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OPTIMUM DESIGN OF STEEL FRAMES CONSIDERING SEMI-RIGID JOINTS

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Abstract. *This paper presents a methodology for optimal design of steel frames considering semi-rigid beam-column connections. The optimization model aims structures of minimum cost by determining the optimal stiffness degree of beam-column connections concomitantly with the classical optimization problem that considers dimensional optimization of beams and columns. Initially, the Structural Analysis module was developed, allowing automatic evaluation of structure's mechanical behavior for different configurations during the optimization process. The Finite Element Method is applied through ANSYS Workbench software to calculate mechanical responses. The structural analysis model considers two distinct finite elements: the classical beam element (representing beams and columns) and the rotational spring element, which enables simulation of semi-rigid connection stiffness. ModeFRONTIER software is used to obtain the structure's minimum fabrication cost. The optimization process is conducted through Genetic Algorithms (MOGAII). Continuous and discrete design variables are used. The design variables are beam-column connections stiffness (continuous variable) and geometry of beams and columns cross sections (discrete variables). Databases of Brazilian commercial structural frames were created using Excel integrated with ANSYS. The design constraints are critical forces and maximum allowed displacements of the structure, according to NBR8800/2008. The validation of proposed methodology is made by comparing results with applications available in literature.*

Keywords: *Semi-rigid Joints, Steel Frame Structures, Structural Optimization*

1 INTRODUCTION

On a structural design, the engineer's goal is to choose a set of structural elements that can sustain the edification while being mechanically and economically efficient, respecting the imposed architecture and the comfort and safety requirements as described in standards as well as construction technology to be employed, in other words, normative requirements and technological limitations. In this context, the optimum structure is the ideal configuration of structural elements, that is, the best possible setting in relation to the objective pursued and the imposed restrictions.

The goal most often used in the classical optimization design of metal structures is the minimization of the manufacturing cost of the structure or the minimization of structural weight. These goals usually are achieved by choosing the optimal cross sections of beams and columns, or alternatively, an important dimension of the section. However, it is noted that there are several parameters of the structure that can also significantly influence the design, such as, for example, the degree of rotational stiffness of beam-column connections, which is object of this study.

In steel frames, during the design considerations, traditionally, the model of structural analysis is simplified by dealing with beam-column connections as pinned or rigid. The pinned joint (or flexible) allows rotation of connected members (φ), not transmitting bending moments between the elements of the frame (Fig. 1 (a)); on the other hand, the rigid connection does not admit the relative rotation between the elements and, consequently, transfers bending moments (Fig. 1 (b)). It is noted, however, that the real behavior of structures is a middle range of the aforementioned joint, with a finite stiffness value (Díaz *et al.*, 2011). These joints are called semi-rigid connections (Fig. 1 (c)).

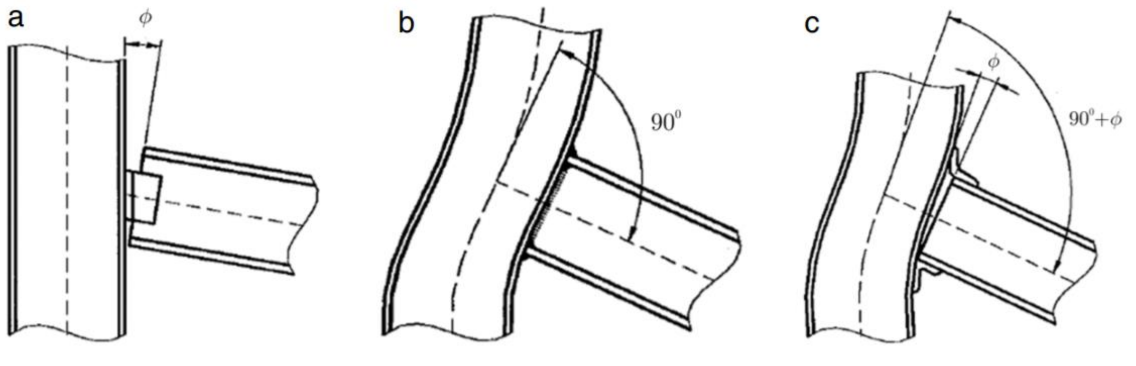

Figure 1. Joint types according to their behavior: (a) pinned; (b) rigid; and (c) semi-rigid (Díaz *et al*, 2011)

In the optimum design of steel structures based on minimization of structural weight, the beam-column connections represent only a small portion of the total weight of the structure and thus usually end up being ignored, however, it is observed that these joints have a high manufacture and assembly cost (Falcon & Montrull, 2014). In general, the cost of the connections directly depends on their degree of rotational stiffness, thus, connections considered as rigid have higher cost due to the need of more robust elements to ensure their mechanical behavior.

In this paper were studied and developed computational methodologies for analysis of the mechanical behavior of planar steel frames with semi-rigid connections and their optimum design, by reducing the manufacturing cost. To this end, were employed three distinct software in an integrated manner: ANSYS Workbench for realization of computational simulations of structural analysis, ModeFRONTIER for the optimization problem management and Excel spreadsheets for implementation of structural profiles database and the design verification as prescribed NBR8800 standard (ABNT, 2008).

2 SRUCTURAL ANALYSIS

The structural analysis module calculates the mechanical responses of the structure considering elastic static linear behavior, based on the Finite Element Method (FEM) using the ANSYS software. A database was created in Excel for automated definition of several configurations for the structure, allowing the use of Brazilian standard structural profiles. The database is composed by 88 laminated profiles sorted in ascending order of linear mass, being 29 "H" shape and 59 "I" shape. Every structural profile was associated with an identifier number (ID) and, through it, the ANSYS is able to collect the geometric data of the chosen profiles and define a particular structure to be studied, thus enabling automatically set of several configurations, needed in the structural optimization process.

For structural analysis were used two distinct finite elements: one representing the behavior of beams and columns and one to represent the beam-column semi-rigid connections. For beams and columns was adopted the linear element BEAM188 as shown in Fig. 2 (a). This element has two nodes with seven degrees of freedom each, which are: three displacements, three rotations and the warping magnitude of the section (Schuwartz *et al.*, 2016). The beam-column connections were represented by a rotational spring element (revolute joint) MPC184. The MPC184 element, shown in Fig. 2 (b), allows the relative rotation between connected members, where its degree of rotation depends on the assigned Initial Stiffness value for the connection (ANSYS, 2013).

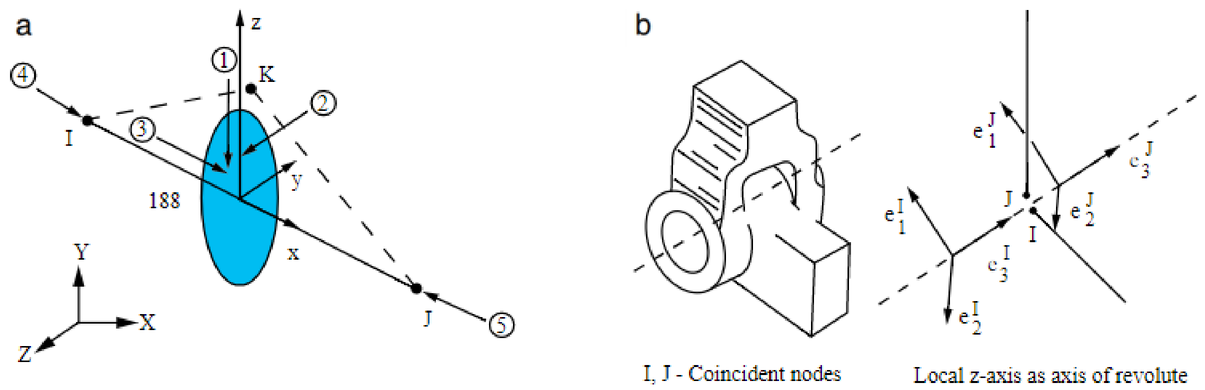

Figure 2. ANSYS elements geometry: (a) BEAM188; (b) MPC184 Revolute Joint (ANSYS, 2013)

3 OPTIMUM DESIGN PROBLEM

The mathematical modeling of the problem starts with the definition of a function of merit, which behaves as an evaluation parameter of the structure quality, appropriate choice of design variables and the definition of restrictions that limit the space of feasible solutions of the project.

For optimal design of structures there are several methodologies that seek to minimize the weight, the manufacturing cost or improve the efficiency of the structural behavior, among other objectives. In this context, it´s proposed a design model that aims to obtain optimal steel frames usually used in edifications considering regulatory and technological constraints.

$$\text{Minimize} \quad C = C_{profiles} + C_{joints} \qquad (1),$$

$$\text{Subject to} \quad g_{\sigma,i} \le 1\,, \quad i = 1, \dots, n$$

$$g_{\delta,j} \le 1\,, \quad j = 1, \dots, m$$

where C is the total cost of the structure, $C_{profiles}$ is the cost of the beams and columns, C_{joints} is the cost of beam-column connections and n and m are the total number of elements and degrees of freedom of the structure, respectively. The inequalities g_{σ} and g_{δ} represents the constraints associated with the resistance elements and nodal displacements, respectively.

The design variables of the adopted model are the structural profiles of commercial beams and columns and, additionally, the Initial Stiffness of beam-column connections. Thus, beams and columns profiles (ID) are represented by discrete variables and the Initial Stiffness of the beam to column connections (S_i) are continuous variables.

In order to generate unbraced frames with semi-rigid behavior were defined limits for the initial stiffness S_i (kN.m/rad) of the beam-column connections. According to NBR8800 (ABNT, 2008) standard settings, the lower limit is the value where the joint is classified as nominally pinned and the upper limit is the value when it's considered as nominally rigid, as represented below:

$$0{,}5\, E \frac{I_b}{L_b} \le S_i \le 25\, E \frac{I_b}{L_b} \qquad (2),$$

where E is the Young's modulus of the steel I_b is the moment of inertia of the cross section of the beam in the plane of the structure and L_b is the length of the beam connected to the joint. It is noted that the range that classifies a semi-rigid connection is changed in accordance with the structural profiles used, since the moment of inertia is a geometric property of profiles.

A fixity factor p_k is introduced to allow better understanding of the behavior of joints, once it value does not change with the profiles properties as happens at Eq. 2. The p_k is given by (Sánchez, 2013):

$$p_k = \frac{1}{1 + (3\, E\, I_b / L_b\, S_i)} \qquad (3),$$

replacing the Eq. 3 in Eq. 2, it is obtained the p_k range where joints are classified as semi-rigid:

$$0{,}143 \le p_k \le 0{,}893 \qquad (4),$$

The total manufacturing cost of the frame is defined by the cost of the beams, columns and beam-column connections. At the linear elements (beams and columns), calculating the cost is simple, since it is proportional to the mass of steel used, however, at the beam-column connections, calculating the cost becomes more complex.

To evaluate the cost of various types of joints, Bel Hadj Ali *et al* (2009) took into account the cost of material, manufacturing costs and installation of each components cost, to compose the total manufacturing cost. Thus, this methodology depends on the availability of all details of the design and manufacture of the joints. However, in this study it was considered only a single global value for the cost of the connections according to their degree of rotational stiffness, based on Sánchez (2013) proposed methodology.

Sánchez (2013) developed a model to estimate the manufacturing cost of semi-rigid connections according to their degree of rotational stiffness, by interpolation between a minimum cost associated with pinned joints and a maximum cost associated with rigid joints. It was proposed the calculation of the equivalent mass (m_{eq}), which corresponds to the cost of the joint divided by the cost of steel per unit mass (c_s). The function proposed for this calculation has as variable the initial stiffness of the joint S_i and its coefficients (linear and angular) receives different values for each type of connection.

For this paper, the extended end plate without stiffeners joint type of joint was selected for all beam-column connections. In this case, accordingly with Sánchez (2013) the corresponding equation is set to:

$$m_{eq} = 43{,}176 + 0{,}000335\, S_i \qquad (5).$$

Once it was set the cost for each joint, the total cost of the frame is calculated through the sum of the cost for each linear element (beams and columns) with the cost of the joints. The cost function, objective of the optimization is therefore defined as:

$$C = c_s \left\{ \sum_{i=1}^{nc} \left(\gamma_i A_i L_{c,i} \right) + \sum_{i=nc+1}^{ne} \left[\gamma_i A_i L_{v,i} + \sum_{k=1,2} \left(m_{eq} \right) \right] \right\} \qquad (6),$$

where nc is the number of columns of the structure, ne is the total number of structural elements, γ_i is the density of the material, A_i is the cross-sectional area of the element and L_c is the length of the column.

The Brazilian standard of steel construction, NBR8800 (ABNT, 2008), contains several requirements to Ultimate Limit State and Serviceability Limit State that the framework should satisfy.

Regarding to the restrictions, mechanical and design standards restrictions were adopted. Bending deflection at the beans, lateral displacements of the structure and maximum forces at the bars were considered.

The displacement constraints for the beams that limits the maximum bending deflection, according to NBR880 (ABNT, 2008), are given by the expression:

$$\frac{\delta}{L_v/350} \leq 1 \qquad (7),$$

where δ is the maximum vertical displacement (bending deflection) on the beam. The maximum horizontal displacement of the structure (in upper nodes of the column) is limited by:

$$\frac{h}{H/400} \leq 1 \qquad (8),$$

where h is the maximum vertical displacement of the framework and H is the total height of the structure.

The mechanical strength constraints are given by the expressions of interaction of internal forces, that considers the simultaneous action of axial force and bending moments. These expressions are defined by the following inequalities:

$$\text{for } \frac{N_{sd}}{N_{rd}} \geq 0{,}2 \;; \qquad \frac{N_{sd}}{N_{rd}} + \frac{8}{9}\left(\frac{M_{sd}}{M_{rd}}\right) \leq 1 \tag{9}$$

$$\text{for } \frac{N_{sd}}{N_{rd}} < 0{,}2 \;; \qquad \frac{N_{sd}}{N_{rd}} + \frac{M_{sd}}{M_{rd}} \leq 1 \tag{10}$$

where N_{sd} is the design normal force in the element, N_{rd} is the design resistant normal force of the element, M_{sd} is the design bending moment and M_{rd} is the design resistant bending moment of the element. As described in the item 5.3 of NBR8800 (ABNT, 2008) the calculation of the normal resistant force N_{rd} was performed considering the occurrence of local and global buckling in the elements.

To avoid displacements in the axis perpendicular to the plane of the frame (z-axis, Fig. 2) side supports were provided at the structural elements, which prevents lateral torsional buckling to occur. The side supports have a minimum spacing length defined by:

$$L_b \leq 1{,}76\, r_y \sqrt{\frac{E}{f_y}} \tag{11}$$

where L_b is the distance between two side supports from the structure (unconstrained length), r_y is the radius of gyration of the section in the axis perpendicular to the axis of the bending and f_y is the yield strength of the steel.

Prevented the occurrence of lateral buckling, the bending moment resistance M_{rd} in each element was then calculated considering only local buckling of wide and flange, according Annexes G and H of NBR8800 (ABNT, 2008).

For the beams, there is still an additional restriction due to shear resistance request, which given by the expression:

$$\frac{V_{sd}}{V_{rd}} \leq 1 \tag{12}$$

where V_{sd} is the maximum shear force in the request element and V_{rd} is the shear resistance of the element calculated according to the item 5.4.3 of NBR8800 (ABNT, 2008).

The process of optimization was implemented using the ModeFRONTIER commercial software. This software provides computational interface between structural analysis and optimization algorithms.

The Genetic Algorithm (GA) are evolutionists optimization algorithms based on the natural evolution of the species. These algorithms are suitable for highly nonlinear problems, once they are able to cover large and complex search regions and allow the definition of the best individuals within a population of solutions candidates. (Díaz et al., 2012). The GA also allow the use of continuous and discrete variables.

The iterative process of optimization using GA involves the following steps: (1) Definition of a population of candidates to solve the problem. At this stage the commercial profiles for beams and columns are selected arbitrarily in the database, as well as the Initial Stiffness of beam-column connections; (2) All the solution candidates generated on each iteration, are analyzed for evaluation of the fitness of the individuals in the population; (3) Creating a new population through reproduction operations, crossover and mutation individuals according to the fitness values calculated previously; (4) the steps 2 and 3 are repeated using a new population in each new iteration, until obtaining the desired convergence.

4 APLICATIONS

For the resolution of the examples presented here, it was used the GA MOGA-II (Multi Objective Genetic Algorithm) available in ModeFRONTIER. The optimization parameters were set to: 50% probability of directional cross-over, classical cross-over probability of 35%, probability of selection of 5% and 10% probability of mutation (Poles, 2003).

With results comparison purposes, all the examples were solved considering static linear behavior, same consideration made by the other authors who studied these applications.

4.1 Framework 1

The first studied framework is a one bay and two storey frame that was studied in Simões (1996). The elements of the frame were divided into 6 groups, where on each storey are defined three groups of distinct elements: beams, columns and stiffness of the joints. For the columns group, were considered all 29 profiles "H" type of the database, and for beams, all 59 profiles "I" type. The loading conditions, the geometry of the structure and the grouping of members are presented in Fig. 3.

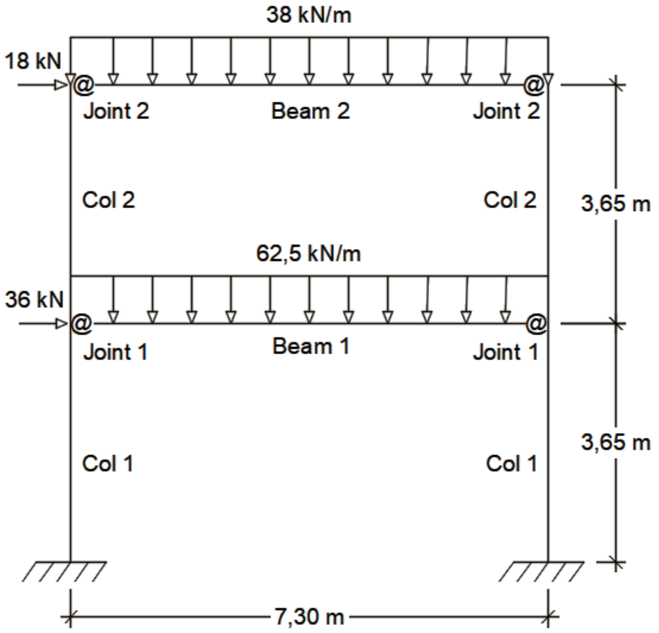

Figure 3. Geometry, loads and variables of the first example

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For all members of the structure the Young's modulus is $E = 206$ GPa, the yield stress $f_y = 250$ MPa, the density of steel $\gamma_i = 7850$ kg/m³ and the steel unit cost $c_s = 3{,}70$ reais/kg. For this example, were made 30 iterations with a population of 40 individuals each, with a total of 1200 individuals.

The following restrictions were considered: horizontal displacements on the top of the columns; bending deflection of beams; interaction of internal forces at beams and columns and shear strength at beams.

Figure 4 presents the history of the cost variation as a function of the optimization iterations, where are presented the average cost and the best cost at each generation. The optimum value was obtained in 23rd iteration, with the total cost of R$ 7247,32, of which 13,2% of it correspond to the cost of the joints. It is observed in Fig. 4 a certain dispersion of average values of each iteration, which occurs due to the probabilistic nature of the GA in the search for optimum value.

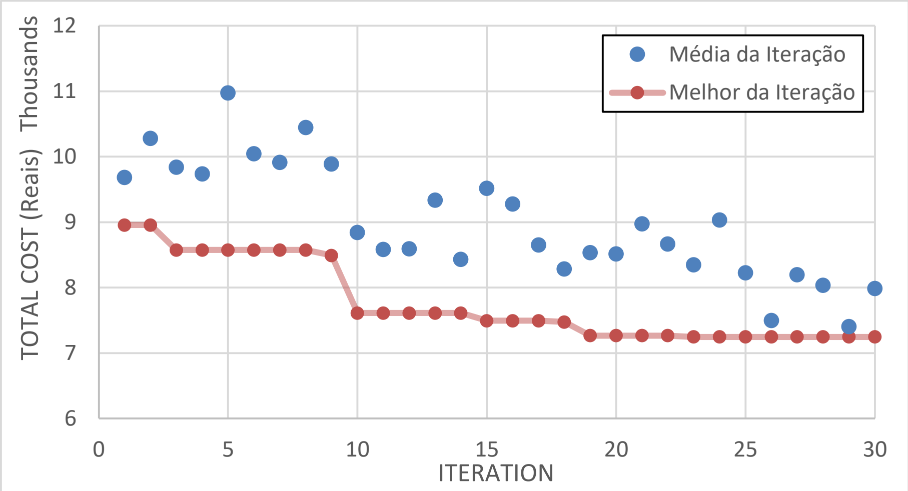

Figure 4. Objective function evolution

The constraints values obtained in the optimum design are given in Fig. 5, where: C1_h, C1_I, C2_h and C2_I are, respectively, the maximum horizontal displacement and interaction equation of internal forces related to columns 1 and 2; B1_δ, B1_I, B1_V, B2_δ, B2_I and B2_V are, respectively, bending deflection in the beans, interaction of internal forces and shear strength in beams 1 and 2.

At the optimum configuration, an active constraint corresponds to the horizontal displacement at the top of the frame (C2_h), limited by Eq. (8), reaching a maximum displacement of 1,82 cm. For the beams, the bending deflections were the closer constraints to the restrictions limit which achieved on the beams 1 and 2, 1,91 cm and 2,03 cm respectively. It is observed that these maximum deformations were decisive in the studied optimum frame design.

The Table 1 presents the results obtained by Simões (1996) and the obtained in this study. It’s presented the design result for all considered groups of profiles and the joints stiffness values.

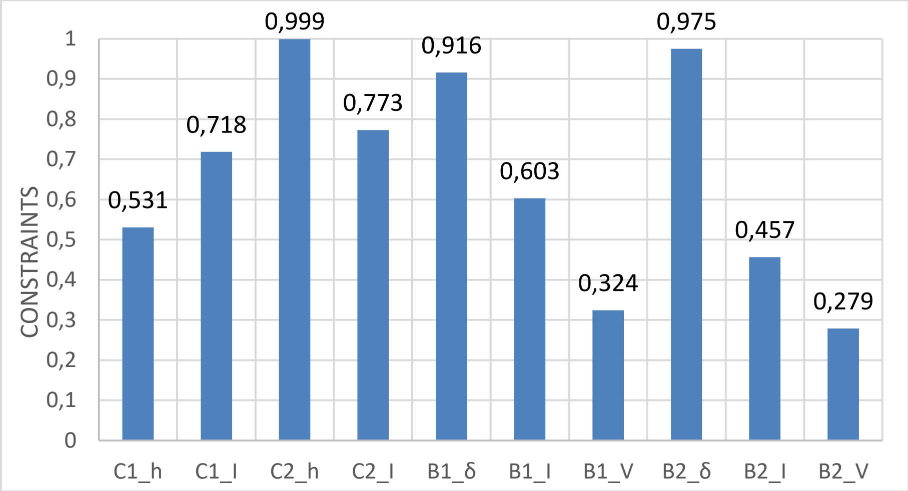

Figure 5. Optimal value constraints

Because of the particular production characteristics of the national structural profiles, the profiles database created in this work slightly differs from the structural profiles used in the works of reference, which prevents your direct comparison, however the comparison of results can still be evaluated by using the linear mass (kg/m) of each of the profiles which are given in parenthesis. The p_k values for the results found in this study are also shown in parenthesis.

Table 1. Optimization procedure results: variables

Variable	Simões (1996)		Rangel (Present)
	Rigid	Semi-rigid	
Colum 1	IPE 450 (77,6)	IPE 400 (66,3)	HP 250 x 62,0
Colum 2	IPE 450 (77,6)	IPE 400 (66,3)	HP 200 x 52,0
Beam 1	IPE 550 (105,0)	IPE 500 (90,7)	W 530 x 66,0
Beam 2	IPE 450 (77,6)	IPE 400 (66,3)	W 460 x 52,0
Joint 1 (kN.m/rad)	Rigid	59.000	56.900 (0,66)
Joint 2 (kN.m/rad)	Rigid	38.000	37.000 (0,68)

The mass of the designed frames through the optimization models are presented in Table 2, which also shows their total costs. For comparison, the costs of the structures obtained by Simões (1996) also were estimated by Eq. 6, since in its results the total cost of the structures were not presented.

Through the presented methodology, it was possible to obtain a reduction in the total cost of the structure of 12,3%, in addition to a reduction of total steel weight of 19,6% in comparison with the consulted references.

Table 2. Optimization procedure results: cost and mass

	Simões (1996)		Rangel (Present)
	Rigid	Semi-rigid	
Total cost (reais)	10339,0	8261,9	7247,3
Mass of members (kg)	2469,6	2114,1	1700,3
Mass of columns (kg)	1133,0	968,0	835,5
Mass of beams (kg)	1336,6	1146,1	864,8

4.2 Framework 2

The second studied framework is a two storey and three bays frame where the elements were divided into 8 groups. The structure's geometry and loading conditions used are shown in Fig. 6, as well as the grouping of members. For groups of columns and beams were considered respectively 29 "H" type profiles and 59 profiles of the "I" type.

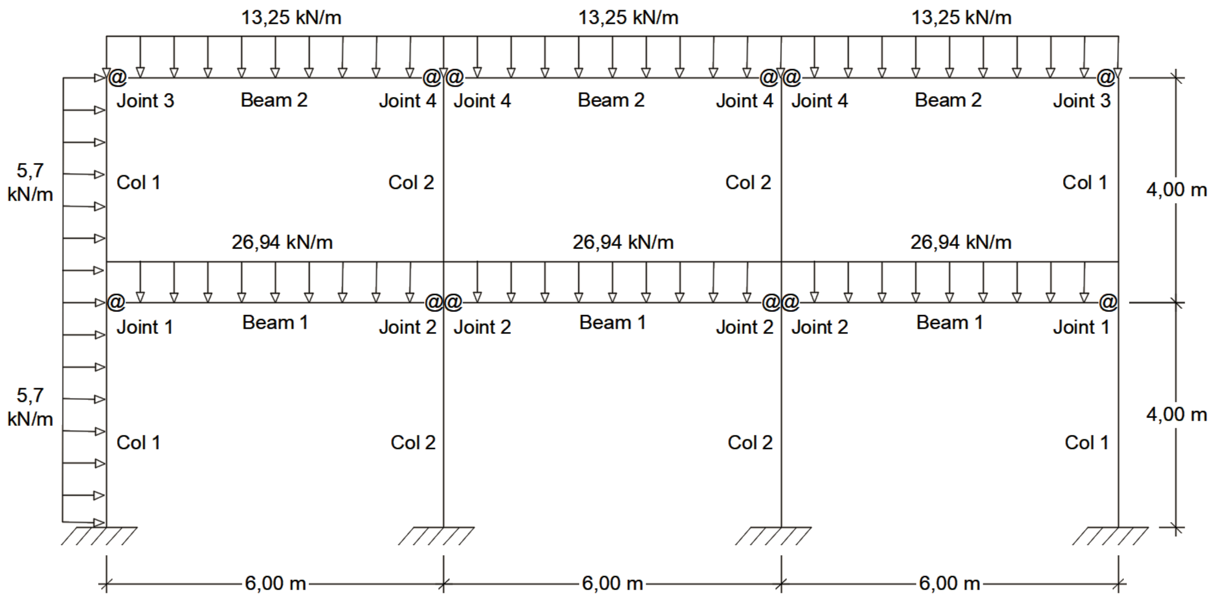

Figure 6. Geometry, loads and variables of the second example

For all the members of the structure were considered: Young's modulus $E = 210$ GPa, yield stress $f_y = 275$ MPa, steel density $\gamma_i = 7.850$ kg/m³ and unit cost of steel $c_s = 3{,}70$ reais/kg. For this example, 40 iterations were made with a population of 35 individuals each, totaling 1400 individuals evaluated.

Figure 7 presents the optimization history, detaching the average and the best cost to each generation.

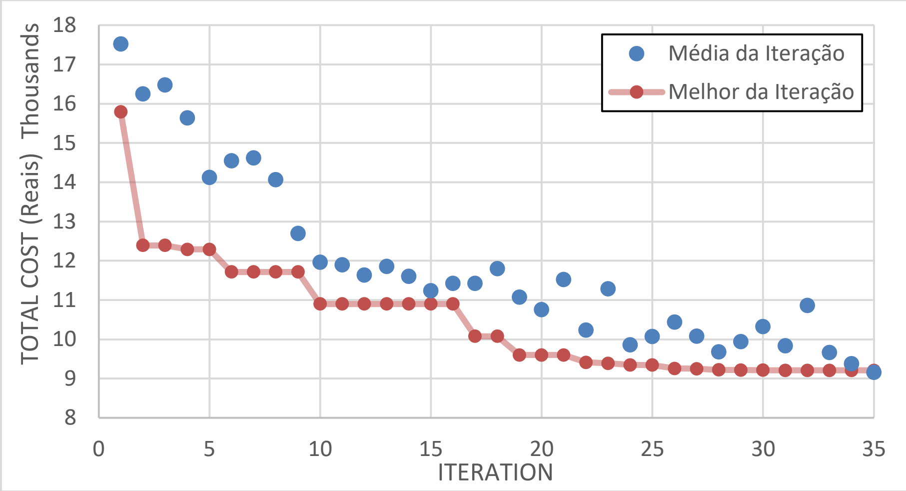

Figure 7. Objective function evolution

The optimum value was obtained in the 31st iteration and presented the cost of R$ 9206,49, while the cost of the joints corresponds at 24,5% of the total cost.

The calculated values for the optimum configuration constraints are presented in Fig. 8. It is observed that the determinants constraints for the optimum design were, again, the restrictions of maximum displacement, which are active or very close to the limit in all considered elements: horizontal displacement at the top of the columns (C1_h e C2_h) and bending deflections in the beans (B1_δ e B2_δ).

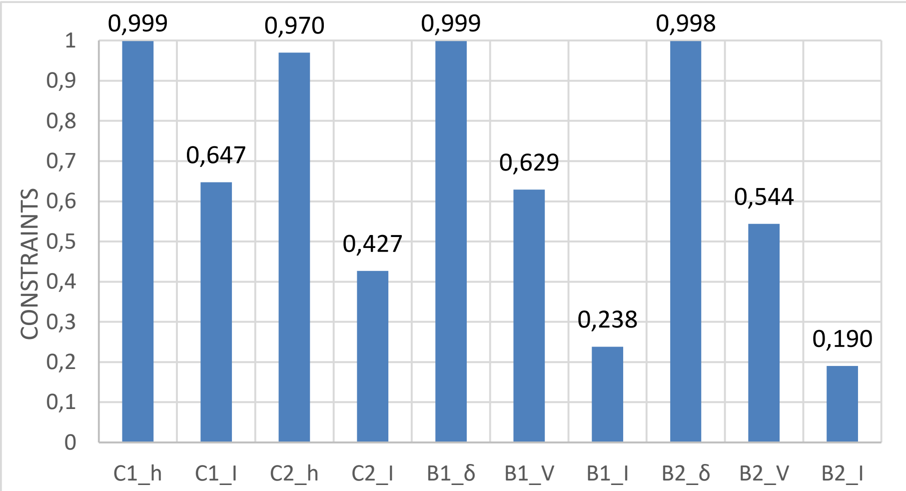

Figure 8. Optimal value constraints

The nodal displacement at the top of the columns reached a maximum displacement of 2,00 cm and the bending deflections observed in the groups of beams 1 and 2 were approximately 1,71 cm.

In relation to structural profiles defined for the optimum configuration, Table 3 presents a comparison between the results obtained in this study and the results available in the literature, which includes the authors Cabrero and Bayo (2005), Bel Hadj Ali et al (2009) and Sánchez and Espín (2013). To make feasible the comparison, the linear mass (kg/m) of European standard profiles is also displayed.

Table 3. Optimization procedure results: variables

Variable	Cabrero and Bayo (2005)	Bel Hadj Ali *et al* (2009)	Sánchez and Espín (2013)	Rangel (Present)
Colum 1	HEB 140 (33,7)	HEB 140 (33,7)	HEB 120 (26,7)	W 150 x 22,5
Colum 2	HEB 160 (42,6)	HEB 140 (33,7)	HEB 160 (42,6)	W 200 x 41,7
Beam 1	IPE 270 (36,1)	IPE 240 (30,7)	IPE 300 (42,2)	W 310 x 28,3
Beam 2	IPE 200 (22,4)	IPE 180 (18,8)	IPE 240 (30,7)	W 250 x 17,9

The stiffness values established for the solution of the problem of optimization are shown as: $\mathrm{Si}_1 = 23400$ kN.m/rad , $\mathrm{Si}_2 = 25000$ kN.m/rad, $\mathrm{Si}_3 = 6500$ kN.m/rad e $\mathrm{Si}_4 = 20000$ kN.m/rad, and their p_k are respectively: 0,802; 0,812; 0,730; 0,893. Other authors presented the dimensions of its main components instead of presenting the results for the Initial Stiffness, which prevents the comparison.

The mass of the designed frames obtained by the optimization models are presented in Table 4. To facilitates the comparison of the results, the unit cost of steel considered for obtaining the results of the present study was changed to $c_s = 1{,}60$ €/kg, same value considered by other authors.

Table 4. Optimization procedure results: cost and mass

	Cabrero and Bayo (2005)	Bel Hadj Ali *et al* (2009)	Sánchez and Espín (2012)	Rangel (Present)
Total cost (€)	4124,6	6878,6	4804,8	3981,2
Mass of members (kg)	2273,8	1969,4	2421,7	1877,5
Mass of columns (kg)	1120,8	1078,4	1108,6	1075,9
Mass of beams (kg)	1053,0	891,0	1313,1	801,6

The comparison of the results shows that the present work obtained the lowest value for the objective, it is possible to notice a cost reduction of 3,5% in relation to the second lowest value given by the proposed methodology, in addition to a reduction of 4,7% in the weight of the structure compared to second smallest value.

5 CONCLUSIONS

Computational tools for the determination of the optimum design of planar steel frames with beam-column semi-rigid joints have been presented.

The proposed methodology makes possible to design structures of lowest cost considering not only costs of beams and columns but also beam-column semi-rigid joints costs. Genetic algorithms with discrete and continuous design variables were used to represent commercial structural profiles and the rotational stiffness of the connections.

The consideration of semi-rigid connections allows for a better redistribution of the internal forces in the structure, which, together with optimization techniques, can result in mechanically more efficient structures and cost reductions due to the optimal choice of the structural profiles as well as the components of the beam-column connections.

The optimal structures obtained satisfies the requirements of NBR8800 (ABNT, 2008) that describes the calculation procedures for definition of critical loads and the maximum allowable displacements.

Analyzing the results of the presented applications it is possible to conclude that the proposed methodology presents significant reductions in manufacturing costs and, in some cases, achieves distinct results while comparing to the consulted references.

This paper also intends to emphasize the importance applying more sophisticated computational techniques for the design of structures.

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