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PRELIMINARY ESTIMATION OF THE ATMOSPHERIC DENSITY FROM THE POSITION, VELOCITY AND ACCELERATION STATES FOR A CUBESAT SATELLITE.

Wantuir Aparecido de Freitas

André Fenili

wantuirfreitas@gmail.com

andre.fenili@ufabc.edu.br

Centro de Engenharia, Modelagem e Ciência Sociais Aplicadas – CECS Universidade Federal do ABC - UFABC

Av. dos Estados 5001 Bloco B, CEP 09210-580, São Paulo, Santo André – SP, Brasil Addres

Abstract. *Winds and neutral density are critical parameters for the understanding of the physics and dynamics of the mesosphere – thermosphere - ionosphere (MTI) system. Measurements of these parameters are restricted to specific geographical locations and usually available for limited altitude. As altitude rises, atmospheric density ranges from a higher concentration at sea level to a more rarefied condition at higher altitudes. Obtaining information on atmospheric density at determined altitude levels enhances the understanding of the MTI system. In this work, a survey of the concentration of the chemical elements which compose the atmosphere in the regions studied, will be carried out by an analysis of the drag generated by the atmosphere on the satellite body. In order to understand the dissipative effect of the drag force, a scenario that represents the influence of such force on the surface of a hypothetical satellite is created. When the satellite is in operation the position, velocity and acceleration are well determined. So, it can be trace and its location determined in any interval of time. Based on information about the dynamics of the satellite, one can infer about the behavior of the atmospheric density. The proposal of this paper is to develop a methodology using a priori knowledge of the position, velocity and acceleration states of a satellite in order to provide a wider horizontal sample of the atmospheric densities. Perigee measurements at low altitudes, deepening into the mesosphere, are highlighted as a capability designed to provide various atmospheric profiles across a broad region of altitudes.*

Keywords: Mesosphere, Thermosphere, Drag, CubeSat, Atmospheric Density

INTRODUCTION

The methodology applied for the determination of the atmospheric density is based on the dissipative force generated by the interaction of a satellite with the atmospheric (drag). The effect of shape drag, resulting from the interaction of atmospheric pressure on the surface of the satellite body, is significant in a small range of the orbit close to the perigee. The satellite being decelerated by the drag close to the perigee will not have the same energy to reach the same position at the subsequent apogee (Kuga, Carrara, and Rao 2011). Thus, the altitude of the apogee decreases, while the altitude of the perigee remains almost constant and the elliptical orbit becomes a circular orbit (Fig. 1).

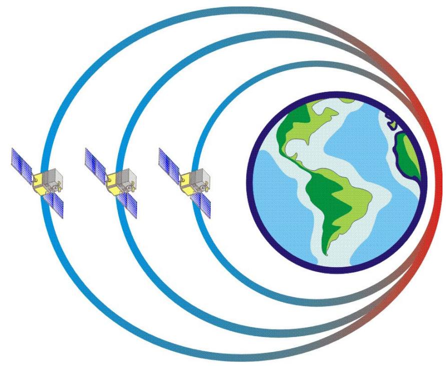

Figure 1. The drag force in an orbit

The atmospheric drag equation is given by:

$$F_a = \frac{1}{2} C_a \rho A V^2 \quad (01)$$

where $\rho \equiv$ atmospheric density

$A \equiv$ projected area

$C_a \equiv$ drag coefficient

$V \equiv$ satellite speed

The drag coefficient C_a is a function of parameters which depend on properties of the satellite surface as the incidence angle, for example.

The effective area *A* is determined by the configuration and size of the vehicle and the angle of attack in relation to the atmospheric flow. This surface is known as the *projected area*, since it is associated with the outer area of the projected satellite in the direction of relative velocity with the atmosphere.

The velocity is calculated assuming that there is no influence of the Earth's rotation, which means that it is being assumed at first that the Earth is at rest and the velocity V is the velocity of the satellite relative to the inertial system. Thus, the final expression for the drag force on the components x, y and z are in the form:

$$F_{ax} = \frac{1}{2} C_a \rho A V \dot{x}$$
$$F_{ay} = \frac{1}{2} C_a \rho A V \dot{y} \,. \qquad (02)$$
$$F_{az} = \frac{1}{2} C_a \rho A V \dot{z}$$

The decay of a satellite orbit at an altitude of less than 1000 km is directly related to the dissipative generated by the interaction of the satellite body with the molecules present in the atmosphere and the force of Newton's universal gravitation.

In an Earth perfectly spherical and homogeneous, the equation describing the orbital motion of a satellite affected by the gravitational field and dissipative forces is given by:

$$\ddot{\vec{r}} = -\mu \frac{\vec{r}}{r^3} + \vec{P} \qquad (03)$$

where $\vec{P}$ represents the dissipative disturbances. In this work, $\vec{P}$ represents only the drag force.

Once in operation, the position, speed and acceleration of the satellite are well determined and can be used to track it and know its location at any time. Based on these information about the dynamics of the satellite, the behavior of the atmospheric density can be inferred. This is an efficient and inexpensive way of obtaining temporal behavioral information of atmospheric density only with software resources.

1 METHODOLOGY

The Cowell's numerical integration method in conjunct with the fourth order Runge Kutta's method (Ruggiero and Lopes, 1988), are used in this work and briefly described below. The great advantage of the Cowell's method is the speed with which it can be used. It consists of writing the equations of the movement for the object, including all disturbances and then integrating them numerically step by step. The disturbances can be included all at the same time. The Runge-Kutta's method is stable and does not require a boot procedure. It is relatively simple, easy to implement, produces a small truncation error, and the step size is easy to change. For the two-body problem (Earth-satellite) with perturbations (Kuga, Carrara, and Rao 2008), the equation of motion in the vector form is given by:

$$\ddot{\vec{r}} + \mu \frac{\vec{r}}{r^3} = \vec{P} \qquad (04)$$

where μ is the geo-gravitational constant, and $\vec{P}$ vector of disturbing accelerations The numerical integration of equation (04) can be reduced to two differential equations of first order:

$$\dot{\vec{r}} = \vec{v}$$
$$\dot{\vec{v}} = \vec{P} - \frac{\mu}{r^3}\vec{r} \tag{05}$$

where $\vec{r}$ e $\vec{v}$ are the position and velocity vectors of a satellite. The equations in (5) can be expressed in the vector components:

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \end{cases} \qquad \begin{cases} \dot{v}_x = P_x - \dfrac{\mu}{r^3}x \\ \dot{v}_y = P_y - \dfrac{\mu}{r^3}y \\ \dot{v}_z = P_z - \dfrac{\mu}{r^3}z \end{cases} \tag{06}$$

where $r = \sqrt{x^2 + y^2 + z^2}$.

1.0 Illustrative example of the Cowell's numerical method, two-body problem.

In this example a scenario with simulated conditions for the space shuttle orbit are constructed. The altitude, mass, cross-sectional area of drag and atmospheric density were obtained from the website:

https://www2.hao.ucar.edu/sites/default/files/users/whawkins/Lab8.pdf.

Data $\mu = 3{,}986\times10^5\ km^3/s^2$, $P = 5.4921\times10^3\ s$ (orbit period), $e = 0{,}08$ (eccentricity); $\rho = 4{,}25\times10^{-11}\ g/m^3$ (atmospheric density); Cd=2 (drag coefficient) ; $S = 3{,}64\times10^{-4}\ km^2$ (cross-sectional area) and m = 91974 kg (Mass of the space shuttle), with initial conditions: x1(1) = 6.728 [km] ; x2(1) = 0 [km/s] ; x3(1) = 0 [km] ; x4(1) = -7.7 [km/s] ; x5(1) = 0 [km] ; x6(1) = 0 [km/s]. Determine the vector displacement, velocity, acceleration and space shuttle orbit, knowing that the dissipative force $\vec{P}$ expressed by equation (1).

Solution: The equation (3) can be reduced to first order differential equations in the form:

$$\begin{aligned}
&\dot{x}_1 = x_2 \\
&\dot{x}_2 = P_x - \frac{\mu}{r^3} x_1 \qquad\qquad P_x = -\frac{1}{2}\rho C_d \frac{S}{m}\sqrt{\dot{x}_2^2 + \dot{x}_4^2 + \dot{x}_6^2}\, x_2 \\
&\dot{x}_3 = x_4 \qquad\qquad\qquad\quad\; P_y = -\frac{1}{2}\rho C_d \frac{S}{m}\sqrt{\dot{x}_2^2 + \dot{x}_4^2 + \dot{x}_6^2}\, x_4 \\
&\dot{x}_4 = P_y - \frac{\mu}{r^3} x_3 \quad \text{where;} \quad P_z = -\frac{1}{2}\rho C_d \frac{S}{m}\sqrt{\dot{x}_2^2 + \dot{x}_4^2 + \dot{x}_6^2}\, x_6 \qquad (07) \\
&\dot{x}_5 = x_6 \qquad\qquad\qquad\quad\; r = \sqrt{{x_2}^2 + {x_4}^2 + {x_6}^2}\,. \\
&\dot{x}_6 = P_z - \frac{\mu}{r^3} x_5
\end{aligned}$$

The equations in (07) represent the system of first order differential equations with constant parameters to be numerically integrated.

1.1 Results obtained by the numerical method

Figure 2 shows the nominal orbit of a hypothetical space vehicle whose atmospheric density was stipulated in $\rho = 4{,}25 \times 10^{-11}\, g/m^3$, to study its influence on the drag force on the satellite.

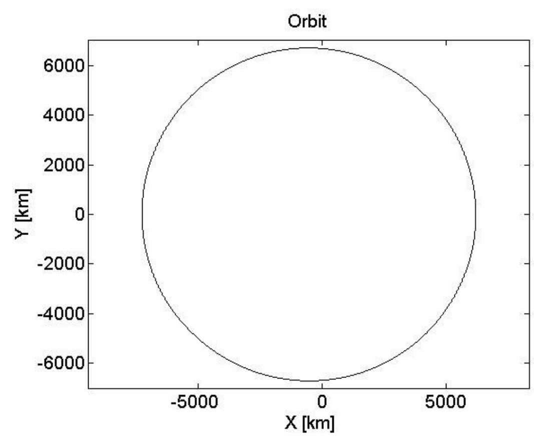

Figure 2. Hypothetical orbit of a hypothetical space vehicle at the X and Y coordinates.

Figure 3 shows the hypothetical trajectory of the space vehicle as a function of time, which corresponds to an orbit around the Earth, at an altitude of 350 km, with atmospheric density stipulated in $\rho = 4{,}25 \times 10^{-11}\, g/m^3$.

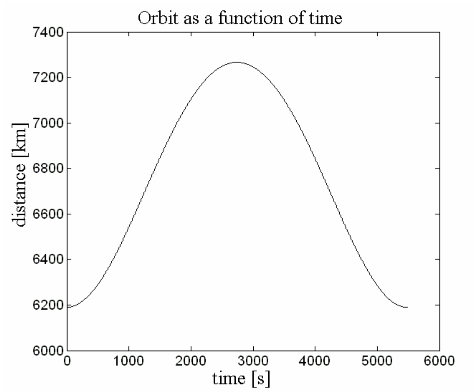

Figure 3. Hypothetical displacement of the space shuttle as a function of time.

Figures 4 and 5 represent the temporal behavior of the space vehicle, velocity and acceleration, which corresponds to an orbit around the Earth, at an altitude of 350 km, with atmospheric density stipulated in $\rho = 4{,}25 \times 10^{-11}\ \mathrm{g/m^3}$.

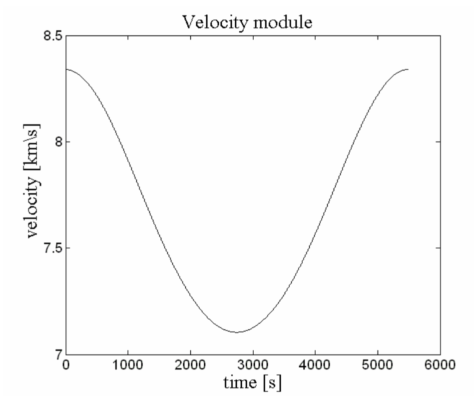

Figure 4. Hypothetical space shuttle velocity.

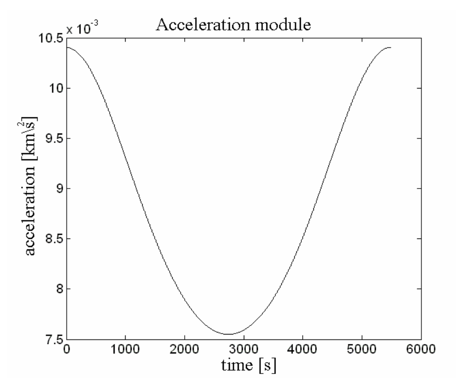

Figure 5. Hypothetical space shuttle acceleration.

In Figures 6, 7, 8 and 9 the atmospheric density is amplified for a better visualization of the influence of the drag effect on the trajectory, velocity and acceleration of the space vehicle, over the hypothetical orbit.

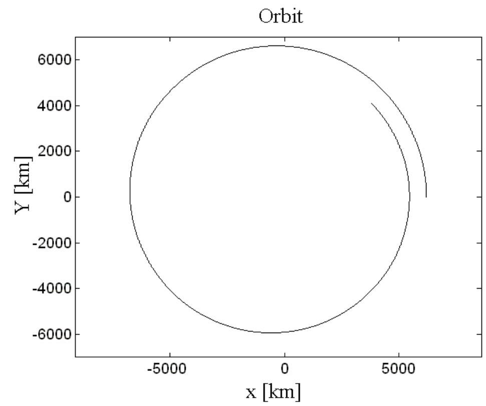

Figure 6. Hypothetical space vehicle orbit at the X and Y coordinates: amplified atmospheric density for a better visualization of the drag effect.

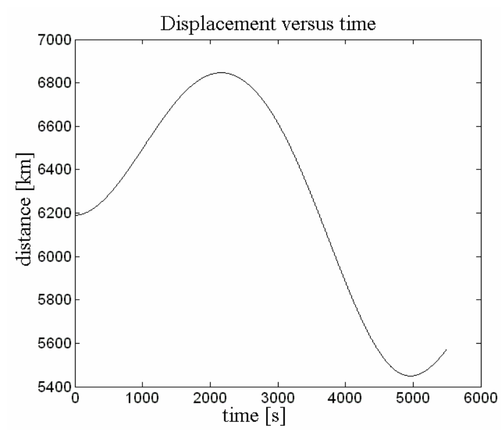

Figure 7. Hypothetical displacement of the space shuttle versus time: amplified atmospheric density for a better visualization of the drag effect.

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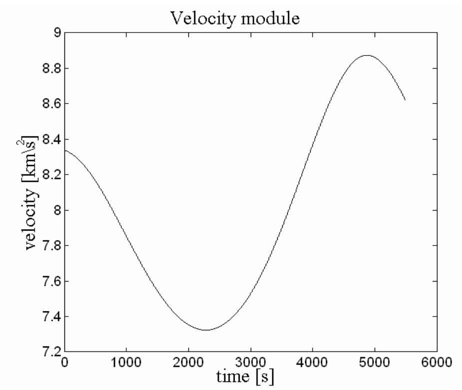

Figure 8. Hypothetical speed of space shuttle versus time: amplified atmospheric density for better visualization of drag effect.

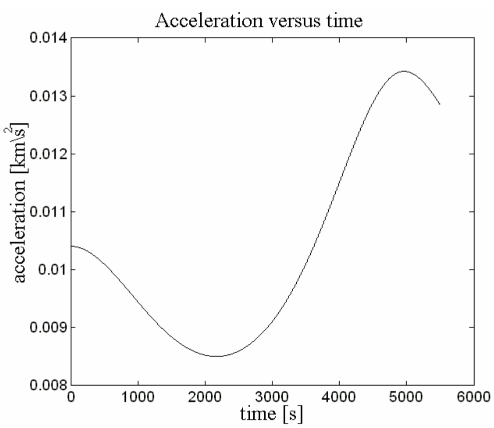

Figure 9. Hypothetical acceleration of space shuttle versus time: amplified atmospheric density for better visualization of drag effect

Figures 10 to 13 show the effect of atmospheric drag on the displacement, speed and acceleration of the space shuttle in a hypothetical orbit with respect to two orbital periods. Over time the space shuttle is decelerated causing the orbit to decrease due to the effect of atmospheric drag as shown in Figures 10 and 11. Consequently the speed and acceleration modules suffer an increase as shown in Figures 12 and 13.

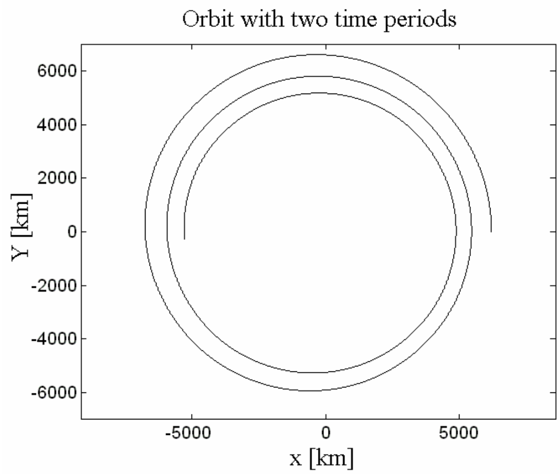

Figure 10. Hypothetical space shuttle orbit in the X and Y coordinates with two orbital periods, amplified atmospheric density for a better visualization of the drag effect.

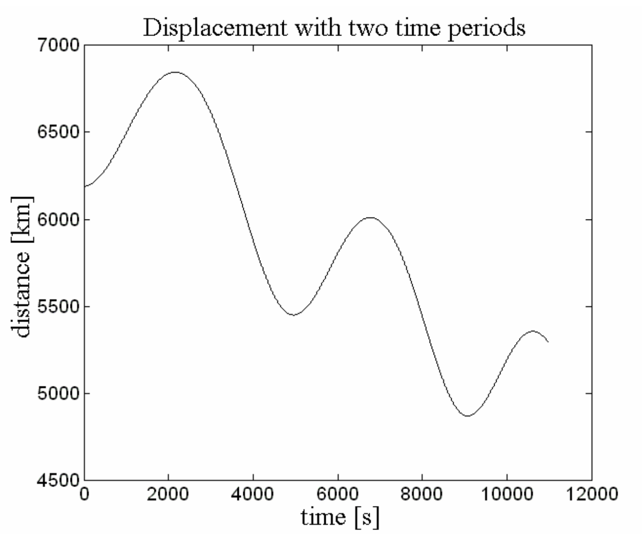

Figure 11. Hypothetical displacement of space shuttle as a function of time with two orbital periods, amplified atmospheric density for better visualization of drag effect.

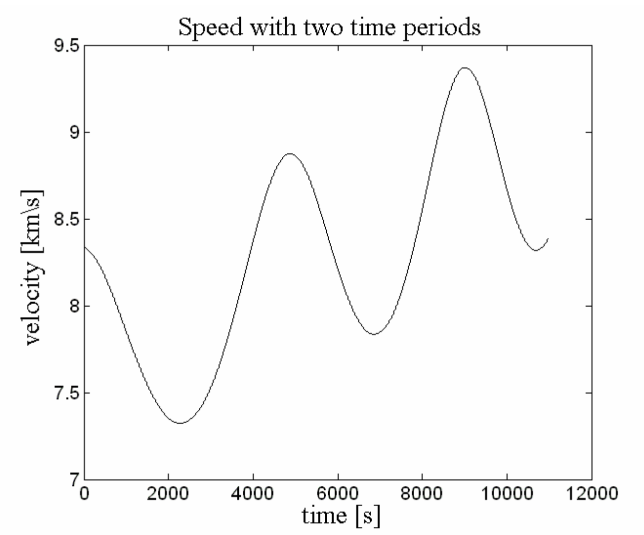

Figure 12. Hypothetical speed of space shuttle versus time with two orbital periods, amplified atmospheric density for better visualization of drag effect.

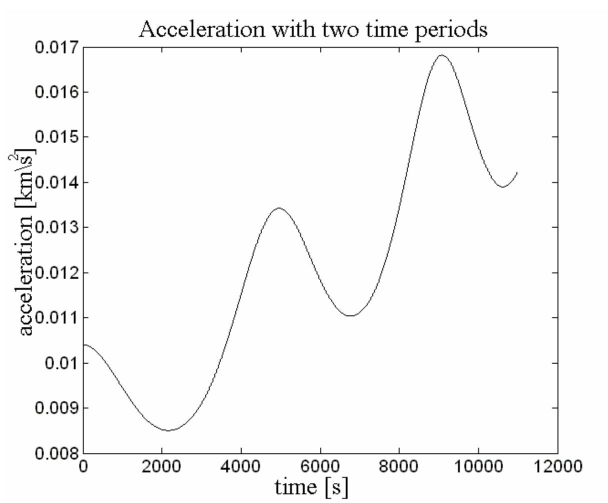

Figure 13. Hypothetical space-time acceleration with two orbital periods, amplified atmospheric density for better visualization of drag effect.

1.1 Inverse process to obtain density

The results obtained in section 1.0 simulate the hypothetical movement of a spacecraft in the orbital plane X and Y relative to Earth. Assuming the mass of the Earth is concentrated in its center, it determines the trajectory of the space vehicle of mass m subject to the action of a central force. The numerical resolution of the system presented in equations (7), represents a direct method to obtain the states of position, speed and acceleration of the space vehicle.

Any satellite in the Earth's orbit is influenced by unwanted disturbing forces that alter its dynamics. One of these forces is the drag force acting on the *cross-section* by decelerating the satellite. The atmospheric density is the parameter related to the environment in which the satellite is immersed, being one of the agents of the deceleration of any object in orbit to an altitude in which its action is relevant. That is, to an altitude of 1000 km, the drag force is predominant in the deceleration of objects in orbit.

In order to obtain the density of the environment in which a satellite is inserted, a methodology based on position, velocity and acceleration states has been developed.

For a better understanding of the dynamic behavior of a spacecraft and the parameter density, stipulated ($\rho = 4{,}25 \times 10^{-11}\, g/m^3$), only the effect of the drag force is taken into account; at first because it is the external force of greater influence in the dynamics of space vehicles.

The method is based on the study of three cases:

- Case1 - The acceleration of the space vehicle is related only to the influence of a central force (gravitational force);
- Case 2 - The acceleration of the space vehicle is related to the influence of the central force and the traction disturbing force acts on the body of the satellite;
- Case 3 - The acceleration of the space vehicle is related to that of the Uniform Circular Movement (UCM).

The equations that define the accelerations for the three cases are:

$$acp_1 = \frac{\mu}{r_1^2}$$

$$acp_2 = \frac{\mu}{r_2^2} + \frac{1}{2}\rho Cd \frac{S}{m} V_2^2 \qquad (07)$$

$$acp_3 = \frac{V_i^2}{r_i} \text{ sendo i=1 e 2.}$$

where acp1 is the acceleration due to the Earth's pulling force, acp2 is the acceleration of Earth's attraction plus the dissipative drag force and acp3 is the centripetal acceleration of uniform circular motion.

The ascertainment of cases 1 and 2 are individually compared to the acceleration of the case3. Each comparison shows that there is an acceleration difference between the combinations of cases, this difference is the acceleration variation: dif_{31}, dif_{32} and dif_{321}.

Comparing the accelerations of case 1 with case 2, find the value of the geo-gravitational constant μ, that is, the value of the geo-gravitational constant is reprocessed using the position and velocity states, as shown in equations (08), (09) and (10).

$$acp_3 - acp_1 = dif_{31} \qquad (08)$$

$$\frac{V_1^2}{r_1} - \frac{\mu}{r_1^2} = dif_{31} \tag{09}$$

$$\mu = V_1^2 r_1 - dif_{31} r_1^2 \tag{10}$$

Analyzing equation (08) we can rewrite the acceleration acp_1 in terms of the acceleration acp_3 and the difference dif_{31}, as shown below:

$$acp_1 = acp_3 - dif_{31} \tag{11}$$

Now comparing the accelerations of case 2 and 3 we have:

$$acp_3 - acp_2 = dif_{32}. \tag{12}$$

In the same way as in equation (11) it can be written that,

$$acp_2 = acp_3 - dif_{32} \tag{13}$$

Substituting the equations of (7) into (11), (12) and (13) we have:

$$acp_1 = \frac{V_i^2}{r_i} - dif_{31}, \text{ para i=1} \tag{14}$$

$$acp_2 = \frac{V_i^2}{r_i} - dif_{32}, \text{ para i=2} \tag{14}$$

Substituting acp2 of (7) into (14) is an equation for the drag force:

$$\frac{\mu}{r_2^2} + \frac{1}{2}\rho Cd \frac{S}{m} V_2^2 = \frac{V_i^2}{r_i} - dif_{32} \tag{15}$$

$$\frac{1}{2}\rho Cd \frac{S}{m} V_2^2 = \frac{V_i^2}{r_i} - \frac{\mu}{r_2^2} - dif_{32} \tag{16}$$

$$ac_{drag1} = \frac{V_2^2}{r_2} - \frac{\mu}{r_2^2} - dif_{32}\text{, para i=2} \tag{17}$$

where ac_{drag} Is the acceleration of the drag force, ie:

$$ac_{dag1} = \frac{1}{2}\rho Cd \frac{S}{m} V_2^2 . \tag{18}$$

An alternative way of writing in an analogous way ac_{drag}, is to use the terms referring to acp_1, acp_3 and dif_{321},

$$ac_{drag1} = \frac{V_1^2}{r_1} - \frac{\mu}{r_1^2} - dif_{321} \tag{19}$$

Equations 18 and 19 are equivalent as shown in Figure 14.

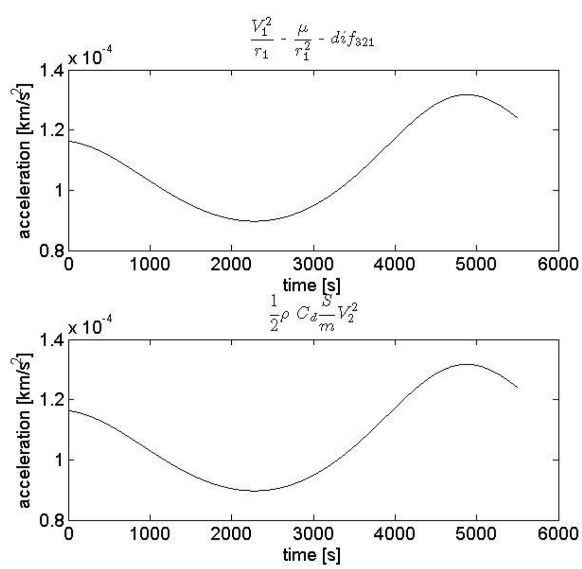

Figure 14. Acceleration generated by drag force due to equations (19) and (18).

Equating the equations (18) and (19), we can isolate the variable ρ, which represents the atmospheric density of the medium where the spacecraft is inserted. The equation for the atmospheric density in modulus is in the form:

$$\rho = \frac{2m}{C_d S V_2^2}\left(\frac{V_1^2}{r_1} - \frac{\mu}{r_1^2} - dif_{321}\right) \tag{20}$$

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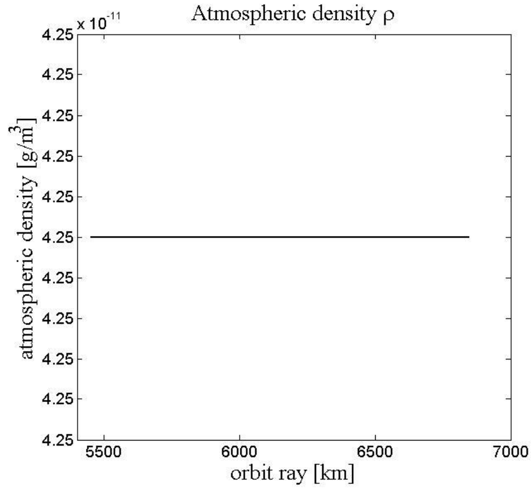

Figure 15. Atmospheric density as a function of the orbit radius.

Table 1 presents the results of atmospheric density obtained using equation (20). It can be inferred that the atmospheric density can be obtained by knowing the states that determine the location of the satellite (position, velocity and acceleration).

Table 1: Hypothetical atmospheric density value in relation to orbital distance for the first ten time intervals.

Density [g/m^3]	Obital distance [km]
4.25e-11	6189,76000000000
4.25e-11	6189,76000416136
4.25e-11	6189,76001664522
4.25e-11	6189,76003745126
4.25e-11	6189,76006657916
4.25e-11	6189,76010402858
4.25e-11	6189,76014979920
4.25e-11	6189,76020389069
4.25e-11	6189,76026630274
4.25e-11	6189,76033703501

1.2 Conclusion

In order to obtain accurate satellite acceleration data in its orbit, an inertial sensor system (accelerometer) is modified and adapted for use on the satellite platform. The expected advantage is the reliability associated with a system that has no mechanical parts. The improvement of the sensitivity of the devices, relative to mechanical sensors, is also seen as important. The acceleration data from the satellite's inertial sensor will be processed using the results of aerodynamic simulation, parameter estimation techniques and state estimation.

The model for drag simulation in dynamic satellite equations based on parameter estimation and state estimation, converts the analysis to an inverse problem and results in a measurement of atmospheric density.

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