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SOFTWARE DEVELOPMENT ON THE MATLAB FOR STRUCTURAL RELIABILITY AND SENSITIVITY ANALYSIS

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Abstract. *In this paper, we present the software for reliability analysis, named RelyLAB. We developed it in the MATLAB environment using the symbolic math tool. Two methods are implemented: the First Order Reliability (FORM) and the Second Order Reliability (SORM) Methods. We assess the random variables with the omission sensitivity factors, which provides the relative importance of the reliability index of each variable in uncertainty terms. In order to enhance decisions about the parameters of mean and standard deviation, we implement its sensitivities, providing the best parameter to change in an efficient design. To prove the software effectiveness, we applied in a steel connection model to enhance the design efficiently. Furthermore, we compared the sensitivity analysis of the analytical model with the changes in the variables to validate them and to show the reliability analysis performed by the software, providing the ideal decision under uncertainty based on sensitivity analysis tools.*

Keywords: *Reliability Analysis, Software Development, MATLAB, Sensitivity Analysis, Steel Connections.*

1 INTRODUCTION AND GOALS

During the design and construction phases of a structure, there are uncertainties about the strength of materials and associated with the loads, which will really act on a structure during the service life. Obviously, these types of uncertainties are reduced gradually as the collection of information is realized, but not eliminated in its entirety. However, structural designs precede the construction, and the existence of such uncertainties should be taken into account.

The uncertainty can be described as the difference between present and complete state of knowledge. In the context of model-based design, uncertainty is the difference between model prediction and reality (Du, 2005). The uncertainties that we have faced in engineering design can be classified into two types, aleatory and epistemic.

A way to situate them is by describing these uncertainties in a bar diagram, Figure 1, where the left side has the total ignorance and in the extreme right we have the complete knowledge about a problem, and at middle is the present state of knowledge, the right side can be divided into the two categories, so called:

• *Aleatory uncertainty*: it is associated with the nature of its origin, this category includes the physical, intrinsic and material uncertainties of nature, describing the inherent variation associated with it. This is an irreducible uncertainty. Examples of such uncertainties are: wind speed, concrete quality, wood resistance, and soil properties;

• *Epistemic uncertainty*: which is associated with the knowledge about random variables. This uncertainty can be reduced as the technical-scientific knowledge are improved and also when more data about it are acquired, thus providing an adjustment of hypotheses and models that are assumed. Examples of such uncertainties are: an insufficient number of observations and simplified calculation methods;

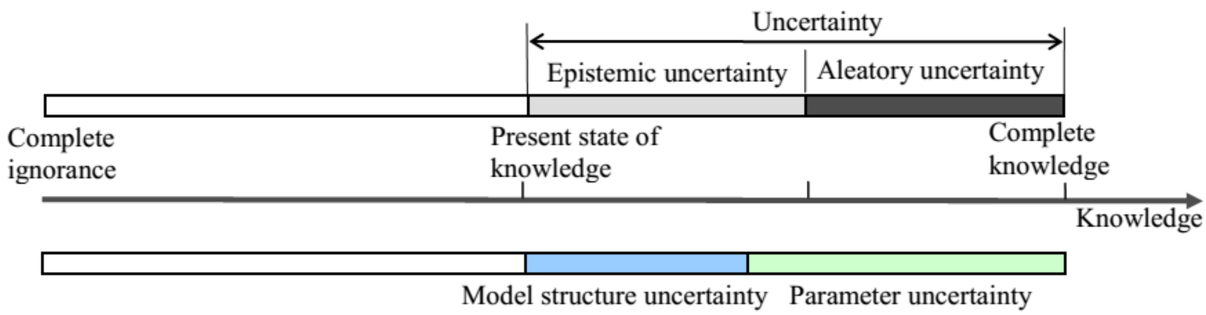

Figure 1: Uncertainty types (Du, 2005).

There are also two others uncertainties that complement these categories. The parameter uncertainty which small fraction is associated with the limited information (epistemic) and most of it is associated with the environment, in which it is estimated the characteristic of a parameter (aleatory). The other is called model structure uncertainty, which is integrally associated with the knowledge.

In our daily lives we are surrounded by numbers, sometimes we have forgotten that are only tools, they are not endowed with soul, only becoming elements of manipulation. A considerable number of our most critical decisions in the modern era are made by computers, the geniality product of humankind capable to digesting numbers, transforming a large amount of disorganized information into useful results for a specific purpose. From an engineering point of view, the software development deals easily the design process and support our decisions quickly.

The goal of this paper is the presentation of software development to assess the uncertainty with reliability analysis, providing a support for an ideal decision based on sensitivity factors, omission sensitivity factors, and parameter sensitivities. The software is called RelyLAB and it is implemented with capacity of application in engineering problems that have analytical formulation, simple systems, independent, and uncorrelated variables.

1.1 Structural Reliability Analysis

According to ISO 2394:2015, reliability is the ability of a structure to meet certain requirements under specified conditions during the working life forecast, for which it was designed. In the quantitative sense, the reliability can be defined as the complement of failure probability.

The ISO 2394, in the definition of reliability, highlights four essentials elements from an engineering point of view, namely:

- Requirements data (performance) - definition of structural failure;
- Time period - evaluation of the working life T;
- Reliability Level - Evaluation the probability of failure p_f;
- Conditions of Use - Limitation of uncertainty inputs.

In practice, the failure is being used in a very general sense, which simply indicates anyone undesirable state of a structure, $e.g.$, collapse or excessive deformation, given by structural conditions.

The most important term in the theory of structural reliability comes from the contribution of statistics, called the probability of failure p_f. In practical terms, one can define p_f in relation to a Limit State Function (LSF) and its basic variables $X = (X_1, X_2, ..., X_n)$, represented by the actions model, material properties, geometry and uncertainty model. The interaction between these basic variables can be established by Equation (1), where R is the set of variables related to resistance and S the set of variables related to the requesting actions.

$$g(R, S) = R - S \tag{1}$$

The structural reliability threshold state function separates the probability space into failure and safe regions, $i.e.$:

$$g(X) > 0\text{, safety domain;}$$
$$g(X) = 0\text{, failure surface;}$$
$$g(X) < 0\text{, failure domain;}$$

Uncertainty analysis consists, basically, in the consideration of variables as random, composed by the probabilities distributions containing the parameters mean (μ) and standard deviation (σ). These probabilities distributions have characteristics values X_k that corresponds to statistical failures. According to Sørensen (2004) correspond to the distribution quantiles, which is 5% for strength variables, 98% for variable actions, and 50% for permanent actions.

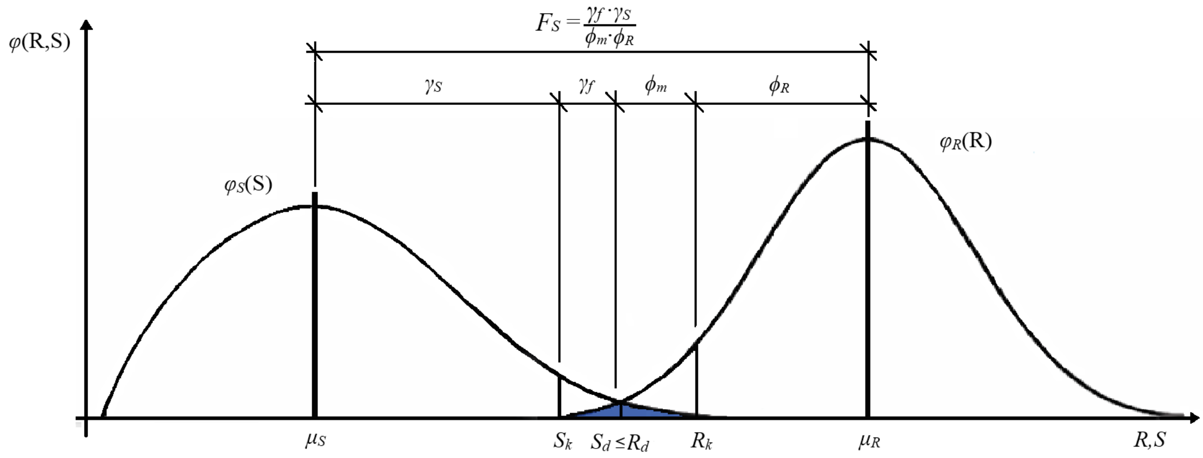

Figure 2: Relationship between resistance and load variables.

Furthermore, the civil structures are provided to meet design values X_d that depend on the type of structure, lifetime, and durability expected for it, which is associated to a small probability of being violated. The Fig. (2) indicates the characteristic and design values on reliability analysis.

A basic requirement for the safe designing is to provide the design resistance greater than or equal to the design load. This condition is obtained through the limit state equation or in general cases by partial factors of safety, reaching an adequate reliability.

The partial factors must make sure that the acceptable failure is guaranteed, which is represented by the characteristic value, and the designer should still guarantee a safety reserve, to ensure that the structure will have an adequate useful life corresponding to its purpose and economic interests of owners. The standards organizations like ABNT[1] in Brazil, ACI[2] and AISC[3] in U.S.A, and Eurocode[4] in Europe have the responsibility of to provide the partial factors designed to the more general cases by means of codes by means of codes.

According to Aoki (2008), the first group of partial factors applied is associated with the variability, and the second with majoring of loads or reducing of resistance, is here that the reliability plays its importance, mainly in terms of safety margin. Furthermore, the product of these partial factors provides the Factor of Safety (F_S) of the structure.

The measure used to determine structural safety is the reliability index represented by Greek letter β. In the structural analysis based on the reliability, it is generally required for approximate methods that the Reliability Index (β) of the structure meets a Design Reliability Index (β_d), Eq. (2), following the requirements of the code to provide the safety:

$$\beta \geq \beta_d \tag{2}$$

This condition is used by designers in probabilistic methods and their design values are defined by standards such as JCSS:2001 and ISO 2394:2015, Table 1 specifies the recommended values for the Ultimate Limit State (ULS), relating them to consequences and costs and for Service Limit State (SLS) associated with the costs.

[1]Brazilian Association of Technical Standards
[2]American Concrete Institute
[3]American Institute of Steel Construction
[4]European Standards

Table 1: Target reliability index β (p_f) related to the 1 year reference period T_0 (Adapted Part I - JCSS, 2001).

Relative cost of safety measure	ULS Failure Consequences Minor	Moderate	Large	SLS[1]
Large (A)	$3.1(10^{-3})$	$3.3(5.10^{-4})$	$3.7(10^{-4})$	$1.3(10^{-1})$
Normal (B)	$3.7(10^{-4})$	$4.2(10^{-5})$	$4.4(5.10^{-6})$	$1.7(5.10^{-2})$
Small (C)	$4.2(10^{-5})$	$4.4(5.10^{-6})$	$4.7(10^{-6})$	$2.3(10^{-2})$

$^{(1)}$: Irreversible Serviceability Limit State.

According to Holický and Marková (2005), these values are recommended as reasonable minimum requirements and they emphasize that β_d is conventionally formal value only, and may not correspond to the current frequency of failure.

El-Reedy (2005) explains that there is a degradation in the concrete structure with time, so that there will be a reduction in the durability of structure until reaching a minimum limit of the reliability index β_{min} as shown in Fig. (3), which is established by standards. Therefore, the reduction of structural reliability will play an important role along time to the planning strategies of maintenance. At this point the repair or rehabilitation of the concrete structure must begin, and then the reliability will (theoretically) return to its original value. However, there are some studies and researches which mention that it is not possible to return at the same reliability value, but to a lower value.

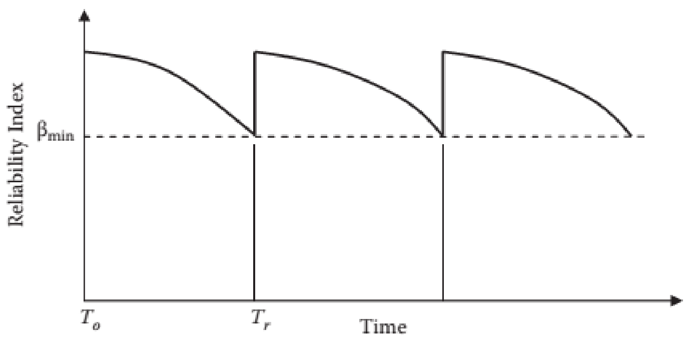

Figure 3: Structural reliability along time (El-Reedy, 2012).

Approximate Reliability Methods

The probability of failure, in principle, can be obtained by integrating the Joint Probability Density Function (jPDF) on the failure region. This integration over all regions of failure by analytical or numerical techniques is a very difficult, if not impractical task, because the domain often has an irregular shape and the jPDF is usually complex. (Melchers, 1999; Du, 2005; El-Reedy, 2012).

In the proposed software, RelyLAB, two approximated methods based on the Taylor series expansion were initially implemented. The first one is how the limit state is approximated at the design point, known as the First Order Reliability Method (FORM) and uses the first term in Taylor series. The another is known as Second Order Reliability Method (SORM) and uses the second term of the Taylor series expansion (Grandhi & Wang, 1999). The basic idea of these methods is to facilitate computational difficulties by simplifying the integral of jPDF and obtain an approximate result of the performance function $g(X)$ on reliability analysis (Du, 2005).

The use of these methods will depend on the complexity of a problem that is related to the Limit State Function nonlinearity. These methods seek to fit on the failure surface and obtain the Most Probably Point (MPP) of failure for standard normal distributions, i.e., mean 0 and standard deviation equal to 1. While FORM approximates the reliability by a linear surface, conversely the SORM approximates the failure surface by the curvature correction as shown in Fig. (4). Geometrically, the β can be represented by the distance of MPP to the mean 0 of standard normal variables. The algorithm implemented to reach the reliability index is based on the extension of the Hasofer-Lind method proposed by Rackwitz and Flessler (β_{HLRF}) for the case of non-normal random variables, which transform each of them into an equivalent normal one.

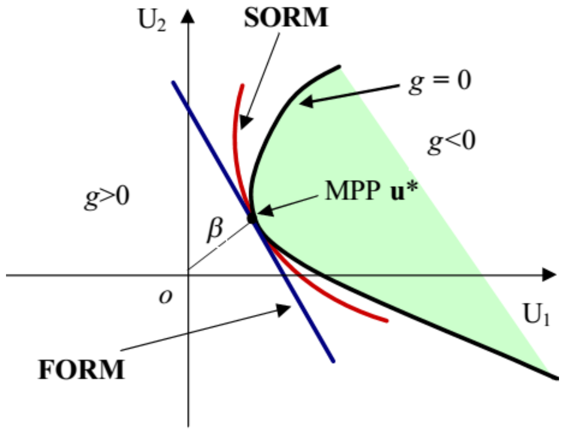

Figure 4: Comparison of FORM and SORM (Du, 2005).

The approximated methods proceed with a simplification achieved by transforming the random variable from their original space into a standard normal space. The original space has the basic random variables $X = (X_1, X_2, ..., X_n)$ known as X-space. Generally, the variables have irregular distribution shapes, then Rosenblatt transformation is performed to obtain all variables in the U-space, $U = (U_1, U_2, ..., U_n)$, with an equivalent standard normal distribution. This transformation is based on the condition that the Cumulative Density Function (CDF) of the basic random variable remains the same as original after its application (Du, 2005).

With the last β at MPP in the point $U = (u_1^*, u_2^*, ..., u_n^*)$ to n variables, the failure probability can be defined by Eq. (3), where $\Phi(.)$ is the CDF of the standard normal distribution or equivalent one.

$$p_f = \Phi(-\beta_{HLRF}) \tag{3}$$

1.2 Sensitivity Analysis

The Sensitivity Analysis is an important tool which allows us to improve the design in most reliable and best decision direction to save money, maintaining reliability and quality at a certain level. The application of this tool can be done in two very useful ways. The first is to obtain the sensitivity factors and it allows to evaluate the variables to reduce the nonlinearity of problem and fit the approximated methods used. The second is to obtain the parameter sensitivities of these variables and it is associated with the best decision regarding possibles changes in the parameters, $i.e$, mean (μ) and standard deviation (σ).

The process of recalculating to obtain new results under alternative assumptions to find the impact of a variable under the sensitivity analysis can be useful for a number of purposes, including:

- Complexity reduction in uncertainty analysis by eliminating insignificant random variables;
- Find variables for optimization of the model to satisfy some criterion;
- Understand and Identify variables with significant uncertainty in the result, leading to greater robustness;
- Try to identify important relationships between observations, variables, and predictions;
- Making recommendations more credible, understandable and persuasive;
- Reduce analysis time with high number of variables;
- Test the robustness of results in a model;
- Simplify the reliability analysis;
- Identify errors in the model.

Variables Sensitivity Factors

According to Grandhi and Wang (1999), the sensitivity factors present the relative importance of each basic random variable to the probability of failure in a structure. As much as higher it is the value, higher is its contribution to the probability of failure.

The sum of sensitivity factors should be equal to the unity, which can be represented by Eq. (4), and each contribution α_i^2 is called as the global sensitivity factor.

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \tag{4}$$

With the calculation of α_i, is obtained the sensitivity of β in relation to u_i, which has two main functions. First, these sensitivity factors are the relative contributions of the random variables to the β or p_f. Second, the sensitivity factor signal provides the relationship between performance function and variables. A positive α_i means that the performance function $g(U)$ decreases as the random variables increase, and a negative factor means $g(U)$ increases as the random variables increase (Grandhi & Wang, 1999). Its geometric representation can be visualized by Figure 5 and interpreted as being the directional cosine of variable i, for $i = 1, 2, ..., n$, also called as the local sensitivity factor.

It is noteworthy that, for the FORM method, the evaluation of sensitivity coefficients does not need additional calculations since this information is obtained during the calculation of

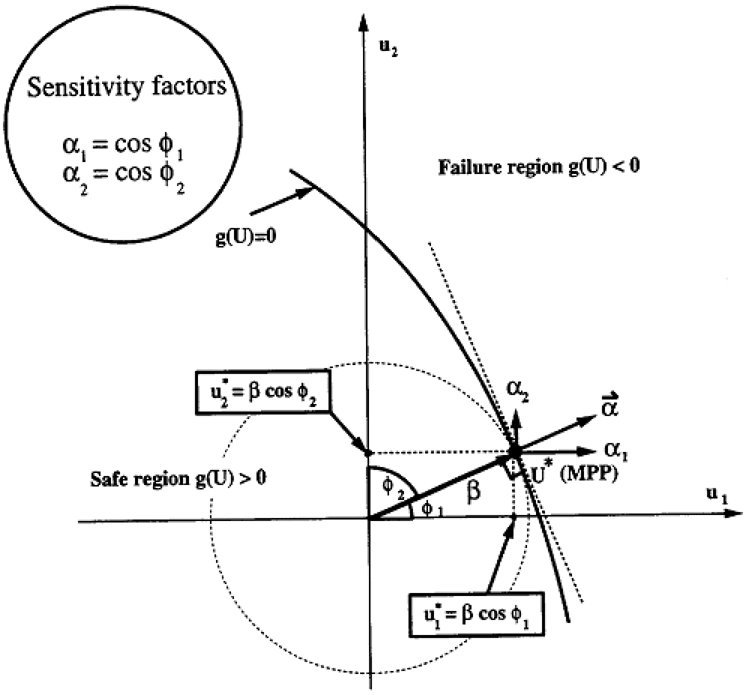

Figure 5: Sensitivity Factors (Grandhi e Wang, 1999).

gradients required to find the design point. It is emphasized that the detail of sensitivity measurement is a linear measure, so it has approximate value for non-linear LSF and non-normal probability distributions.

Omission Sensitivity Factors

The global sensitivity factor provides the statistical uncertainty effect of the basic random variables. We could better interpret them in terms of the omission sensitivity factor, defined by Sørensen (2004) as a measure of impact on the β if a random variable is kept fixed, assuming the variables are independents. This factor can be understood in the definition of author Madsen (1988) as the relative error of β when a basic random variable is considered as a deterministic.

Obtaining this factor depends only on the sensitivity factor and it can be obtained as follows by the Eq. (5) and Eq. (6):

$$\beta_i' = \frac{\beta - \alpha_i u_i^0}{\sqrt{1-\alpha_i^2}} \tag{5}$$

$$\xi_i = \frac{\beta_i'}{\beta} = \frac{1 - \alpha_i u_i^0/\beta}{\sqrt{1-\alpha_i^2}} \tag{6}$$

The omission sensitivity factor is defined in Eq. (7), when the special case of variable i in the normalized space, $u_i^0 = 0$, is chosen, which implies in the consideration of variable i as deterministic on the reliability analysis, then we have:

$$\xi_i = \frac{1}{\sqrt{1-\alpha_i^2}} \tag{7}$$

Parameter Sensitivity

The sensitivity calculation of the p_f or the β for small changes in basic random variables usually provides valuable information in the study of response to statistical variation.

The software inputs are the deterministic variables and the distribution parameters of each random variable. The measure of reliability sensitivity associated with the changes in these parameters is important for an easy and faster assessment of the reliability in case of structural design changes. Regarding the optimization procedure aiming at minimum total costs, the sensitivities can be used in the iterative solution methods. (Ditlevsen & Madsen, 2007).

According to Du (2005), when applying the reliability analysis in a given design and its response is unsatisfactory, $i.e.$, reliability below that required, there will be several means to improve the reliability. We can mention at least three, but for the sake of simplicity, in this paper only alternatives 1) and 2) below will be dealt with.

1) Change the mean;

2) Reduce variance ($i.e.$, Standard Deviation); and

3) Truncate the distribution of the random variable.

In the reliability analysis, when the number of random variables is relatively large, it is difficult or even uneconomical to control all the basic random variables. The sensitivity analysis applied in the main parameter, standard deviation or mean, provides the mathematical basis for more efficient decisions.

Du (2005) describes this application as Reliability Sensitivity Analysis, constituted in the method to finding the rate of change in the probability of failure or reliability index due to the change in the distribution parameters.

For a distribution of the parameter p of the normalized random variable u_i, the sensitivity is defined as the partial derivative of Eq. (3) related to the parameter (p) by Eq. (8):

$$s_p = \frac{\partial p_f}{\partial p} \tag{8}$$

Evaluating the partial derivative Eq. (9), we have the following Eq. (10) with the parameter sensitivity:

$$s_p = \frac{\partial p_f}{\partial p} = \frac{\partial \Phi(-\beta)}{\partial p} = \frac{\partial \Phi(-\beta)}{\partial \beta}\frac{\partial \beta}{\partial p} \tag{9}$$

$$s_p = -\phi(-\beta)\frac{\partial \beta}{\partial p} \tag{10}$$

Thus the partial derivative of the reliability index in relation to the parameter distribution is given by the Eq. (11) associated to a random variable u_i^*, where $i = 1, 2, ..., n$:

$$\frac{\partial \beta}{\partial p} = \frac{\partial \beta}{\partial u_i^*}\frac{\partial u_i^*}{\partial p} \tag{11}$$

Where we can find the Eq. (12):

$$\frac{\partial \beta}{\partial u_i^*} = \frac{\partial \sqrt{\sum_{j=1}^{n}(u_j^*)^2}}{\partial u_i^*} = \frac{u_i^*}{\sqrt{\sum_{j=1}^{n}(u_j^*)^2}} = \frac{u_i^*}{\beta} \tag{12}$$

The normalized variable point u_i^* is given by the Eq. (13) obtained by the standard normal distribution $F_{X_i}(x_i^*)$, using the Rosenblatt Transformation:

$$u_i^* = \Phi^{-1}[F_{X_i}(x_i^*)] = w(p) \tag{13}$$

Substituting the above expressions, Eqs. (11), (12), and (13), into the sensitivity Eq. (10), we have the following expression in Eq. (14):

$$s_p = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial p} \tag{14}$$

Calculating the sensitivities of mean and standard deviation, it is obtained the Eq. (15) and Eq. (16), respectively:

$$s_{\mu_i} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial \mu_i} \tag{15}$$

$$s_{\sigma_i} = -\phi(-\beta)\frac{u_i^*}{\beta}\frac{\partial w}{\partial \sigma_i} \tag{16}$$

The differential term of the Eq. (14) will be obtained as follows using the Rosenblatt Transformation, which in this case is done for a normal or equivalent normal distribution of Eq. (17):

$$w(\mu_i, \sigma_i) = \Phi^{-1}[F_{X_i}(x_i^*)] = \Phi^{-1}[\Phi(\frac{x_i^* - \mu_i}{\sigma_i})] = \frac{x_i^* - \mu_i}{\sigma_i} \tag{17}$$

Applying the partial derivative in relation to the μ_i, then we obtain the Eq. (18):

$$\frac{\partial w(\mu_i, \sigma_i)}{\partial \mu_i} = -\frac{1}{\sigma_i} \tag{18}$$

Applying the partial derivative in relation to the σ_i and substituting expression of the variable normalized u_i^* at the design point, then we obtain the Eq. (19) as follows:

$$\frac{\partial w(\mu_i, \sigma_i)}{\partial \sigma_i} = \frac{\mu_i - x_i^*}{\sigma_i^2} = -\frac{x_i^* - \mu_i}{\sigma_i}\frac{1}{\sigma_i} = -\frac{u_i^*}{\sigma_i} \tag{19}$$

In this way, we can then obtain the value of parameter sensitivity by the expressions in Eq. (20) and Eq. (21):

$$s_{\mu_i} = \phi(-\beta)\frac{u_i^*}{\beta\sigma_i} \tag{20}$$

$$s_{\sigma_i} = \phi(-\beta)\frac{(u_i^*)^2}{\beta\sigma_i} \tag{21}$$

2 METHODOLOGY

2.1 Process in Reliability Analysis Software

A model of the execution process can be seen in Fig. (6), obtained with the software *BizAgi*®, which describes the process of reliability analysis with RelyLAB. Initially, the process performed by the reliability analysis will have data inputs as the mean and deviation parameters of each random variable, distributions types, the definition of deterministic variables, determination of target reliability, and definition of the LSF. Following to the analysis step, the methods are applied to find the reliability β and parameter sensitivity. Passing this algorithm operation, the evaluation is done in terms of sensitivity and reliability, if it is unsatisfactory, the procedure is repeated iteratively. In the condition of satisfactory, the results are used for the generation of graphs and tables, which are used in reports, at this stage the process is manual with the use of spreadsheet and text editor software.

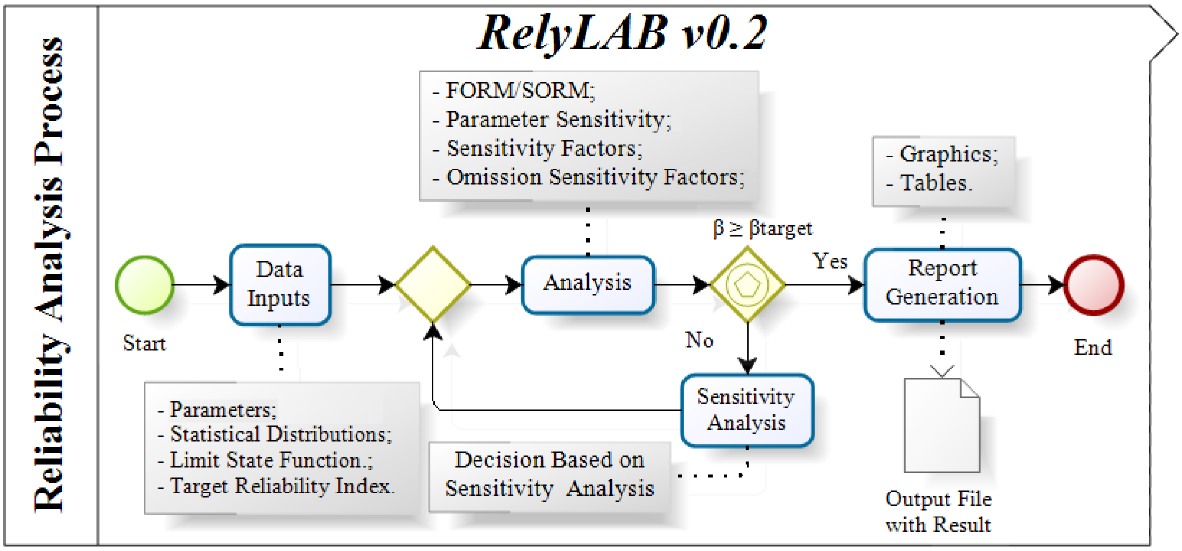

Figure 6: Process of software $Rely_{\mathbf{LAB}}$.

The convergence criterion shown in Eq. (22) was used after the first iteration, where j represent the iteration number:

$$\epsilon_j = \frac{|\beta_j - \beta_{j-1}|}{|\beta_{j-1}|} \tag{22}$$

In order to obtain the design point in the reliability analysis, we choose for the software convergence condition at $\epsilon_j \leq 10^{-10}$, defined as satisfactory for the resolution of software analysis.

The software was developed to execute reliability analysis with the following distributions: Normal, Lognormal, Exponential, Rayleigh, EV (Extreme Value) Type I - Max (Gumbel), EV Type II - Max (Fréchet), EV Type III - Min (Weibull), and Weibull.

2.2 Parameter Sensitivity of Steel Connection with End Plate

To prove the software applicability, the analysis of a steel connection with rectangular cross-section profile on an end plate is carried out.

The analytical model chosen for the analysis is described in Wheeler *et al.* (1997) and the variables used on the model are shown in Fig. (7).

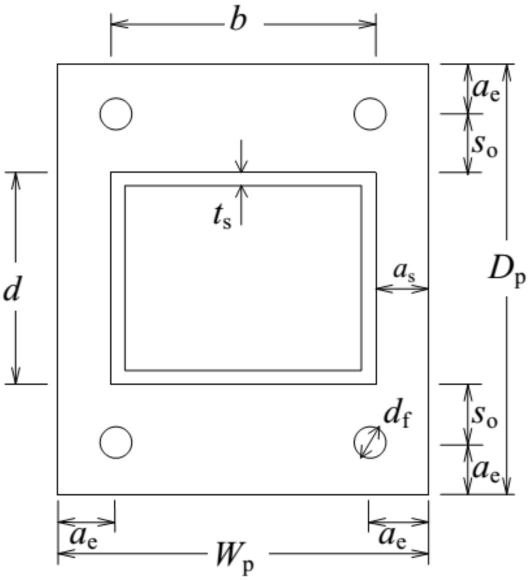

Figure 7: End plate model parameters (Wheeler *et al.*, 1997).

Where:

W_p: plate width;
D_p: plate height;
d: profile height;
b: width of the profile;
d_f: hole diameter ($d_b + 2mm$);
a_e: distance from hole to edge;
a_s: distance from profile to edge in horizontal direction;
s_o: distance from hole to profile in vertical direction;
t_s: profile thickness;
a_s: distance from profile to edge.

The connection between profile beam and end plate is by weld (s), in this way the force act on the center of weld throat. Such that the load effects are applied at the new geometry parameters as presented in Eqs. (23) and (24).

$$d^{'} = d + \frac{s}{\sqrt{2}} \tag{23}$$

$$s_o^{'} = s_o + \frac{s}{\sqrt{2}} \tag{24}$$

The type of connection chosen for the sensitivity analysis can be composed of rectangular and/or square hollow section profile. This type of steel connection presents 3 failure modes, as presented in Fig. (8), which were studied by Wheeler *et al.* (1997). Our demonstration of the sensitivity tool consists in assess the equation presented by these authors for failure Mode 2.

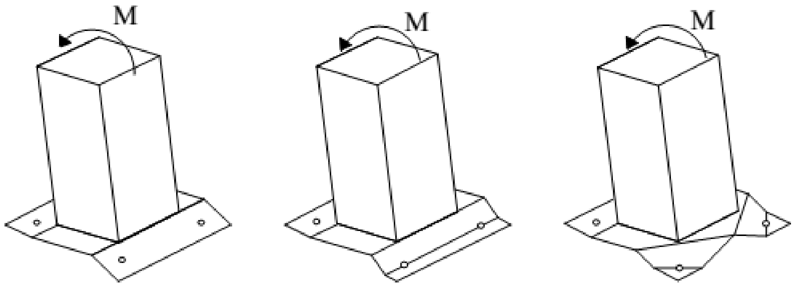

Figure 8: Yield Line Modes of Failure (Wheeler *et al.*, 1997).

In this equation used in for LSF, the yield of the end plate occurs horizontally between the bolts in tension, is described as follows in Eq. (25):

$$M_{2,yl} = \left(\frac{2 \cdot (d' + s_o') \cdot W_p}{s_o' \cdot d'} - \frac{n \cdot d_f}{s_o'}\right) \cdot m_p \cdot (d - t_s) \tag{25}$$

Where:

$$m_p = \frac{t_p^2 \cdot f_y}{4} \tag{26}$$

2.3 Reliability of the Steel Connection

We assumed the values of basic random variables in accordance with Part III of the JCSS (2001), that is include the dimensional variables and the estimating yield resistance for structural steel. In the case of the random variable of moment load effect, we assumed as a single load to simplify the analysis and focus on the components of steel connection. All the random variables are detailed in Table 2.

Table 2: Basic Random variables.

i	Variable	Mean	Standard Deviation	CoV	Unit	Distribution
1	d	200	1.0	0.0050	mm	Normal
2	t_s	9	1.0	0.1111	mm	Normal
3	t_p	16	1.0	0.0625	mm	Normal
4	s	8	1.0	0.1250	mm	Normal
5	s_o	35	1.0	0.0286	mm	Normal
6	f_y	368.7	25.81	0.0700	N/mm^2	Lognormal
7	M_E	3.5×10^7	5.25×10^6	0.1500	$N.mm$	EV Type I - Max

In a pre-analysis, we identified some variables with low sensitivity factors or/and low parameter sensitivity, which influenced our decision of defining them as deterministic. The deterministic variables are in Table 3.

Table 3: Deterministic variables.

i	Variable	Nominal Value	Unit
1	a_e	30	mm
2	b	100	mm
3	W_p	160	mm
4	d_f	22	mm
5	n	2	$Un.$

The declaration in the RelyLAB of all these variables presented in Table 2 and 3 must be as shown below:

```
%========================================= DATA INPUTS ==========================================
% Variables and Parameters Names
labelvar = {'d';'t_s';'t_p';'s';'s_o';'f_y';'M_s'};
labelpar = {'\mu_d';'\mu_{ts}';'\mu_{tp}';'\mu_s';'\mu_{so}';'\mu_{fy}';'\mu_{M_E}'; ...
'\sigma_d';'\sigma_{ts}';'\sigma_{tp}';'\sigma_s';'\sigma_{so}';'\sigma_{fy}';'\sigma_{M_E}'};
% Basic Random Variables and its parameters:  - Units:  N, mm
%-----1---2---3--4-5--6-----7
%-----d---ts--tp-s-so-fy----ME
mu = [200 9.0 16 8 35 368.7 3.5*10^7];
%--------d---ts--tp--s---so--fy----ME
sigma = [1.0 1.0 1.0 1.0 1.0 25.81 5.25*10^6];
%-----d--------ts-------tp-------s-----------so-------fy----------ME
dist={'Normal';'Normal';'Normal';'Normal';'Normal';'Lognormal';'VE Tipo I - Máx'};
% Deterministic Variables:  - Units:  N, mm
ae = 30;  b = 100;  Wp = 160;  df = 22; n = 2;
```

The next step is the introduction of the LSF using the symbolic math tool of MATLAB® which the user can declare any equation, entirely or in a series of split equations as shown below:

Limit State Function, Eq. (27):

$$g(d, t_s, t_p, s, s_o, f_y, M_E) = (\frac{2 \cdot (d' + s_o') \cdot W_p}{s_o' \cdot d'} - \frac{n \cdot d_f}{s_o'}) \cdot m_p \cdot (d - t_s) - M_E \tag{27}$$

With the variables d', s_o', and m_p defined by Eqs. (22), (23) and (25), respectively. The declaration of LSF in MATLAB® proceeds as shown below:

```
%================================== LIMIT STATE FUNCTION ===================================
%LSF: used in FORM;
%Equations:
mp = 1/4*X{3}^2*X{6};    dl = X{1}+X{4}/2^0.5;    sol = X{5} - X{4}/(2^0.5);
Mr = (2*(dl+sol)*Wp/(sol*dl)-n*df/sol)*mp*(X{1}-X{2});
%Limit State Function:
LSF = Mr - X{7};

%LSFU: used in SORM
%Equations:
mpu = 1/4*U{3}^2*U{6};    dlu = U{1}+U{4}/2^0.5;    solu = U{5} - U{4}/(2^0.5);
Mru = (2*(dlu+solu)*Wp/(solu*dlu)-n*df/solu)*mpu*(U{1}-U{2});
%Limit State Function:
LSFU = Mru-U{7};
% Note:  U{i} is substituted by -> Xi = mu(i) + Ui*sigma(i)
```

3 RESULTS AND ARGUMENTS

3.1 Result Tables

The first information obtained is in Table 4, which contains a summary of the results with local (α_i) and global (α^2) sensitivity factors, the omission sensitivity factors (ξ_i), and the parameter sensitivities of the mean (s_μ) and standard deviation (s_σ).

Table 4: Results for sensitivities.

i	Symbol	α_i	α_i^2	ξ_i	$s_{\mu i} \times 10^{-3}$	$s_{\sigma i} \times 10^{-3}$
1	d	-0.0191	0.0004	1	1.86	-0.06
2	t_s	0.0220	0.0005	1	-2.08	-0.08
3	t_p	-0.5585	0.3119	1.21	54.48	-51.07
4	s	-0.0839	0.0070	1	8.18	-1.15
5	s_o	0.1216	0.0148	1.01	-11.86	-2.42
6	f_y	-0.2941	0.0865	1.05	-1.11	-0.55
7	M_E	0.7608	0.5788	1.54	0	0

With the parameters inserted, the reliability index of analysis is about 1.678 in FORM, which is below of the necessary to guarantee the building safety, considering a minor failure consequence to a Large (A) relative cost of safety, $\beta_{target} = 3.1$ at Tab. (1). To reach the target, it is important to change efficiently the parameters, save time in the analysis, and allocating economically the resources, we reach this aim using the parameter sensitivity tool. We reach this aim using the parameter sensitivity tool.

3.2 Sensitivity Factors and Omission Sensitivity Factors

The next results to be analyzed are the global sensitivity factors of the variables shown in the pie chart, Fig. (9). The presentation of this information is complemented by their respective values of omission sensitivity factors, providing the measure of relative error in each variable on reliability when it is being considered as deterministic. By the result presented in Fig. (9), the variable with highest impact is the Moment Effect, following by the end plate thickness (t_p) and its yield strength (f_y). It is important to note that the use of this factor is very useful when there is a high number of variables with the use of approximate methods such as SORM and FORM, to reduce the nonlinearity of failure surface in the analysis.

3.3 Parameter Sensitivity

Another important measure of sensitivity is related to the statistical parameters for each variable. It is indispensable to enhance the structural reliability. The Table 4, initially presented in the section, presents this information about its values. For the Failure Mode 2 equation in the steel connection, the software RelyLAB gives the following result expressed in a bar chart, Fig. (10).

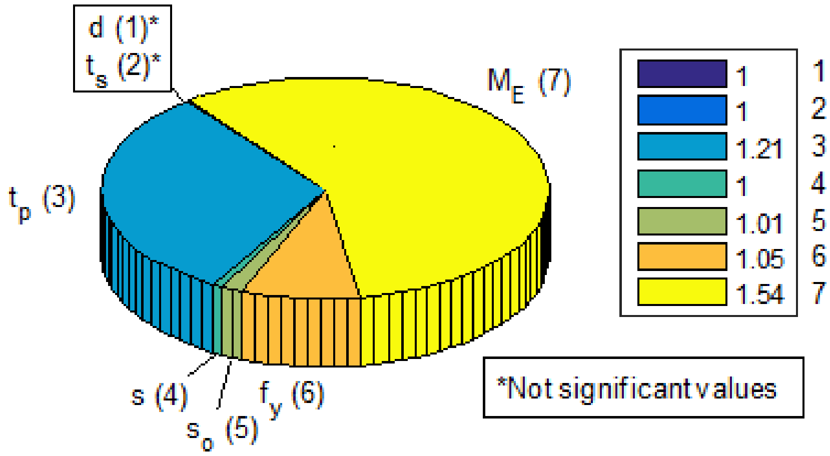

Figure 9: Global Sensitivity Factors and Omission Sensitivity Factors.

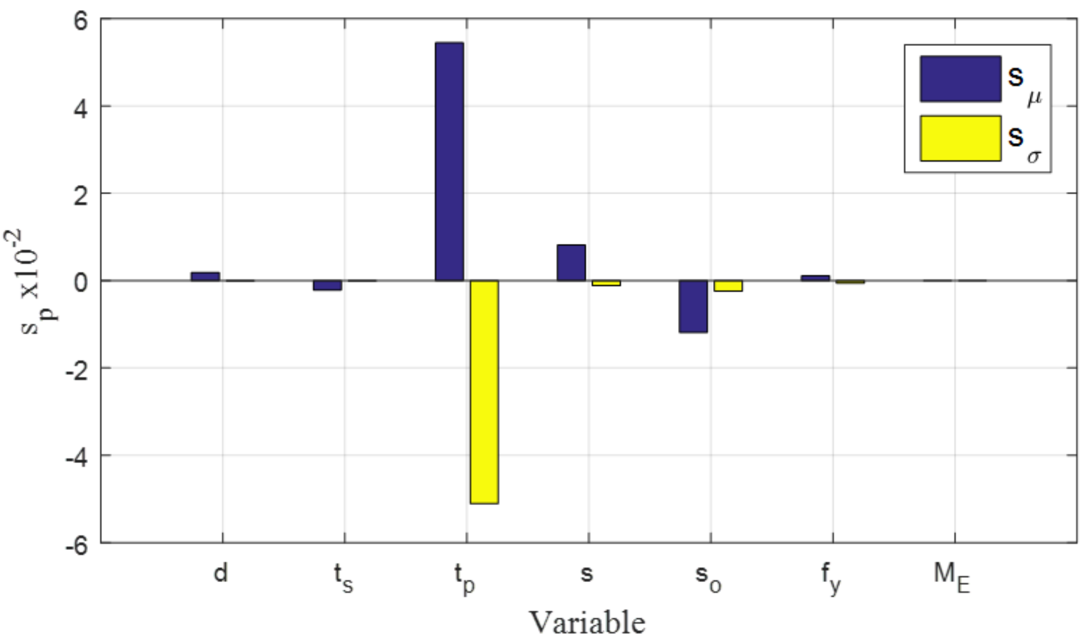

Figure 10: Parameter Sensitivity.

Evaluating the result of the graph above, it is noticeable that the mean thickness of the end plate (t_p) is the most sensitive parameter among the others with its standard deviation, respectively.

The impact assessment of the parameters must be guided by the sign of each one, which is related to the change caused in the reliability index (see Table 2). Moreover, this value may change depending on the nonlinearity that an LSF may exposes in the analysis.

In fact, it is not always possible to control some variables, leading to focus on those which are closer to our decision-making possibilities. For this example, increasing the mean of end plate thickness is one that would give greater reliability and consequently the resistance. With most important parameter known, a new analysis is processed and the results are in Tab. (5) with β_{New} to $\mu_{t_p} = 19.05\ mm$.

Table 5: Parameter Sensitivity Applied.

Method	β_{Old}	β_{New}
FORM	1.678	3.175
SORM	1.681	3.173

3.4 Sensitivity Analysis Varying the Variables

The assessment of results is also based on the parameters variation of steel connection with the analytical equation provided in Eq. (25) using the nominal values of the variables. In this way is obtained some points that provides the possible of modeling the data, which is applied the regression by Least Square Method (LSM) and its resulting equations are derivatives at the initial point for the nominal value of the analytical model, this results can be visualized in the Figures from 12 to 17 at the end of this paper, changing each variable in a defined range.

Based on the comparison of results between mean sensitivity obtained initially and that provided by the variation of nominal values in the analytical model, it is notable that the sum of absolute difference about its contribution is low and acceptable.

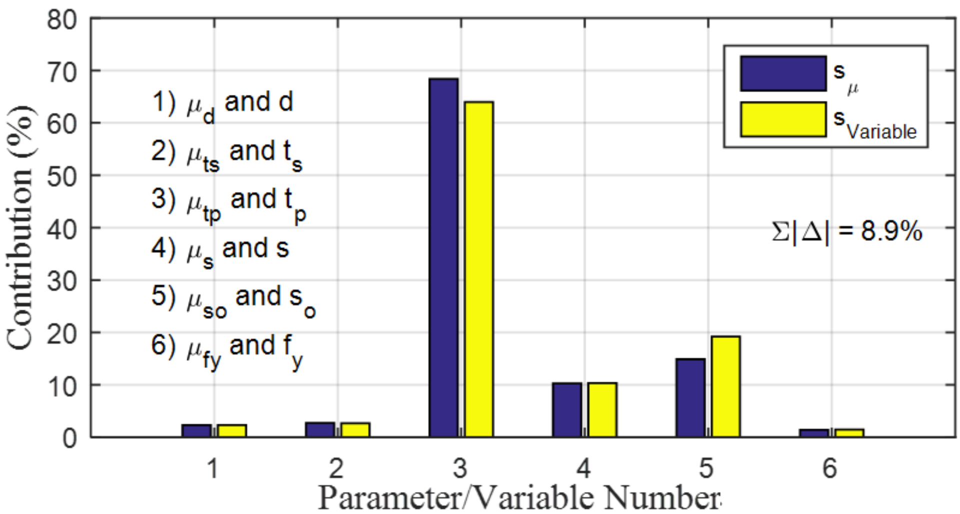

Figure 11: Sensitivity Comparison.

4 CONCLUSION

The development of the software presented here is relevant in the actual scenario of engineering, but it should be remembered that it is only a mathematical manipulation tool providing numbers, which without proper knowledge about the subject do not provide possibilities for ideal and rational decisions. In fact, knowledge is not replaced by results presented in tables and graphs, these should be used to aggregate value in our assumed hypotheses and orienting the future decisions to those more rational in light of theory and experiences.

The sensitivity analysis implemented in the software RelyLAB proved to be an efficient tool to find the most important parameters of the LSF, in order to reduce the time spent in an analysis for the enhancing the design, in which, normally, the variation of a parameter is performed to obtain the sensitivity in response. This can ideally fit when the number of variables is high, and the variation of each of these parameters consumes time in an analysis. Regarding the use of this tool, the task came to simplify and quickly, delegating time to important activities which have the necessity of Engineering judgment.

The software development on the MATLAB® environment proved to be advantageous since it was thus possible to take advantage of tools, function libraries and matrix manipulation, as well as the quality provided in the presentation of results such as the 2D graphics.

The sensitivity analysis provides less subjectivity in the decisions to be made for the design of structures, making the justifications concisely and coherent from the economic point of view.

In general terms, reliability presents an opportunity to develop analyzes of importance in the current scenario, which has been true for decades in European countries and in North America. The sensitivity analysis allows a critical evaluation of the problems, allocating resources focused on the best decision based on the parameter sensitivities of the engineering variables.

REFERENCES

Aoki, N., 2008. Dogma do Fator de Segurança, *Seminário Engenharia de Fundações Especiais e Geotecnia - SEFE VI.*

El-Reedy, M. A., 2012. *Reinforced Concrete Structural Reliability*, CRC Press: Boca Raton.

Du, X., 2005. *Probabilistic Engineering Design*, University of Missouri-Rolla.

Ditlevsen, O. & Madsen, H. O., 2007. Structural Reliability Methods. Chichester, West Sussex, England: John Wiley and Sons Ltd.

Grandhi, R. V. & Wang, L., 1999. Structural Reliability Analysis and Optimization: Use of Approximations, *NASA/Contractor Report - 1999-209154*, Wright State University.

Holický, M. & Marková, J., 2005. Chapter III - Reliability Differentiation, *Handbook 2 - Implementation of Eurocode*, Czech Technical University at Prague: Czech Republic.

ISO 2394:2015. *General principles on reliability for structures*, Switzerland.

Joint Committee on Structural Safety - JCSS, 2001. *The JCSS Probabilistic Model Code*, *in Part I - Basis of Design*.

Joint Committee on Structural Safety - JCSS, 2001. *The JCSS Probabilistic Model Code*, *in Part III - Material Properties*.

Madsen, H. O., 1988. Omission Sensitivity Factors. *Structural Safety*, vol. 5, n. 1, pp. 35-45.

Melchers, R. E., 1999. *Structural Reliability Analysis and Prediction 2nd Edition*. John Wiley & Sons Ltd.

Sørensen, J. D., 2004. *Notes in Structural Reliability Theory and Risk Analysis*, Institute of Building Technology and Structural Engineering: Aalborg University.

Wheeler, A. T., Clarke, M. J., Hancock, G. J., & Murray, T. M, 1997. *Design Model for Bolted Moment End Plate Connections using Rectangular Hollow Sections*, Research Report, No. R745, Department of Civil Engineering, University of Sydney.

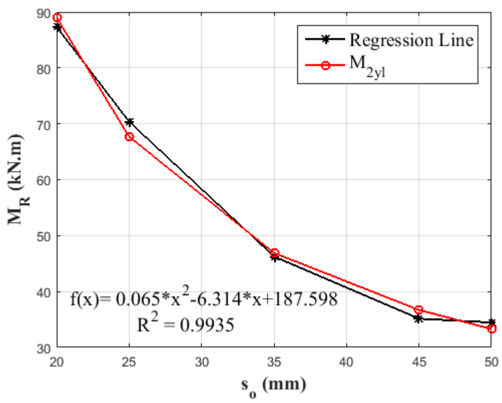

Figure 12: Sensitivity of Hole Distance to Profile Surface in vertical direction.

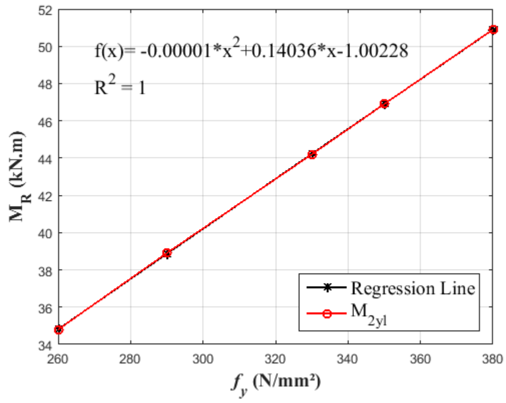

Figure 15: Yield Resistance of End Plate.

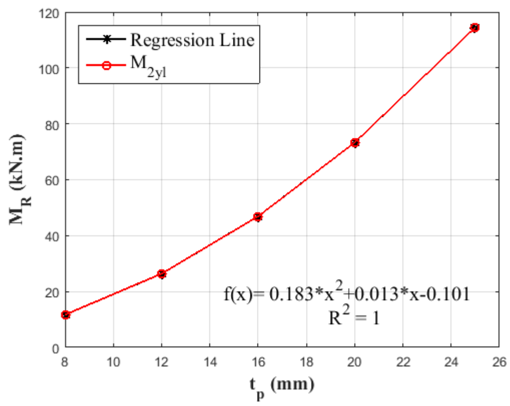

Figure 13: Thickness Sensitivity of the End Plate.

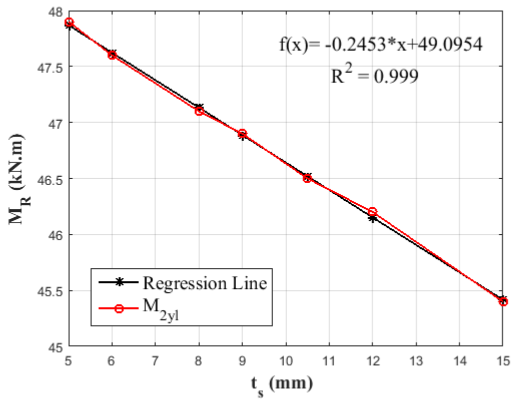

Figure 16: Profile Thickness Sensitivity.

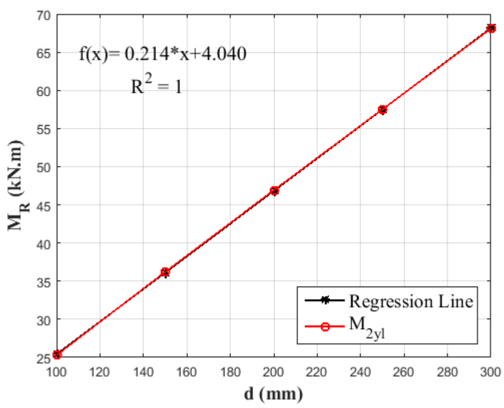

Figure 14: Profile Height Sensitivity.

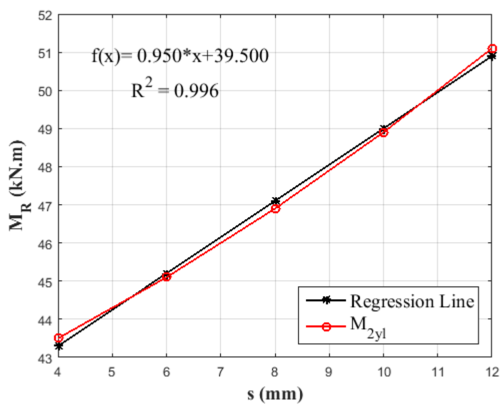

Figure 17: Weld Sensitivity.