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ANALYSIS OF VERTICAL LOADS INDUCED BY PEDESTRIANS ON FOOTBRIDGES THROUGH BIODYNAMIC MODELS

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Abstract. *The simulation of loads induced by walking people in civil engineering structures is still challenging, being the focus of considerable research worldwide in the recent decades due to the increasing number of reported vibration problems in pedestrian structures. Because of the complexity of the pedestrian mechanisms involved in these problems, there are still some gaps in knowledge and more reliable pedestrian models are needed to be investigated. One of the most important key in the designing of slender structures is the Human-Structure Interaction (HSI). How moving people interact with the structures and affects their dynamical properties are questions not even well understood. In this direction, several authors have proposed biodynamics models to represent the pedestrian, however, which of these models provides a consistent approximation to physical reality still needs to be studied. This paper presents a bi-dimensional bipedal pedestrian walking model and the equations of motion of the HSI system applied to a footbridge. The numerical model was implemented in MATLAB. Preliminary results obtained are presented in order to validate the model with experimental measures that were carried out in a prototype footbridge in the Structures Laboratory of COPPE at Federal University of Rio de Janeiro, where pedestrians were asked to walk on the footbridge and the applied force and structural response in vertical direction were recorded with accelerometers placed at strategic points of the structure.*

Keywords: *Human-Structure-Interaction, vertical vibrations, biodynamic models, pedestrian loads*

1 INTRODUCTION

Zivanovic $et~al.$, 2005 [13] related that the amplitude of the spectrum force measured on a rigid surface is not equal to the one measured on a flexible surface and indicated the importance in considering the *Human-Structure-Interaction* (HSI) in analytical and numerical models.

The interaction of walking people with the structure is an essential phenomenon to be simulated. Several authors [1], [2], [3], [5], [6], [7], [8] have proposed biodynamic models to represent the pedestrian. These models consider the human body as a *Spring-Mass-Damper* (SMD) system coupled with the structure.

The dynamic human properties are also difficult to obtain, however there are some estimates of these parameters in literature, for example, a group of people was modelled as a single degree of freedom SMD model by Shahabpoor $et~al.$, 2016 [7] and their parameters were identified analytically using experimental data from tests carried out in the Structures Laboratory at University of Sheffield.

A more sophisticated biodynamic model is the classic inverted pendulum model, for instance, Whittintong and Thelen, 2009 [11] developed a SMD model consisting of a point mass, two massless limb springs of stiffness and two massless circular roller feet of radius R. This kind of model is an useful framework for understanding gait transitions. Dang and Zivanovic, 2013 [4] studied different approaches for modelling walking pedestrian and concluded that the inverted pendulum model replicates the kinematics of the body's center of mass more closely than moving-oscillator- actuators.

Qin $et~al.$, 2013 [5] presented the mathematical development of a *Human-Structure-Interaction* (HSI) system using a 2D inverted pendulum-model and showed that dynamic interaction increases at larger vibration levels of the structure.

The aim of this paper is to present a bipedal walking model based on the previous model proposed by Qin $et~al.$, 2013 [5], but considering constant walking velocity and to validate it with experimental measures from tests with people walking on a prototype footbridge.

2 Experimental tests

A prototype footbridge located at the Structures Laboratory of COPPE (LABEST) was used to perform the experimental tests. The structure consists of a reinforced concrete slab with a length of 12.20 m, a width of 2.20 m and a thickness of 0.10 m and steel beams located beneath the slab in longitudinal and transverse directions. The footbridge is simple supported with a maximum free span length of 11.5 m (Figure 1 and Figure 2).

Six accelerometers were fixed on the steel beams to measure the structural response in the vertical direction (measurement range $\pm 1g$, being g=9.81 m/s^2 the acceleration of gravity) in the positions of L/2, L/4 and L/6, being L the length of the structure. The positions of the sensors were design with the purpose of measuring the first four modes of the structure.

The vertical applied load was measured using instrumented force platforms with dimensions of 0.90 m x 0.90 m, which covered the complete surface area of the structure. The instrumentation can be observed in Fig. 3.

Figure 1: Prototype footbridge

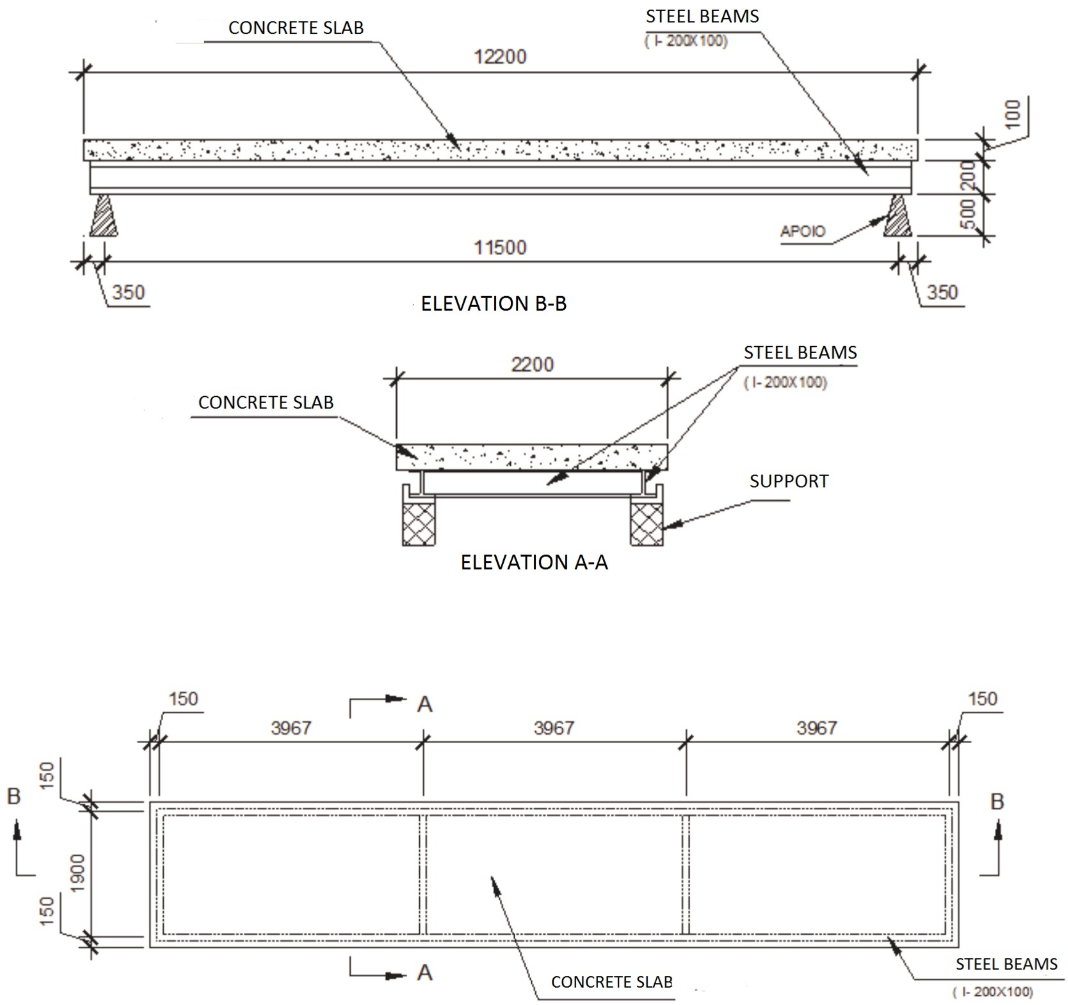

Figure 2: Plant view and elevations of the footbridge

2.1 Modal properties of the empty structure

The modal properties of the structure were identified applying an impact at a specific point over the structure, approximately at L/4. The tests were recorded until the footbridge vibration

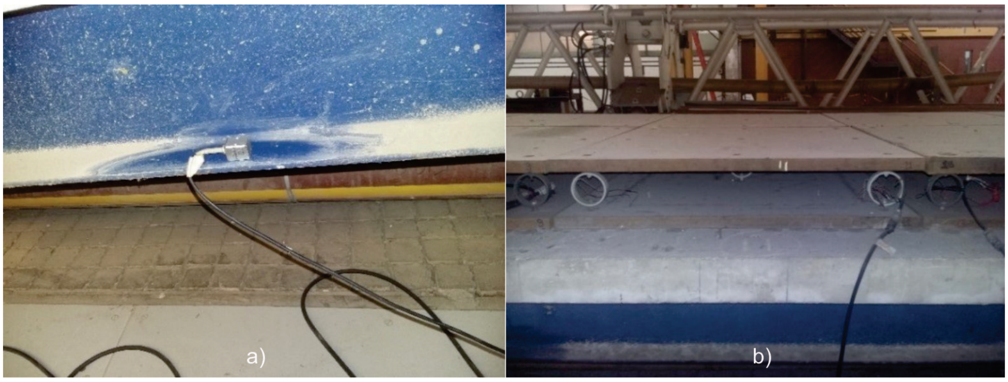

Figure 3: Sensors details: a)Accelerometer fixation; b)Force platforms

ends and were repeated three times.

The first four natural frequencies of the vertical modes were identified through the estimation of the frequency spectrum and the corresponding damping ratios by applying *Short-Time-Frequency-Technique* (STFT). Two bending modes and two torsion modes were identified looking at the phases between parallel accelerometers. The experimental values of modal parameters will be presented in the next section in a comparison with the numerical values.

2.2 Tests with people walking

Experimental tests with people walking were carried out on the prototype footbridge described in section **2**. Participants were asked to walk along the structure at 1/4 of the span, as illustrated in Fig.4 for one person walking. The applied vertical force was measured by thirteen force platforms and response acceleration was recorded by accelerometers (Figure 3). More details concerning to these experimental tests can be found in previous authors works [9], [10].

Figure 4: A typical walking path

3 Numerical model of the structure

The footbridge was modelled in finite elements, using MATLAB. A simplified model formed by 62 space frame elements and 39 nodes was used [9], [10], considering simple supports in the end nodes. In Fig.5 it is shown the simplified structure scheme in finite elements.

The material properties were determined considering the equivalent properties of the homogenized section (Table 1) and were distributed proportionally for longitudinal and transverse elements.

In Table.1 E is the Young's modulus, I_x, I_y, I_z and I_r are de the moments of inertia with respect to the axes x, y, z and the polar moment of inertia respectively, A is transversal section area, ρ is the material density and G is a transverse modulus.

In Table 2 are presented the natural frequencies obtained numerically in a comparison with the experimental values and there are also presented the estimated damping ratios for each mode from experimental tests.

Table 1: Mechanical properties of the homogenized section steel-concrete used in the model

Element	E (N/m^2)	I_x (m^4)	I_y (m^4)	I_z (m^4)	I_r (m^4)	A (m^2)	ρ (kg/m^3)	G (MPa)
Longitudinal	$2e^{11}$	$3.33e^{-5}$	$4.2e^{-2}$	$1.32e^{-4}$	$4.2e^{-2}$	$1.7e^{-2}$	17450	$7.69e^{10}$
Transversal	$2e^{11}$	$1.68e^{-4}$	$4.2e^{-2}$	$3.33e^{-5}$	$4.2e^{-2}$	$2.33e^{-5}$	17450	$7.69e^{10}$

Table 2: Modal properties of the structure

Natural frequencies (Hz)	mode 1	mode 2	mode 3	mode 4
Experimental	3.15 ± 0.01	9.43 ± 0.01	12.06 ± 0.00	22.63 ± 0.00
Numerical	3.11	9.51	12.42	21.84
Damping ratios (%)	0.6 ± 0.01	0.7 ± 0.00	1.5 ± 0.01	1.2 ± 0.01

Vibration modes obtained numerically are shown in Fig. 6. As it is observed the numerical model represents the structure adequately.

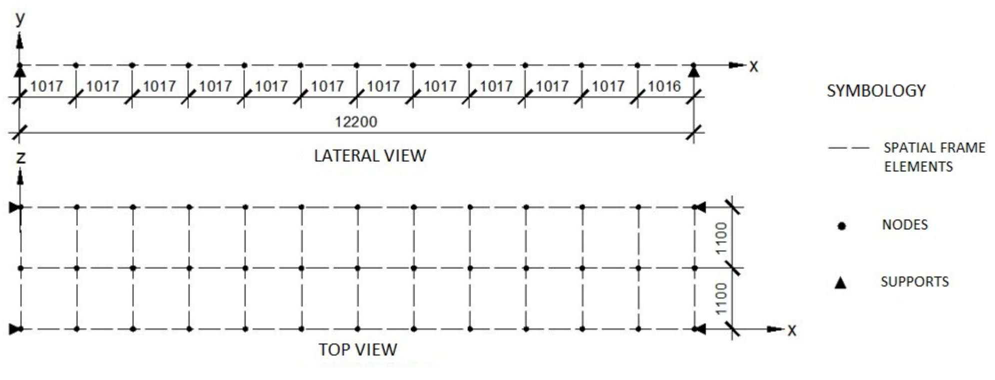

Figure 5: Schematic finite elements structure (dimensions are given in millimeters).

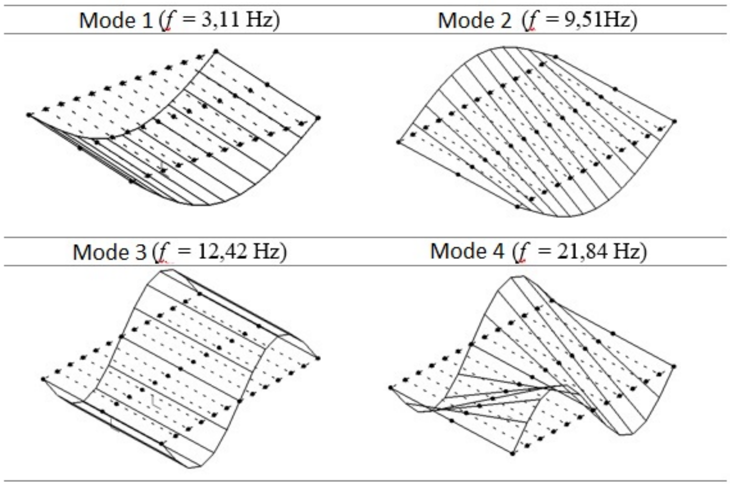

Figure 6: Vibration modes obtained by free vibration analyse.

4 Pedestrian model

The model used to represent the pedestrian body is known in the literature as an inverted pendulum model, it consists in a lumped mass at its center of mass and two massless and compliant legs with a stiffness, a time variant-damper and equal rest length L_0, as shown in Fig.7. In the model is neglected the roller feet. A step cycle is defined between two successive

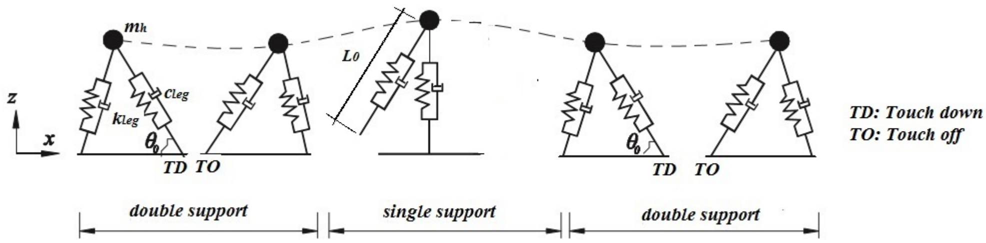

Figure 7: Schematic of the pedestrian biomechanical walking model (Adapted from [5])

heel impacts of the same foot, and is divided in a single support phase, when only one foot is in contact with the ground and a double support phase, when the two feet are in contact with the ground.

The double support phase begins when the unsupported foot, from now on referred to as the trailing leg, touches down the floor and ends when the supported foot, from now on referred to as the leading leg, touches off. That means that the trailing leg is repositioned ahead of the body's center of mass and becomes the leading leg for the next step cycle.

The single support phase ends when the trailing leg hits the ground. The angle with which the leg touches down at the initial moment of the double support phase is called as angle of attack θ_0.

4.1 Equation of motion of the pedestrian body

The longitudinal and vertical displacements of the center of mass are expressed in Cartesian coordinates x, z. The walking velocity is considered constant in order to simplify the mathematical model for this specific case that is focused in vertical direction loads. Therefore, the pedestrian motion can be described by a simple linear equation in the x-direction and by a non-linear equation in the z-direction. The equilibrium equation for the pedestrian in the z-direction is:

$$m_h\ddot{z} + (F_{s,l} + F_{d,l})\sin\theta_l - (F_{s,t} + F_{d,t})\sin\theta_t + m_h g \quad = \quad 0 \tag{1}$$

where $F_{s,l}, F_{s,t}$ are the elastic forces, $F_{d,l}, F_{d,t}$ the damping forces, corresponding to leading and trailing legs respectively, $m_h\ddot{z}$ is the inertial force and $m_h g$ the gravitational force, corresponding to the center of mass.

The elastic forces are given by

$$\begin{aligned} F_{s,l} &= k_{leg}\left(L_l\left(t\right) - L_0\right) \\ F_{s,t} &= k_{leg}\left(L_t\left(t\right) - L_0\right) \end{aligned} \tag{2}$$

and the damping forces can be expressed as

$$\begin{aligned} F_{d,l} &= \alpha(t)c_{leg}v_l \\ F_{d,t} &= (1-\alpha(t))c_{leg}v_t \end{aligned} \tag{3}$$

where $L_l(t)$ and $L_t(t)$ are the lengths of the leading and trailing legs at a given time t, c_{leg} and k_{leg} are the leg damping and leg stiffness, and v_l and v_t are the axial spring velocities of the legs.

The leg lengths can be determined by

$$\begin{aligned} L_l(t) &= \sqrt{\left(n_s l_s - x\right)^2 + z^2} \\ L_t(t) &= \sqrt{\left(x - \left(n_s - 1\right)l_s\right)^2 + z^2} \end{aligned} \tag{4}$$

where l_s is the step length and n_s is the step number, $\alpha(t)$ is the damping coefficient and goes from zero at the beginning of the double support phase to one at the beginning of the single support phase.

During the double support phase $\alpha(t)$ can be calculated as:

$$\alpha\left(t\right) \quad = \quad \frac{L_t(t) - L_t(0)}{L_0 - L_t(0)} \tag{5}$$

The axial spring velocities can be obtained from the derivative of the equations of the leg lengths as

$$\begin{aligned} v_l &= \frac{d}{dt}L_l(t) = \frac{-(n_s l_s - x)\dot{x} + z\dot{z}}{\sqrt{(n_s l_s - x)^2 + z^2}} = -\dot{x}\cos\theta_l + \dot{z}\sin\theta_l \\ v_t &= \frac{d}{dt}L_t(t) = \frac{(x - (n_s - 1)l_s)\dot{x} + z\dot{z}}{\sqrt{(x - (n_s - 1)l_s)^2 + z^2}} = \dot{x}\cos\theta_t + \dot{z}\sin\theta_t \end{aligned} \tag{6}$$

By substituting Eq. 4 in Eq. 2 and Eq. 6 in Eq. 3:

$$\begin{aligned} F_{s,l} &= k_{leg}\left(\sqrt{(n_s l_s - x)^2 + z^2} - L_0\right) \\ F_{s,t} &= k_{leg}\left(\sqrt{(x - (n_s - 1)l_s)^2 + z^2} - L_0\right) \end{aligned} \tag{7}$$

$$\begin{aligned} F_{d,l} &= \alpha(t)c_{leg}(-\dot{x}\cos\theta_l + \dot{z}\sin\theta_l) \\ F_{d,t} &= (1-\alpha(t))c_{leg}(-\dot{x}\cos\theta_t + \dot{z}\sin\theta_t) \end{aligned} \tag{8}$$

and substituting Eq. 2 and Eq. 8 in Eq. 1 we obtain the equation of motion of the pedestrian in the z direction

$$\begin{aligned} 0 &= m_h\ddot{z} + k_{leg}(L_l - L_0)\sin\theta_l + \alpha(t)c_{leg}(-\dot{x}\cos\theta_l + \dot{z}\sin\theta_l)\sin\theta_l \\ &\quad + k_{leg}(L_t - L_0)\sin\theta_t + (1-\alpha(t))c_{leg}(-\dot{x}\cos\theta_t + \dot{z}\sin\theta_t)\sin\theta_t + m_h g \end{aligned} \tag{9}$$

4.2 Equations of motion of the HSI system

The equations of motion of the HSI system can be obtained by solving the Lagrangian equations of motion of the system. The development of these equations is not the focus of this paper, for further details, the reader is addressed to [5].

The matrix system of equations of the HSI for this case is

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F}(t) \tag{10}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are the mass, damping and stiffness matrices and $\ddot{\mathbf{U}}$, $\dot{\mathbf{U}}$, $\mathbf{U}$ and $\mathbf{F}(t)$ are acceleration, velocity, displacement and force vectors respectively and can be expressed by:

$$\mathbf{M}_{(n+1,n+1)} = \begin{bmatrix} m_1 & 0 & \dots & 0 \\ 0 & m_2 & \dots & 0 \\ \dots & \dots & \dots & 0 \\ 0 & 0 & \dots & m_h \end{bmatrix}$$

$$\mathbf{C}_{(n+1,n+1)} = \begin{bmatrix} c_{1,1} & c_{1,2} & \dots & c_{1,n+1} \\ c_{2,1} & c_{2,2} & \dots & c_{2,n+1} \\ \dots & \dots & \dots & 0 \\ c_{n+1,1} & c_{n+1,2} & \dots & \alpha(t)c_1 + (1-\alpha(t))c_2 \end{bmatrix}$$

$$\mathbf{K}_{(n+1,n+1)} = \begin{bmatrix} w_1^2 m_1 & 0 & \dots & -k_{t,v}\phi_1(N-1) - k_{l,v}\phi_1(N) \\ 0 & w_2^2 m_2 & \dots & -k_{t,v}\phi_2(N-1) - k_{l,v}\phi_2(N) \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & k_{t,v} + k_{l,v} \end{bmatrix}$$

$$\begin{aligned} \mathbf{F}(t)_{(n+1)} &= \begin{bmatrix} f_1, & \dots, & f_n, & -(m_h g + (1-\alpha(t))c_4 - \alpha(t)c_3) \end{bmatrix} \\ \mathbf{U}_{(n+1)} &= \begin{bmatrix} Y_1, & \dots, & Y_n, & z \end{bmatrix} \end{aligned}$$

where

$$\begin{aligned}
c_{i,i} &= 2\xi_i w_i m_i + (1-\alpha(t))c_2\phi_{i,i}(N-1) + \alpha(t)c_1\phi_{i,i}(N) && (i=1,2,...,n)\\
c_{i,j} &= c_{j,i} = (1-\alpha(t))c_2\phi_{i,j}(N-1) + \alpha(t)c_1\phi_{i,j}(N) && (i \neq j \leq n)\\
c_{i,n+1} &= c_{n+1,i} = -\alpha(t)c_1\phi_i(N) - (1-\alpha(t))c_2\phi_i(N-1) && (i=1,2,...,n)\\
f_i &= \alpha(t)c_3\phi_i(N) - (1-\alpha(t))c_4\phi_i(N-1) && (i=1,2,...,n)
\end{aligned}$$

$$c_1 = c_{leg}\sin^2\theta_l, \quad c_2 = c_{leg}\sin^2\theta_t$$

$$c_3 = c_{leg}\sin\theta_l\cos\theta_l, \quad c_4 = c_{leg}\sin\theta_t\cos\theta_t$$

$$k_{l,v} = k_{leg}\left(1-\frac{L_0}{L_l}\right)\left(1-\frac{y_{(N)}}{z}\right), \quad k_{t,v} = k_{leg}\left(1-\frac{L_0}{L_t}\right)\left(1-\frac{y_{(N-1)}}{z}\right)$$

$$\phi_{i,j}(N) = \phi_i(N)\phi_j(N)$$

where the coefficients c_1 and c_2 express the effective vertical damping, c_3 and c_4 the effective horizontal damping, $k_{l,v}$ and $k_{t,v}$ are the effective vertical stiffness of the two legs, ϕ_i is the modal vector corresponding to i^{th} vibration mode, y is the vertical displacement of the structure and N, $N-1$ denote the positions of the contact points of the two feet over the structure during the N^{th} step cycle.

The procedure implemented in MATLAB to obtain the time response of the HSI system is described in the following steps:

1 Through a free vibration analysis are obtained the frequencies $w_1, w_2, \ldots, w_n$, the modal masses $m_1, m_2, \ldots, m_n$, the modal vectors $\phi_1, \phi_2, \ldots, \phi_n$ and the damping ratios $\xi_1, \xi_2, \ldots, \xi_n$ of the structure alone. In this case the damping ratios were estimated experimentally.

2 The geometrical and physical parameters of the pedestrian, such as: the weight of the human body, the leg stiffness, the damping leg, the step length and the rest length of the leg are established.

3 The initial conditions of the structure and the center of mass of the pedestrian as: the initial displacement and velocity of the structure and the pedestrian, the touch down position for the first leg, the leg lengths at the initial time $L_l(0)$ and $L_t(0)$, the initial step number $n_s = 1$ and the time increment Δ_t are set.

4 The overall mass, stiffness and damping matrices of the system at time $t = 0$ are assembled.

5 The initial acceleration of the system is computed using the equilibrium force equation: $\ddot{\mathbf{U}}_0 = \mathbf{M}^{-1}\left(\mathbf{F}_0 - \mathbf{C}\dot{\mathbf{U}}_0 - \mathbf{K}\mathbf{U}_0\right)$

6 For each time step the following time integration procedure is performed:

6.1 The position of the two feet over the structures is determined by:

$$\mathrm{x}_l = n_s l_s - x_0, \qquad x_t = (n_s - 1)l_s - x_0$$

where x_0 is the distance between the edge of the footbridge and the first heel impact point.

6.2 The effective vertical stiffness and damping are then computed.

6.3 The overall mass, damping, stiffness matrices and force vector of the system are assembled.

6.4 The displacement, velocity and acceleration at the end of the time increment are determined applying the 4^{th} order Runge-Kutta method.

6.5 The step cycle is checked by comparing the length of the trailing leg $L_t(t)$ with the rest length of the leg L_0. If the length of the trailing leg is larger than the rest length of the leg the cycle is in the single support phase, otherwise the cycle is in the double support phase.

6.6 If $L_t(t) > L_0$, the distance between the center of mass and the contact point with the structure for the next step cycle is computed. When the distance between the center of mass and the next contact point equals the rest length of the leg, the step number is set to $n_s = n_s + 1$.

6.7 The procedure described in steps 6.1 to 6.7 is repeated to obtain the dynamic response of the system for the next time step until the end of the time history.

5 Numerical simulation of the pedestrian walking on rigid level ground

The walking process equations were implemented in MATLAB, considering a pedestrian walking on rigid level ground in a straight line. The equations of motion of the pedestrian were validated considering the parameters of the human body according to [5] and [12]. The step length l_s and the angle of attack θ_0 have been considered the same for each step.

The numerical values of the human properties used in the simulation are: the leg stiffness $k_{leg} = 20$ kN/m , the damping ratio of the leg $\xi_{leg} = 6\%$, the pedestrian mass $m_h = 72$ kg, the rest length of the leg $L_0 = 1$ m, the walking speed $w_s = 1$ m/s, the angle of attack $\theta_0 = 72°$ and the step length $l_s = 0.6$ m.

The vertical motion of the pedestrian's center of mass obtained considering rigid level ground are showed in Fig.8 a-c. The graphs are similar to those obtained by Qin $et\ al.$,2013 [5].

The influence of the parameters leg stiffness, leg damping ratio and walking velocity on the ground reaction force was studied varying k_{leg}, ξ_{leg} and w_s and keeping the all other parameters fixed at the same values mentioned above. The results are shown in Fig. 8 d-f. It is noted that an increase in any of the three parameters results in an increase of the ground reaction force peak. These results are also in agreement with [5].

6 Numerical simulation of the pedestrian walking on the footbridge

The test carried out with one pedestrian walking across the footbridge was numerically simulated using the 2D bipedal walking model. The equations and procedure described in **4.2** were implemented in MATLAB. Modal parameters of the structure were obtained from the numerical model presented in section **3**. The adopted pedestrian parameters as walking velocity $w_s = 1.17$ m/s , step length $l_s = 0.66$ m, overlap time of step $t_{overlap} = 0.1$ s, rest length of leg $L_0 = 1.1$ m and pedestrian mass $m_h = 72$ kg have been estimated from the experimental test average values.

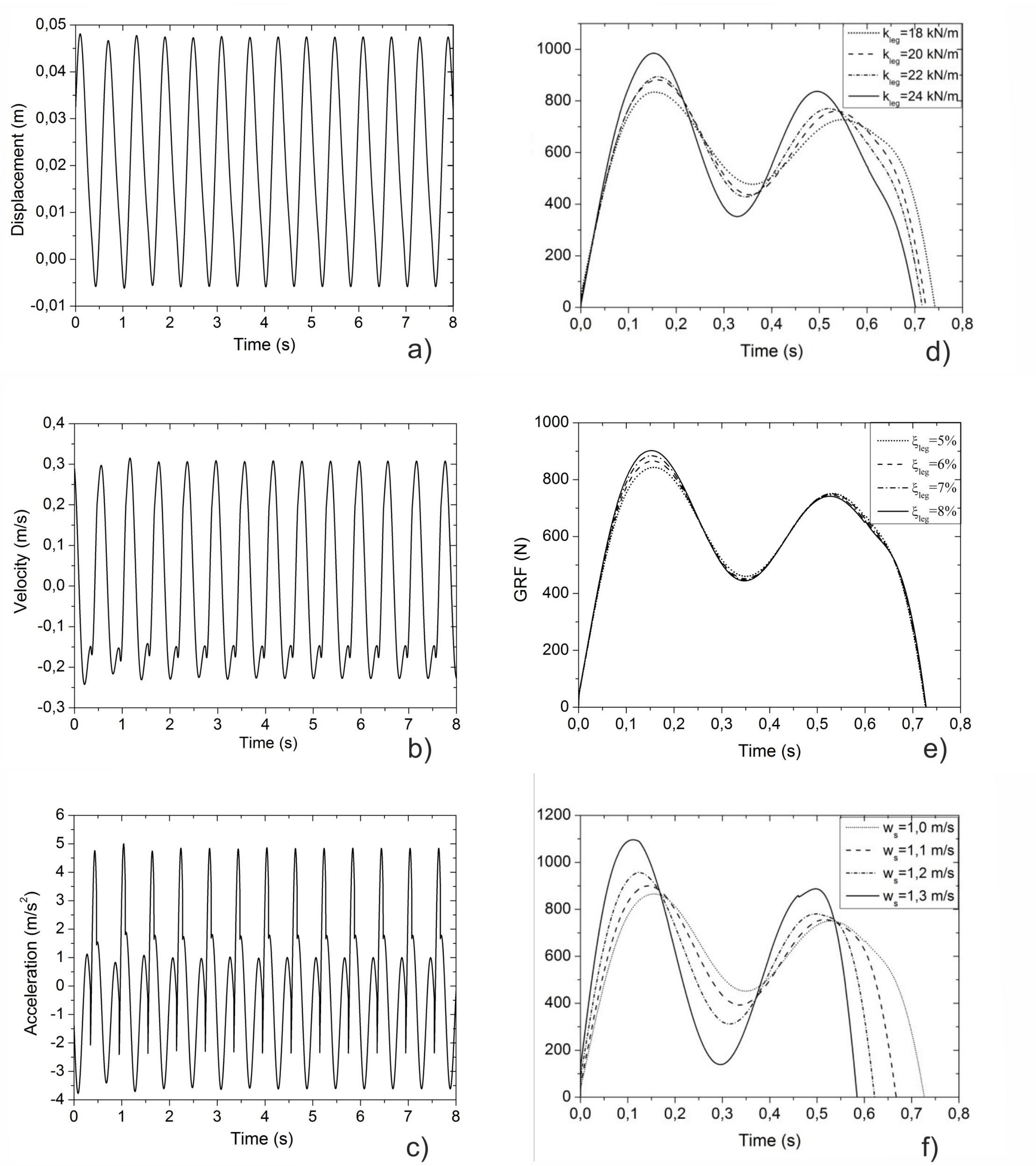

Figure 8: Vertical motion of the pedestrian's center of mass walking on a rigid level ground: a) Displacement, b) Velocity and c) Acceleration. Effect of: d) k_{leg}, e) ξ_{leg} and f) w_s on the vertical GRF.

In Fig.9 is presented a comparison of the structural response measured at mid span of the footbridge with its numerical simulation. The best correlation was obtained for k_{leg} =18 kN/m and ξ_{leg} =6 %.

Considering that in realistic human walking every step is different from each other the comparison between experimental and numerical pedestrian force is only possible taking a set

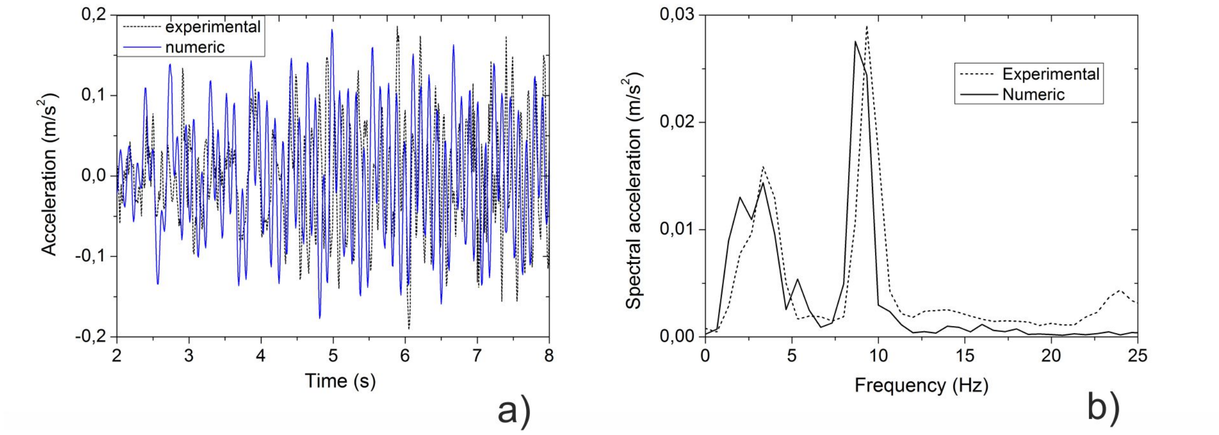

Figure 9: Numerical-experimental comparison of the structure response due to a walking pedestrian: a) Acceleration at mid span b) Frequency spectrum (Adopted pedestrian parameters: $w_s = 1.17$ **m/s,** $l_s = 0.66$ **m,** $t_{overlap} = 0.1$ **s,** $L_0 = 1.1$ **m,** $m_h = 72$ **kg,** $k_{leg} = 18$ **kN/m and** $\xi_{leg} = 6$ **%).**

of parameters that reproduce an specific step. In this case has been selected a representative number of experimental step forces that presented similar patterns and the step parameters considered in the model have been established in order to reproduce these force patterns.

In Fig.10 a) is presented a comparison of the vertical force produced by the bipedal walking model with four step forces measured from force platforms that have similar patterns. It is noted that the amplitude of the response is in the same range for both cases, however more time is needed for signal analysis to identify the frequency peaks with greater accuracy.

In Fig.10 b) is shown a comparison of the vertical force produced by the pedestrian on a rigid level ground with that produced by walking on the footbridge for a specific set of parameters. Small changes in force amplitude are perceived, meaning that the effect of the interaction for this specific case is not so relevant, however the model seems capable to reproduce the interaction phenomenon.

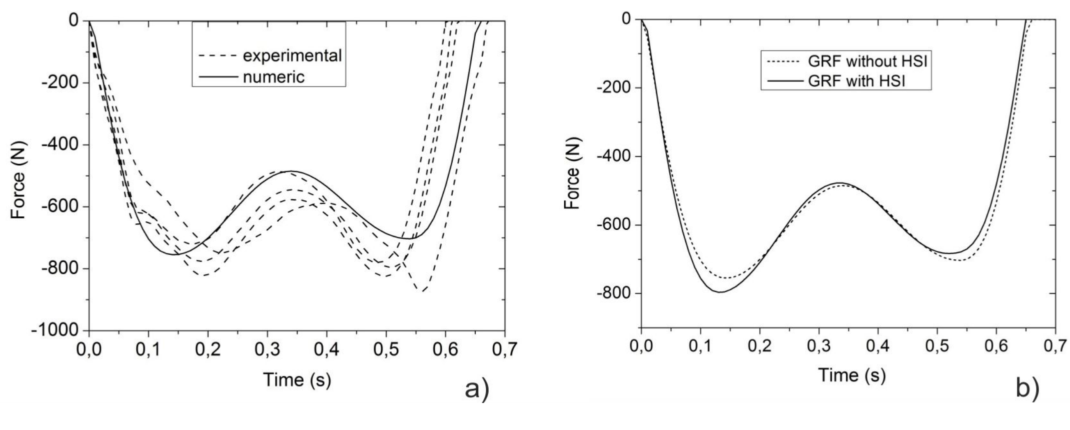

Figure 10: Vertical force applied on the footbridge by the bipedal walking model in a comparison with: a) Experimental measures b) The vertical force by the model on rigid level ground.

7 Conclusions and final comments

The presented 2D bipedal walking model is able to reproduce the human walking gait cycle and provides a framework to understand the HSI problem.

It was showed that human parameters have an important influence in the ground reaction force, however a greater number of simulations is needed to obtain a better understanding of the effect of these parameters.

The model was able to reproduce properly the vertical force applied by a walking pedestrian when compared with experimental measures and the numerical response of the walking pedestrian simulation was consistent with that recorded by accelerometers during the test.

In order to get a more realistic simulation of the human motion, the accelerations and decelerations of the center of mass in longitudinal direction should be considered in the model.

Situations with greater number of people could be done using this model and validated with the corresponding experimental results.

The present model is a basic framework for the development of more sophisticated models.

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