# VERIFICATION OF PUNCHING SHEAR STRENGTH USING THE SURFACE OF MINIMUM SHEAR RESISTANCE METHOD

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***Abstract.*** *The Surface of Minimum Shear Resistance (SMSR) method presents an iterative method that checks the possible failure surfaces by varying their slope ($\theta$) in order to determine the minimum shear resistance ($V_{R,cs}$), which is composed by the concrete resistance ($V_{R,c}$) and the shear reinforcement strength ($V_{R,s}$), both as a function of $\theta$. This paper presents a numerical analysis in Visual Basic for Applications that does what is proposed on the SMSR method. In order to estimate more accurately the force in each layer of the shear reinforcement, the parcel of strength from the steel $V_{R,s}$ was determined by the CCD Method. The routine was applied in a database with 35 slabs reinforced with studs or stud rails with symmetric loading. Besides the SMSR Method, the NBR 6118:2014 and the Eurocode 2:2014 were also used to predict the ultimate punching shear resistance for the slabs. The results from these 3 methods were compared with the ultimate rupture load verified experimentally.*

***Keywords:*** *Reinforced Concrete, Flat Slabs, Punching Shear, Shear Reinforcement, SMSR*

## 1. INTRODUCTION

The constructive system in which slabs are directly supported by column is called flat slabs. This system offers a number of advantages such as simplification in geometry and execution of forms, possibility of reducing the cost of labor, greater ease in electrical, hydraulic and air conditioning installations. It is worth mentioning some of the disadvantages the flat slab system offers: the reduction in overall rigidity mainly when subjected to horizontal stresses, and the control of vertical displacement demand more attention when compared to conventional system, because of the absence of beams that would promote greater rigidity to the structure considering a structural model in space portico. The main disadvantage, however, is the possibility of localized punching shear failure.

Punching shear can be characterized as a brittle failure caused by shear stresses due to the action of a concentrated charge on an area. In design situations, this phenomenon is commonly observed in slab-column connections, connections between column and flexible footings or in the vicinity of a concentrated force. The punching shear failure is similar to one of a beam, such as having a shear crack extending from the top face of the slab the ends of the column, giving rise to a cone-shaped failure surface.

There are possible solutions to avoid that this type of failure occurs in flat slabs, for example, the adoption of concrete with greater compression resistance, or the increase in the rate of flexural reinforcement. Tests in reinforced slabs show that such measures are not very effective as they do not significantly increase the slab punching shear resistance. Other possible measures are the increase of the slab thickness, which could increase the overall cost, or the adoption of capitals in the slab-column connection, which may not be compatible with the architecture of the building because of the height restriction of the floor. The most efficient way to increase the ductility and the strength capacity of a flat slab is through the use of shear reinforcements, which, when effectively and correctly positioned, increase the ductility of the slab-column connection, thus increasing its strength capacity.

It is possible to distinguish three modes of punching shear failure when having shear reinforcement: the crushing of the concrete compressed diagonally near the column face Fig. 1 (a); The reach of the diagonal tensile strength of the slab-column connection when the failure occurs within the shear reinforcements region Fig. 1 (b); Finally, the failure may occur outside the shear reinforcement region resembling the failure of slabs without shear reinforcement. Fig. 1 (c).

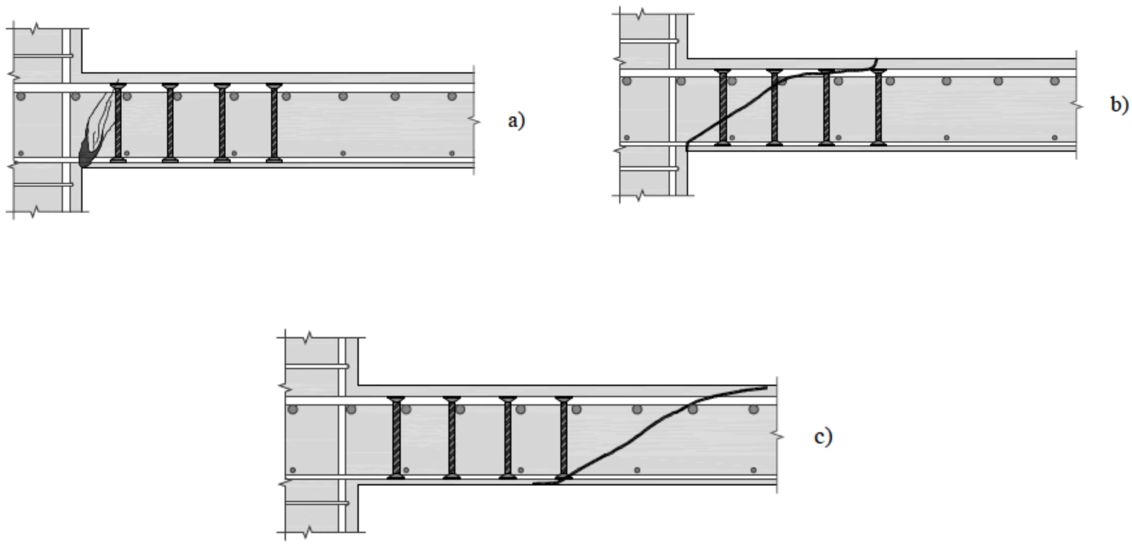


**Figure 1 – Modes of punching shear failure with shear reinforcement - Ferreira (2010)**

Ferreira (2010) states that one of the factors that relate to punching shear resistance is the slope of the failure surface formed by a shear crack, since, considering the classical mechanisms of cutting efforts, a slight sloping failure surface decreases the contribution of concrete to the bond strength. Standards such as Eurocode 2: 2014 and CEB-FIP MC90: 1993 does forecast an average slope of failure surface around 26.6° in slabs without shear reinforcement based on experimental results.

## 2. THEORETICAL BACKGROUND

### 2.1. Parameters that influence the punching shear resistance

Several parameters influence the punching shear resistance. Among the main ones, there is the concrete compressive strength ($f_c$), the tensile flexural reinforcement ratio ($\rho$), column size and geometry, size effect ($\xi$) which takes into account the influence of the effective depth of the concrete slab ($d$), besides the presence or not of the shear reinforcement.

Shear failure is governed, among other factors, by concrete tensile strength ($f_{ct}$), which is commonly related to the concrete compressive strength, and also by its own material compression resistance since there is a possibility of failure by crushing the concrete diagonally adjacent to the column.

The tensile flexural reinforcement ratio, which is the ratio of the tensile flexural reinforcement area ($A_s$) to the concrete area ($A_c$), influences the punching shear resistance mainly in the absence of shear reinforcement. Regan (1981) asserts that the increase of the amount of flexural reinforcement has the effect of increasing the compressed zone and, consequently, the area of uncracked concrete available to resist shear stresses, besides contributing to the dowel action.

According to Oliveira (2003), the column geometry and dimensions also affect the slab strength, as they determine the way the stress are distributed in the slab-column connection. Vanderbilt (1972) concluded that slabs with square column have less resistance than slabs



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with circular column, since in the circular column a uniform distribution of stresses was observed, while in the rectangular column there was stress concentration in the corners of the square section of the column, showing thus, the influence of the column geometry. Hawkins et al. (1971) state the shear stress decreases with the increase of the ratio between the largest and the smallest side of the column ($c_{\max}/c_{\min}$), when it is bigger than 2.

Regarding the relation to the size effect, it can be said that the first to warn that the nominal shear strength could vary in a way not proportional to the thickness of the slabs were Graf (1938) and Richart (1948). The CEB-FIP standards MC90: 1993 and EUROCODE 2: 2004 recommend that the size effect should be estimated by Eq. (1). However, Eurocode 2 limits the result of this expression to a maximum of 2.0. The effect of this limitation is to reduce the increment of the punching shear resistance estimates of flat slabs with effective depth less than 200 mm by limiting the value of ξ. There is, however, no sound experimental basis for such limitation.

$$\xi = 1 + \sqrt{200/d} \tag{1}$$

None of the parameters presented above is as effective as the use of shear reinforcement. More shear reinforcement layers are activated when the slope of the rupture surface decreases. This makes the failure more ductile. It is important that these shear reinforcement used in the slab-column connection are well anchored so that the steel can reach the flow stress, which can make the shear reinforcement more efficient. Often it is not possible to reach the flow stress, since slabs are normally thin elements and do not have sufficient anchoring length to develop stresses at the flow stage.

## 2.2. Regan (2000)

Regan (2000) studied the effects of shear reinforcement on flat slabs and states that their role in these slabs is similar to that of other concrete elements. The shear reinforcement provides vertical or inclined stresses to maintain vertical equilibrium with inclined forces in the concrete, while the horizontal balance is maintained by components of tensile and compression bending. However, this model is inadequate when it comes to punching shear resistance in flat slabs with shear reinforcement.

For a slab with a small amount of well-anchored shear reinforcement, the punching shear resistance capacity is approximately equal to the sum of the slab strength as if it had no shear reinforcement ($V_{R,c}$), which grows with the slope of the rupture surface, and with the yield strength of the shear reinforcement ($V_s$), which decreases with the inclination of the failure surface.

$$V_{R,c} = \left(\frac{2 \cdot d}{a_i}\right)^n \cdot u \cdot d \cdot v_c \leq u \cdot d \cdot v_c \tag{2}$$

$$V_s = \sum A_{sw} \cdot f_{ys,w} \tag{3}$$

Where:

$a_i$ is the radial projection of the surface considered;
$u$ it is the perimeter that the surface reaches the layer of reinforcement;
$v_c$ is the ultimate stress for a slab without shear reinforcement;
$n$ is a coefficient here taken as 1.25;
$V_s = \sum A_{sw} \cdot f_{ys,w}$ is the sum of the forces that cross the failure surface $f_{sv} \leq f_{yv}$.

In this way, the punching shear resistance will be the minimum between and corresponds either to the surface with $a_i \leq 2 \cdot d$ within the region of the shear reinforcement or to that with $a_i = 2 \cdot d$.

Determination of the stress developed on the steel bars that are cut by the failure surface is a problem for thin slabs. The stress has an absolute limit equal to the capacity of the lowest anchorage above or below the critical shear crack. The most common criterion adopted is conical failure.

### 2.3. Fuchs *et.al* (1995)

In the context of conical failures, Fuchs et. al. (1995) created the Concrete Capacity Design Method. Thus, when subjected to a tensile load, the ultimate strength of a connector is calculated assuming a slope of 35 ° between the failure surface and the surface of the concrete member. The tensile strength of a concrete connector with no edge influence or overlapping concrete cones close to uncracked concrete regions is given by Eq. (4).

$$N_u = 17,33 \cdot \sqrt{f_c} \cdot h_{ef}^{1,5} \qquad (4)$$

Where:
$N_u$ is the failure load of the concrete cone (N);
$f_c$ is the compressive strength of the concrete according to cylindrical specimen test (MPa);
$h_{ef}$ is the development length of the connector (mm).

### 2.4. Ferreira (2010)

It is known that shear strength varies as a function of the declination of the failure surface because there is less contribution of the concrete in the punching shear resistance as the declination of the failure surface decreases, but also because more layers of shear reinforcement are activated, making them effective in combating shear stresses. Figure 2 shows the variation of shear strength as a function of the angle of the failure surface in a beam with stirrups.

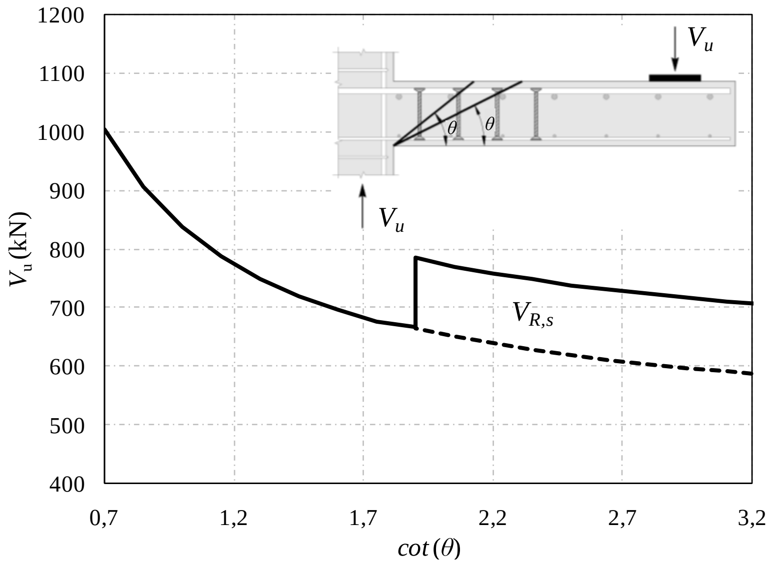



**Figure 2 – Shear strength as a function of the declination of the failure surface - Ferreira (2010)**

The author proposed the Eq. (5) to express the punching shear strength of the slab-column connection. The equation represents the sum of the concrete strength given by Eq. (6) and the steel strength given by Eq. (7).

$$V_{R,cs} = \eta_c \cdot V_{RC} \cdot \left( \frac{2 \cdot d}{a_i} \right) + V_{R,s} \tag{5}$$

$$V_{R,c} = 0{,}18 \cdot \xi \cdot \left( 100 \cdot \rho \cdot f_c^{'} \right)^{1/3} \cdot u_i \cdot d \tag{6}$$

$$V_{R,s} = \sum A_{sw,cam} \cdot f_{ys,w} \tag{7}$$

Where:

$\eta_c$ is a coefficient assumed equal to 0.75, due to the cracking of the concrete;

$V_{R,c}$ is the contribution of the concrete obtained by Equation 5;

$d$ is the effective depth;

$a_i$ is the horizontal projection of the failure surface investigated;

$V_{R,s}$ is the portion of contribution of the shear reinforcement effectively anchored and cut by the failure surface;

$\xi$ is the *size effect* determined as $\xi = 1 + \sqrt{200/d} \leq 2$, with $d$ in mm;

$\rho$ is the flexural reinforcement ratio, defined as $\rho = \sqrt{\rho_x \cdot \rho_y} \leq 0{,}02$;

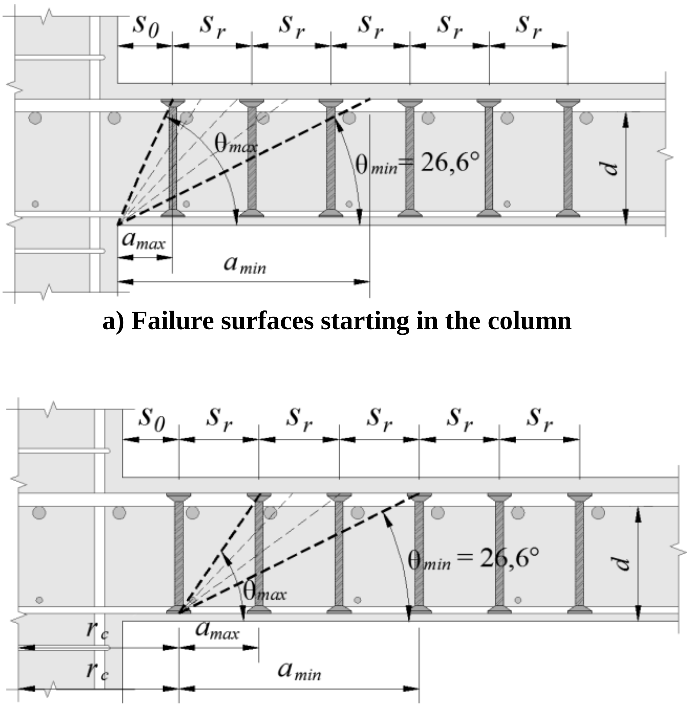
$f_c$ is the compression strength of the concrete;

$u_i$ is the critical perimeter with geometry equal to EC2 (2004), but defined at a distance $a_i$ from the face of the column or the lower point of the failure surface investigated;

$A_{sw,cam}$ is the steel area by layer of the shear reinforcement;

$f_{ys,w}$ is the yielding stress of the shear reinforcement.

The author states that each possible failure surface must be investigated in order to obtain a reasonable representation. It is observed from Fig. 3 (a) that, assuming that the critical crack begins at the end of the column, the maximum declination that it can reach is determined by a line drawn from the face of the column to the anchorage of the first layer of reinforcement. In this situation only concrete contributes to punching shear strength. Such line theoretically varies from the maximum declination, which is a function of the distance from the first layer of reinforcement to the face of the column, to a minimum declination given by the shear surface of a slab without shear reinforcement, which is experimentally given in accordance with EC2 (2004). This line can also start at the base of the shear reinforcement layers as in Fig. 3 (b).

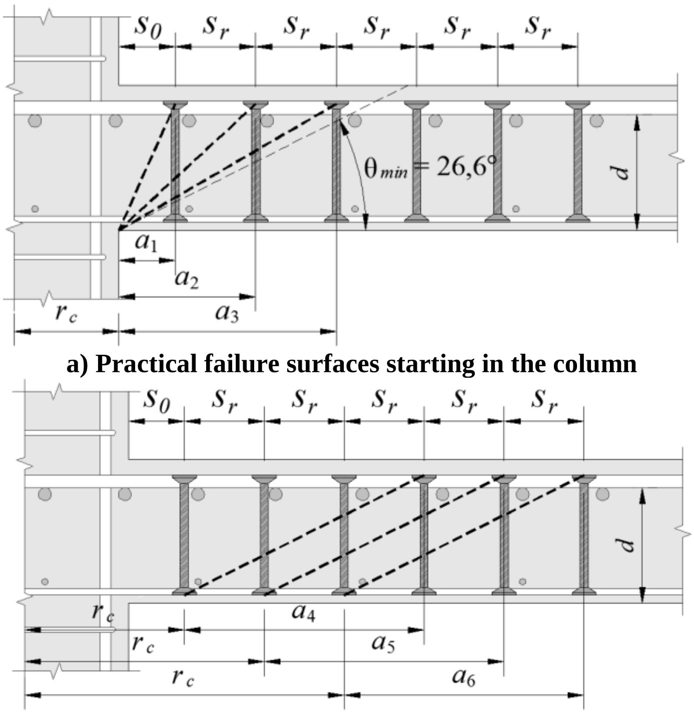



**a) Failure surfaces starting in the column**

**b) Failure surfaces starting in the layers**

**Figure 3 – Theoretical failure surfaces within the region of shear reinforcement – Ferreira (2010)**

Ferreira (2010) simplified this investigation, establishing that the failure surfaces will always seek the ends of the shear reinforcements. Thus, the total number of surfaces to be investigated will always be equal to the number of layers adopted, as shown in Fig. 4 (a) and (b). This method was called Surface of Minimum Shear Resistance (SMSR). The author considered that the use of this method may be interesting from the design point of view because it can attribute greater sensitivity to the designers to punching shear resistance.




**a) Practical failure surfaces starting in the column**



**b) Practical failure surfaces starting in the layers**

**Figure 4 – Practical failure surfaces within the region of shear reinforcement – Ferreira (2010)**

## 2.5. Oliveira (2013)

Oliveira (2013) evaluated the results of his tests and also of other tests done by several authors that compose a database dedicated to the study of punching shear in flat slabs. Based on these results, the author was able to present a modification in the SMSR presented by Ferreira (2010). One of the proposed modifications reduces the strength capacity of the concrete, replacing the factor of Eq. (4) of Ferreira (2010) for cases with and without transfer of bending moment. Thus, the punching shear strength of a slab-column connection with shear reinforcement for a symmetric loading using the SMSR modified by Oliveira (2013) is determined by Eq.(8).

$$V_{R,cs} = \eta_c \cdot 0{,}15 \cdot \xi \cdot \left(100 \cdot \rho \cdot f_c\right)^{1/3} \cdot u_i \cdot d \cdot \left(\frac{2 \cdot d}{a_i}\right) + \sum A_{sw,cam} \cdot f_{ys,w} \tag{8}$$

## 2.6. Eurocode 2 (2014)

In the case of slabs with shear reinforcement, the standard also recommends the verification of three possible failure modes: the first deals with shear failures within the reinforcement region, considering the maximum strength due to the combination of shear and concrete ($V_{R,cs}$ expressed by Eq.(10)). The second considers the rupture occurring outside the region of the shear reinforcement ($V_{R,out}$ according to Eq. (11)); and, finally, the verification of the strength of the compressed strut near the ends of the column ($V_{R,max}$ obtained by Eq.(12)).

$$V_{R,c} = 0{,}18 \cdot \xi \cdot \left(100 \cdot \rho \cdot f_c\right)^{1/3} \cdot u_1 \cdot d \tag{9}$$

$$V_{R,cs} = 0,75 \cdot V_{R,c} + \left( 1,5 \cdot \frac{d}{s_r} \cdot A_{sw} \cdot f_{yw,ef} \right) \tag{10}$$

$$V_{R,out} = 0,18 \cdot \xi \cdot \left(100 \cdot \rho \cdot f_c\right)^{1/3} \cdot u_{out} \cdot d \tag{11}$$

$$V_{R,max} = 0,3 \cdot f_c^{'} \cdot \left( 1 - \frac{f_c}{250} \right) \cdot u_0 \cdot d \tag{12}$$

Where:
$u_0$ is the perimeter of the column;
$u_1$ is the control perimeter taken to be at a distance of $2{\cdot}d$ from the column face;
$u_{out}$ is the control perimeter taken to be at a distance of $1,5{\cdot}d$ from the last layer of the shear reinforcement, respecting a limit of $2{\cdot}d$ for the maximum distance between two concentric lines of concentric *studs*. In case of this limit doesn't be attended, must be used the effective external control perimeter ($u_{out,ef}$);
$s_r$ is the distance between the layers of the shear reinforcement;
$A_{sw}$ is the area of the shear reinforcement by layer;
$f_{yw,ef}$ is the effective stress in the shear reinforcement, which should be calculated as $f_{yw,ef} = 1,15 \cdot \left(250 + 0,25 \cdot d\right) \leq f_{ysw}$, given in MPa and with $d$ in mm.

In its recent update, the Corrigendum AC - Eurocode 2 (2010), there are no major changes in its recommendations for punching shear design, presenting modifications, in relation to the previous version, only in the adjustment of the model of strut and tie by changing the angle of 45º to 26.5º, proposed by BERTAGNOLI and MANCINI (2008). The crushing strength of the strut with adjustments can be obtained by Eq. (13). The recent update from 2014 brings a restriction to $V_{R,cs}$ as shown in Eq. (14).

$$V_{\mathrm{Rmax.EC2-10}} = 0,4 \cdot f_c \cdot v \cdot u_0 \cdot d \tag{13}$$

$$V_{R,cs} = 0,75 \cdot V_{R,c} + V_s \leq 1,5 \cdot V_{R,c} \tag{14}$$

### 2.7. ABNT NBR 6118:2014

ABNT NBR 6118 (2014), same to the Eurocode 2 (2014), has its recommendations based on the text of CEB-FIP MC 90 (1993). Therefore, the verification of the shear strength without shear reinforcement ($V_{R,c}$) should be performed by Equation 10, but without the limitations for the flexural reinforcement ratio and the size effect recommended in Eurocode 2.

Another important difference between the two normative recommendations refers to the geometry and length of the control perimeter outside the region of the shear reinforcement. For the control perimeter $u_{out}$, the brazilian standard recommends that it should be adopted as $2{\cdot}d$ away from the last layer of reinforcement and that should be circular, in the case of radial arrays, and not polygonal as in Eurocode 2: 2004. In the case of $u_{out,ef}$, the geometry is the same as that used in Eurocode 2: 2004, but this perimeter must also be spaced $2{\cdot}d$ from the last layer of reinforcement and not $1,5{\cdot}d$ as in the previous standard.

## 3. COMPUTATIONAL ROUTINE

Based on what was proposed by Ferreira (2010) and Oliveira (2013), this paper proposes a VBA (Virtual Basic for Applications) computational routine to evaluate all possible failure surface inside shear's reinforcement zone, starting at the column to the first layer of shear reinforcement, varying the angle from $\theta_{\max}$ to $\theta_{\min}$ equal to 26.6°, as a function of height of $S_0$ or $S_r$ and of the stud's height.

The programming routine works with 3 chained loops ranges. First of all, the program input data is entered in an Excel worksheet. The input data is: effective depth (d), reinforcement ratio for longitudinal reinforcement ($\rho$), number of lines and layers of shear reinforcement, distance of column board to first shear reinforcement layer ($S_0$), distance between layers of shear reinforcement ($S_r$), column dimensions ($C_1$ e $C_2$), characteristic compressive cylinder strength of concrete ($f_{ck}$), stud's height ($h_{stud}$) and bottom cover($c_{\inf}$). Then iteration begins at $i_i$, which means the position of failure surface and $i_0$ the possible surfaces starting at column, $i_1$ the possible surfaces starting at the base of the first shear reinforcement layer. The routine calculates, using $i_i$, $\theta_{\min}$ and $\theta_{\max}$, the lower and upper limits of the next loop $\theta_i$, which means the failure surface decline. For each $\theta_i$, a horizontal projection of the failure surface ($a_i$) is calculated. Thus it becomes possible to calculate the perimeter $u_i$, as shown in Fig. 5, and it can be obtained $V_{R,c}$, given by Eq. (15).

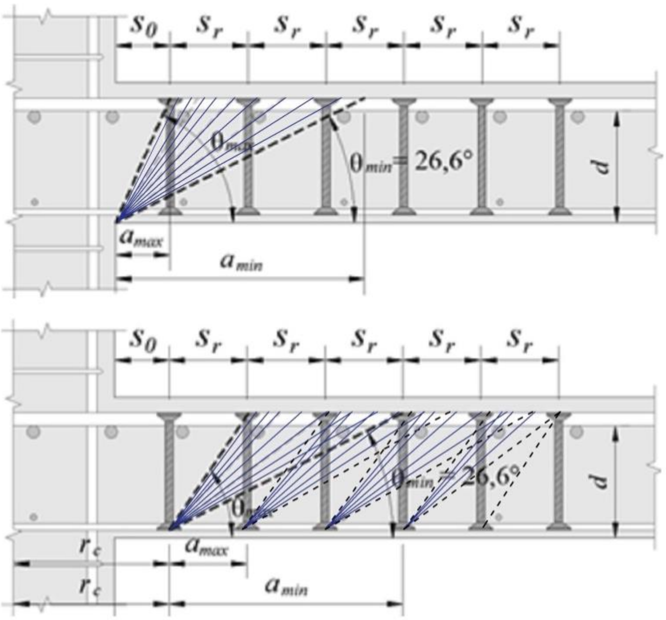



**Figure 5 – Failure Surfaces checked – Adapted from Ferreira (2010)**

$$V_{R,c} = 0{,}18 \cdot \xi \cdot (100 \cdot \rho \cdot f_c)^{1/3} \cdot u_i \cdot d \tag{15}$$

Inside this loop, another loop starts using the pointer $j_i$, varying from 1 to 3 and counting the 1st, 2nd and 3rd shear reinforcement layers. $\theta_i$ and $j_i$ determine the position $x_i$ where each of the shear reinforcement layers has been cut by the failure surface. The value $x_i$ will be the development length of CCD Method by Fuchs et al. (1995), which will be applied in order to find the strength in each shear reinforcement, as already shown in Eq. (4).

$$N_u' = 17{,}33 \cdot \sqrt{f_c} \cdot h_{ef}^{1{,}5} \tag{4}$$

This coefficient 17,33 is used for uncracked concrete regions, which could be the slab bottom region. Applying this formulation to cracked concrete regions, it is necessary to use 70% of the coefficient value (Eligehausen e Balogh -1995) resulting in Eq. (16).

$$N_u'' = 12{,}13 \cdot \sqrt{f_c} \cdot h_{ef}^{1{,}5} \tag{16}$$

The development length to Eq. (16) is the complement of $x_i$ with the stud total size $\left(h_{stud} - x_i\right)$. Obtaining the strength at the stud, the smaller among $N_u'$ and $N_u''$ will be called $p_i$. Therefore, the Eq. (17) gives the share of steel resistance:

$$V_{R,s} = (p_1 + p_2 + p_3) \cdot \mathrm{n}_{\mathrm{lines}} \tag{17}$$

Lastly, shear resistance of the failure surface with declination $\theta_i$ at the position $i_i$ is calculated by Eq. (18).

$$V_{R,\mathrm{cs}} = \eta_c \cdot V_{R,c} \cdot \left( \frac{2 \cdot d}{a_i} \right) + V_{R,s} \tag{18}$$

The coefficient $\eta_c$ is a reduction factor about the concrete portion because of the fissure opening and the value used was 0.75.

In the very beginning it was attributed an extremely high value for $V_{\min}$ , in a way that even in the first iteration, this value is compared to $V_{R,cs}$ and the variable $V_{\min}$ receives its value. Therefore, in the following iteration, the result $V_{R,cs}$ is again compared to $V_{\min}$. In the end of the iterations, $V_{\min}$ would have the lowest value for $V_{R,cs}$, which will be the method result $V_{SMSR}$. Fig. 6 shows the flowchart routine.

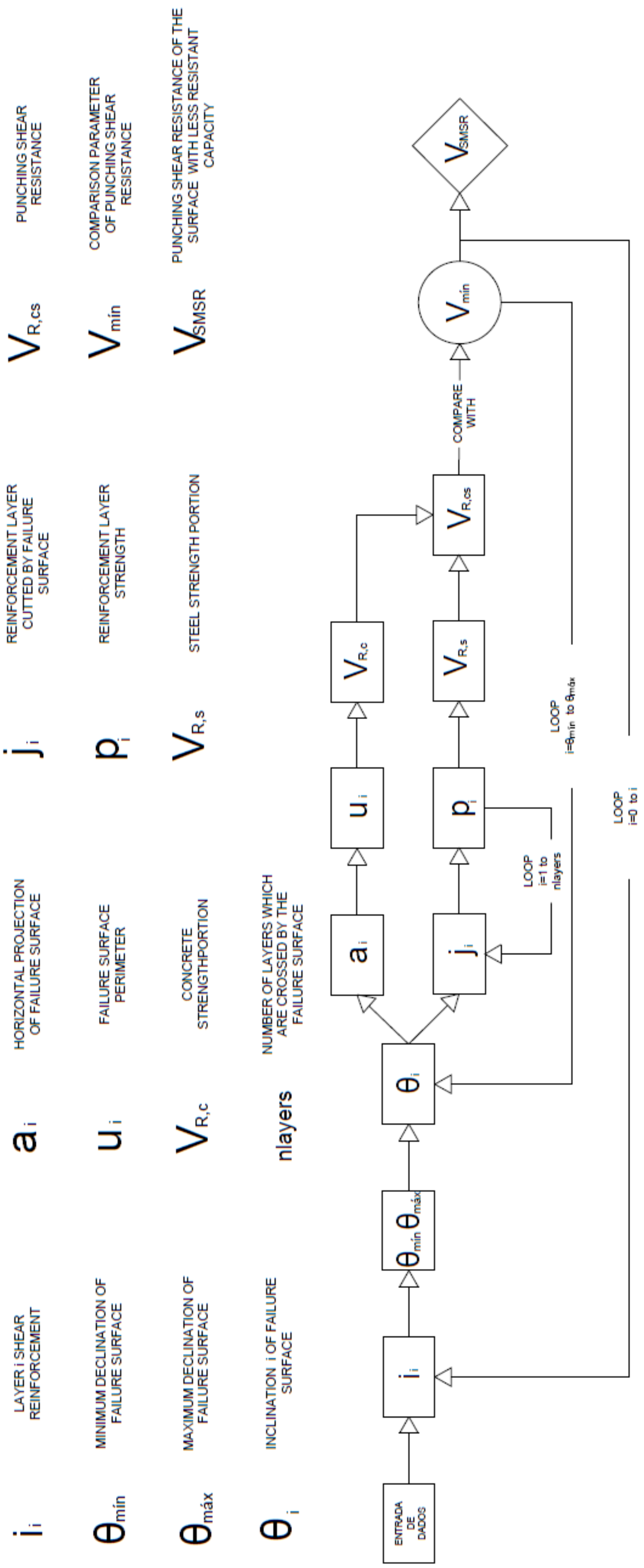



**Figure 6 – Flowchart of the VBA programming routine for the SMSR for slabs subjected to symmetric loading**

## 4. ANALYSIS OF THE SMSR METHOD

It was collected the experimental results from 35 slabs of 9 different authors, with the following peculiarities: the shear reinforcement was of the Double-Headed studs or stud rails, they were subjected to symmetrical loading and the failure mode was within the region of shear reinforcement. Three models were used to evaluate the punching shear resistance of these slabs: the building code recommendations of Eurocode 2 (2014) and NBR 6118 (2014) and the SMSR Method using or not the limitation of the size effect.

The comparison of the results obtained by these methods were made by using statistical parameters such as the average of the results, standard deviation, coefficient of variation and percentile of 5 per cent.

For NBR and Eurocode analysis, it’s important to consider, besides to overall result, the diagonal tensile strength, which is the one is related to the failure that happens within the region of shear reinforcement. This kind of rupture is what happened to the slabs from the database and the SMSR Method checks just within the region of shear reinforcement.

The models were also evaluated by an adaptation of the Demerit Points Classifications - DPC proposed by Collins (2001). This criterion takes into account aspects of safety, accuracy, dispersion and economy. It proposes a classification that is made by means of a demerit scale according to Table 1. The penalty of the models is calculated from the sum of the products of the score corresponding to the interval in which it is related. The greater the sum of the penalties, the worse the model estimate. Table 2 presents the characteristics of the slabs that compose the database.

Table 1. Demerit scale

| $V_u/V_{R.teo}$ | **Classification** | **Penalty** |
|---|---|---|
| < 0,50 | Extremely Dangerous | 10 |
| [0,50 - 0,65[ | Dangerous | 5 |
| [0,65 - 0,85[ | Low security | 2 |
| [0,85 - 1,30 [ | Appropriate Safety | 0 |
| [1,30 - 2,00[ | Conservative | 1 |
| ≥ 2,00 | Extremely Conservative | 2 |

Table 2. Characteristics of the slabs from the database

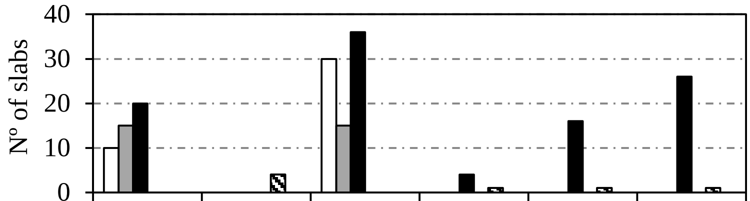
| Author | d (mm) | Cinf (mm) | ρ (%) | $Ø_w$ (mm) | hstud (mm) | No. Lin. | No. Layer | $S_0$ (mm) | $S_r$ (mm) | $C_1$ (mm) | $C_2$ (mm) | $f_c$ (MPa) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Oliveira (2013) | 143 | 10 | 1,6 | 6,3-8,0 | 150 | 14 | 4-6 | 70 | 100 | 400 | 200 | 54-56 |
| Ferreira (2010) | 140-144 | 10 | 1,5-2,0 | 10,0 | 150 | 10-12 | 4-7 | 55-70 | 80-100 | 270-450 | 270-450 | 47-50 |
| Regan (2009) | 150-160 | 10-20 | 1,5-1,6 | 10,0-20,0 | 160 | 10-12 | 4-5 | 60-80 | 120 | 240-300 | 240-300 | 26-62 |
| Birkle (2004) | 124-260 | 20 | 1,1-1,5 | 9,5-12,7 | 104-240 | 8 | 5-6 | 45-95 | 90-195 | 250-350 | 250-350 | 29-35 |
| Regan and Samadian (2001) | 160 | 20 | 1,3-1,6 | 10,0-12,0 | 160 | 8 | 4 | 80 | 120 | 200 | 200 | 38-43 |
| Broms (2007) | 141-151 | 15 | 1,2-1,3 | 12,0 | 145 | 12 | 6-9 | 60 | 75-110 | 300 | 300 | 38-39 |
| Lips, Ruiz and Muttoni (2012) | 201-343 | 20 | 1,6 | 10,0-22,0 | 215-365 | 8-16 | 5-7 | 80-130 | 160-260 | 260-440 | 260-440 | 33-35 |
| Markouk and Jiang (1997) | 120 | 15 | 1,1 | 11,3-15,0 | 110 | 12 | 3-4 | 60 | 90 | 250 | 250 | 60 |
| Mokhtar *et al.* (1985) | 116 | 6 | 1,4 | 9,5 | 140 | 12 | 5-8 | 58 | 87 | 250 | 250 | 23-40 |

## 5. EVALUATION OF THE METHODS TO PREDICT THE PUNCHING SHEAR RESISTANCE

Table 3 presents the average, the standard deviations, the coefficient of variation and the percentile of 5% of the ratios between the failure load ($V_u$) and the estimated diagonal tensile strength ($V_{R,cs}$), the estimated strength of the compressed strut ($V_{\max}$) and the estimated shear strength outside the region of shear reinforcement ($V_{out}$) for EC and NBR. Figures 8 and 9 present the DPC distribution and penalization.

Table 3. Comparison between the SMSR Method, EC2 and NBR 6118

| OBS: SMSR* with limitation of size effect / SMSR** without | Failure mode | EC2 | | | | | NBR 6118 | | | | | SMSR* | SMSR** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Vu/ Vrcs | Vu/ Vout | Vu/ Vmax | Vu/ VEC2 | Fail | Vu/ Vrcs | Vu/ Vout | Vu/ Vmax | Vu/ VNBR | Fail | Vu/ Vsmsr* | Vu/ Vsmsr** |
| | AVER | 0,88 | 1,10 | 0,74 | 1,14 | (5) in | 0,78 | 0,98 | 0,48 | 1,02 | (7) in | 0,99 | 0,92 |
| | SD | 0,20 | 0,20 | 0,18 | 0,16 | (3) max | 0,18 | 0,17 | 0,11 | 0,13 | (0) max | 0,17 | 0,17 |
| | COV | 0,22 | 0,18 | 0,24 | 0,14 | (27) out | 0,23 | 0,18 | 0,22 | 0,13 | (28) out | 0,17 | 0,18 |
| | PER | 0,48 | 0,71 | 0,45 | 0,93 | | 0,40 | 0,67 | 0,29 | 0,81 | | 0,73 | 0,71 |




| | Vrcs Ec | VEC2 | Vrcs Nbr | VNBR | Vsmsr * | Vsmsr ** |
|---|---|---|---|---|---|---|
| <0,5 | 10 | 0 | 30 | 0 | 0 | 0 |
| [0,5-0,65[ | 15 | 0 | 15 | 0 | 0 | 0 |
| [0,65-0,85[ | 20 | 0 | 36 | 4 | 16 | 26 |
| [0,85-1,30[ | 0 | 0 | 0 | 0 | 0 | 0 |
| [1,30-2,00[ | 0 | 4 | 0 | 1 | 1 | 1 |
| >2,00 | 0 | 0 | 0 | 0 | 0 | 0 |

**Figure 8 – Distribution of the slabs in each interval from DPC criteria**

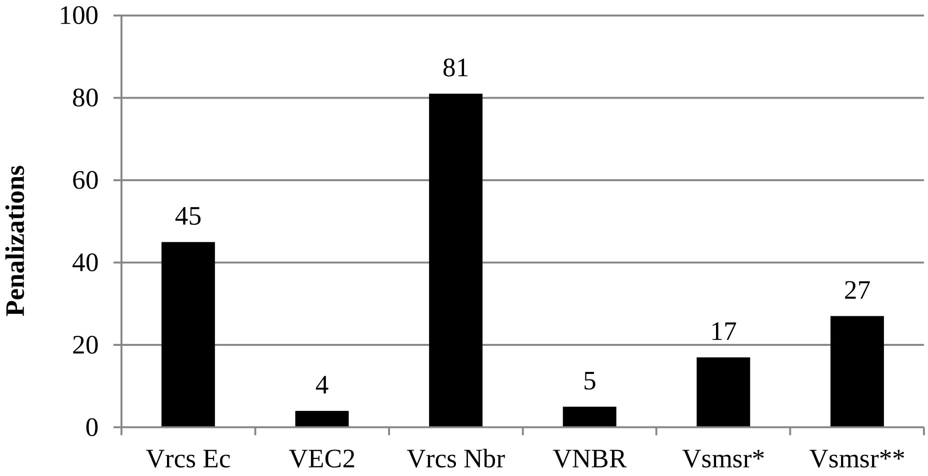



**Figure 9 – Penalizations from DPC criteria**

From Table 3, it can be said that the SMSR has shown much better results when compared to the diagonal tensile strength ($V_{R,cs}$) from both building codes, showing results with less penalizations according to DPC criteria, better average, smaller standard deviation and coefficient of variation and a higher percentile of 5%. Besides that, most part of the results from the building codes is placed in the interval called ‘low security’. When compared to the overall results from the building codes, which are the smallest between $V_{R,cs}$, $V_{\max}$, $V_{out}$, the Eurocode’s and NBR’s results are less disperse and more secure than the SMSR method’s results. However, Table 3 shows that most part of the failure modes from these predictions were outside of the region of shear reinforcement, which did not happen experimentally, since all the slabs from the database had the failure mode inside the region of shear reinforcement. So, though the building codes predicts good results for the ultimate rupture load, they didn’t say the right position of the failure surface, which can be defined by the SMSR Method.

# 6. CONCLUSION

## 6.1. Computational Routine

The programming language used to automate the Surface of Minimum Shear Resistance Method was chosen in order to make simple the process of calculating the punching shear resistance using Microsoft Excel, which is already widely used. In this way, the functions created in VBA can be called in Excel, making it possible to use this method for any layman in programming.

## 6.2. Evaluation of the methods to predict the punching shear strength

The results from this article show that the SMSR Method is a better way to estimate the diagonal tensile strength for slabs with shear reinforcement, since it can predict the position of the failure surface due to the computational routine implemented in the method developed by Ferreira (2010). It is an advance since the building codes are not accurate neither to determine the position of the failure surface neither to the diagonal tensile strength.

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