



CROSS-SECTION OPTIMIZATION OF CONCRETE GRAVITY DAMS USING GENETIC ALGORITHMS

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Abstract. Concrete gravity dams are the most commonly used hydraulic structures in engineering, for the designer of these structures, the overall stability (external forces) must be guaranteed as well as the internal stresses that the element is subjected to. The scaling mechanisms of these structures are presented in the manual *Critérios de Projeto Civil de Usinas Hidroelétricas* (2003) from Eletrobras, which is strongly influenced by the U.S. Bureau of Reclamation (1976). After an analysis of the proposed design forms, it was observed a tendency to oversize these structures, in order to reduce this oversizing and to obtain slender forms of dam, the present work brings an optimization study for the cross sections of concrete gravity dams, this optimization will be made by means of routines used in Matlab software, the genetic algorithm technique will be used to reduce the cross-section of this type of dam and consequently reduce the volume of concrete used in its construction.

Keywords: Optimization, Gravity dams, Cross sections, Genetic algorithm

1 INTRODUCTION

A concrete gravity dam is a structure that acquire stability by her own weight. A solid concrete structure so designed and shaped that it weight is sufficient to ensure stability against the effects of all imposed forces (Usace,1995). This dams generally has trapezoidal cross sections (Gutstein et al., 2011).

However, design this structure is not a simple thing. Although the geometry of a gravity dam is very basic, the structural analysis of a such mass concrete structure is relatively complex, duo to the no-linear material behavior and the variety of static and dynamic load acting on the structure (Durieux,2009). For each case of loads there are several possible configurations that satisfy such requirements. Therefore, many solutions are considered acceptable.

The codes, (ICOLD,2003), (USB,1976) and (Usace,1995) brings the minimum requirements to ensure the stability of concrete gravity dams, both make use factors of safety, in order to guarantee the overall safety of the structure. That way, to ensure the massif equilibrium, must be analyzed, the global balance in flotation, toppling and sliding. However, sometimes, in order to guarantee this safety, dam designers make over-sized structures that produce excessive use of concrete.

In the current context in which engineering is, reducing costs is a priority, in this order, the reduction in the consumption of materials is necessary. One way to reduce costs and consumption of materials is to make optimal design (Vianna, 2003), that is, the best design for the structures studied.

Within the techniques of optimization, the technique of genetic algorithms created by John Holland (Holland, 1975) and popularized by David Goldberg (Goldberg, 1989) gains force in detriment of others commonly used, this is happening because of the facility that this tool can solver problems.

The principle of the genetic algorithm (GA) is simple, it follows the evolutionary theory proposed by Charles Darwin, where the fittest individuals within a given population survive time, while the less fit individuals will be eliminated (Bastos, 2004).

It was observed that among the works in the area of concrete gravity dams, that the main focus of the researchers is the stresses suffered in the rocky foundations, having few works focused on the analysis of rigid body of the concrete mass.

The purpose of this work is to apply an optimization by genetic algorithms inside the software Matlab, reducing the area of trapezoidal cross sections of concrete gravity dams, consequently reducing the volume of concrete used in the construction of such structures. To elucidate the work efficiency, an example is presented.

2 METHODS

The present work makes use of numerical simulations, elaborated in the software Matlab. The research was elaborated from routines in proper language to the software used. However, with due changes, the routines can be applied on other platforms, not being exclusive by Matlab software.

2.1 Genetic Algorithm

The main idea of GA follows Darwin's theory of evolution, the individuals that better adapt to its environment, will survive and have more offspring. Therefore, the individuals in optimization problems are points of solution, the GA generate a population of these points (the best solutions), after that, the GA modifies the population, choosing the individuals more feasible, then, GA repeatedly modifies a population of individual solutions until achieve the best point of solution. (Bastos, 2004).

At each step (iteration), the GA selects individuals at random from the current population to be parents and uses them to produce the children for next generation. Over successive generations, the population evolves toward an optimal solution.

As in the evolutionary process of species, genetic algorithms manipulate a population of individuals, each with an associated fitness value, to a new generation of individuals, using the Darwinian principles of reproduction and survival of the fittest, performing genetic operations of selection, crossover and mutation. Each individual of the population in each generation represents a possible solution.

Therefore, basically what the genetic algorithm does is to seek from the set of solutions of the search space, always towards the global optimum, the individual with the highest fitness (Bastos, 2004).

The GA can be used to solve problems of unconstrained and constrained optimization. Unconstrained problems consider a problem of minimizing (or maximizing) an objective function that depends on real variable, with no restrictions on their values. Constrained problems of optimization have boundaries that limit their values. In this paper we are treating with a constrained problem (Venkataraman, 2009).

To solve some constrained complex optimization problems, the GA in its simplest form is not enough to solve these problems. Then, one solution to these is increase GA with a penalty function.

The penalty function goal is to convert constrained problems into unconstrained by introducing an artificial penalty for violating the constraint. The penalty function approach involves a number of penalty parameters which must be set right in any problem to obtain feasible solutions (Deb, 2000).

2.2 Factors of safety

In order to guarantee the overall stability of a concrete dam, the codes refer to maintaining the equilibrium of the rigid body of the dam, to have this equilibrium satisfied. It is necessary to guarantee the stability of the structure regarding the phenomena of floating, toppling and sliding. Therefore, using a factor of safety for each of these phenomena, might achieve a security design (ICOLD, 2003).

Floating factor of safety (FSF), has the function to guarantee the stability of structure against the forces that tend to make the body float. The floating factor of safety is defined as the ratio between the sum of the forces gravitational forces, and the sum of the forces of underpressure (ICOLD, 2003). The FSF is here calculate using the expression (1).

$$FSF = \frac{\sum V}{\sum U} \quad (1)$$

FSF is the floating factor of safety, $\sum V$ is the sum of vertical forces and $\sum U$ is the sum of polypression.

The factor of safety against toppling (FST), deals with the stability of the structure for the moments to which it is subject. The factor of safety for toppling in any direction is defined as the ratio between the stabilizing moments and the toppling moments, over a point or an effective line of rotation (ICOLD, 2003). FST is here calculate using the expression (2).

$$FST = \frac{\sum Me}{\sum Mt} \quad (2)$$

FST is safety factor against toppling, $\sum Me$ is the sum of stabilizing moments and $\sum Mt$ is the sum of toppling moments.

Sliding factor of safety (FSD), guarantees safer margins of structural stability against translational movements. In order to verify the stability of structures to sliding (slip), rupture surfaces should be selected to include all low resistance, or those subjected to critical stresses in the structure, and in the foundation-structure contact on which the dam may suffer sliding movement as rigid body (ICOLD, 2003). The expression for FSD is (3).

$$FSD = \frac{(\mu * (\sum w * \sum U) * F\varphi) + (\tau cA / Fc)}{\sum Fh} \quad (3)$$

FSD is the sliding factor of safety, $\mu = \tan(\varphi)$ is coefficient of internal friction of the material (varies from 0.65-0.75 for the concrete), $\sum w$ is the sum of the dam's self-weight, $\sum U$ is the sum of polypression, $F\varphi$ is the partial factor of safety in relation to friction, τcA is the cohesion of the material or shear stress admissible in the plane considered (1.4 N / mm for concrete), Fc is the partial factor of safety in relation to cohesion and $\sum Fh$ is the sum of horizontal forces.

2.3 Construction of the problem and the solution

The problem here studied is the over sizing in design of concrete gravity dams. An explanation for this tendency to oversize, its the nature of the problem, its a great volume of concrete inside the water. Concrete is a non-linear material of difficult prediction, however, this structures have a great size, so that the massif acquire stability by self-weight.

To guarantee the dam security, it's necessary to analyses global stability and the stresses in foundation. Here is studied the process involving global stability of the dams. The stresses in foundation are not studied here, there are many works that analyzed this, such as (Gutstein, 2003), (Gutstein, 2011), (Ribeiro, 2006), (Araujo, 2005).

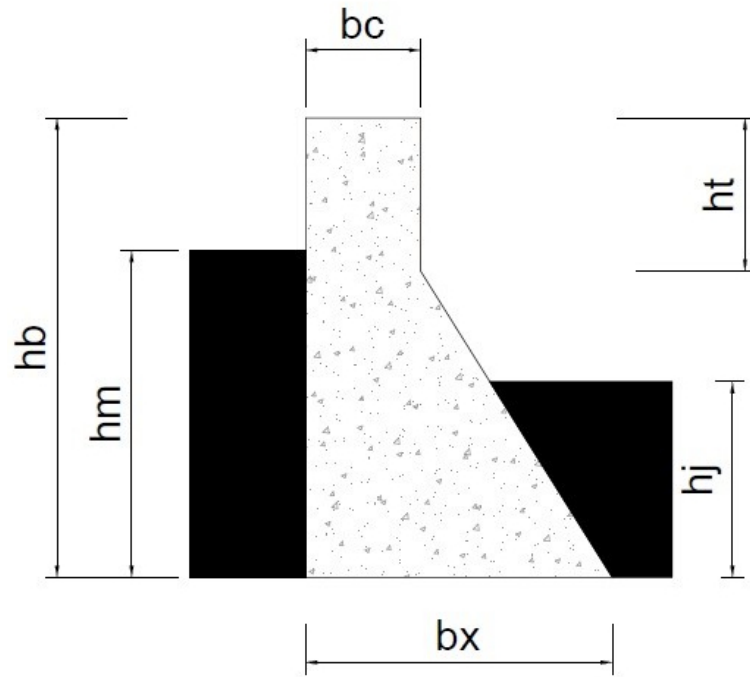


Figure 1: Generic scheme of the studied cross section

To guarantee the global stability of a concrete gravity dams, have to be reached, values for the factors of safety higher than the minimum required by design standards. In this way, considering normal loading conditions, the values for factors of safety are 1.3 for FSF , 1.5 for FST and 1 for FSD (ICOLD, 2003).

In order to calculate the overall stability of the structure, a main routine was developed serving as a starting point for the entire study. This routine calculate body equilibrium of the dam from conditions provided by the designer. This algorithm returns the values factors of safety, calculated by (1), (2) and (3) from the arrangement that efforts assume.

Using the genetic algorithm option that Matlab brings, it was formulated an optimization mechanism for the cross-sections of the dams. It consists of a discretization of the problem in parts.

First, an objective function was defined, that is, the function to be optimized or just fitness function. This fitness function refers to the cross-sectional area of a concrete gravity dam, resembling the area of a trapezoid, the function is shown in equation 4.

$$f(A) = (hb * bc) + \frac{(bx - bc) * (hb - ht)}{2} \quad (4)$$

A is the cross-section area of the dam, $f(A)$ is the function that represent the area of the dam, hb is the height of the dam, bx width of the dam, bc is width of the dam crest and ht is tardoz height. Other two inherent parameters upstream hm and downstream water column hj were defined. The parameters are illustrated in scheme of figure 1.

To calculate the effects, on which the dam is subject, it was necessary to define some constants. These constants are shown in table 1.

Table 1: Constants adopted

Constant	Nomenclature	Value
Specific weight of water	γ_w	$10(kN/m^3)$
Specific weight of concrete	γ_c	$25(kN/m^3)$
Saturated soil specific weight	γ_{sat}	$20(kN/m^3)$
Dry weight specific weight	γ_{sat}	$18(kN/m^3)$
Concrete-rock mean cohesion	C_r	$800(kN/m^2)$
Rock friction angle	φ_r	$\frac{\pi}{4}$
Partial friction safety factor	f_{at}	1
Internal friction coefficient	A	0.7

Defining the function to be optimized, we proceeded to delimit the constrained conditions. So that, calculating FSF (eq. 1), FST (eq. 2) e FSD (eq. 3), the constraints are defined equations (5), (6) and (7).

$$1.3 - FSF \leq 0 \quad (5)$$

$$1.5 - FST \leq 0 \quad (6)$$

$$1.0 - FSD \leq 0 \quad (7)$$

Having the objective function and the constraints, the optimization was performed with the GA. The GA option that fits the problem analyzed makes use of a penalty function that is defined (eq. 8).

$$F(\vec{x}) = \begin{cases} f(\vec{x}) & \text{if } x = g_j(\vec{x}) \forall j = 1, 2, 3, \dots, m, \\ f_{\max} + \sum_{j=1}^m \langle g_j(\vec{x}) \rangle & \text{otherwise.} \end{cases} \quad (8)$$

This penalty function combined with binary tournament selection to select individual for subsequent generation. The $F(\vec{x})$ of a member of a population is $f(x)$ if the member is feasible. Whether the the member is infeasible, the $F(\vec{x})$ is the maximum fitness function, $f_{\max}$, among feasible member of the population plus the sum of the constraint violation of the point $\sum_{j=1}^m g_j^i$ (Chipperfield, 1995). If no feasible solution exists in a population, $f_{\max}$ is set to zero (Deb, 2000).

With the fitness function programed inside a script, the constraint in other one, already can optimize the section of interest. To do that, it is necessary to define the other parameters to feed the GA.

Within the parameters, the dimensions of the analyzed dam will be included, and the measurements determined in the pre-demise will be used as maximum border values for the studied function.

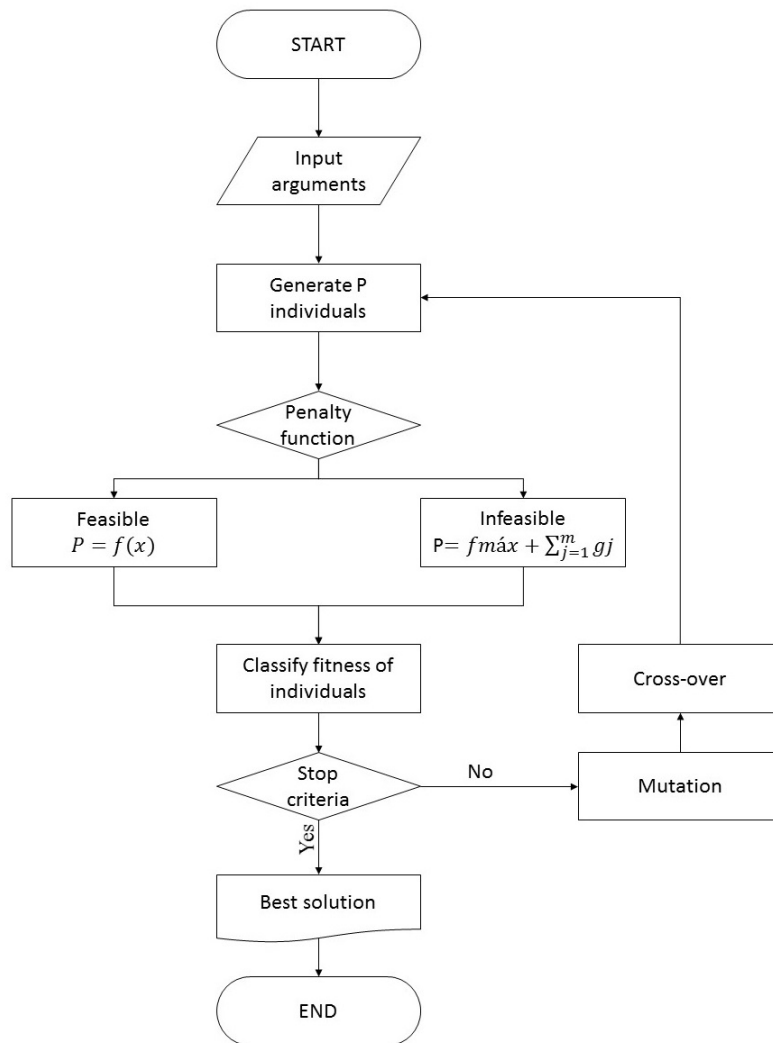


Figure 2: Genetic algorithm functioning

The designer must arbitrate lower dimensions for the dam, these dimensions must follow the logic of the problem studied. For example, the height of the dam must satisfy the maximum. With the defined lower boundaries, the GA will work within the range of these lower boundaries and the dimensions defined in the pre-dimensioning.

With the dimensions of the dam already defined. One of the dimensions is arbitrated as integer variable. In figure 2 is presented the operation of the optimization routine.

3 EXAMPLE

In order to demonstrate what the cross-sectional optimization can do for the best design of a concrete gravity dam, an example has been defined to be studied, this example treats a hypothetical dam that follows the molds of conventional dam structural designs.

Table 2: Initial dimensions of the dam

Study range	Nomenclature	Value
Area of the dam	A	$4182.5(m^2)$
Height of the dam	hb	$100(m)$
Width of the dam	bx	$70(m)$
Dam crest	bc	$21(m)$
Tardoz height	ht	$15(m)$
Upstream water	hm	$90(m)$
Downstream water	hj	$60(m)$
Factor of safety Float	FSF	1.9785
Factor of safety toppling	FST	1.6191
Factor of safety sliding	FSD	1.5982

The (Usace, 1995) as well as other bibliographies diffused worldwide, recommend the width of the dam bx as being about 0.7 of the height of the dam hb (Gutstein, 2003), aiming at the initial sizing case follows this recommendation, as well as another widespread recommendation in which the dam crust bc is about 0.3 from the width of the dam bx .

To make the optimization, it was defined that the initials dimensions will be the upper bounder of the GA, and, it was arbitrated a lower bounder, that lower bounder as being $hb = 93(m)$, $bx = 50(m)$, $bc = 10(m)$, $ht = 5(m)$. In addition to the optimization workspace, for GA operation it is necessary to arbitrate parameters for the GA, these parameters are presented in table 3.

Finally, optimization of the cross-section of the dam can be performed. However, it was defined that the optimization would be done by fixing one of the dimensions in the best available integer, this value being chosen by GA, making the other parameters vary successively until the desired optimum section is obtained. Thus, an optimization was performed for each of the four dimensions of the dam, these results will be presented separately. The stop criteria is the number of iterations (100).

3.1 Optimization

In the process of optimization of the sections, it was adopted as a constant one dimension in relation to the others, so that, in the first optimized section, the height of the dam was fixed, followed by width, crest and tardoz.

First it was analyzed the behavior that the function area presents within GA, that way, can be analyzed the how that the choice of the dimension to be fixed implies in the form of convergence. The behavior for the different optimization options adopted is presented in figures 4, 5, 6 and 7.

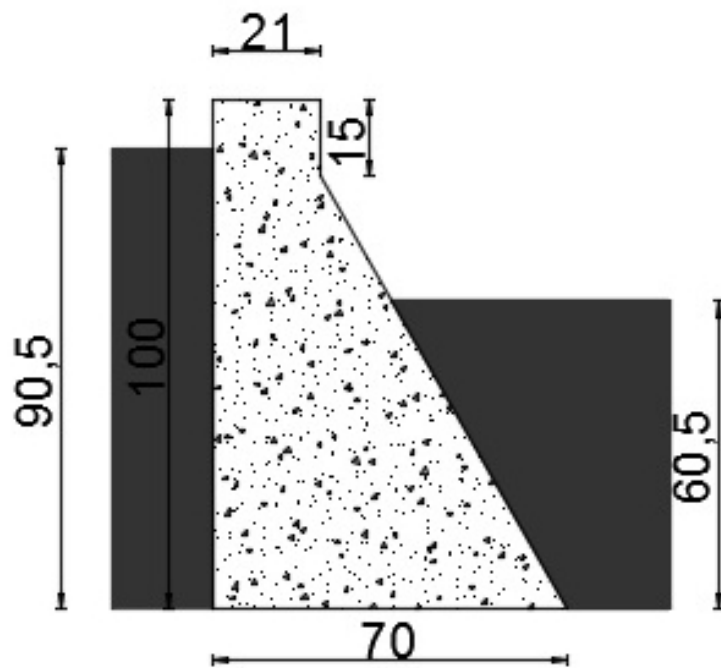


Figure 3: Initial dimensions of the dam

Table 3: Parameters of the GA

Parameter	Adopted
Population type	Double vector
Population size	10^3
Maximum number of iterations	10^2
Function tolerance	10^{-6}
Cross-over fraction	0.8
Mutation function	Constraint dependent
Cross-over function	Constraint dependent
Migration direction	Forward
Migration fraction	0.2
Migration interval	20
Code	Real

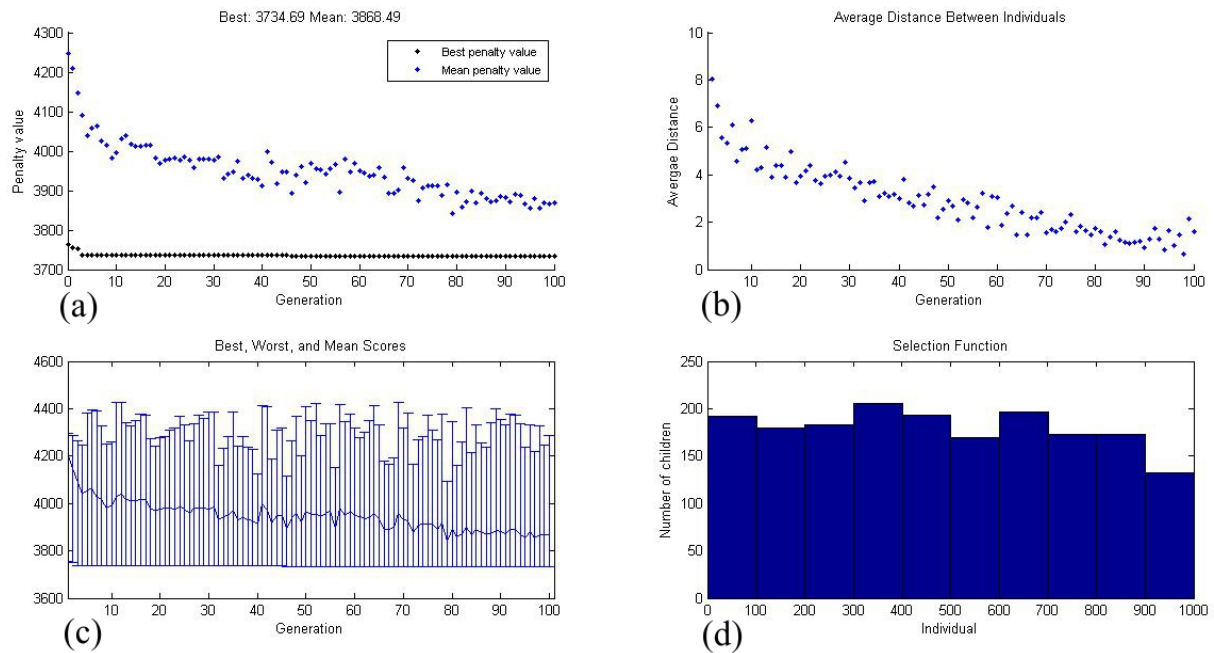


Figure 4: Optimum section-fixed height

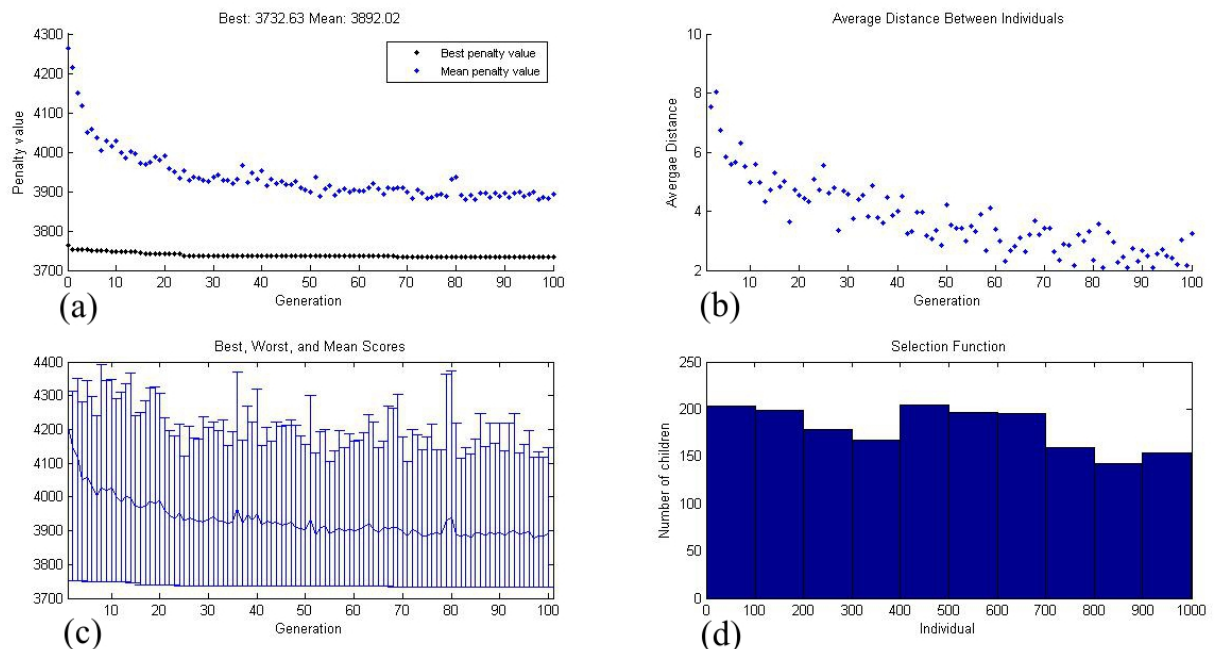


Figure 5: Optimum section-fixed Width

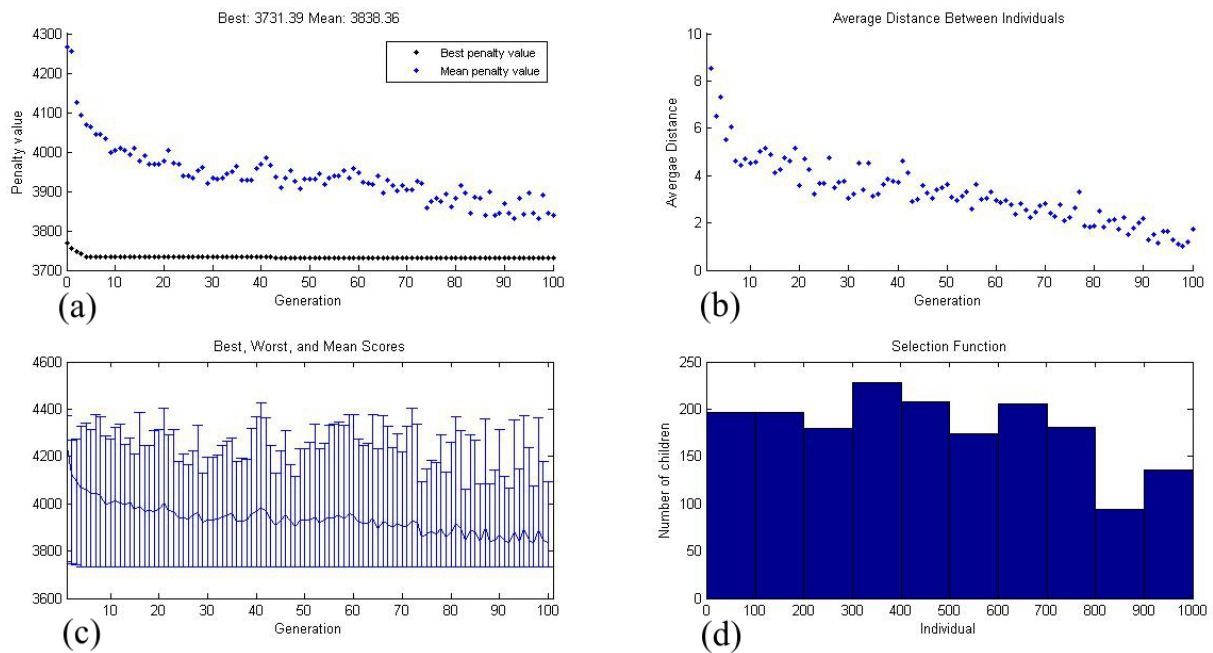


Figure 6: Optimum section-fixed Crest

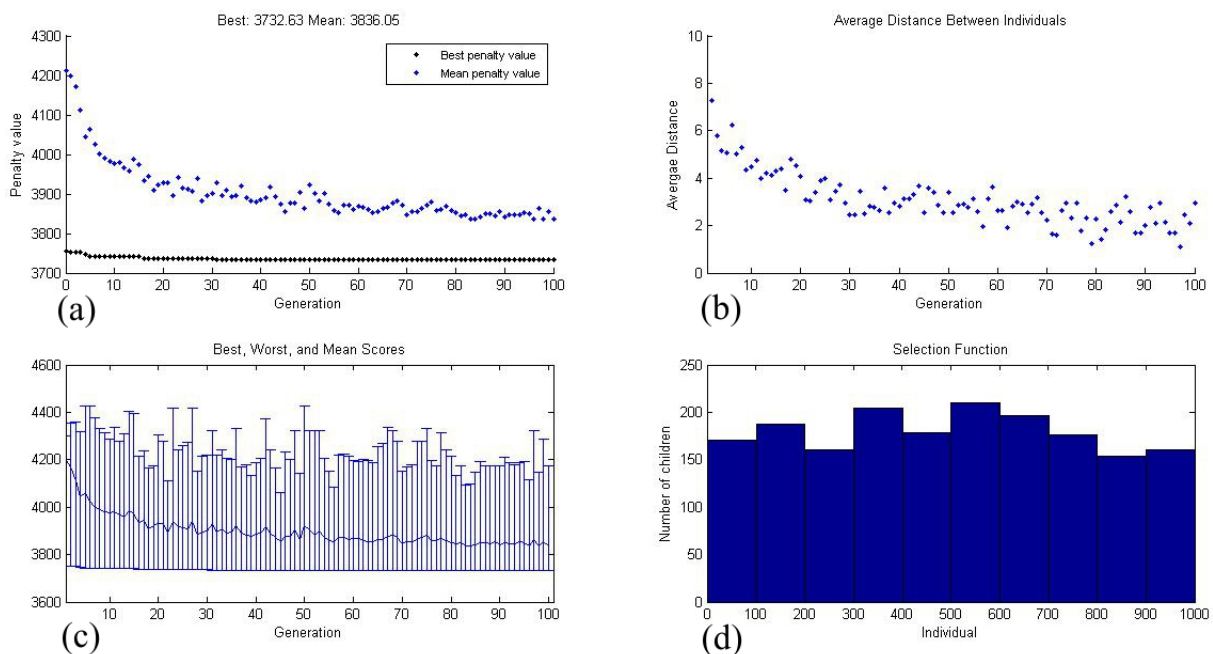


Figure 7: Optimum section-fixed Tardoz

Table 4: Optimum section-fixed height

Study range	Nomenclature	Value
Area of the dam	A	$3734.7(m^2)$
Height of the dam	hb	$100(m)$
Width of the dam	bx	$63.5279(m)$
Dam crest	bc	$17.7846(m)$
Tardoz height	ht	$14.4693(m)$
Upstream water	hm	$90(m)$
Downstream water	hj	$60(m)$
Factor of safety Float	FSF	1.9466
Factor of safety toppling	FST	1.5000
Factor of safety sliding	FSD	1.4032

Graphs (a) in figures 4, 5, 6 and 7, it shows the best penalty function found in each iteration, besides, it shows the mean values for the penalty functions for each of the iterations.

Can be observed that the best values found, in all cases, already presents in the first interaction, values that bring savings of satisfactory area. However, the mean values show values distant to those expected by the algorithm.

In the graphs (b), in figures 4, 5, 6 and 7, the distances between the individuals are analyzed, in these graphs it was evidenced a tendency that when the width of the dam is fixed, the individuals tend to be closer in the future generations.

The other three optimizations are closely similar, given that have a similar medium distance. It can be concluded that by this configuration, when fixing the width of the dam the GA tends to converge faster in relation to the other proposals.

Graphs (c) in figures 4, 5, 6 and 7, show a comparison between the worst, the best and the average values obtained. It should be noted that in all cases analyzed, the best values are far from the worst ones, which shows even more the advantage of optimizing a project. Since even the mean values found are far from the desired values.

In graphs (d) in figures 4, 5, 6 and 7, the behavior that the cross-over brings to as different variants of optimization are analyzed, being that all the configurations presented a procreation condition very close to each other. The results of the dimensions optimized are presented in the tables 4, 5, 6 and 7.

It was identified that the choice of which of the dimensions to be fixed, in terms of area savings, implies little in the results obtained.

Thus, it is up to the designer choose which of the dimensions will be fixed, in search of the optimum section that best suits your case. This choice may be influenced by external factors, such as a topographical configuration of the terrain or climatic conditions of the region.

Table 5: Optimum section-fixed Width

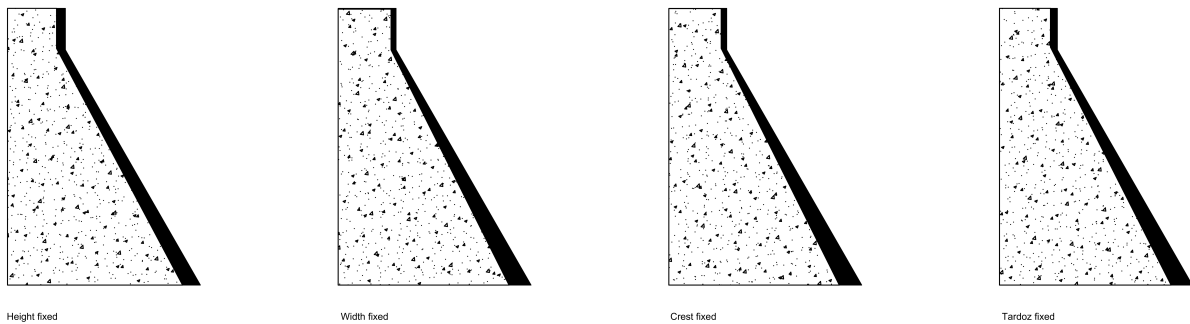
Study range	Nomenclature	Value
Area of the dam	A	$3732.6(m^2)$
Height of the dam	hb	$99.7039(m)$
Width of the dam	bx	$62(m)$
Dam crest	bc	$19.1937(m)$
Tardoz height	ht	$14.7190(m)$
Upstream water	hm	$90(m)$
Downstream water	hj	$60(m)$
Factor of safety Float	FSF	1.9935
Factor of safety toppling	FST	1.5000
Factor of safety sliding	FSD	1.4373

Table 6: Optimum section-fixed Crest

Study range	Nomenclature	Value
Area of the dam	A	$3731.4(m^2)$
Height of the dam	hb	$99.9113(m)$
Width of the dam	bx	$61.8585(m)$
Dam crest	bc	$19(m)$
Tardoz height	ht	$14.3700(m)$
Upstream water	hm	$90(m)$
Downstream water	hj	$60(m)$
Factor of safety Float	FSF	1.9974
Factor of safety toppling	FST	1.5000
Factor of safety sliding	FSD	1.4396

Table 7: Optimum section-fixed Tardoz height

Study range	Nomenclature	Value
Area of the dam	A	$3732.6(m^2)$
Height of the dam	hb	$99.9965(m)$
Width of the dam	bx	$62.2791(m)$
Dam crest	bc	$18.5047(m)$
Tardoz height	ht	$14(m)$
Upstream water	hm	$90(m)$
Downstream water	hj	$60(m)$
Factor of safety Float	FSF	1.9846
Factor of safety toppling	FST	1.5000
Factor of safety sliding	FSD	1.4307

**Figure 8: Cross-sections optimized**

To better observe the economy that optimization offers, in figure 8 a scheme is presented, in this scheme the optimized section is materialized inside the initial.

A comparison is made between the cases studied. From this comparison, it can be seen how the values obtained are close to each other.

The choice of a section to be fixed makes no difference to the desired result. However, all the optimized sections have a relevant area saving and, consequently, a desirable saving in concrete volume.

4 CONCLUSIONS

The present work brought a methodology of optimization of cross-sections of concrete gravity dams. The process presented here was based on genetic algorithms in Matlab function to produce economic cross-sections.

The technique of genetic algorithms proved to be an important ally of the process of optimization programming, being able to be applied in different cases and extend to many areas of

engineering.

The optimization produced using GA satisfactory results, producing transversal sections with concrete saving about 10 percent. It is observed, how optimization work can bring benefits to concrete economy. Being that, if applied in large scale dams, the saving of concrete can reach thousands of m^3 .

In a design of great magnitude and high complexity such as dam, codes tend to be conservative in favor of safety. However, studies of optimization such as the one presented here show us that we can do more economic designs while maintaining the safety of the structure.

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