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FAILURE PROBABILITY MINIMIZATION OF STRUCTURES WITH MULTIPLE FRICTION TUNED MASS DAMPERS UNDER SEISMIC EXCITATIONS

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Abstract. *Optimum design of tuned mass dampers (TMDs) and multiple tuned mass dampers (MTMDs) in vibration mitigation due to earthquakes is an important research topic. A literature survey reveals that the controlled system is often considered linear and a deterministic approach is applied. Moreover, we could not find applications of global design optimization of multiple friction tuned mass dampers (MFTMDs) which considers a robust reliability based approach. Hence, this paper presents a novel application on robust optimization of nonlinear MFTMDs to control vibrations in structures submitted to seismic excitations, in order to minimize the* *failure probability of these structures. The nonlinear solution of this stochastic dynamical system under uncertainties is carried out through a statistical linearization technique in the state space, which requires considerably fewer computational effort than a classical time domain analysis. A hybrid global local scheme composed of the algorithms Firefly (FA) and Nelder Mead (NMA) is applied for the optimization. A numerical example on a ten-story planar steel building frame with one, two and five dampers is demonstrated. The results point out that the proposed methodology can reduce significantly the failure probability of civil structures under seismic excitations.*

Keywords: *Earthquakes, MFTMDs, Robust optimization, Failure probability, Statistical linearization*

1 INTRODUCTION

In the last decades, the interest of researchers in the study of mechanisms that mitigate the undesirable effects of human and nature (wind and earthquake) induced vibrations on structures has gradually increased (Saaed et al., 2015). In this context, passive energy dissipation systems have been widely used for control vibrations in civil structures submitted to seismic excitations (Curadelli and Amani, 2014). Within this purpose, the tuned mass damper (TMD), initially presented by Frahm (1909), stands out, but its performance in seismic engineering is still under discussion (Giuliano, 2013). This device is basically composed of a spring-mass-damper system, wherein the damper behavior is usually considered linear (e. g. viscous damping).

Although the TMD concept is relatively simple, the parameters of this device must be obtained through robust optimization procedures to provide an appropriate control. Some classical works present expressions of optimum stiffness and damping of a TMD attached to an undamped single degree of freedom (SDOF) system under harmonic load (Den Hartog, 1956) and under white noise random excitations (Warburton, 1982). More recently, many researches showed that the multiple tuned mass dampers (MTMDs) are more effective than a single TMD (Xu and Igusa, 1992; Yamaguchi and Harnpornchai, 1993; Abe and Fujino, 1994). In this sense, in the last years, optimization studies of MTMDs and considering uncertainties have gained prominence (Joshi and Jangid, 1997; Hadi and Arfiadi, 1998; Hoang and Warnitchai, 2005; Lee et. al, 2006; Taflanidis, Beck and Angelides, 2007; Leung and Zhang, 2009; Marano, Greco and Sgobba, 2010; Mohebbi et al., 2013; Lopez, Fadel Miguel and Beck, 2014; Beck, Kougioumtzoglou and Santos, 2014; Fadel Miguel et. al, 2016). It is important to mention that some papers considering TMDs with nonlinear viscous damping are also found in the literature (Rudinger, 2007; Chung et. al, 2009; Chung et. al, 2013).

However, a literature survey reveals that the most TMDs and MTMDs studies is associated with the hypothesis of linear behavior for the structure and for the dampers, as well as considers the excitation as the only source of uncertainty. These limitations in the structure and in the energy dissipation systems models needs to be better evaluated (Gewei and Basu, 2010). In this sense, researches involving some non linearity of this devices, such as the consideration of a friction damping, which is the case of the friction tuned mass dampers (FTMDs), are significantly reduced. It should be noted that in most applications, a nonlinear behavior must be taken into account, since it may be associated with the structural system. In addition, a nonlinear damper behavior can provide better performance compared to a linear one and may represent a simplification for construction and maintenance issues (Pall and Rashimi Pall, 1996).

In this scenario, the FTMDs have been studied by some authors. Inaudi and Kelly (1995) investigated a FTMD attached in an undamped SDOF system, considering white noise random excitation. Ricciardelli and Vickery (1999) evaluated the behavior of a FTMD connected to a SDOF system submitted to harmonic load. Analytical expressions for the optimum FTMD parameters were proposed for this scheme. Gewei and Basu (2010) assessed the behavior of a FTMD attached to a SDOF system under base acceleration. An analytical expression was obtained for the displacement of the system under harmonic load by the harmonic balance method and the root mean square value of the system displacement was evaluated under white noise random excitation. Numerical simulations showed that the performance of the FTMD is similar to the TMD. Barbosa and Ramadhan (2014) evaluated the response of a 72 story building subjected to real seismic excitations with one and two FTMDs attached to it via numerical time integration. The results showed that the use of MFTMDs is more efficient in controlling the structure response than the use of a single FTMD. C. Lin, L. Lin and Lung (2014) developed a prototype of a system with three FTMDs to control vibrations in tall buildings, in that the damping derive from the friction between sliding blocks and rails. Shaking table tests and numerical simulations were successfully carried out. Pisal and Jangid (2014) studied the performance of MFTMDs in reducing the structural response of a five story building. The motion equations were solved numerically via state space method and optimum MFTMDs parameters were obtained for four real seismic records through a parametric study. It was observed that the MFTMDs are more efficient in controlling the vibration of the structure than a single FTMD. Mantovani et al. (2017) presented a study on global optimization of MFTMDs to control vibrations in structures subjected to seismic excitations through nonlinear analysis in the time domain. The use of MFTMDs proved to be more efficient than the use of a single FTMD and the proposed approach demonstrates to be a convenient alternative to provide the optimum MFTMDs parameters.

Within this context, this paper proposes a global robust optimization of MFTMDs in structures submitted to seismic excitations modeled as a stationary stochastic process via statistical linearization through state space. In Section 2, the process used to determine the dynamic response and the failure probability of structures with MFTMDs, as well as the the hybrid optimization scheme, which uses the Firefly algorithm (FA) and the Nelder-Mead algorithm (NMA), sequentially, are presented. To take into account uncertainties present in the system and in the excitation, some of their parameters are modelled as random variables. Section 3 shows a numerical example in order to verify the efficiency of the proposed methodology. Section 4 presents the conclusions obtained in this paper.

2 METHODOLOGY AND PROBLEM FORMULATION

The nonlinear equation that governs the motion of a system composed of a structure with n_S degrees of freedom equipped with n_F FTMDs arranged in parallel in the top story (Fig. 1), submitted to seismic excitation, represented by the ground acceleration $\vec{\ddot{x}}_g(t)$ ($d \times 1$ vector, in which d represents the quantity of ground movement directions), can be written as

$$\mathbf{M}\vec{\ddot{x}}(t) + \mathbf{C}\vec{\dot{x}}(t) + \mathbf{K}\vec{x}(t) + \vec{F_F}(t) = \mathbf{M}\Gamma\vec{\ddot{x}}_g(t), \tag{1}$$

where $\vec{x}(t)$, $\vec{\dot{x}}(t)$ and $\vec{\ddot{x}}(t)$ are, respectively, the displacement, velocity and acceleration system vectors relative to base, of size $(n_S + dn_F) \times 1$. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ denotes, respectively, the mass, damping and stiffness matrices of the system, of size $(n_S + dn_F) \times (n_S + dn_F)$. Γ represents the $(n_S + dn_F) \times d$ influence matrix of ground motion coefficients. $\vec{F_F}(t)$ is the friction force vector between the last structure story n and the FTMDs, of size $(n + n_F) \times 1$.

For the sake of simplicity, the ground motion is considered unidirectional ($d = 1$). Then, the matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are expressed by Eqs. (2), (3) and (4), with $\mathbf{M}_F = diag[m_{F1}, m_{F2}, ..., m_{F_{nF}}]$. m_{Fj}, k_{Fj} and f_{Fj} are, respectively, the mass, the stiffness and the friction force magnitude of the j-th FTMD. $\mathbf{M}_S$, $\mathbf{C}_S$ and $\mathbf{K}_S$ are the submatrices of the structure and $\mathbf{0}$ is the null matrix.

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_F \end{bmatrix} \tag{2}$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{3}$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_S + \sum_{j=1}^{n_F} k_{Fj} & -k_{F1} & -k_{F2} & \cdots & -k_{Fn_F} \\ -k_{F1} & k_{F1} & 0 & \cdots & 0 \\ -k_{F2} & 0 & k_{F2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_{Fn_F} & 0 & 0 & \cdots & k_{Fn_F} \end{bmatrix} \tag{4}$$

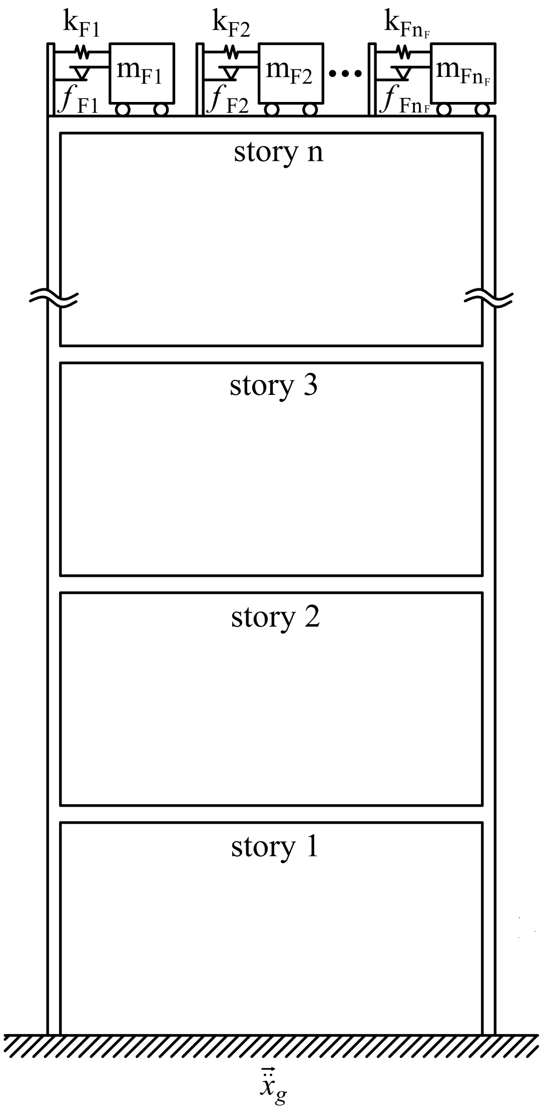

Figure 1: *n*-story structure equipped with n_F FTMDs

The Coulomb friction force, developed between the j-th FTMD and the last story of the structure, is given by

$$F_{Fj}(t) = \mu_j m_{Fj} g sgn[\dot{x}_{Fj}(t) - \dot{x}_{n_S}(t)] = f_{Fj} sgn[\dot{y}_{Fj/n_S}(t)], \tag{5}$$

in which μ_j is the dynamic friction coefficient (assumed constant and equal to the static), g is the gravity acceleration, $sgn[.]$ is the signal function and $\dot{y}_{Fj/n_S}(t)$ is the relative velocity between each FTMD ($\dot{x}_{Fj}(t)$) and the last structure story ($\dot{x}_{n_S}(t)$).

From Eq. (5), it could be noted that the friction force depends on the direction of the relative velocity $\dot{y}_{Fj/n_S}(t)$. The control problem problem presented is characterized by recurrent changes in the direction of this velocity, which combined with the stick slip effect, results in discontinuities in the friction force, characterizing the nonlinear behavior of the system and thus making its analysis even more difficult. Given these assumptions and considering the balance of forces of the system shown in Figure 1, the friction force vector is given in Eq. (6).

$$\overrightarrow{F_F}(t) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -\sum_{j=1}^{n_F} F_{Fj}(t)_{n_S} \\ F_{F1}(t) \\ F_{F2}(t) \\ \vdots \\ F_{F_{nF}}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -\sum_{j=1}^{n_F} f_{Fj} sgn[\dot{y}_{Fj/n_S}(t)] \\ f_{F1} sgn[\dot{y}_{F1/n_S}(t)] \\ f_{F2} sgn[\dot{y}_{F2/n_S}(t)] \\ \vdots \\ f_{F_{nF}} sgn[\dot{y}_{F_{nF}/n_S}(t)] \end{bmatrix} \tag{6}$$

The seismic excitation was simulated by a zero mean Gaussian stationary stochastic process, $\overrightarrow{\ddot{x}_0}(t)$, filtered through the Kanai-Tajimi (Kanai, 1961; Tajimi, 1960) model, with power spectral density function expressed by

$$S(\omega) = S_0 \left[\frac{{\omega_f}^4 + 4{\omega_f}^2{\xi_f}^2\omega^2}{(\omega^2 - {\omega_f}^2)^2 + 4{\omega_f}^2{\xi_f}^2\omega^2} \right], \tag{7}$$

where S_0 is the constant spectral density, ω is the angular frequency and, ξ_f and ω_f are, respectively, the damping ratio and the natural frequency of the filter (soil parameters).

To evaluate the failure probability of the controlled system, it is first necessary to evaluate its dynamic response in terms of its mean square values. With this purpose, a statistical linearization technique is applied. For more details, the reader is referred to Caughey (1963) and Roberts and Spanos (1990). In this context, it is proposed to substitute the nonlinear terms of Eq. (1) by equivalent linear viscous terms, keeping the premise that the excitation and the system response are stationary stochastic processes. Then, it is obtained the following equivalent linear differential equation

$$\mathbf{M}\ddot{\vec{x}}(t)+(\mathbf{C}+\mathbf{C}_{EQ})\dot{\vec{x}}(t)+\mathbf{K}\vec{x}(t)=\mathbf{M}\Gamma\ddot{\vec{x}}_g(t), \tag{8}$$

in which $\mathbf{C}_{EQ}$ is the equivalent viscous damping matrix relative to the dampers, which should be determined simultaneously with the equivalent linear system response, for a given set of friction force magnitudes f_{Fj}. In this sense and after some mathematical manipulations, the minimization the square of the Euclidean norm of the difference between Eqs. (1) and (8), leads to

$$\frac{\partial E[[f_{Fj}sgn[\dot{y}_{Fj/n_S}(t)]-c_{Fj}\dot{y}_{Fj/n_S}(t)]^2]}{\partial c_{Fj}}=0, \tag{9}$$

where $E[.]$ is the expected value operator and c_{Fj} is the equivalent viscous damping coefficient of the *j*-th FTMD, with $c_{Fj}\in\mathbf{C}_{EQ}$. From Eq. (9), for a zero mean Gaussian stationary stochastic process, it can be shown that

$$c_{Fj}=\frac{f_{Fj}E[\dot{y}_{Fj/n_S}(t)sgn[\dot{y}_{Fj/n_S}(t)]]}{\sigma^2_{\dot{y}_{Fj/n_S}}}=\sqrt{\frac{2}{\pi}}\frac{f_{Fj}}{\sigma_{\dot{y}_{Fj/n_S}}}, \tag{10}$$

where $\sigma^2_{\dot{y}_{Fj/n_S}}=E[\dot{y}_{Fj/n_S}(t)^2]$ is the mean square value of $\dot{y}_{Fj/n_S}$, obtained by the numerical resolution of the equivalent linear system. To solve this system, the Eq. (8) is rewritten using the state space representation, and then we get the following system of first-order differential equations

$$\dot{\vec{z}}(t)=\mathbf{S}\vec{z}(t)+\vec{e}(t), \tag{11}$$

in which $\vec{z}(t)$ is the state vector, $\mathbf{S}$ is the augmented system state matrix and $\vec{e}(t)$ is the excitation vector.

Defining $\mathbf{C}_{ZZ}$ as the covariance matrix of $\vec{z}(t)$, it can be demonstrate that the subsequent equation is verified (Lutes and Sarkani, 2004)

$$\mathbf{S}\mathbf{C}_{ZZ}+\mathbf{C}_{ZZ}\mathbf{S}^T+\mathbf{B}=\mathbf{0}, \tag{12}$$

in which $\mathbf{B}$ has all null elements except $B_{2(n_S+n_F)+2,2(n_S+n_F)+2}=2\pi S_0$. The Eq. (12) is known as the Lyapunov equation. Extracting from $\mathbf{C}_{ZZ}$ the covariance matrix of the system velocity relative to the base, called $\mathbf{C}_{\dot{x}}$, the mean square value of each relative velocity, necessary to evaluate Eq. 10, is obtained by $\mathbf{T}\mathbf{C}_{\dot{x}}\mathbf{T}^T$, in wich $\mathbf{T}$ is a square matrix composed of elements 0, -1 and 1. As c_{Fj} also $\in\mathbf{S}$, used to evaluate the system response $\mathbf{C}_{ZZ}$ and depends on the statistic of that same response, an iterative scheme is applied for the determination of c_{Fj} and $\mathbf{C}_{ZZ}$.

2.1 Robust reliability based optimization of MFTMDs

As reported by some researchers (Papadimitriou, Katafygiotis and Au, 1997; Scruggs, Taflanidis and Beck, 2006; Lopez, Fadel Miguel and Beck, 2013), the minimization of some mean square value of the response of a controlled system does not necessarily correspond to the optimum design in terms of reliability. In this sense, it is proposed a robust reliability based optimization of the structural control system presented (MFTMDs), with respect to the uncertainties present in the excitation and in the properties of the structure-dampers system (mass, damping and stiffness).

The procedure to evaluate the probability of failure of the structure is based in Melchers, 1999, Lopez, Fadel Miguel and Beck (2014) and Beck, Kougioumtzoglou and Santos (2014). Thus, it is assumed that during a specific seismic event with duration t_E, a specific response of the structure, such as its maximum displacement, should not exceed some barrier level b associated with some limit state. It is assumed, for a stationary excitation and a given barrier level b, time independent, that the probability of failure P_f is given by the Poisson model,

$$P_f(b,t_E) = 1 - exp(-2\nu_u^+(b)t_E), \tag{13}$$

where $\nu_u^+(b)$ represents the barrier crossing rate. For a linear system excited by a Gaussian process with zero mean, this rate can be evaluated by the following equation 14,

$$\nu_u^+(b) = \frac{\sigma_{\dot{u}}}{\sigma_u}\frac{1}{2\pi}exp\left(-\frac{b^2}{2\sigma_u^2}\right), \tag{14}$$

in which σ_u e $\sigma_{\dot{u}}$ represent, respectively, the standard deviation of the displacement and of the velocity of the response for a determined quantity, depending on the barrier adopted. Modeling the occurrence of seismic events as a Poisson process, considering a design life of the structure t_D, the probability of failure can be expressed by

$$P_f(b,t_D) = \sum_{i=1}^{\infty} P_f(b,t_E|i)p_i(t_D), \tag{15}$$

where $P_f(b,t_E|i)$ represents the conditional failure probability, given the occurrence of i events during t_D, and $p_i(t_D)$ represents the probability of there being i events (Poisson distribution).

In this context, the robust optimization problem is formulated in terms of the maximization of the reliability index of the controlled structure, given by $\beta = -\Phi^{-1}(P_f)$ (objective function), where $\Phi[.]$ is the standard Gaussian cumulative distribution function. The total mass of the FTMDs is fixed as a percentage of the mass of the structure. The adopted design variables are the stiffness and the friction force of each FTMD, both considered continuous and constrained. The optimization procedure is evaluated for predetermined quantities of dampers located on the last story of the structure. Uncertainties are considered in the parameters of the controlled system and of the excitation, modelled as random variables. Therefore, the optimization problem proposed can be expressed by

$$
\begin{aligned}
&\text{Find} && \vec{w} = [k_{F1}, k_{F2}, ..., k_{F_{nF}}, f_{F1}, f_{F2}, ..., f_{F_{nF}}],\\
&\text{Maximize} && J(\vec{w}) = \beta,\\
&\text{Subject to} && G_1 : k_{Fj}^{min} \leqslant k_{Fj} \leqslant k_{Fj}^{max}, \ \forall \ FTMD\ j,\\
& && G_2 : f_{Fj}^{min} \leqslant f_{Fj} \leqslant f_{Fj}^{max}, \ \forall \ FTMD\ j,\\
& && G_3 : \sum_{j=1}^{nF} m_{Fj} = \theta \times m_t^S, \quad \begin{array}{c} \forall \ FTMD\ j \\ m_{F1} = m_{F2} = \cdots = m_{F_{nF}}, \end{array}
\end{aligned}
\tag{16}
$$

in which $\vec{w}$ is the design vector, $J(\vec{w})$ is the objective function, G_1, G_2 e G_3 are the constraints, k_{Fj}^{min} and k_{Fj}^{max} are, respectively, the lower and upper limits of stiffness, f_{Fj}^{min} and f_{Fj}^{max} are, respectively, the lower and upper limits of the magnitude of the friction force, θ is the mass ratio between the FTMDs and the structure and m_t^S is the total mass of the structure. As a barrier level b, a limit displacement is considered at the top story of the structure. The calculation of the standard deviations presented in Eq. (14), necessary for the evaluation of the probability of failure, is performed by statistical linearization through state space, as already presented. A certain number of random samples of the variables that guarantees the convergence of these standard deviations and, consequently, of the reliability index, is considered.

Hybrid optimization algorithm FA-NMA

To solve the optimization problem presented, due to the nonconvex nature of the objective function when MFTMDs are considered, it is proposed to use a hybrid stochastic/deterministic algorithm composed by the Firefly algorithm (FA), stochastic part of global search, and by the Nelder-Mead algorithm (NMA), deterministic part of local search. For further details about the FA and the NMA, the reader is referred to Yang (2009) and Nelder and Mead (1965). The hybrid optimization algorithm utilized, previously applied by Fadel Miguel et al. (2016) for conventional TMD optimization, is based on the use of the NMA inside the FA to speed up the global search convergence, i. e., the FA provides an initial design vector near to the global solution, so that the NMA search starts from this vector. The local search is performed by the NMA every $it_{FA,max}$ iterations of the FA, that is, it is initialized the global search by the FA an stop it at $it_{FA,max}$ iterations. At each stop of the FA, the best design vector is used as a starter point for the local search by the NMA, whose stopping criterion is a determined number of iterations $it_{NMA,max}$. The local search result is then introduced into the FA population and the optimization procedure is restarted. The optimization scheme is repeated until a number of objective function evaluations (OFE) is reached. The consequent combination of these algorithms enhance the search performance.

3 NUMERICAL EXAMPLE

In this section, a numerical example is evaluated in order to check the efficiency of the presented methodology, which consists in the robust optimization of MFTMDs connected to the top story of structures submitted to seismic excitations, via statistical linearization. In this sense, a common residential building with one, two and five dampers is studied. The example is conducted to three different configurations concerning the amount of FTMDs considered: one, two and five, which correspond to scenarios S1, S2 and S3, respectively.

The structure is modeled by a planar steel building frame, with ten storys (37.42 m high) and three spans (23.77 m wide), taken from Bertero and Kamil (1975). Figure 2 illustrates the studied building. The finite element model is discretized in 70 elements and 44 nodes, with 132 degrees of freedom. The structural analysis adopted for the frame is linear elastic. The geometric properties of the steel profiles were obtained from the Manual of Steel Construction (1973). An additional mass is considered per story, due to slabs and some eventual overload. The mass matrix is considered consistent. The structure damping is treated as viscous (Rayleigh proportional damping matrix).

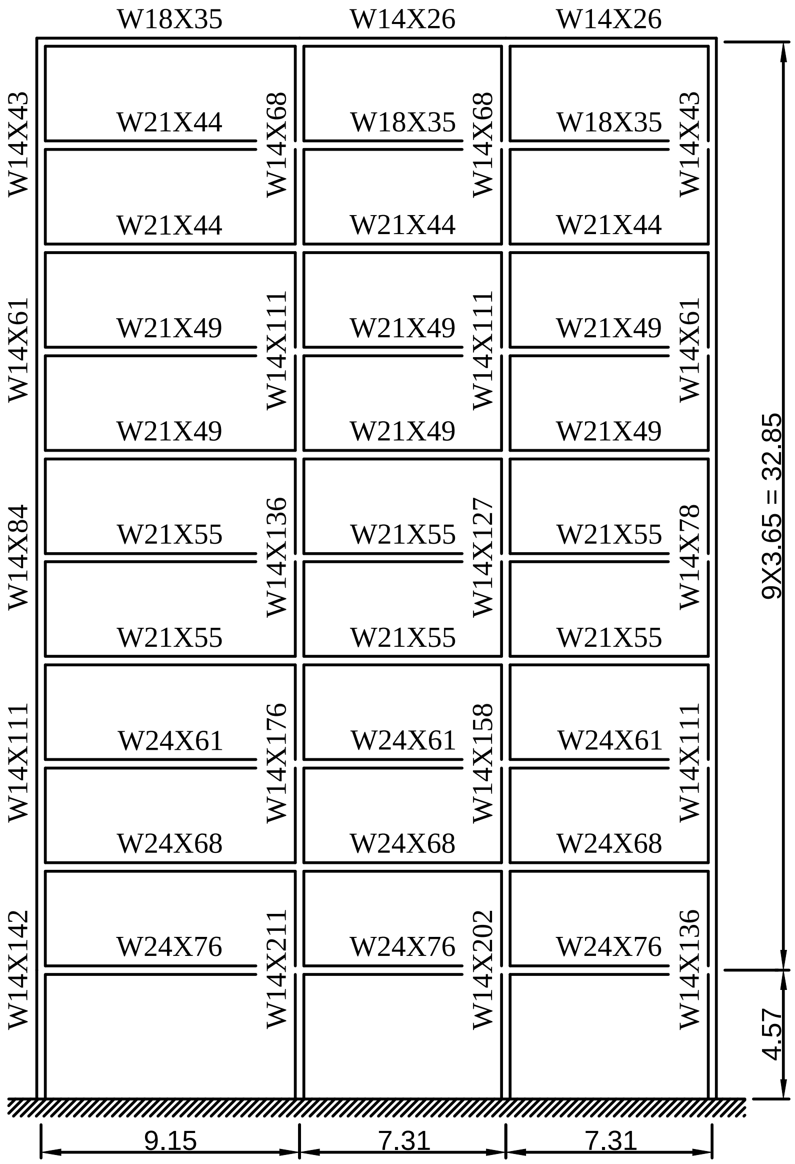

Figure 2: Planar steel frame (dimensions in m): Adapted from Bertero and Kamil (1975)

The mass ratio is considered equal to 3%. For the lower and upper bounds of the stiffness and of the friction force magnitude of each FMTD, it is assumed, respectively, $k_{Fj}^{min} = 0$, $k_{Fj}^{max} = 400$ kN/m, $f_{Fj}^{min} = 0$ and $f_{Fj}^{max} = 20$ kN. For the structure, the elasticity modulus and the specific mass of the steel, the additional mass per story and the damping ratio of its first two vibration modes are assumed as random variables. For the FTMDs, as random variables, it is assumed the mass, the stiffness and the magnitude of the friction force. Finally, for the excitation, the filter parameters and the peak ground acceleration (PGA), in terms of the gravity acceleration g, are considered as random variables. All random variables are considered independent. In this context, the statistical information of these variables is presented in Table 1. It was necessary 100 samples of this variables to get the convergence of the reliability index.

Table 1: Statistical information about the random variables

Building frame			
Random variable	Probability distribution	Mean value	Coefficient of variation (%)
Elasticity modulus (steel)	Gamma	200 GPa	5.0
Specific mass (steel)	Gamma	7500 kg/m^3	5.0
Additional mass per story	Gamma	44 t	5.0
Damping ratio	Gamma	0.05	10.0
FTMDs			
Random variable	Probability distribution	Mean value	Coefficient of variation (%)
Mass	Gamma	$(\theta \times m_t^S)/n_F$	5.0
Stiffness	Gamma	-	10.0
Friction force magnitude	Gamma	-	10.0
Seismic excitation			
Random variable	Probability distribution	Mean value	Coefficient of variation (%)
Natural frequency of the filter	Normal	37.3 rad/s	20.0
Damping ratio of the filter	Normal	0.30	20.0
PGA	Log-normal	0.500g	20.0

From a modal analysis using the mean values related to the structure mass and stiffness, it is obtained that the frame natural frequencies for the first three modes of vibration are, respectively, 0.60 Hz, 1.63 Hz and 2.72 Hz, and the corresponding modal participation factors are 75.87%, 12.98% and 5.32%. These factors indicate that the first mode of vibration has an effective mass much larger than the other modes and, consequently, tends to have a significant participation in the response. The limit displacement of the last story of the building was considered equal to 0.467 m (barrier level b), obtained by Curadelli and Amani (2014) from a static nonlinear analysis (pushover). This value corresponds to the elastic limit of the pushover curve. The reliability index calculated for the uncontrolled structure is 2.87, which corresponds to a failure probability equal to 2.05×10^{-3}.

For the optimization scheme, an initial population with 10 design vectors was considered, with the NMA called at each $it_{FA,max}$ = 100 iterations, with $it_{FA,max}$ = 1000 additional iterations, for all scenarios evaluated. Table 2 shows the optimum parameters obtained as well as the tuning (frequency ratio between the FTMD and the first structure mode of vibration) of each FTMD, the reliability index and the failure probability for all scenarios evaluated.

Table 2: Optimum FTMDs parameters, reliability indexes and failure probabilities

FTMD	k_{Fj} (kN/m)	f_{Fj} (kN)	Tuning	β	P_f
Scenario S1: 1 FTMD					
1	158.84	9.90	0.88	5.58	1.15×10^{-8}
Scenario S2: 2 FTMDs					
1	66.35	3.92	0.80	5.60	1.05×10^{-8}
2	97.44	4.71	0.98		
Scenario S3: 5 FTMDs					
1	46.93	1.79	1.07	5.69	6.29×10^{-9}
2	24.23	1.34	0.77		
3	34.63	1.56	0.92		
4	26.18	1.45	0.80		
5	36.10	1.64	0.94		

For the scenarios S1, S2 and S3, the optimum parameters were found after 1500 OFE, 2000 OFE and 6000 OFE, respectively. It was required a computational time of one day for the scenarios S1 and S2, and of three days for the scenario S3. It can be noted that the optimum tunings obtained are very close to the unit, which is consistent with the modal participation factors, as already related.

It is noted that the presence of FTMDs considerably reduces the failure probability of the structure. The structural control becomes more efficient with increasing quantity of FTMDs. The reduction of this failure probability provided by the FTMDs, relative to the uncontrolled structure, was greater than 99%, for all the scenarios evaluated.

For illustration purposes, the displacement standard deviation, relative to the ground, for all structure storys, is presented in Fig. 3. As expected from the results presented for the failure probability, it is observed that the biggest reduction obtained for this standard deviation was for the scenario S3, reaching approximately 30%.

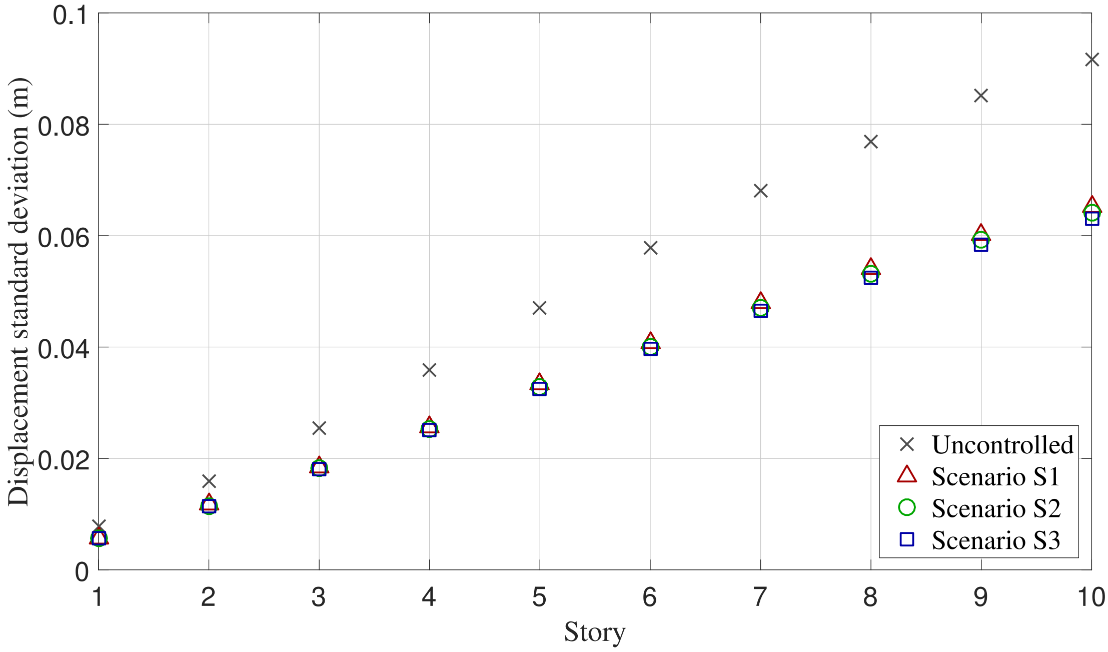

Figure 3: Structure displacement standard deviation relative to the ground for scenarios S1, S2 and S3

4 CONCLUSIONS

This paper presented a global optimization scheme of multiple friction tuned mass dampers (MFTMDs), with the purpose of mitigating vibrations in structures under seismic excitations, through a robust reliability based approach. The seismic excitations were represented by a stationary stochastic process and the structure response was obtained by a statistical linearization technique. The number and location of the MFTMDs in the structure were previously defined. The optimization procedure was performed by a hybrid algorithm which uses the Firefly and the Nelder-Mead algorithms sequentially. The numerical example presented showed that the presence of MFTMDs can significantly reduce the probability of failure of structures submitted to seismic excitations. The use of five FTMDs was more effective in reduzing this failure probability than the use of one and two FTMDs. It is also important to emphasize that the applicability of the presented methodology is not restricted to civil structures, being able to be flexible for the control of diverse structural systems, as well as adapted to other types of excitations.

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