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A STUDY OF THE INFLUENCE OF THE MECHANICAL PROPERTIES IN THE SUBSIDENCE AND RESERVOIR COMPACTION IN HETEROGENEOUS MEDIA

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Abstract. *Production of hydrocarbons in reservoirs produces changes in the stress and strain fields generating compaction in the reservoir and subsidence at free surface. Analytical methods are frequently used for a first estimation of the response of displacements and stresses generated by the production of reservoirs. Geertsma's method calculates the displacement and stress fields for a reservoir under uniform depletion in a linear elastic medium based on the poroelastic approach. Geertsma's method demands the use of a single set of properties for the entire medium. However, real problems are characterized by media with layered stratigraphy. In this work, the influence of the mechanical properties of a three-layer linear elastic medium over the response of displacements due to a uniform depletion in a cylindrical reservoir is assessed. For this, parametric analyses through an axisymmetric finite element model are carried out varying the relation of Young's moduli between the layers. Finally, three criteria for the determination of mechanical properties to be used in Geertsma's method are proposed.*

Keywords: *Reservoir compaction, Surface subsidence, Geertsma's method, Heterogeneous model.*

1 INTRODUCTION

The production of hydrocarbons in reservoirs introduces changes in the stress and strain fields generating compaction in the reservoir and subsidence at free surface. Field cases of subsidence due to reservoir production have been reported in the Dan field in Denmark, studied by Hatchell et al. (2007), and more recently in the lower Liaohe plain in China studied by Sun et al (2017). In addition, changes in stress field can cause geological problems, such as fault reactivation, and induced seismicity, as pointed out by Barkved (2005), Kartveit (2015), and Verdon et al (2016).

Analytical methods are frequently used as first estimate of the response of displacements and stresses generated by production of reservoirs. One of the first studies involving analytical methods is the work published by Geerstma (1973), who formulated displacement and stress solutions for an elastic homogeneous cylindrical reservoir under uniform depletion. This study was based on the nucleus of strain approach in analogy with the theory of thermo-elasticity (Mindlin, 1950). The nucleus of strain approach has been employed in several studies, as in Geertsma and van Opstal (1973), Tempone et al (2010), and more recently, in Mehrabian and Abousleiman (2015), and in Paullo and Roehl (2017).

Geertsma's method demands the use of a single set of properties for the entire medium. However, real problems are characterized by media with layered stratigraphy. Some researchers have formulated analytical methods to analyse the effect of multilayer media. Some studies on heterogeneous media can be found in Fokker and Orlic (2006), and Mehrabian and Abousleiman (2015). Approximated methods are also proposed to evaluate that effect, as it can be found in Rudnicki (1999), who calculates the approximate stress field considering a horizontal layered medium.

Some researchers have proposed methodologies to work with a single set of properties to obtain an equivalent behavior in heterogeneous problems. Those methodologies are known as upscaling techniques. In reservoir compaction, Al-Ruwaili (2012) and Settari et al (2013) calculated an equivalent Young's modulus for the analysis of interbedded shales within reservoirs based on the uniaxial compaction theory. The upscaling technique may be used even in the modelling of coupled problems as can be found in Yang (2013).

In this work, it is assessed the influence of the mechanical properties of a three-layered linear elastic medium in the response of displacements in a cylindrical reservoir under uniform depletion. For this, parametric analyses through an axisymmetric finite element model are carried out varying the relation of the Young's moduli of each layer. Finally, three criteria for the determination of mechanical properties are proposed to be used in Geertsma's method.

2 PROBLEM DESCRIPTION

This work studies the influence of the material property variation in the displacements of a cylindrical reservoir contained in a three layered elastic semi-infinite medium. The reservoir is under the effect of uniform pressure changes. The three layers are defined by three horizontal planes: the free surface, the top and the bottom reservoir planes, as shown in Figure 1.

The cylindrical reservoir has a radius of R = 500m and a uniform thickness of h = 100m. Figure 1 also shows the geometric and properties of the problem. In addition, the reservoir is depleted uniformly generating a change of pore pressure of $\Delta P = -10MPa$.

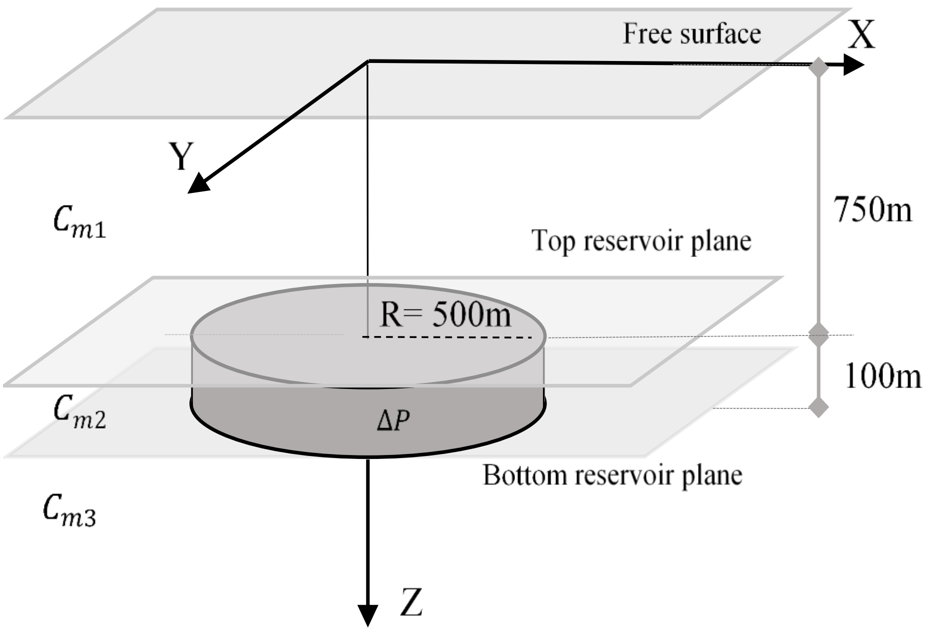

Figure 1. Geometry and properties of the problem

3 NUMERICAL SOLUTION VIA FINITE ELEMENT METHOD

The calculation of the displacement solution is obtained numerically through the use of the Finite Element Method (FEM). For the simulations the software ABAQUS v. 6.14 was employed.

The modelling of the problem considers the following conditions: linear elastic material behavior, small strains and displacements, and impermeable caprocks. It is important to highlight that in all numerical simulations only the mechanical effect generated by the pressure change is considered. In this way, coupled phenomena are not assessed.

The problem described in section 2 is modelled considering axisymmetric conditions. In the model, the fixed borders are located at an appropriate distance to reproduce the semi-infinite condition. Figure 2 shows the FE mesh and the geometry of the model.

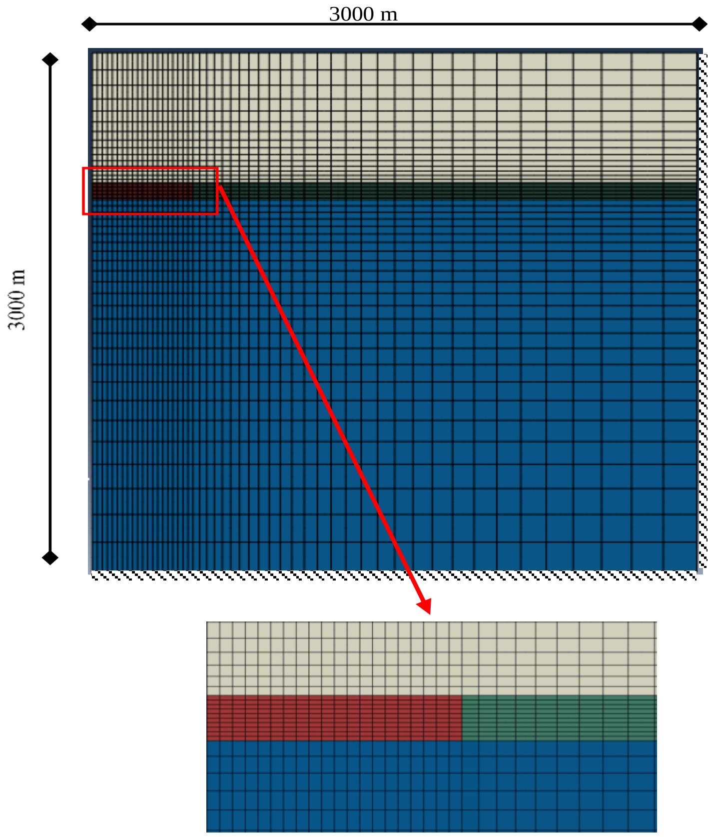

Figure 2. Finite element mesh and geometry of the axisymmetric model.

4 ANALYTICAL SOLUTION

The analytical solution is obtained through the hypothesis of homogeneous elastic material and a uniform pressure change throughout the reservoir. That hypothesis also consider small strains and displacements.

This section presents the basic equations to evaluate the displacement field generated by uniform pressure changes analytically. The studies of Tempone et al. (2010) and Quispe et al. (2015) provide more details on the derivation of the analytical expressions.

4.1 Solution for linear elastic homogenous case

Using Geertsma's approach, Tempone et al. (2010) obtain the three components of displacement generated by the uniform increment of pressure in an arbitrary shape reservoir. In that study, it is considered that the reservoir is in a homogeneous semi-infinite medium. In addition, the reservoir is discretized in regular cells. According to Tempone et al. (2010), the displacements at a generic point (see Figure 3) are given by:

$$
\begin{aligned}
u_x &= A\frac{x}{r}\left[\int_0^{\infty}\lambda e^{-\epsilon\lambda(z-c)}\mathrm{J}_1(\lambda r)d\lambda + \int_0^{\infty}(3-4\nu-2z\lambda)\lambda e^{-\lambda(z+c)}\mathrm{J}_1(\lambda r)d\lambda\right] \\
u_y &= A\frac{y}{r}\left[\int_0^{\infty}\lambda e^{-\epsilon\lambda(z-c)}\mathrm{J}_1(\lambda r)d\lambda + \int_0^{\infty}(3-4\nu-2z\lambda)\lambda e^{-\lambda(z+c)}\mathrm{J}_1(\lambda r)d\lambda\right] \\
u_z &= A\left[\int_0^{\infty}\epsilon\lambda e^{-\epsilon\lambda(z-c)}\mathrm{J}_0(\lambda r)d\lambda + \int_0^{\infty}(3-4\nu+2z\lambda)\lambda e^{-\lambda(z+c)}\mathrm{J}_0(\lambda r)d\lambda\right]
\end{aligned}
\tag{1}
$$

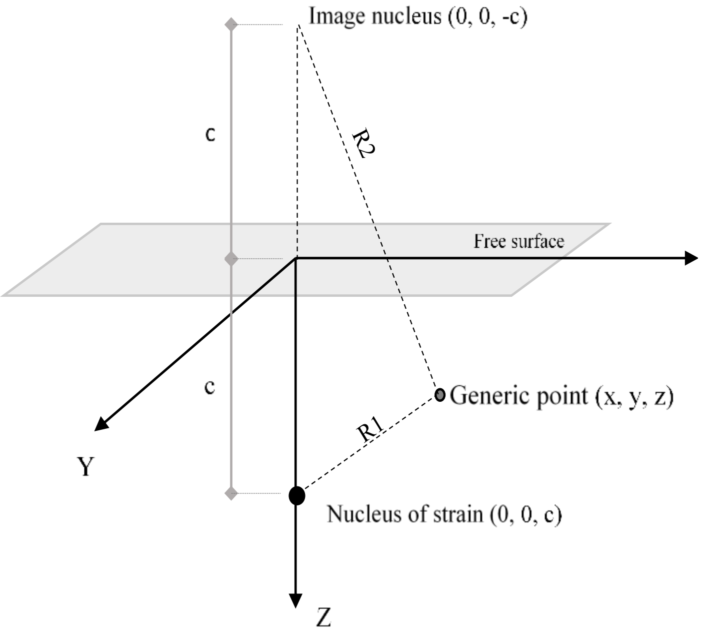

Figure 3. Nucleus of strain scheme.

where ν is the Poisson coefficient, $r = \sqrt{x^2 + y^2}$, c is the depth of the reservoir cell, $\epsilon = 1\ if\ (z-c) > 0,\ \ \epsilon = -1\ if\ (z-c) < 0$, and A is a constant that involves the deformability of the cell, and which is defined as:

$$A = \frac{C_m V}{4\pi}\Delta P \tag{2}$$

where ΔP is the change of pressure, V is the volume, and C_m is the unidimensional compaction coefficient given by:

$$C_m = \frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \tag{3}$$

The solution given in equation Eq. (1) is not adequate for the evaluation of the displacement inside the reservoir (Tempone et al., 2010). To determine the displacement components inside the reservoir in a cell with uniform thickness, the solution given by Quispe et al. (2015) and Paullo and Roehl (2017) is adopted. In the latter, the solution is obtained by the integration of

Eq. (1) along the thickness of the cell. The total solution of displacement at a certain point can be calculated by the superposition of the solution of all the cells considered in the discretization,

$$\begin{aligned}\boldsymbol{u_x} &= \sum_{i=1}^{N} u_{x,i}\\ \boldsymbol{u_y} &= \sum_{i=1}^{N} u_{y,i}\\ \boldsymbol{u_z} &= \sum_{i=1}^{N} u_{z,i}\end{aligned} \tag{7}$$

where $u_{x,i}$, $u_{y,i}$ and $u_{z,i}$ are the solutions corresponding to each cell, and N is the number of cells considered in the discretization.

4.2 Equivalent Young's Moduli

The present work investigates three criteria for the determination of the upscaled or equivalent Young's modulus. In the first criterion, the equivalent Young's modulus is calculated as the simple average of the layers:

$$E_{eq} = \frac{\sum_{i=1}^{NL} E_i}{NL} \tag{8}$$

where E_{eq} is the equivalent Young's modulus, E_i is the Young's modulus of the $i-th$ layer, and NL is the number of layers. The second criterion obtains the equivalent Young's modulus through the expression given in Settari et al. (2013), which is based on the unidimensional compaction case. In this case, the equivalent modulus for a single depleted layer is given by:

$$E_{eq} = \frac{H}{\sum_{i=1}^{NL} \frac{1}{E_i} h_i} \tag{9}$$

where H is the total thickness of the medium and h_i is the thickness of the i-th layer. Note that this calculation requires the definition of a thickness value for the basement reservoir underburden (layer 3). For this criterion, the basement thickness is assumed as being 2.5 times the sum of the thickness of the other layers.

The third criterion considers that the Equivalent Young modulus is given by the following expression:

$$E_{eq} = \left(\frac{1}{\left(1-\sum_{i=1}^{NL} \alpha_i\right)}\right) E^{resv},\ \alpha_i = \frac{u_{zmax}(E^i)}{u_{zmax}(E^{resv})} - 1 \tag{10}$$

where E^{resv} is Young's modulus of the reservoir layer and α_i is a coefficient obtained from the parametric analysis.

5 NUMERICAL RESULTS

5.1 Parametric study with FEM

This section presents a parametric study assessing the influence of the variation of the Young's modulus value of each layer in the subsidence and reservoir compaction. Young's modulus is taken as the parameter with most influence in the deformability of the medium. This choice is based on the fact that in several real cases, stratigraphy with a big variation of this parameter can be found. On the other hand, Poisson's ratio is generally found with values that range from 0.20 to 0.50.

For this section, Young's modulus in each layer varies proportionally to a reference value defined as E* = 3333.33MPa. In addition, is considered a Poisson's ratio of ν = 0.25 for the three layers. Fig. 4 shows the vertical and radial displacements at the free surface (subsidence) considering 5 values for the Young modulus's of the first layer (overburden), and taking *E2* = *E3* = *E**. It can be observed that both displacements increase with the reduction of *E1*, showing an inverse relation between *E1* and the subsidence magnitude variations.

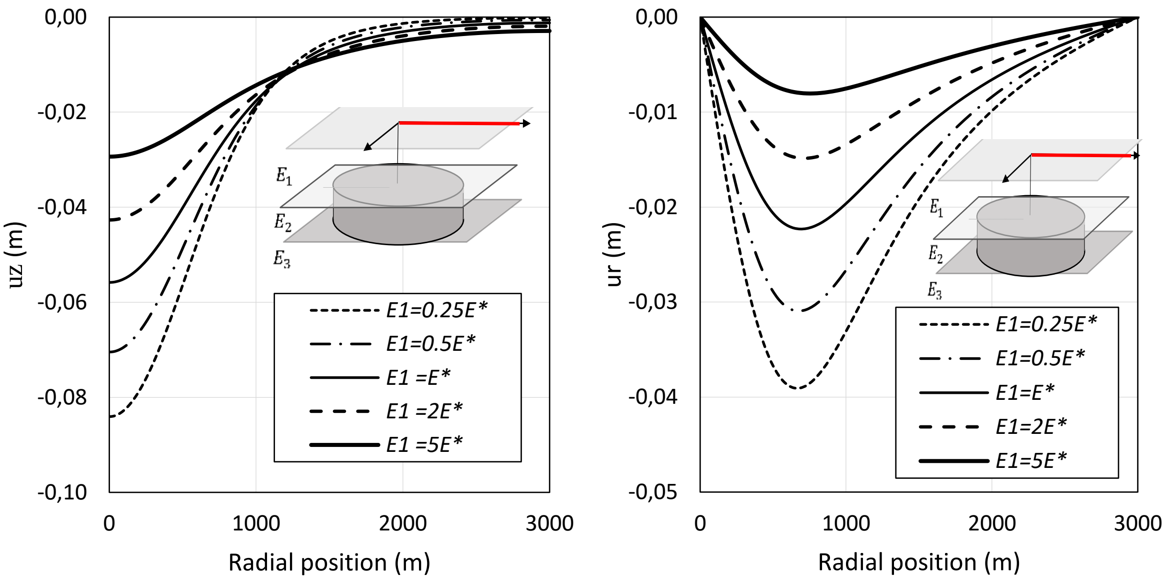

Fig. 4. Vertical and radial displacement at the free surface, *E2* = *E3* = *E.**

Fig. 5 shows the vertical, uz, and radial displacement, ur, at free surface (subsidence) considering variation of Young's modulus in the third layer, *E3* (underburden), and *E2* = *E3* = *E**. In this case, the magnitude of both displacements rises with the increase of Young's modulus of the underburden *E3*, excepting the case for *E3=0.2E5**. For this value of *E3*, the results do not follow the expected behavior, and exhibit a possible numerical instability. For the other case of *E3*, it is showed that the variations of *E1* and *E3* have the opposite effect.

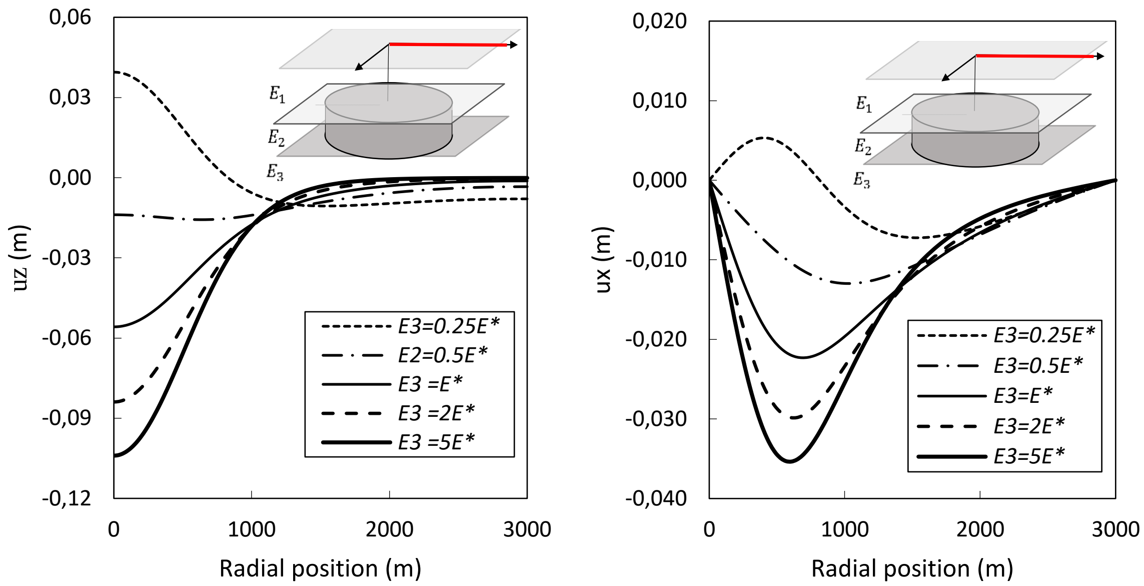

Figure 5. Vertical and radial displacement at free surface, *E1 = E2 = E*.*

Fig. 6 shows the vertical and radial displacements at the free surface (subsidence) considering the variation of Young's modulus in the reservoir layer, *E2*, remaining *E1 = E3 = E**. It can be observed a similar behavior to the case of the first layer property variation, since the reduction of *E2* produces the increase of the subsidence magnitude but with a stronger intensity. For instance, for E2=0.25E*, the maximum value of uz is around 5.6 times greater than the maximum value of uz for E2=E* (homogeneous case). On the other hand, a similar variation of E1, produces a value of uz around of 1.5 times the result of the homogeneous case (E1=E*).

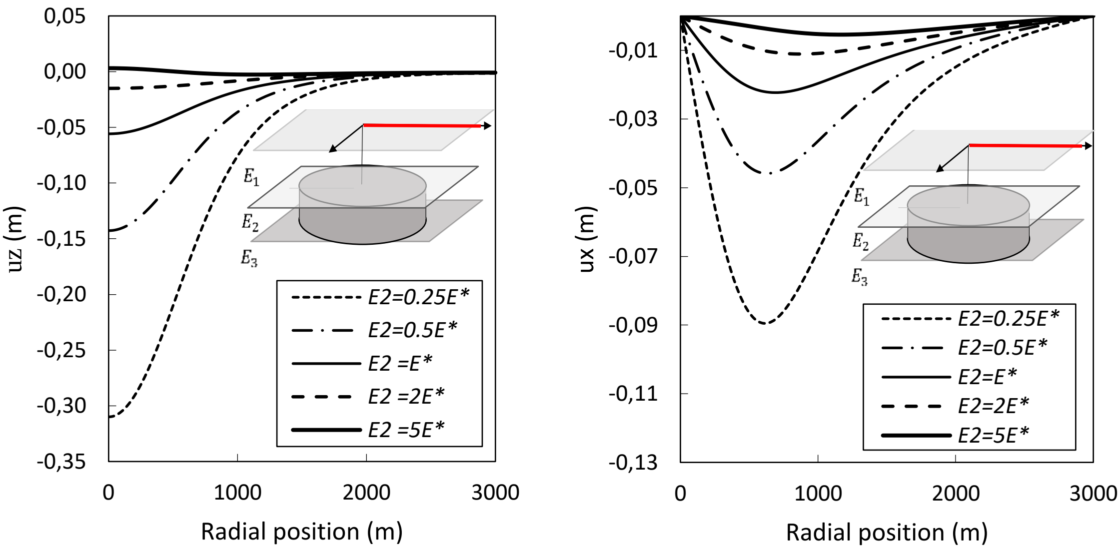

Figure 6. Vertical and radial displacement at free surface, *E1 = E3 = E*.*

As mentioned in the previous paragraph, it can be observed that the influence of the variation of *E2* (reservoir property) is larger than the variation of the other layer properties (*E1* and *E3*). This condition is depicted in Fig. 7, which shows the relation between the maximum vertical displacement, Uzmax, normalized by the homologous result of the homogenous case (*E1* = *E2* = *E3* = *E**) and the ratio *E/E** for the three layers. It can be observed that the variation of E2 has the most important effect in the subsidence and, consequently, in the deformability of the medium. Figure 7 also shows a nonlinear behavior for the variation of Young's modulus of all the layers, as well as, an asymptotic behavior when Young's moduli increases.

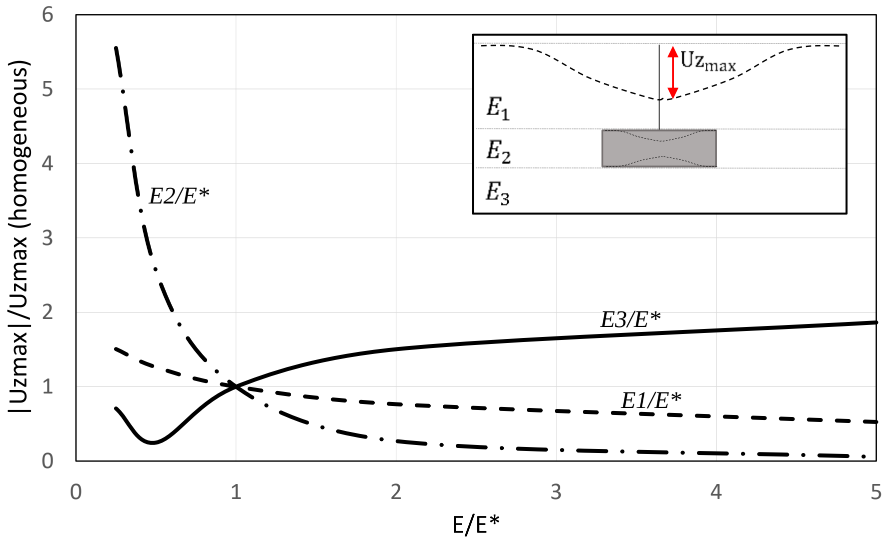

Figure 7. Maximum normalized subsidence vs normalized Young's modulus.

Figures 8, 9 and 10 show the vertical displacement along the radii at reservoir top and bottom planes, for the variation of E1, E3 and E2, respectively, to show the influence of these variations in the reservoir compaction. Figure 8 shows that the increase of Young's modulus of the fist layer produces the growth of the vertical displacements at the reservoir top plane and the reduction of the vertical displacements at the bottom, indicating the existence of an upwards displacement of the reservoir.

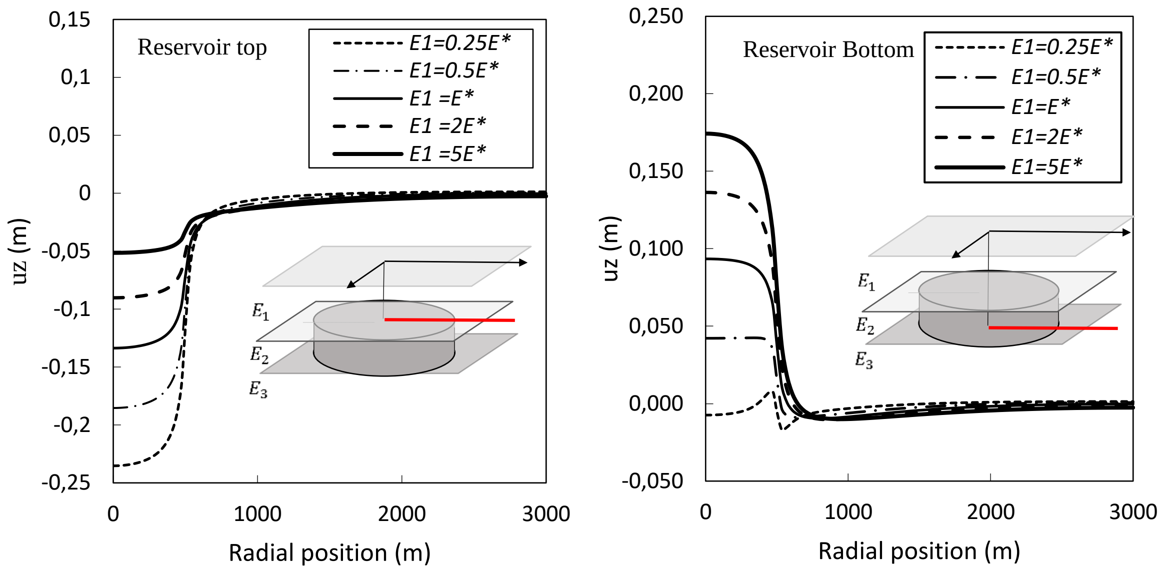

Figure 8. Vertical displacement at reservoir top and reservoir bottom surfaces, *E2* = *E3* = *E**.**

On the other hand, Figure 9 shows that the increase of Young's modulus of the third layer produces both the reduction of the vertical displacement at the reservoir top plane and the increase of the vertical displacements at the reservoir bottom plane. This indicates the existence of a downward displacement of the reservoir beyond the compaction effect. Finally, Figure 10 shows that the effect of the variation of Young's modulus in the layer which contains the reservoir (*E2*) is different from the effect observed in the variation of *E1* and *E3*. In this case, the increase of *E2* produces the rise of the vertical displacements at the reservoir top and bottom planes. This behavior indicates the increase of the compaction with the growth of *E2*.

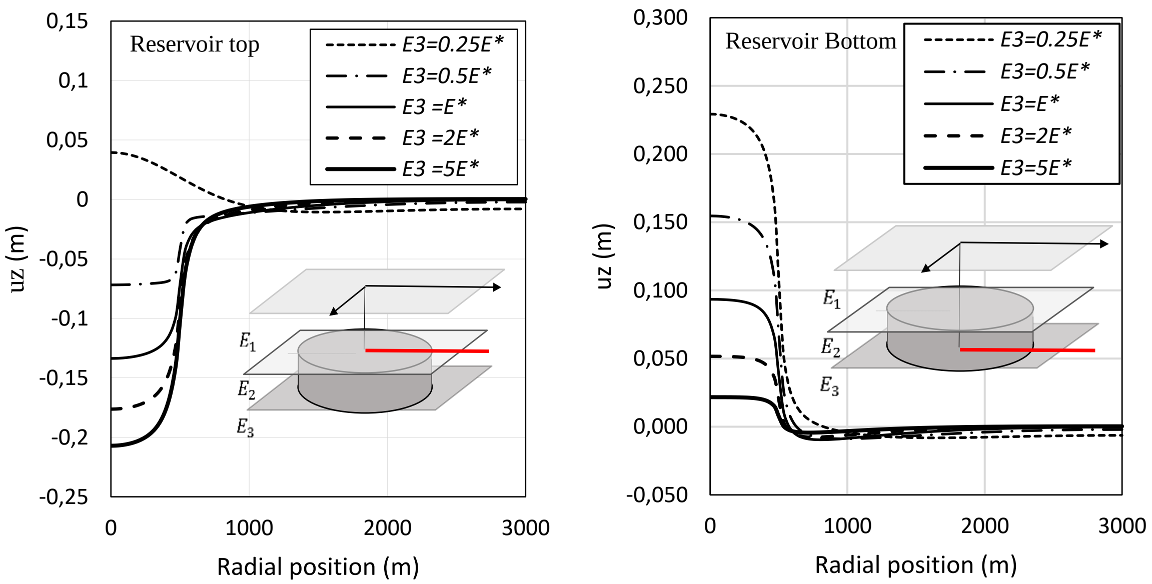

Figure 9. Vertical displacement at reservoir top and bottom surfaces, *E1*= *E2* = *E**.**

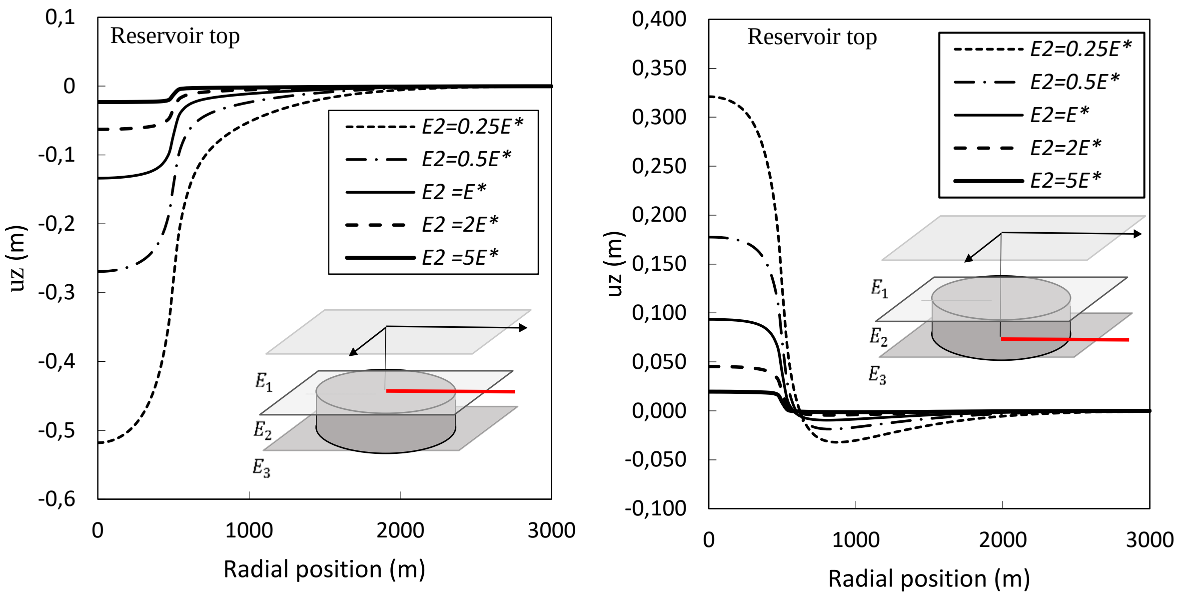

Figure 10. Vertical displacement at reservoir top and bottom surfaces, *E1* = *E3* = *E.**

Figure 11 shows the relation between the maximum vertical reservoir compaction (located at the reservoir center) normalized by the homologous result of the homogeneous case (*E1* = *E2* = *E3* = *E**). It can be noted that the variation of the normalized values of *E1* and *E3* does not produce noticeable changes in the normalized maximum reservoir compaction in comparison with the homogeneous case. On the other hand, the effect of the variation of *E2* can be easily noticed exhibiting a nonlinear behavior in which the compaction increases with the reduction of *E/E** and decreases with the growth of *E/E**. This behavior shows once more the major incidence of the reservoir layer properties in the deformability of both the medium and the reservoir. These results can be useful to determine the set of mechanical properties used in analytical solutions, once the upscaling techniques can be adapted to consider the incidence of the layers.

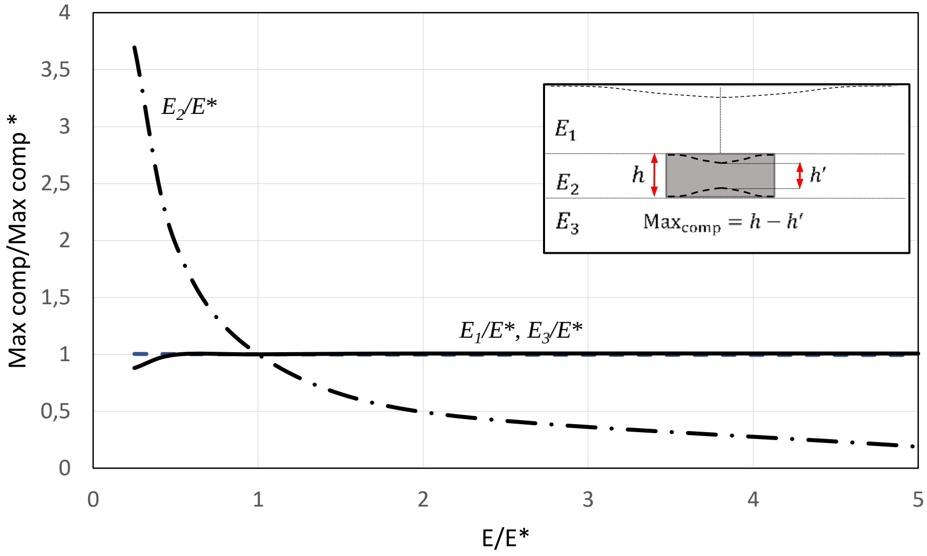

Figure 11. Normalized maximum compaction vs. normalized Young's modulus.

5.2 Analytical surface subsidence with upscaled Young's modulus

In this section the vertical displacement at free surface (subsidence) is calculated analytically through the modified Geerstma solution given in Eq. (5). For this, an upscaled Young's modulus value is adopted considering the three criteria given in Equations (8)- (10). In this example, it is considered the following Young's modulus values: $E1 = 1.5E*$, $E2 = E*$, $E3=2E*$, E*=3333.33MPa. It is also considered that all layers have the same Poisson coefficient of $\nu = 0.25$. Thus, It has the following values of Equivalent (Upscaled) Young's modulus: ${E_{eq}}^{average} = 1.5E^*$, ${E_{eq}}^{Settari.1} = 1.74$ (considering the three layers), ${E_{eq}}^{Settari.2} = 1.41E^*$ (considering the overburden and the reservoir layers) and ${E_{eq}}^{superposition} = 0.753E^*$ with $\alpha_1 = 0.5,\ \alpha_2 = 0$ and $\alpha_3 = -0.17$.

Figure 12 shows the vertical displacement at the free surface (subsidence) calculated with the analytical solution using all the upscaling criteria to calculated the Young modulus. The analytical solutions are compared with the results obtained via FEM. It can be observed that the superposition criterion agrees better with the FEM solution specially in the region of maximum subsidence. This effect is by the fact the coefficients used in the upscaling are related to the maximum subsidence. On the other hand, the other three criteria appear far from the FEM solution showing bad performance for this example. In the case of the results obtained with the criterion of Settari, the bad performance can be explained by formulation of the technique. In this method, it is considered interleaved layers of reservoir and medium, differently from the example here assessed, in which the reservoir is concentrated only in a continuous layer.

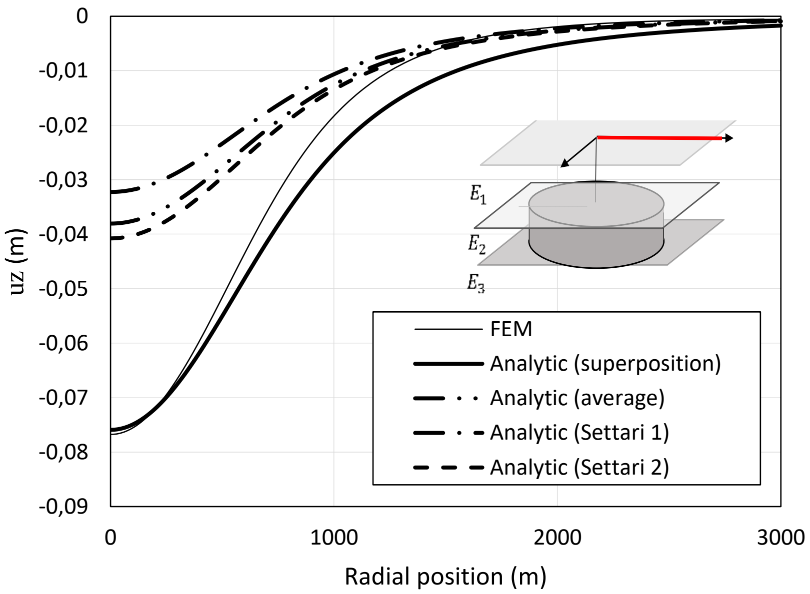

Figure 11. Vertical displacement at free surface.

6 CONCLUSIONS AND REMARKS

The influence of Young's modulus in the response of a cylindrical reservoir in a three-layer medium under uniform depletion was assessed.

The parametric study shows that the Young modulus of the reservoir layer has the largest influence in subsidence. In this case the maximum subsidence exhibits an exponential behavior with the variation of the reservoir Young modulus.

The parametric study also shows that the variation of the overburden and underburden does not influence the maximum compaction. In this case the maximum compaction only varies with the variation of Young's modulus of the reservoir layer.

The upscaling criterion based on the parametric study shows significantly better results for subsidence prediction in comparison with the other two criteria, the simple average and the unidimensional compaction approach (Settari, 2013).

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