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POSTBUCKLING OPTIMIZATION OF A REINFORCED COMPOSITE PANEL USING DISCRETE FIREFLY ALGORITHM

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Abstract. *Laminated composite materials have high strength-to-weight and stiffness-to-weight ratios. These characteristics are important in areas such as automotive and aeronautics, where reducing mass of components is essential. This also implies on manufacturing thin-walled parts, which are usually subjected to structural instability. In addition, optimization techniques can be applied to improve the structural design. Thus, this work aims to optimize a composite panel in the postbuckling regime, using the firefly algorithm. The problem studied here consists of a flat panel of laminated composite material, with six reinforcements (stiffeners) subjected to in-plane shearing load. The objective of the optimization is to find the best positions of the stiffeners and the best stacking sequence that maximize the postbuckling load without failure according to the Tsai-Wu criterion. Two cases of optimization are investigated. The first one considers only the positions of the stiffeners as design variables, and the second one both, positions and orientations of the plies, are the design variables. The structural response of the panel is obtained through finite elements in a model built in ABAQUS code. Both the position of the stiffeners and the ply orientations assume discrete values, thus, the firefly algorithm was adapted to this situation.*

Keywords: *Optimization, Laminated composite materials, Postbuckling*

1 INTRODUCTION

Laminated composite materials are increasingly being used in a variety of engineering applications, especially in the aerospace, aeronautics and automotive industries. This is due to the fact that they have high stiffness-to-weight and strength-to-weight ratios when compared to other materials. For example, in the aeronautical industry, stiffened composite panels are widely used in aircraft, especially in the composition of wings and fuselages (Wang et al., 2010).

Composite panels are thin-walled structures, usually subjected to structural instability. This implies that buckling studies must be considered in their design. In addition, the study of postbuckling should also be considered, as these structures can support loads in the postbuckling regime significantly higher than the buckling load. Several studies on the stability of laminated composites can be found in the literature. Considerations and revisions of several theories involved in determining the behavior in buckling and postbuckling of composite plates can be found in Leissa (1983). Balamurugan et al. (1998) investigated the buckling behavior of composite plates subjected to shear loads. Mallela and Upadhyay (2006) conducted a study of the buckling behavior of shear loaded composite stiffened panels using the finite element method (FEM) and verified which are the parameters that enhances the buckling load.

For an improvement in the design, optimization techniques can be applied. In the case of stiffened panels, the geometric parameters of the stiffeners and the panel stacking sequence affect the structural performance. Usually, for practical reasons, composite panels are restricted to symmetrical laminates and the discrete values of $0°$, $-45°$, $45°$ e $90°$ are considered for the plies orientation (Koide, 2016). This implies a high number of possible combinations, which has led several authors to apply optimization methods to find optimum design in recent years.

Some of the most used methods for optimization of composites are genetic algorithms (GA), simulated annealing (SA), particle swarm (PSO), ant colony (ACO). Le Riche and Haftka (1993) used a GA to maximize the buckling load, having discrete ply angles as design variables. Nagendra et al. (1996) also used an genetic algorithm to minimize the weight of a composite stiffened panel under buckling constraint having the stacking sequence and the stiffener height as design variables. Wang et al. (2010) used the ant colony algorithm to find the maximum buckling load on a stiffened panel subjected to bi-axial compressive load, using the stacking sequence and geometric parameters of the stiffeners as design variables. A more detailed study of the state of the art of composite materials optimization can be found in (Ghiasi et al., 2010).

The firefly algorithm is a heuristic algorithm based on the social behavior of the fireflies and it was originally proposed by Yang (2010). Fireflies send flashes of light to attract partners and move in the direction of those who have the biggest attractiveness. The attractiveness is related to the intensity of the light emitted and it is proportional to the distance between them. In the algorithm, the luminosity intensity is determinated by the value of the objective function. Koide (2016) applied the discrete form of FA to solve composite optimization problems using the stacking sequence as design variable. However, no work using geometric parameters of stiffeners as design variables was found in the literature.

In the present paper, the discrete firefly algorithm was used to optimize the ultimate load of a stiffened composite panel in the postbuckling regime subject to a in-plane shearing under the Tsai-Wu failure criteria constraint. The finite element method was used to obtain the structural response of the panel and a Python code was developed for the implementation of the discrete firefly algorithm. Two cases of optimization were investigated. In the first one only the positions of the stiffeners were considered as design variables, and the second one both, the positions and the stacking sequence of the skin were optimized.

2 PROBLEM DEFINITION

The characteristics of the stiffened composite panel studied here was taken from Stroud et al. (1984). The structure is composed of a flat square shell of 762 mm x 762 mm and six stiffeners with also 762 mm long and 34.34 mm high, which are paralely positioned as shown in figure Fig. 1. The blade-stiffened panel is simply supported on all edges and is subject to in-plane shear loading.

The structural response was obtained through the finite element method using the software ABAQUS (Simulia, 2012), in order to determine the values of the buckling loads and the maximum load supported in the postbuckling regime without failure according to the Tsai-Wu criterion (the formulation of the Tsai-Wu criterion can be found in Jones (1999)).

2.1 Validation of the FEM model

At first, the numerical model was validated considering a linear buckling analysis by comparing the values of the buckling load factor obtained in this study with those obtained by Stroud et al. (1984). For the validation, different load cases are considered combining uni-axial compression in the x direction (N_x) and in-plane shearing load (N_{xy}). In this case, the mesh, shown in Fig. 1, was discretized using 1296 ABAQUS S8R elements, which is an eight-node shell element with six degrees of freedom per node.

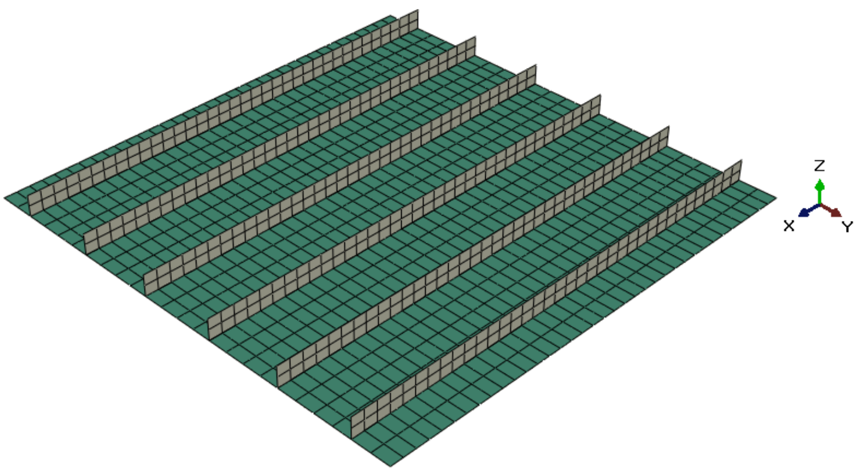

Figure 1: Finite element model of the stiffened panel.

The material properties of the laminate used in the validation model are: E_1=131 GPa, E_2=132 GPa, ν_{12} = 0.38, G_{12} = 6.41 GPa. The stacking sequences of the skin and the stiffeners are [45/-45/-45/45/0/90]$_s$ and [45/-45/-45/45/0]$_s$ respectively.

The results obtained are shown in Table 1 and it can be noted that there is a good agreement for the values of buckling loads in all analyzes, with the largest difference being 2.56% for pure shear and the lowest 0.49% for pure compression.

Table 1: Results of the validation FEM model

Load (N/mm)		Buckling load factor		
N_x	N_{xy}	Strout et al. (1984)	Present study	Difference (%)
0	175.1	1.553	1.513	2.56
-87.6	175.1	1.206	1.185	1.74
-175.1	175.1	0.840	0.832	0.88
-175.1	0	1.003	0.998	0.49

2.2 FEM model used in the optimization

For the purpose of this study, some adaptations in the initial model were performed. Because some mechanical strength properties are required to apply the Tsai-Wu criterion, the material was changed to laminate IM7 / 8552, composed of unidirectional graphite fibers and epoxy matrix (Araico et al., 2010), for which the elastic and strength properties are presented in Table 2. In addition, the number of plies increased from 12 to 32, all with the same thickness of 0.150 mm.

Table 2: IM7/8552 carbon unidirectional tape material properties

Property	Value	Property	Value
E_{11} (GPa)	145	X_T (MPa)	2414
E_{22} (GPa)	8.9	X_C (MPa)	1365
ν_{12}	0.33	Y_T (MPa)	51
G_{12} (GPa)	5.6	Y_C (MPa)	269
G_{13} (GPa)	5.6	S_{12} (MPa)	120
G_{23} (GPa)	4.48		

To avoid the use of contact elements, the skin and the stiffeners were tied by the use of *TIE command in ABAQUS, which function is to make the translations and rotations of the elements between two surfaces equal, resulting in a single part. As a consequence, in order to increase the possibilities in positioning the reinforcements, the panel mesh between the stiffeners was refined, as shown in Fig. 2.

The finite element analysis were divided into two stages. First, a linear buckling analysis was performed to find the buckling modes. Thereafter, the first three modes of buckling are introduced into the model through geometric imperfections with amplitudes of 10 %, 1 % and 0.1 % of the thickness of the laminate. Then, a nonlinear analysis in large displacements is performed to obtain the maximum load without failure according to the Tsai-Wu criterion. In both cases, prescribed displacements were introduced to the model and the corresponding load was computed by the panel reactions.

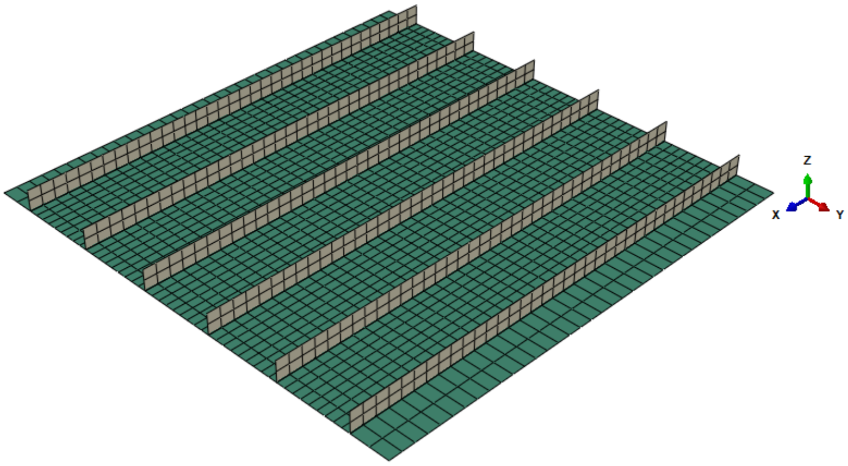

Figure 2: Model adapted for the optimization problem.

3 FIREFLY ALGORITHM (FA)

The firefly algorithm (FA) is a metaheuristic inspired by the luminous behavior of fireflies and was proposed by Yang in 2007 (Yang, 2010). The formulation of FA is based on the luminous intensity, $I(r)$, which is associated with the evaluation of the objective function. The intensity decreases with the square of the distance, r^2, between two fireflies, due to the fact that the absorption of light by the air intensifies with the increase of the distance. For the formulation of the FA, some idealizations of the characteristics of fireflies are considered.

The following idealizations of the characteristics of fireflies are considered to formulate the FA:

- Fireflies are unisex, meaning any firefly can be attracted to another.
- Attractiveness is proportional to brightness and decreases with distance.
- The brightness of a firefly is determined by the evaluation of the objective function.

A pseudo-code of the algorithm is presented in Fig. 3, where nr_fly is the number of fireflies, nv is the number of design variables, γ the light absorption coefficient and β the attractiveness.

3.1 Discrete firefly algorithm (DFA)

Originally, the firefly algorithm was proposed for the optimization of continuous variables. However, there are several situations where the domain of optimization is given by non-continuous variables. According to Tilahun and Ngnotchouye (2017), there are two ways of modifying the space. The first one is to use a continuous domain in the optimization and then the obtained values are updated to discrete numbers using some discretization mechanism. The other alternative is to modify the algorithm so that continuous and non-continuous domain exchanges are not necessary.

In the present study, the modification used is in the second option and was initially proposed by Durkota (2011), who developed the discrete form of firefly algorithm to solve the Quadratic

Input: $f(\boldsymbol{x}), \boldsymbol{x} = (x_1, x_2, \ldots, x_{nv})$; // objective function
$nr\_fly, I_0, \gamma, \alpha$; // defined initial constants
Output: $\boldsymbol{x}^{min}$; // objective function optimization value

For $i \leftarrow 1$ **to** nr_fly **do**
  $\boldsymbol{x}^i \leftarrow$ Initial_Solution();
End
While $criterion\ not\ satisfied$ **do**;
  $min \leftarrow arg_{i\in\{1,\ldots,m\}}\min\left(f(\boldsymbol{x}^i)\right)$;
  For $i \leftarrow 1$ **to** nr_fly **do**
    For $j \leftarrow 1$ **to** nr_fly **do**
      If $f(\boldsymbol{x}^i) < f(\boldsymbol{x}^j)$ **then** // move $\boldsymbol{x}^i \rightarrow \boldsymbol{x}^j$
        $r_{i,j} \leftarrow$ Distance$(\boldsymbol{x}^i, \boldsymbol{x}^j)$;
        $\beta \leftarrow$ Attractiveness$(I_0, \gamma, r_{i,j})$;
        $\boldsymbol{x}^i \leftarrow (1-\beta)\boldsymbol{x}^i + \beta\boldsymbol{x}^j + \alpha\left(Random(\,) - \frac{1}{2}\right)$; // movement
      End
    End
  End
  $\boldsymbol{x}^{min} \leftarrow \boldsymbol{x}^{min} + \alpha\left(Random(\,) - \frac{1}{2}\right)$; //best firefly moves randomly
End

Figure 3: Pseudo-code of the firefly algorithm.

Assignment Problem (QAP). Koide (2016) applied the same DFA to solve composite materials optimization problems. In this algorithm, the continuous functions of attractiveness, distance and motion were modified to discrete functions.

The generation of the initial firefly population is done using a random permutation between the possible values that the design variables can adopt, belonging to the search space S_{nv}. The distance is computed using the Hamming distance. The Hamming distance between two vectors is given by the number of non-corresponding values in a sequence. For example, for the permutations π_1, π_2 and $\pi_3 \in S_{vn}$:

$$
\begin{aligned}
\pi_1 &= [1\ 2\ 3\ 4\ 5\ 6];\\
\pi_2 &= [1\ 2\ 4\ 3\ 6\ 5];\\
\pi_3 &= [1\ 2\ 4\ 5\ 6\ 3];
\end{aligned}
\tag{1}
$$

The Hamming distance between π_1 and π_2 is 4, as well as between π_1 e π_3. The only elements that they have in common are in the first two positions.

The movement due to the attraction between the fireflies is divided into two steps and done sequentially, β and α. In β-step, the distance between the fireflies of the iteration decreases due to the movement of one towards the other. In order to do so, we first evaluate what the permutations have in common and extract it. For example, π_1 = [4 9 3 7 6 8 2 1 5] is attracted by π_2 = [4 1 3 2 6 5 9 7 8], resulting in $\pi_{1\rightarrow 2}$:

$$\pi_{1\to 2} = [4\_\ 3\_\ 6\_\ \_\ \_\ \_]. \tag{2}$$

The gaps in $\pi_{1\to 2}$ are filled with values of the previous permutations, depending on the value of β, which is calculated as a function of the Hamming distance (d_{π_1,π_2}) and the light absorption coefficient γ.

$$\beta = \frac{1}{1+\gamma {d_{\pi_1,\pi_2}}^2}. \tag{3}$$

β ranges from 0 to 1 and, once calculated, it is compared to a value obtained in the random() function, which returns a random value also from 0 to 1. If β> random(), the gap is filled with the value of π_2, otherwise the value remains π_1.

The α step is determined by the random motion of the firefly and characterizes a global search. The value of α means the number of elements that are changed. A random position is picked and the element is replaced by another from the search space. After this, the movement of the firefly in one iteration is completed.

4 OPTIMIZATION CASE STUDIES

4.1 Case 1: Stiffeners position optimization

The first case studied was the optimization of the stiffeners positions. In this problem, the positions of the four internal stiffeners are optimized and the two at the outer positions are fixed at a distance of 63.5 mm from the edges. The domain of the stiffeners positions is discretized so that there is no overlap between them and each one has 11 possibilities to be assumed.

Then the optimization problem is formulated as follows:

find	$X_k, X \in (1,2,3,...11), k = 1,...,n_s$
maximize	$N_{xy_{max}}$ (Maximum shearing load in postbuckling regime)
subjected to	Tsai-Wu failure criterion

where X_k represents the position of the stiffeners, k the sequence index and n_s the number of stiffeners to be optimized.

For this problem, the population is composed of 10 fireflies and the algorithm parameters γ and α were set as 0.8 and 1, respectively. A total of 1000 evaluations of objective function were computed per optimization.

The composite layup of the skin is $[\pm 45\ \mp 45\ 0\ 90\ 0_2\ \pm 45_2\ 90_2\ 0_2]_s$.

The obtained results are shown in the Table 3 and are compared with the configuration used in the validation of the model, i.e, all stiffeners equally spaced. In addition, a study of the convergence of the algorithm was performed by comparing the results obtained in five different runs of the algorithm, as shown in the Fig. 4.

It can be observed from the results that the algorithm tends to converge to an optimal value in a reasonably low number of evaluations of the objective function, since in this case there are a total of 14641 possibilities. In addition, only optimizing the stiffeners positions was possible to obtain a significant increase in the postbuckling load by around 14 % compared to the baseline configuration.

Table 3: Optimization results of case 1

Description	Stiffeners positions	$N_{xy_{max}}$ (N/mm)
Baseline configuration	[9 7 5 3]	513.245
Optimized configuration	[11 9 4 2]	584.702

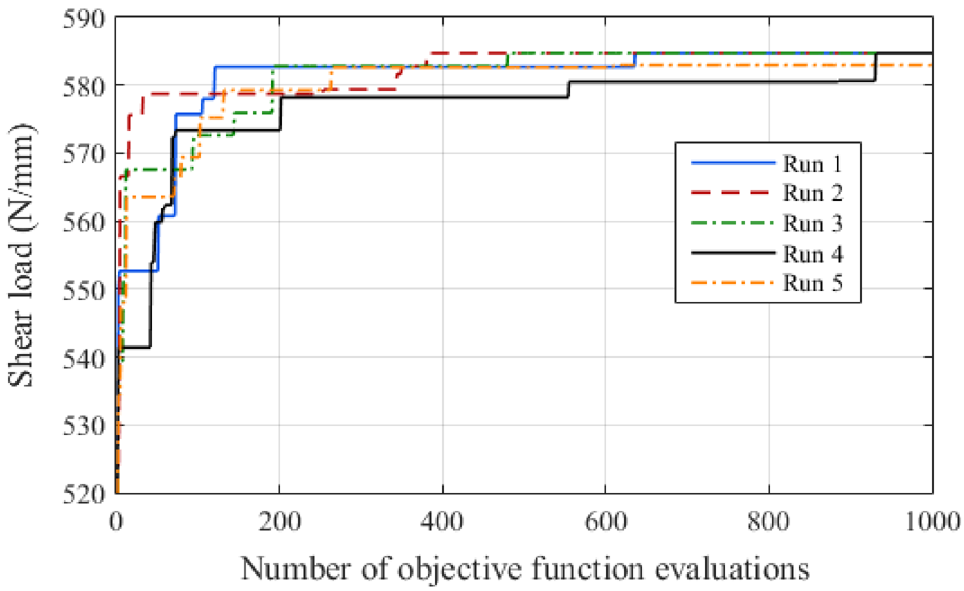

Figure 4: Convergence study of firefly algorithm: Maximum postbuckling load versus number of objective function evaluations.

4.2 Case 2: Stiffeners and stacking sequence optimization

In this case, the stacking sequence of the skin is also optimized. This implies a high number of possibilities, making the problem more complex, so the number of fireflies has been increased to 50, as recommended by Gandomi et al. (2011), and the number of objective function evaluations was raised to 1500. The other parameters (α and γ) have remained the same as the case 1. Then, the optimization is formulated as:

find	$X_k, X \in (1, 2, 3, ...11), k = 1, ..., n_s$
	$\theta_j, \theta \in (0°,\ \pm45°,\ 90°), j = 1, ..., n_l$
maximize	$N_{xy_{max}}$ (Maximum shearing load in postbuckling regime)
subjected to	Tsai-Wu failure criterion

where θ_j is the ply orientation and n_l the number of plies that are optimized. As the laminate was considered to be symmetric and balanced, the number of angles that needs to be set was reduced from 32 to 8. As a design constraint, the values that can be assumed for the angles are $0°$, $45°$ and $90°$. The stiffeners position options remained the same as those presented in case 1. The results obtained for the case 2 are shown in Table 4.

It can be noted that, for the optimized configuration, the postbuckling load increased by around 36% compared to the baseline configuration.

Table 4: Optimization results of case 2

Description	Stacking sequence	Stiffeners positions	$N_{xy_{max}}$ (N/mm)
Baseline configuration	$[\pm45\ \mp45\ 0\ 90\ 0_2\ \pm45_2\ 90_2\ 0_2]_s$	[9 7 5 3]	513.245
Optimized configuration	$[90_4\ \pm45\ 90_2\ \pm45_3\ 0_2]_s$	[11 8 3 2]	701.540

5 CONCLUSIONS

In this study, the firefly algorithm in its discrete form was applied in the optimization of the postbuckling load of stiffened composite panels with the Tsai-Wu failure criterion as a constraint. The objective was to optimize the positions of the stiffeners and the stacking sequence of the panel. Two cases were analyzed. In case 1, only the positions of the stiffeners were considered as design variables and the results obtained showed a 14 % increase in the maximum postbuckling load value when compared to a baseline configuration. For case 2, both the positions of the stiffeners and the stacking sequence were optimized and, compared to the same baseline configuration, a 36 % increase was found in the maximum postbuckling load.

Thus, the algorithm proved to be a good technique for the resolution of this class of problems and it was possible to conclude that, in the case of in-plane shear loading, the use of stiffeners in optimum positions can increase the performance and consequently decrease the weight of this category of structures. For a even better design, combining ply angles and the stiffeners positions as design variables higher values for the postbuckling load can be obtained.

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