CILAMCE 2017
Florianópolis - Santa Catarina - Brazil

A PARAMETRIC STUDY OF MIXED-MODE FRACTURE PROPAGATION IN CONCRETE USING XFEM

Renan Marks de Oliveira Pereira

Eleazar Cristian Mejia Sanchez

Deane Roehl

renanmarks@tecgraf.puc-rio.br

crisms@tecgraf.puc-rio.br

deane@tecgraf.puc-rio.br

Tecgraf Institute and Department of Civil Engineering, PUC-Rio

Rua Marques de São Vicente, 225, Gávea, 22453900, Rio de Janeiro, RJ, Brasil.

Abstract. *Cracking is a fundamental characteristic of quasi-brittle materials and should be taken into consideration while predicting its ultimate-load capacity. However, modelling crack propagation in finite element simulations is a cumbersome task owing to its mesh dependency to the fracture geometry. The Extended Finite Element Method (XFEM) is a technique which combines the FEM with enrichment functions to represent discontinuities in the displacement field. Besides, there is a complexity for numerical models to represent mixed-mode fracture. In this work, a parametric study is presented for modelling a concrete beam under mixed-mode loading. Different parameters were taken into account such as mesh refinement, damage model, crack initiation and propagation and solution control techniques. The study was carried out using an XFEM approach based on the Phantom Node Method in conjunction with a cohesive law. The purpose of this research was to elucidate about the strengths and weaknesses in using the XFEM in a mixed-mode situation. The numerical results were compared to experimental data, presenting excellent agreement. The present work emphasizes the importance of using adequate parameters to obtain realistic fracture trajectories in the numerical simulations using XFEM.*

Keywords: *Extended finite element method (XFEM), Mixed mode, Concrete fracture, Numerical analysis*

1 INTRODUCTION

Several well-known characteristics such as high compression strength with economy, availability, durability, applicability and resistance to temperature and weathering, make concrete the most used material in the world in civil construction. However, unlike other quasi-brittle materials, concrete fracture is evidenced by the formation of a micro-cracked zone prior to visible cracks (Bažant & Oh, 1983). Under further loading, the process of micro-cracking and coalescence begins, provoking its inelastic deformation, and therefore, the propagation of a macroscopic crack.

Huge effort has been dedicated to developing computational methods for modelling fracture processes in quasi-brittle materials such as concrete, ceramics and rocks. Additionally, when fracture involving the tensile stress (mode I) and the shear retention (mode II) occurs, mixed-mode crack propagation is developed. That kind of fracture generates a complex fracture process. There is also some divergence about mode II fracture energy. While some researchers concluded it to be about eight to ten times mode I energy (Swartz, et al., 1988), others find it to be about twenty-five times larger (Bažant & Pfeiffer, 1986).

From experimental tests of notched beams based on Iosipescu geometry (Iosipescu, 1967; Arrea & Ingraffea, 1982; Bažant & Pfeiffer, 1986), a novel test procedure was developed by Gálvez et al. (1998) to study mixed mode fracture behavior in concrete under proportional and non-proportional loading. The author provides several experimental results for different specimen sizes, such as crack trajectories and load-displacement curves. Those data were used as a reference to validate the numerical predictions. In addition, simulations using cohesive crack model (Barenblatt, 1962) of the same notched beam were analyzed by Cendón et al. (2000) and Machado et al. (2015), with good matches to experimental results. Moreover, other techniques could be used to simulate crack propagation in concrete such as element elimination, inter-element crack method and extended finite element method (Bendezu, et al., 2015).

The Extended Finite Element Method (XFEM) is a mesh independent approach (Moës et al., 1999; Dolbow et al., 2001), which is one of the most outstanding techniques in this category for modelling structures with strong discontinuities. The method allows representing discontinuities in the displacement fields due to the enrichment of the elements with a Heaviside step function (Moës & Belytschko, 2002). The Phantom Node Method, which is equivalent to XFEM (Song, et al., 2006), enables to model discontinuities by "duplicating" the nodes of an existing element (Hansbo & Hansbo, 2004). In this formulation, the displacement jump on the crack is computed as the difference between the displacement fields of the overlapping elements. The crack growth follows a simple approach, in which the crack propagates element by element, enforcing its tip to remain at the edge of the element. The phantom nodes method is easier to implement into existing finite element programs. However, in relation to the traditional formulation, the phantom nodes method presents some drawbacks such as the addition of extra degrees of freedom in the element and the consideration of just one model crack inside of the element.

The purpose of this work is to evaluate the impact of some parameters using XFEM on the crack trajectory and in the structure load response, under mixed mode behavior. Different parameters were taken into account such as analysis type, solution control techniques, mesh refinement, type of cohesive crack model, crack initiation and propagation criteria. The examples were carried out on the finite element software ABAQUS 6.14. The numerical results such as fracture path and Load-Crack Mouth Opening Displacement (CMOD) were compared to the experimental data provided by Gálvez et al. (1998).

2 THE EXTENDED FINITE ELEMENT METHOD

The Extended Finite Element method is based on the partition of unity approach in which the standard finite element is enhanced by means of enrichment functions (Eq. 1).

$$u^h(x) = u_{FEM}(x) + u_{ENR}(x) = \sum_{i\in I} N_i(x)u_i + \sum_{j\in J} N_j\left[H(x) - H(x_j)\right]a_j \quad (1)$$

Where J indicates the set of nodes whose support domain is completely cut by the crack, and thus, enriched with the Heaviside function H(x). Moreover, u_i is the standard degrees of freedom, while a_j indicates the vectors of additional nodal degrees of freedom for modelling crack faces.

This form correlates with the phantom nodes theory proposed by Hansbo & Hansbo (2004). According to Areias & Belytschko (2005), this formulation is an alternative approach of the XFEM displacement field, which can be written as follows (Song, et al., 2006):

$$u(x) = u_{FEM}(x) + u_{ENR}(x) = \sum_{I\in\Omega_A} N_I^A(x)u_I H(-f(x)) + \sum_{I\in\Omega_B} N_I^B(x)u_I H(f(x)) \quad (2)$$

As we can see in Fig. 1, when a finite element is cut by a crack at Γ_c, it is divided into two subdomains Ω_A and Ω_B. The original element is then replaced by two new independent overlapping elements (indicated as A and B).

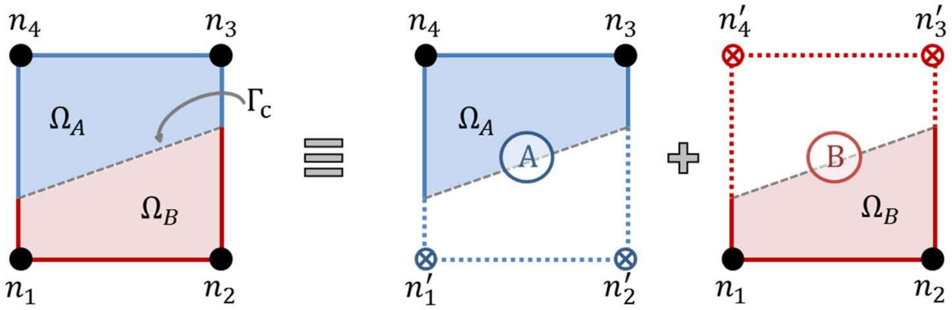

Figure 1. Connectivity and subdomains of an enriched element in phantom node method

Both elements A and B, which contain the real nodes (n_i) and the phantom nodes (n_i'), are only active in their subdomains, as shown in Fig. 1. In addition, the displacement field is computed separately for each element, using its nodal displacement values and its standard shape functions (Eq. 3).

$$u(x) = \begin{cases} N(x)u_A, & x \in \Omega_A \\ N(x)u_B, & x \in \Omega_B \end{cases} \quad (3)$$

Therefore, the crack aperture is represented by the difference in the displacement field of elements A and B (Eq. 4).

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

$$[\![u]\!](x) = N(x)(u_A - u_B), \qquad x \in \Gamma_c \tag{4}$$

The XFEM available in ABAQUS is based on the phantom nodes method (ABAQUS, 2014). This method, in conjunction with a cohesive crack model, permits simulation of the degradation and failure of the enriched element. The fracture propagation mechanism involves a specific crack initiation and damage evolution criteria.

3 FRACTURE PROPAGATION MECHANISM

3.1 Criterion of crack initiation

Crack initiation indicates the beginning of degradation of the cohesive behavior at an enriched element. A new crack segment is incorporated into an element when the fracture propagation criterion, f, is satisfied. Several criteria based on stresses or strains, such as maximum principal stress/strain, maximum nominal stress/strain and quadratic traction/separation, are proposed. In this work, the numerical simulations were carried out considering the maximum principal stress criterion, which is given by,

$$f = \left\{\frac{\langle \sigma_{max} \rangle}{\sigma_{max}^{o}}\right\}, \qquad 1 \leq f \leq 1 + f_{tol} \ \therefore \ \text{new crack segment} \tag{5}$$

$$\sigma_{max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad , \quad \theta_p = \frac{1}{2} arctan\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right) \tag{6}$$

where $\boldsymbol{\sigma_x, \sigma_y, \tau_{xy}}$ are the normal and shear stresses at a specific region around the crack tip; $\boldsymbol{\sigma_{max}^0}$ is the maximum allowable principal stress ; $\boldsymbol{\sigma_{max}}$ and $\boldsymbol{\theta_p}$ are the maximum principal stress and the its rotation angle, respectively.

3.1.1 Region of evaluation

Regardless of the crack initiation criterion adopted, there are several options about the location where it is evaluated. The location considered can be classified as local or nonlocal. The former considers an element ahead of the crack tip to determine if the criterion is satisfied, while the latter takes into account a region ahead of the crack tip to improve the accuracy of the direction of crack propagation.

In the local option, the crack initiation criterion and the direction of crack propagation can be evaluated either at the crack tip or at the element centroid or still a combination of both, as shown in Fig 2.a and 2.b. In the centroid case, the stresses/strains are considered at the centroid ahead of the crack tip in order to verify the criterion and its direction of crack propagation. At the crack tip, the analysis is made based on the extrapolated stresses/strains. Finally, when combining them, the criterion is evaluated at the crack tip and the direction of propagation at the centroid.

In nonlocal option, the stresses/strains inside a specified radius r_c are averaged to evaluate the crack propagation criterion, as shown in Fig. 2.c. Moreover, the stress/strain field

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

can be smoothed with a specific weight function, which is based on the distance from the crack tip to its integration points r.

Figure 2. Damage initiation criteria a) Centroid b) Crack tip c) Nonlocal

Different weight functions ω, such as uniform, Gaussian given by Eq. 7 and Cubic Spline given by Eq. 8, are used in order to improve the accuracy of the direction of crack propagation.

$$\omega(r) = \frac{1}{(2\pi)^{3/2} r_c^3} \cdot e^{\frac{-r^2}{2r_c^2}} \tag{7}$$

$$\omega(r) = \begin{cases} 4\left(\frac{r}{r_c} - 1\right)\left(\frac{r}{r_c}\right)^2 + \frac{2}{3}, & 0 < r < \frac{r_c}{2} \\ \frac{4}{3}\left(1 - \frac{r}{r_c}\right)^3, & \frac{r_c}{2} \leq r \leq r_c \\ 0, & r_c < r \end{cases} \tag{8}$$

3.2 Damage Evolution

Two different behaviors are attributed to the material, before and after its fracture propagation. Once the criterion of crack propagation is reached, the damage evolution defines the degradation rate of its stiffness. There are two different ways to define the damage evolution; either by the traction-displacement relation, or by the fracture energy G_f. Anyhow, the area under the traction-separation curve is equivalent to the fracture energy. This is the amount of energy released when the material is fully damaged.

According to the descending shape of the traction-separation curve, several extrinsic cohesive zone models with softening such as linear, bi-linear or exponential are defined. This procedure couples the strength of the cohesive zone model to the benefits of the extended finite element method. The linear cohesive zone model was adopted in this paper, as shown in Fig 3.

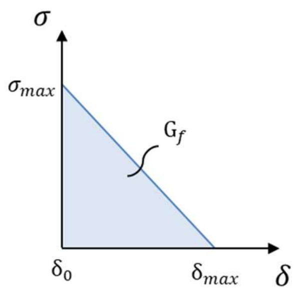

Figure 3. Extrinsic cohesive zone model with a linear softening

Since the structure is under a mixed-mode loading condition, a proper damage criterion should be taken into account. It is done considering the interaction between both the normal and the shear components of the traction energy rates, G_I and G_{II}, respectively. Some criteria for mixed-mode, such as power law (Eq. 9) (Camanho & Dávila, 2002), BK law (Eq. 10) (Benzeggagh & Kenane, 1996) and Reeder law for three-dimensional model (Reeder, et al., 2002), are widely used to deal with this type of problem. In this paper, the power law was used, where G_{IC} and G_{IIC} are the critical conjugate energy, and the parameter η is found through fitting of the experimental data of the energy toughness under different mixed-mode ratios.

$$\left(\frac{G_I}{G_{IC}}\right)^{\eta} + \left(\frac{G_{II}}{G_{IIC}}\right)^{\eta} = 1 \tag{9}$$

$$G_{IC} + (G_{IIC} - G_{IC}) \cdot \left(\frac{G_{II}}{G_I + G_{II}}\right)^{\eta} = G_f \tag{10}$$

3.3 Solution Techniques

A numerical analysis can be solved statically or dynamically. In the last decades, different solution techniques for nonlinear quasi-static analysis, such as load and arc-length, are proposed. However, depending on the problem, each one of them has its vantages and disadvantages. Load control cannot decrease the load increment; therefore, after the highest value reached, it cannot describe the softening behavior. On the other hand the arc-length control can face convergence problems after trespassing the peak of the curve while the displacement control does not handle snap-back problems.

4 FOUR-POINT BENDING TEST

4.1 Experimental test

A four-point bending test conducted on a notched concrete beam under mixed-mode loading was chosen as an experimental reference for this study (Gálvez, et al., 1998). To provide different paths of fracture, the stiffness of the spring at point B was varied between null (Type 1) and infinite (Type 2). Three different proportions of concrete beams, were tested. The sizes of D were 75, 150 and 300mm, respectively associated as D1, D2 and D3. Figures 4.a and 4.b below detail the geometry, the boundary conditions and the loads of the experimental setup.

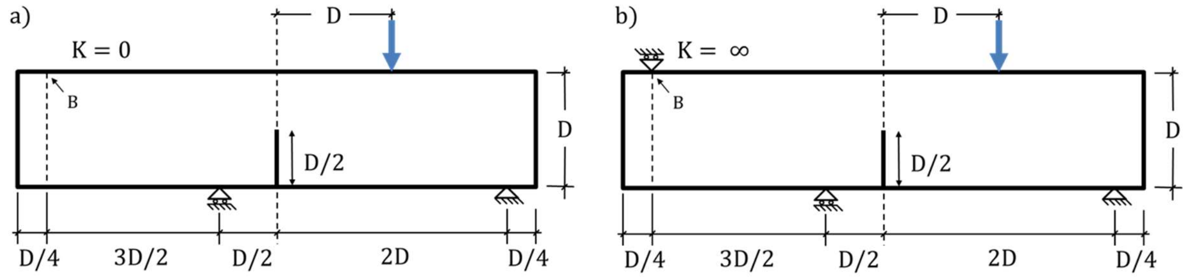

Figure 4. Schematic of the beam and its experimental setup a) Type 1 b) Type 2

For the numerical models, the bulk material is considered linear elastic until it reaches the maximum tensile strength of $f_t = 3.0\ MPa$. After that, a strong discontinuity governed by an extrinsic cohesive law with linear softening is added. The numerical simulation is carried out considering the following material properties: Young Modulus $E = 38\ GPa$, Poisson's coefficient $\nu = 0.2$ and critical fracture energy $G_f = 69\ N/m$.

4.2 Numerical model

The models were discretized using linear quadrilateral elements with full integration under plane stress conditions. Three different finite element meshes were considered in order to study the mesh effect on the XFEM results, as shown in Fig. 5. In addition, the notch was modelled geometrically into the finite element mesh by the enrichment of an initial crack. The numerical solution was performed using displacement control and arc-length control.

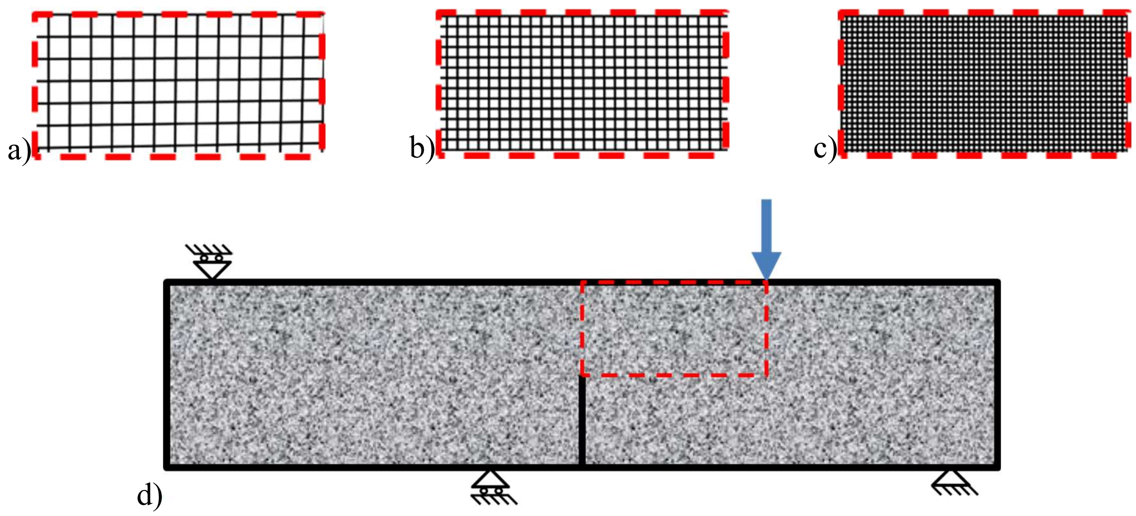

Figure 5. Concrete beam with a a) coarse (Mesh 1 – 741 elements) b) intermediate (Mesh 2 – 2,938 elements) and c) fine (Mesh 3 – 11,350 elements) finite element meshes

5 NUMERICAL RESULTS AND DISCUSSION

A parametric study of crack beam modelling under mixed-mode loading was performed. The effect of the crack initiation criteria, the solution control technique, the mesh refinement and the region of criterion evaluation are discussed below.

5.1 The effect of the crack initiation criteria and solution control

For the following parametric study, the maximum principal stress and the maximum principal strain were adopted as the crack initiation criteria. Both criteria were controlled through displacement control and arc-length control with an adaptive increment size. Furthermore, the simulations were carried out considering an intermediate finite element mesh (Mesh 2) and a local criteria based on crack tip stress evaluation. Figure 6 shows the obtained fracture propagation path and load-CMOD curve.

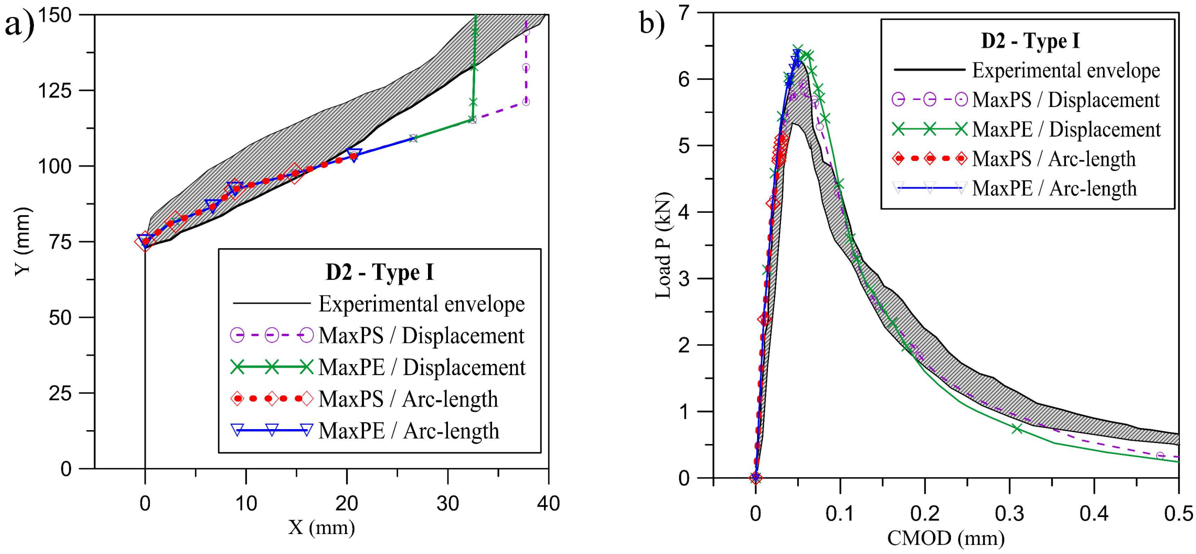

Figure 6. Effect of the crack initiation criteria and solution techniques: a) Fracture propagation path and b) load P *vs.* CMOD curve.

According to the results shown in Figure 6, the fracture propagation path and load-CMOD curve for the maximum principal stress criterion conducted with displacement control exhibit good agreement with the experimental data. It was also observed convergence complications to trespass the peak of the curve while using arc-length control. Additionally, propagation problems are usually encountered as the crack approaches the edge of the beam (Khodabakhshnejad, 2016).

The criteria based on the nominal stress/strain and the quadratic interaction of traction/separation were also tested. However, none of those criteria presented satisfactory results.

5.2 The effect of the criterion evaluation location

Considering the best configuration of the previous results, a new parametric study was carried out in order to analyze the effect of the location to evaluate the crack initiation criterion and its direction. The numerical simulations were also conducted considering an intermediate finite element mesh (Mesh 2) and displacement controlled analysis.

For the nonlocal analysis, a semi-circular zone with a radius r_c equal to three times the element characteristic length was assumed.

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

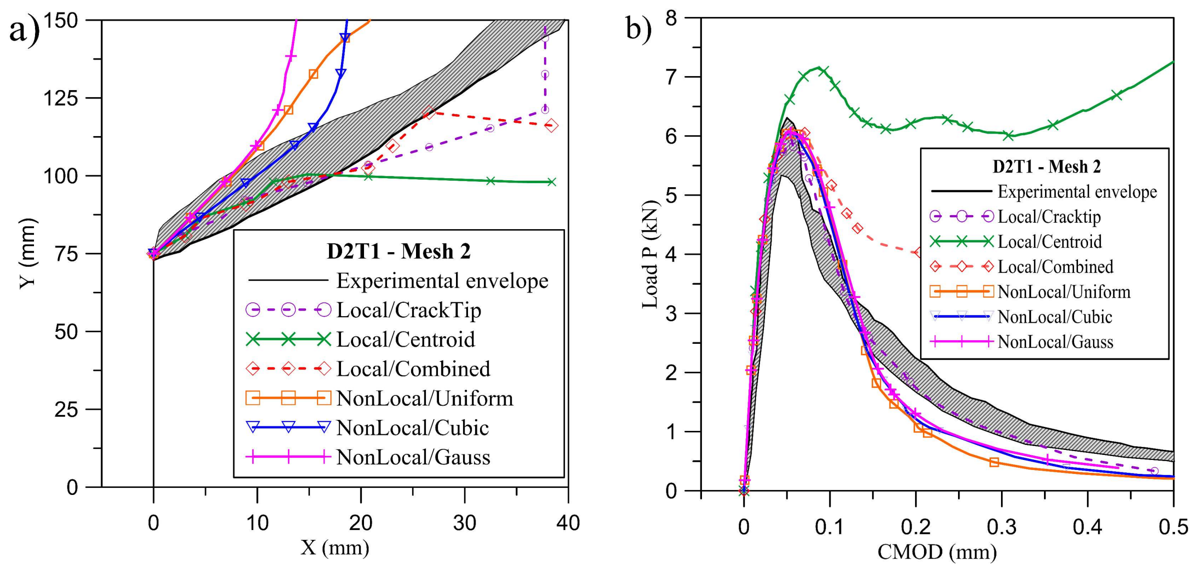

Figure 7. Effect of the local and nonlocal criteria: a) fracture propagation path b) load-CMOD curve

The crack initiation criterion evaluated at the crack tip presented better results in the load-CMOD curve, as shown in Fig. 7.b. However, its predicted crack path went out of the experimental envelope, as in Fig. 7.a. On the other hand, the criterion evaluated at nonlocal zone with cubic spline stress smoothing predicted an acceptable tendency for the crack path and load-CMOD curve.

5.3 The mesh refinement effect

In order to understand the effect of the mesh in the fracture trajectory and Load-CMOD curve, a new study considering three different finite element meshes was performed.

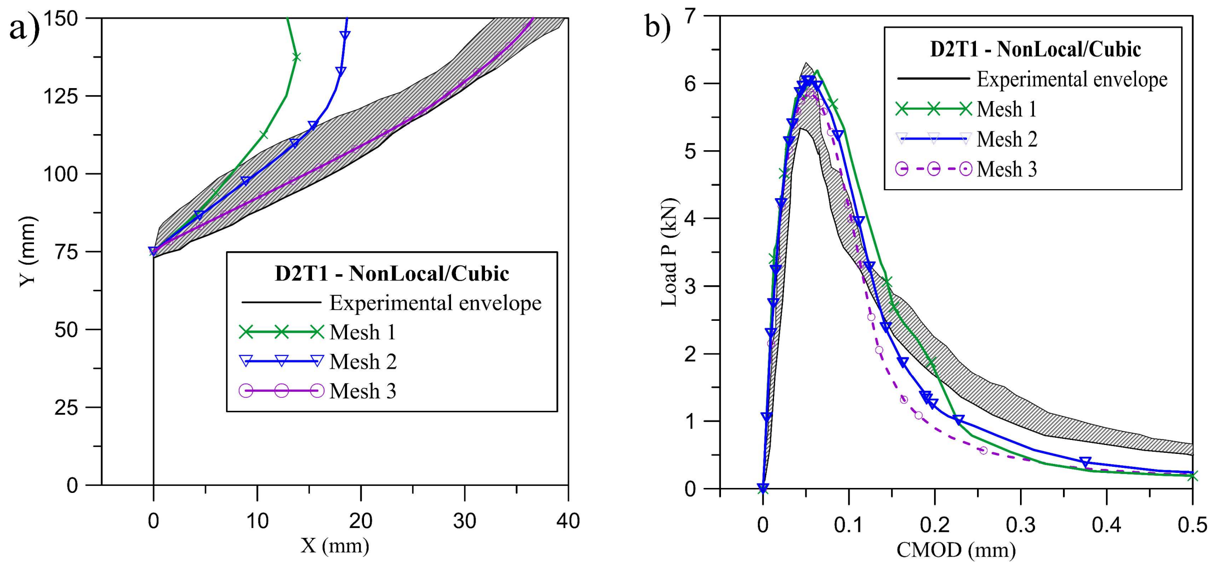

Figure 8. Effect of the mesh refinement: a) fracture propagation path b) load-CMOD curve

The simulations were carried out considering the criterion of maximum principal stress evaluated at nonlocal zone and displacement controlled analysis. The fine mesh presented better results in the predicted crack path and load-CMOD curve, as shown in Fig.8. Finally, for validation purposes, the medium size model for type 1 and type 2 were simulated in the same way (Fig. 9)

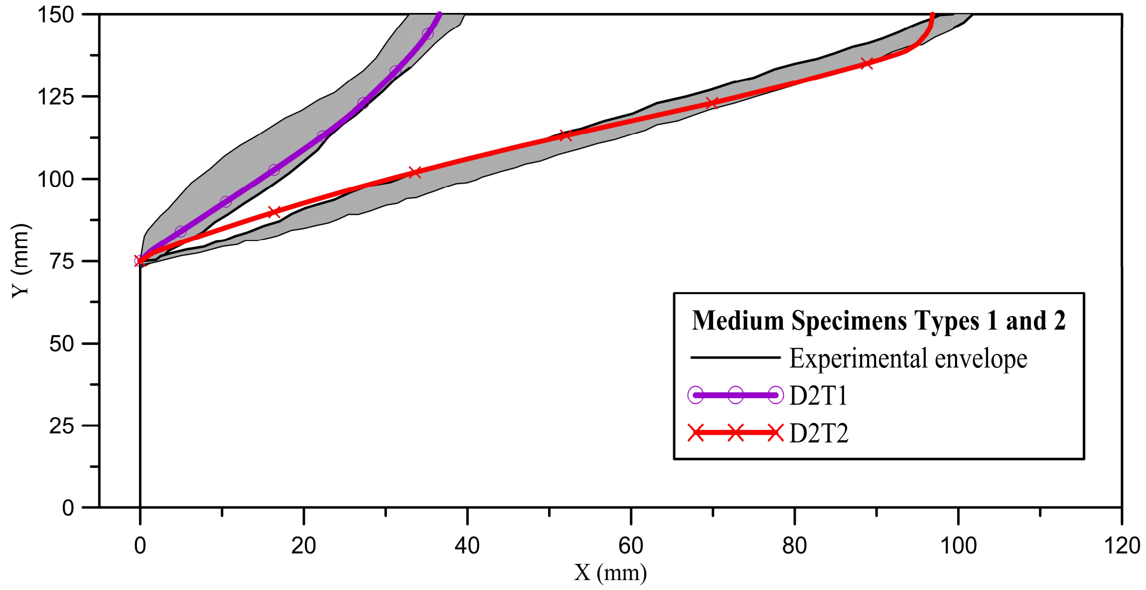

Figure 9. Fracture propagation path for a medium specimen size (D2) in test Types 1 and 2

6 CONCLUSIONS

The experimental data presented in this paper were used as a reference to validate the XFEM simulations for mixed-mode crack propagation. The numerical models allied to the correct parameters were able to predict the fracture trajectories. Furthermore, matching results for load-CMOD curves were found, with the ultimate-load capacity within the experimental envelope limits.

The displacement control proved to be very effective circumventing the problems of numerical convergence near the peak load. Yet, it was noticed that analyzes made with arc length control presented best convergence results for the beams with notches modelled explicitly, instead of those where the notch was modelled through the addition of an initial fracture imposed on the enriched elements. However, in the case of the use of prescribed displacement, there was no difference between the two controls.

The location where the crack initiation criterion is evaluated has proved to be of paramount importance for an accurate analysis. The strategy of analyzing a nonlocal finite element region allows the determination of a coherent fracture trajectory. This is due to a better stress average, and better path computation, so that high stresses at isolated gauss points do not govern the trajectory as a whole. However, a sensitivity analysis of the range radius has proved to be necessary, since this parameter affects the simulation as a whole. Good results were found for a radius equal to three times the characteristic element size.

In the finest mesh (Mesh 3), all weight functions were able to find good fracture trajectories, according to the proper radius used. However, better results were found using the cubic spline stress smoothing. Thus, the nonlocal evaluation criterion has proven to exhibit mesh dependency. Besides, it has been noticed that the more refined the mesh, the more susceptible it was to the radius length employed.

Finally, when compared to cohesive interface elements, XFEM demonstrates practicality and efficiency in situations where the path of the fracture is unknown. Moreover, the method evidenced to be fully effective in modelling of structures in situations under mixed mode loading.

ACKNOWLEDGEMENTS

This research was carried out with support of CAPES and Shell Brazil.

REFERENCES

ABAQUS, 2014. *Abaqus 6.14 Documentation.* Pawtucket, RI, USA: Dassault Systèmes.

Areias, P. & Belytschko, T., 2005. A comment on the article: a finite element method for simulation of strong and weak discontinuities in solid mechanics.. *Computer Methods in Applied Mechanics and Engineering,* pp. 1275-1276 .

Arrea, M. & Ingraffea, A., 1982. *Mixed Mode Crack Propagation in Mortar and Concrete,* s.l.: Department of Scructural Engineering, Cornell University.

Barenblatt, G. I., 1962. The mathematical theory of equilibrium crack in brittle fracture. *Advanced Applied Mechanics,* Volume 7, pp. 55-129.

Bažant, Z. P. & Oh, B. H., 1983. Crack band theory for fracture of concrete. *Materials and Structures,* 16(3), pp. 155-177.

Bažant, Z. P. & Pfeiffer, P. A., 1986. Shear fracture tests of concrete. *Matériaux et Constructions,* 19(110), pp. 111-121.

Bendezu, M. A. L., Romanel, C. & Roehl, D., 2015. *A comparative study on finite element method for crack propagation in concrete.* Rio de Janeiro, XXXVI Ibero-Latin American Congress on Computational Methods in Engineering.

Benzeggagh, M. L. & Kenane, M., 1996. Measurement of mixed-mode delamination fracture toughness of unidirectional glass/epoxy. *Composites Science Technology,* Volume 56, pp. 439-449.

Camanho, P. & Dávila, C., 2002. Materials, Mixed-Mode Decohesion Finite Elements for the Simulation of Delamination in Composite. *NASA/TM-2002-211737*, June, pp. 1-37.

Cendón, D., Gálvez, J., Elices, M. & Planas, J., 2000. Modelling the fracture of concrete under mixed loading. *International Journal of Fracture,* Volume 103, pp. 293-310.

Dolbow, J., Moës, N. & Belytschko, T., 2001. An extended finite element method for modeling crack growth with frictional contact. *Comput. Methods Appl. Mech. Engrg,* Volume 190, pp. 6825-6846.

Gálvez, J. C., Elices, M., Guinea, G. & Planas, J., 1998. Mixed mode fracture of concrete under proportional and nonproportional loading. *International Journal of Fracture,* Volume 94, pp. 267-284.

Hansbo, A. & Hansbo, P., 2004. A finite element method for the simulation of strong and weak discontinuities in solid mechanics. Volume 193, pp. 3523-3540.

Iosipescu, N., 1967. New accurate procedure for single shear testing of metal. *Journal of Materials,* Volume 2, pp. 537-566.

Khodabakhshnejad, A., 2016. *An extended finite element method based modeling of hydraulic fracturing,* s.l.: University of Southern California.

Machado, W. G., Mejia Sanchez, E. C. & Roehl, D., 2015. *Numerical modelling of four-point bending test using cohesive fracture models.* Rio de Janeiro, XXXVI Ibero-Latin American Congress on Computational Methods in Engineering.

Moës, N. & Belytschko, T., 2002. Extended finite element method for cohesive crack growth. *Engineering Fracture Mechanics,* pp. 813-833.

Moës, N., Dolbow, J. & Belytschko, T., 1999. A finite element method for crack growth without remeshing. *International Journal for Numerical Methods in Engineering,* pp. 131-150.

Reeder, J., Song, K., Chunchu, P. B. & Ambur, D. R., 2002. *Postbuckling and Growth of Delaminations in Composite Plates Subjected to Axial Compression.* Denver, Colorado, American Institute of Aeronautics and Astronautics, p. 10.

Song, J.-H., Areias, P. M. A. & Belytschko, T., 2006. A method for dynamic crack and shear band propagation with phantom nodes. *International Journal for Numerical Methods in Engineering,* Volume 67, pp. 868-893.

Swartz, S., Lu, L., Tang, L. & Refai, T., 1988. Mode II fracture parameter estimates for concrete from beam specimens. *Experimental Mechanics,* Volume 28, pp. 146-153.