer surfaces of the cylindrical body are adiabatic. The total volume of the body and the total volume of the fins are fixed, but the lengths of the fins can vary, i.e., there are two degrees of freedom: the ratio between the lengths and the ratio between the thicknesses of the fins. The fins and the body have different thermal conductivities. The initial simulations demonstrated that the optimal performance is achieved when the tributaries shape becomes slender and the stem thicker.*

Keywords: *Constructal design, cylindrical solid body, T-shaped assembly of fins, heat removal, convection heat transfer.*

1 INTRODUCTION

With the rapid growth of technologies linked to modern industry, it has become convenient to improve heat transfer techniques. Examples of heat transfer include biological systems, common household appliances, residential and commercial buildings, industrial processes, electronic devices, food processing, including also heat exchangers and cooling systems. An additional motivation for approaching the study of problems related to heat transfer is becoming increasingly important as they play a crucial role in the design of various elements of everyday life such as automobiles, power plants, Refrigerators, electronics, among others.

Among the existing heat transfer mechanisms, thermal convection is present in most of the examples above, where in these applications the high temperature is present even in power cycles, cooling systems, engine cooling as well as in the cooling of electronic components. The strong fin design has been significantly important in the field of engineering and has a range of applications, including cooling systems from combustion engines to heat exchangers used in domestic systems. Thus, if the study of fins is a very significant field in engineering the study of the geometric optimization of finned systems also represents an important contribution to engineering.

Connected to the study of heat exchangers, finned cooling systems can use Constructal Theory, where the mental visualization of the generation of flow structures that we see throughout nature (river basins, lungs, atmospheric circulation, vascularized tissues, etc.) is based on a principle of evolution of access flow in time. This principle allows us to state: "For a finite flow system to persist in time (to live) it must evolve in such a way as to make easy the access of currents flowing through it, (Bejan, 1997a).

The optimization of geometry with the objective of maximizing heat transfer in cavities and fins using the Constructal Theory is a problem that has been receiving the attention of researchers in the last decade (Bejan and Almogbel, 2000, Biserni et al., 2004, Rocha et al. Lorenzini et al., 2010, Lorenzini et al., 2014, Rocha et al., 2012).

The Constructal Method, "Constructal Design", is based on the Constructal law (Bejan, 1997b, Bejan and Lorente, 2008) and allows improve the performance of systems where there is flow. What flows is not relevant in this case: heat, fluid, stresses, people, goods, etc. However, for the method to be applied, one must define the performance indicator and the constraints to which the system is subjected. Usually in the study of cavities and fins, the performance indicator is conductance or thermal resistance between the system and the vicinity. Examples of constraints, i. e. constants to which the system must be subjected, are the volume of the system and the volume of the fins or cavities.

It should be noted that the numerical work published to date addresses the study of cavities and fins with the application of the Constructal Design Method (Bejan and Almogbel, 2000, Biserni et al., 2004, Rocha et al., 2005, Lorenzini e Rocha, 2006), were limited to the two-dimensional analysis, that is, considering that the third dimension was sufficiently long so that the problem could be treated as two-dimensional.

This paper documents numerically the relationship between maximizing overall performance and the architecture of a flowing system. The aim of the present study was to discover numerically, through the Constructal method associated to exhaustive search method, the configuration of the optimum geometry of a set with two fins connected to a solid cylindrical body with uniform internal heat generation. The sets of fins are bathed by a current with constant temperature and convective heat transfer, while the solid body has adiabatic conditions on the outer surface. The total body volume and the total volume of the fins are fixed, but the lengths of the fins are free to vary. Optimized geometry and performance are reported graphically as a function of the ratio of the thickness as well as the ratio between the lengths of the fins.

2 MATERIAL AND METHODS

Consider the domain shown in Figure 1. There is a cylindrical body with adiabatic generation of constant internal heat per unit volume q''' *(W/m³)* and constant thermal conductivity k *(W/mK)*. Attached to the cylinder are two T-shaped assembly of fins. The configuration is two-dimensional, with the third dimension *(W)* sufficiently long compared to the cylinder diameter. The heat transfer coefficient h is uniform throughout all the exposed surfaces of the T-shaped fisns and the temperature of the fluid (T_∞) is known. The maximum temperature (T_{max}) occurs inside the cylinder and varies according to the geometry of the two T-shaped assembled of fins.

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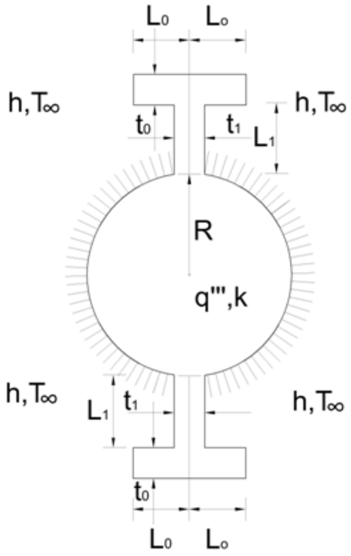
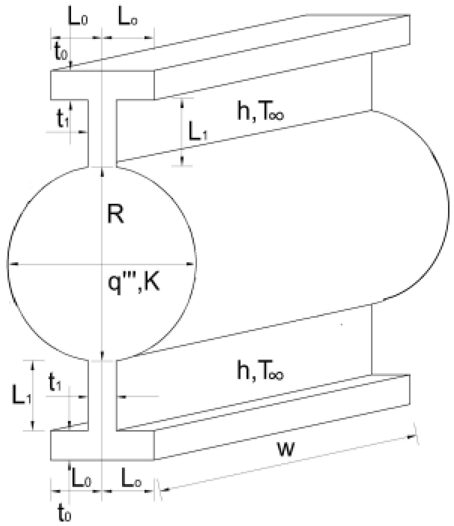


(a)



(b)

Figure 1. (a) Computational domain; (b) Cylindrical body with two T-shaped assembly of fins.

The objective of the analysis is to determine the optimal geometry *(L_1/L_0, t_1/t_0)* which is characterized by the maximum excess temperature *(T_{max} - T_∞)/(($q'''A$)/k_b)* where k_b is the thermal conductivity of the body. According to the Constructal design, this optimization can be submitted to constants, that is, a restriction on the cross-section area of the cylindrical body, given by Equation (1).

$$A = \pi R^2 \tag{1}$$

The area constraint of fin material,

$$A_f = N(2L_0t_0 + L_1t_1) \tag{2}$$

Where N is the number of fins, L is the length and t is the thickness of the fin. The aproximate area that circunscribes two T-assembly fins is given by.

$$Ac = 2L_0L_1 \tag{3}$$

Equations (2) and (3) can be expressed as the T-assembly of fins area fraction

$$\phi = \frac{A_f}{A} \tag{4}$$

and the area that circuncribes two T-assembly of fins, the area fraction is given by

$$\psi = \frac{A_c}{A} \tag{5}$$

The analysis that provides the maximum temperature excess as a function of the geometry consists of numerically solving the heat conduction equation over the entire domain. The solid body is governed by the heat conduction equation with the heat generation constant, shown in Equation (6)

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} + 1 = 0 \tag{6}$$

while the heat conduction equation without heat generation is applied in the T-assembly of fins

$$\frac{\partial^2\theta}{\partial\tilde{x}^2} + \frac{\partial^2\theta}{\partial\tilde{y}^2} = 0 \tag{7}$$

where the dimensionless variables are

$$\theta = \frac{T - T_\infty}{q'''A/k_b} \tag{8}$$

and

$$\tilde{x}, \tilde{y}, \tilde{L}_0, \tilde{L}_1, \tilde{t}_0, \tilde{t}_1, \tilde{R} = \frac{x, y, L_0, L_1, t_0, t_1, R}{A^{1/2}} \tag{9}$$

The outer surfaces of the solid cylindrical body are insulated and the boundary conditions are given by

$$\frac{\partial\theta}{\partial\tilde{n}} = 0 \tag{10}$$

where ñ is the direction perpendicular to the fin surface.

The boundary conditions on the surfaces of the fins are given by

$$-\frac{\partial \theta}{\partial \tilde{y}} = \lambda\theta \qquad \text{ou} \qquad -\frac{\partial \theta}{\partial \tilde{x}} = \lambda\theta \tag{11}$$

and the parameter λ is the defined by

$$\lambda = \frac{hA^{1/2}}{k_f} \tag{12}$$

and k_f is the fin conductivity thermal.

The dimensionless form of Equations (1), (2), and (5) are

$$1 = \pi\tilde{R}^2 \tag{13}$$

$$\phi = N\left(2\tilde{L}_0\tilde{t}_0 + \tilde{L}_1\tilde{t}_1\right) \tag{14}$$

$$\psi = 2\tilde{L}_0\tilde{L}_1 \tag{15}$$

The maximum excess of temperature, $\theta_{\max}$, according to Equation (8) is given by

$$\theta_{max} = \frac{T_{max} - T_\infty}{\dfrac{q''' A^{1/2}}{k_b}} \tag{16}$$

3 NUMERICAL RESULTS

It is not possible to express the overall objective function analytically in terms of the geometric parameters of the solid cylindrical body connected to the T-shaped fins. This function can be determined numerically by solving the temperature field in all assumed configurations to verify that the maximum excess temperature can be minimized by varying the geometry.

Equation (6) and (7) were solved using a finite element code, based on triangular elements, developed in $MATLAB^{®}$, precisely PDE (partial differential equations) of the toolbox. Further details on the optimization procedure, numerical method, mesh refinement and validation can be found in (Rocha et al., 2010).

The structure shown in Figure 1 has two degrees of freedom: L_1/L_0, t_1/t_0. We start the simulations by varying the ratio t_1/t_0 and maintaining constant the degree of freedom L_1/L_0. Figure 2 shows that there is an optimal $(t_1/t_0)_o$ ratio which minimizes the maximum excess temperature, θ_{max}, when the ratio L_1/L_0 is fixed and also the parameters ϕ, ψ e λ were kept fixed as well as the fin thermal conductivity $\tilde{k}_f = \frac{k_f}{k_b} = 200$. It is a minimum for small values of the L_1/L_0 ratio that becomes more pronounced when the L_1/L_0 ratio decreases.

The results of Figure 2 were summarized in Figure 3, which shows the once minimized maximum excess temperature, $(\theta_{max})_m$, and the resulting optimal $(t_1/t_0)_o$ ratio as function of the L_1/L_0 ratio. This figure indicates that $(\theta_{max})_m$ diminishes approximately 59,75% and $(t_1/t_0)_o$ increase dramatically when the L_1/L_0 ratio decreases from 5 to 0.1. The work presented in Figs. 2 and 3 is repeated for several values of the thermal conductivity $\tilde{k}_f$. Figure 4 presents the once optimized $(L_1/L_0)_o$ ratio, the twice optimized $(t_1/t_0)_{oo}$ ratio, and the twice minimized $(\theta_{max})_{mm}$ using the fin thermal conductivities $\tilde{k}_f = 10, 50, 100, 200$ and 300. The parameters ϕ, ψ e λ are fixed. The best shapes of Figure 4 are shown in Figure 5 which demonstrated that the fin set improved when the tributaries became thinner and the stem became thicker. Figure 4 also illustrates that, for the best shapes, the T-shaped fins temperature distribution (L_1/L_0 = 0.1 and $(t_1/t_0)_o$ = 14) as well as the temperature distribution in the body are more homogeneous than the temperature fields obtained for the other studied cases. This behavior is in accordance with the constructal principle of "optimal distribution of imperfections".

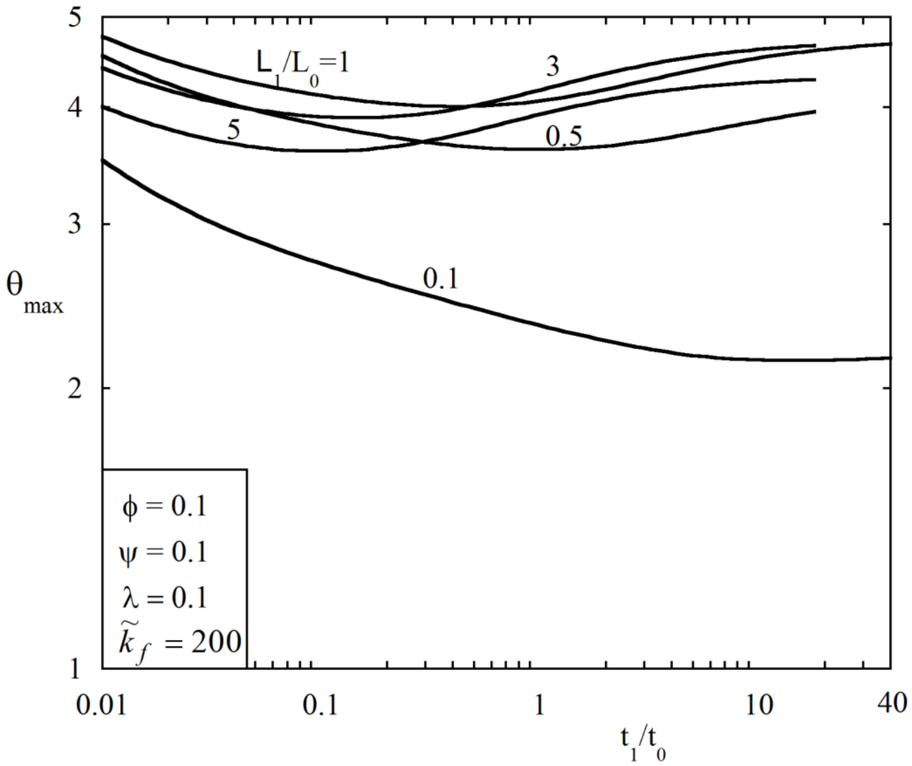


Figure 2. Optimization of the maximal excess of temperature as function of the ratio t_1/t_0 for several values of the ratio L_1/L_0

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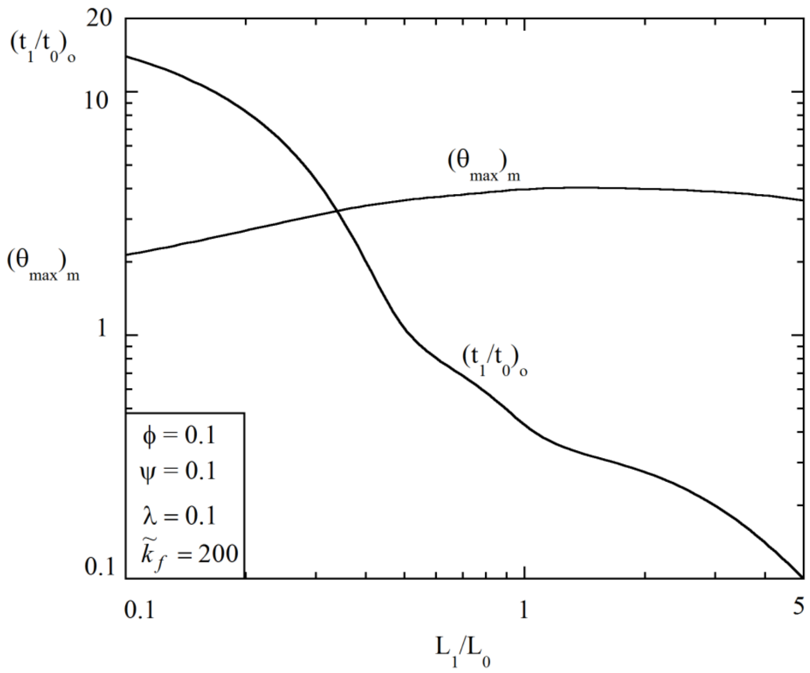

Figure 3. The bahavior of the once minimized global maximal excess of temperature $(\theta_{max})_m$ and its corresponding optimal ratio $(t_1/t_0)_o$ as function of ratio L_1/L_0

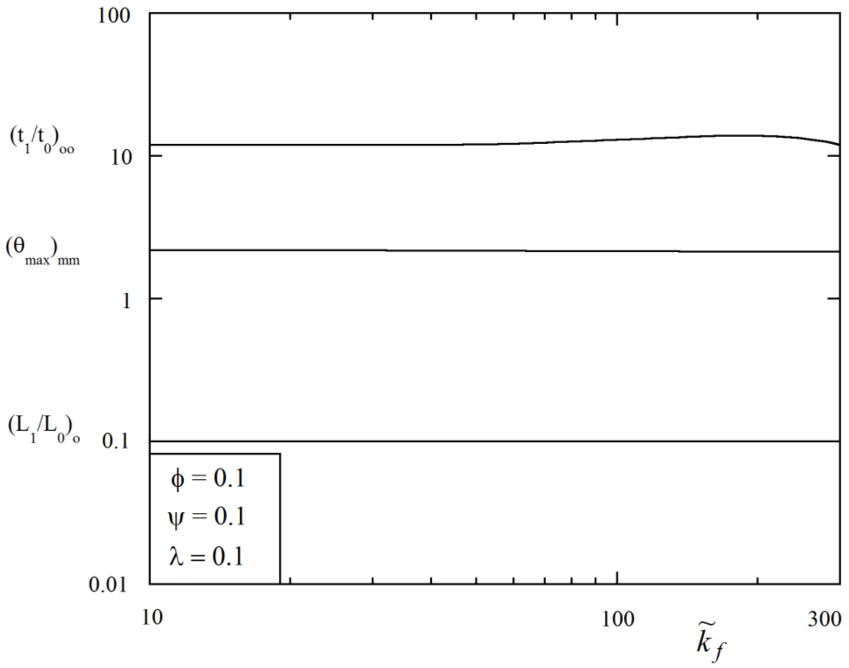

Figure 4: Optimization of once minimized global maximal excess of temperature that corresponds to two optimized the ratio $(t_1/t_0)_{oo}$ and its corresponding optimal ratio $(L_1/L_0)_o$ as a function of the boundary condition of the fin $\tilde{k}$

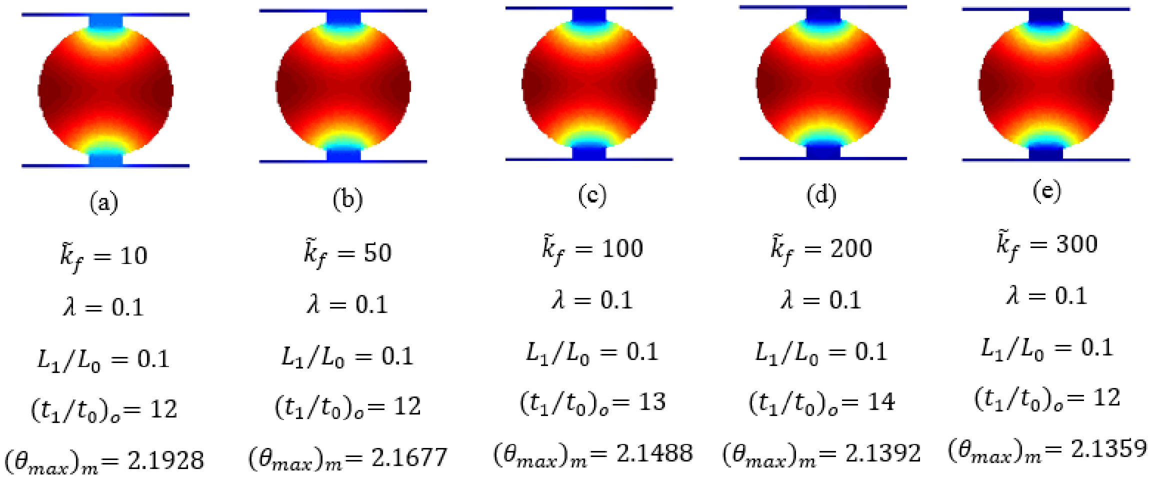

Figure 5. The best configurations of Figure 4.

4 CONCLUSION

This work applies the constructal method to obtain the ideal architecture of two T-shaped fin geometry of a solid cylindrical body with uniform internal heat generation. The sets of fins are bathed by a constant current at constant ambient temperature and heat transfer coefficient, while the solid body has adiabatic conditions on the outer surface.

The results indicate that there is an optimal relation $(t_1 / t_0)_{oo}$ that minimizes the maximum excess temperature, $(\theta_{max})_{mm}$ when the relation of $(L_1/L_0)_o$ is fixed with the parameters, ϕ, ψ and λ. It is also important to note that the maximum excess temperature minimized $(\theta_{max})_{mm}$ decreases approximately 59.75% and the optimum ratio (t_1/t_0) increases significantly as the L_1/L_0 ratio decreases from 5 to 0.1.

Future work will consist in to study the effect of the number of fins on the thermal performance of the assembly of fins. It is also interesting to notice that we expect that the optimal geometry of the assembly of fins will vary depending on the dimensionless thermal conductivity of the fins and the thermal conditions of the fluid that bath the assembly of fins.

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