



INFLUENCES OF CURVE MOMENT ROTATION MODELS IN STATIC AND DYNAMIC ANALYSIS OF STEEL STRUCTURES WITH BEAM-COLUMN AND COLUMN-BASE SEMI-RIGID CONNECTIONS

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Abstract. *An important idealization admitted in the structural analysis is the connections between the elements. Beam-column and column-base joints are usually assumed to be either perfectly rigid or ideally pinned in relation to the requesting efforts. Focusing on the behavior of the connections, experiments have demonstrated that connections classified between the extremes of fully rigid connections and frictionless pinned connections, known as semi-rigid connections, behave non-linearly. This means that a finite degree of joint flexibility exists in these connections, representing the semi-rigid behavior. This real connection behavior is usually modeled by null length rotational springs with moment-rotation relations described by several mathematical models presented in the literature. Thus, the main objective of this paper is to analyze the influence of different non-linear moment-rotation formulations in static and dynamic analysis of steel structures with beam-column and column-base semi-rigid connections. The results obtained by non-linear models and multilinear models of beam-column connections are also compared in order to verify their efficiency. The analyzes were performed using the software NIDA (Non-linear Integrated Design and Analysis) which considers the hybrid element proposed by Chan & Chui (2000) formed by coupling the connection stiffness in the element global stiffness matrix.*

Keywords: *Semi-rigid connection, Column-Base connection, Non-linear analysis, Multi-linear model.*

1 INTRODUCTION

An important idealization allowed in the structural analysis is the connections between the elements, which are traditionally considered to be perfectly rigid or ideally pinned (flexible) in relation to the requesting efforts (Ribeiro, 1998). However, there is a wide variety of connections, in addition to the diversity of configurations, means and connection devices. On the other hand perfect coupling and perfect flexible joints cannot be obtained once the connections introduce local effects and imperfections which may induce a partial joint stiffness behavior: semi-rigidity (Maggi, 2000). In structural designs the consideration of semi-rigidity in the design stage has received increasing attention, due to its capacity of energy dissipation and manufacturing efficiency (Fang et al., 2013).

The connections represent a small share of the weight of the structure, however, they have high manufacturing and assembly costs. Therefore, the consideration of semi-rigid joints still in the structural design can generate more economical solutions, although its consideration is more laborious in the steps of design and structural analysis (Ramires, 2010). According to Nguyen and Kim (2014) the real behavior of the connections is non-linear and generally it can be represented by rotational springs that have null lengths and a specific moment-rotation relation.

In order to describe the semi-rigid behavior, it is necessary to know the rotational response of the connection by means of the moment-rotation behavior ($M-\theta$), usually obtained by experimental means or by theoretical, mathematical, empirical or semi-empirical models. This behavior, in turn, should be incorporated into the structural analysis to obtain more accurate information on the performance of the structure (Bessa, 2009).

In addition to the traditional studies on semi-rigidity covering beam-column links, another field that deserves attention is the column-base connection, which influences structural behavior of isolated columns and frames (Lau et al., 2003; Hayalioglu & Degertekin, 2005). However, the column bases are a little remembered when discussing the influence of the semi-rigidity of the links. In this work, the consideration of the partial stiffness of the column-base connections will be approached based on literature studies, since well-calibrated $M-\theta$ curves are not available. Several papers present studies on the consideration of semi-rigidity in column-base joints: Jaspert & Vandegans (1996), Wald et al. (2000), Wald (2000), Hensman & Nethercot (2001), Degertekin & Hayalioglu (2004), Wald et al. (2008) and Bučmys & Dališanskis (2012).

The technological advance has allowed the improvement of structural analysis tools and the development of new techniques and procedures that seek to represent the real behavior of the structure, from the consideration of semi-rigid joints.

In the present work, the simple framed structure proposed by Chan & Chui (2000) will be analyzed. First and second order analyzes considering the variation of stiffness of beam-column and column-base connections will be performed using the NIDA computational program. The rigidity of the beam-column connection varies according to the mathematical models presented in this work. The rigidity of the column-base joint is determined from the formulation presented by Chan & Chui (2000), which considers the stiffness of the joint as linear. The results will show the influence of these different models in the static and dynamic behavior of the structure.

2 BEHAVIOR AND STRUCTURAL ANALYSIS

The objective of the structural analysis is to determine tensions, deformations, internal stresses and displacements for a certain structure that presents certain load and defined boundary conditions. Structural engineering, understood as a science, allows designing structures with stability and resistance assurance, taking advantage of their capacity to withstand larger loads (considering effort redistribution), minimizing costs and material (weight, time and processes) and, in order to achieve all this, using numerical resources with computer science (Alvarenga, 2010). The procedure for analysis can be performed in several ways, and in this work two groups will be considered when considering stability: first order analyzes and second order analyzes.

2.1 First order elastic linear analysis

Currently linear analyzes are the most used among designers, and the results obtained in these analyzes are used as a basis for calculation of displacements, deformations and stresses to be used in structural design.

In considering linear analysis, three basic conditions must be verified: compatibility, equilibrium and constitutive relations. Chan & Chui (2000) complement that the deflection of the structure is assumed to be very small and the second order (geometric) effects are ignored. The stiffness of the structural members is constant and independent of the presence of axial forces, ignoring the $P-\Delta$ and the $P-\delta$ effects.

One of the disadvantages of linear elastic analysis has been its inability to reflect the real behavior of structures when unusual loads or boundary loads are operating, since all structures behave non-linearly before reaching their resistance limits (Pinheiro, 2003). In these cases the basic consideration of analysis does not allow an adequate evaluation of the stability or the ultimate strength of the structure, for which it is necessary to increase the efficiency of the analysis and to take into account the second order effects.

2.2 Second order analysis

During the linear analysis of structures, the force is assumed linearly proportional to the displacement and the principle of superposition of the effects can be applied to obtain the final diagram of bending moments and stresses. This technique of overlapping effects cannot be applied to a nonlinear analysis because the structural response is affected by the interaction between loads and deformations and therefore they cannot be isolated from each other (Chan & Chui, 2000).

Nonlinear analysis is sought to improve the simulation of a structure's behavior in some respects, i.e., a more realistic modeling from the proper consideration of the effects related to nonlinearities that affect the structural behavior. Thus, two considerations of non-linearity are addressed in the analyzes. The first consists of the non-linearity of the physical material, resulting from the change in stiffness of the elements due to the presence of axial forces and deflections along the length of the element. The second class consists of geometric nonlinearity, the $P-\Delta$ effect, which is produced by finite deformations caused by a certain applied load.

The main benefit of second-order analysis is to be responsible for the redistribution of internal forces after the member resistance has been overcome (Chen et al., 1996). Thus, the second order analysis, although more complex, makes structural behavior more real and economic (Chen & Sohal, 1995). Figure 1 shows the redistribution of the internal stresses of a metallic section, where P_y is the applied load; σ_y the yield stress of the steel; σ_{rc} is the maximum residual stress of the section; and P_p is the load on the plastic boundary.

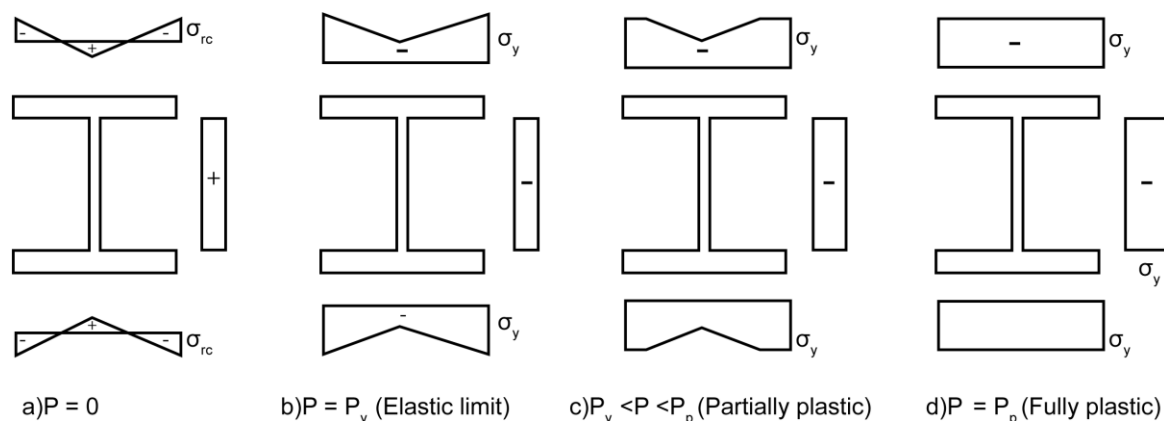


Figure 1. Redistribution of section efforts: a) residual stresses; b) stresses at the beginning of the plastic state (or yield elastic limit); c) elasto-plastic tensions; and d) tensions in the limit plastic state (Chen & Sohal, 1995)

The application of the analyzes presented here will be demonstrated in Section 5. In Section 3 will be described the behavior and the mathematical models for modeling the stiffness of the connections.

3 SEMI-RIGID CONNECTIONS

Conventionally in the analysis and design of steel structures, beam-column connections are idealized as perfectly rigid: it is assumed that the beam and the column have the same rotation, allowing full transmission of the bending moment; or ideally pinned or flexible: it is accepted that the joint is so flexible that the value of the bending moment transmitted is negligible, with no rotational continuity occurring.

However, it is observed that the nonlinear curve that characterizes the real behavior of the link is in a position intermediate to the traditional idealizations, presenting the semi-rigid behavior of the links. Assuming the semi-rigid behavior of the joints, a better approximation of the reality can be obtained by modifying the moment distribution, Figure 2.

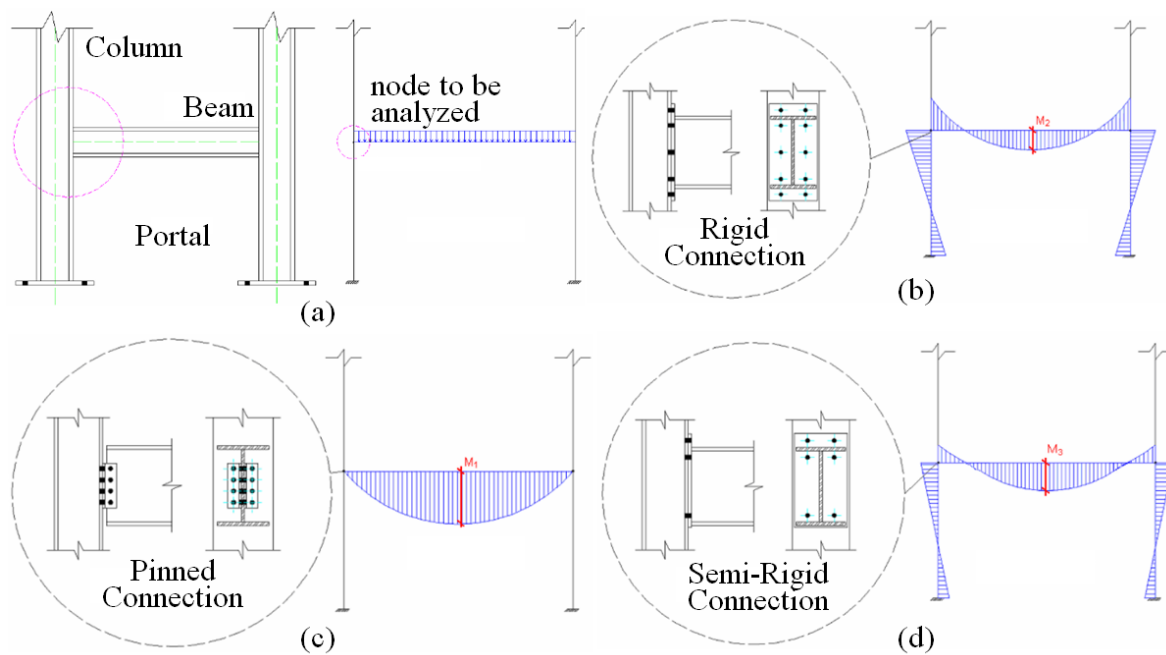


Figure 2. a) Frame and its respective model for analysis; b) Moment diagram for the rigid beam-column connection; c) Flexible beam-column connection and d) Semi-rigid beam-column connection (Barbosa, 2006)

Focusing on the behavior of the joints, experiments demonstrated that they behave non-linearly between the two ends of rigid and flexible connections (Fig. 3). This means that there is a finite degree of joint flexibility in the connections (Chan & Chui, 2000). The nonlinear trajectory of the behavior of the connections depends on the type of connection used, thus, different connections have different non-linear trajectories (Fig. 3).

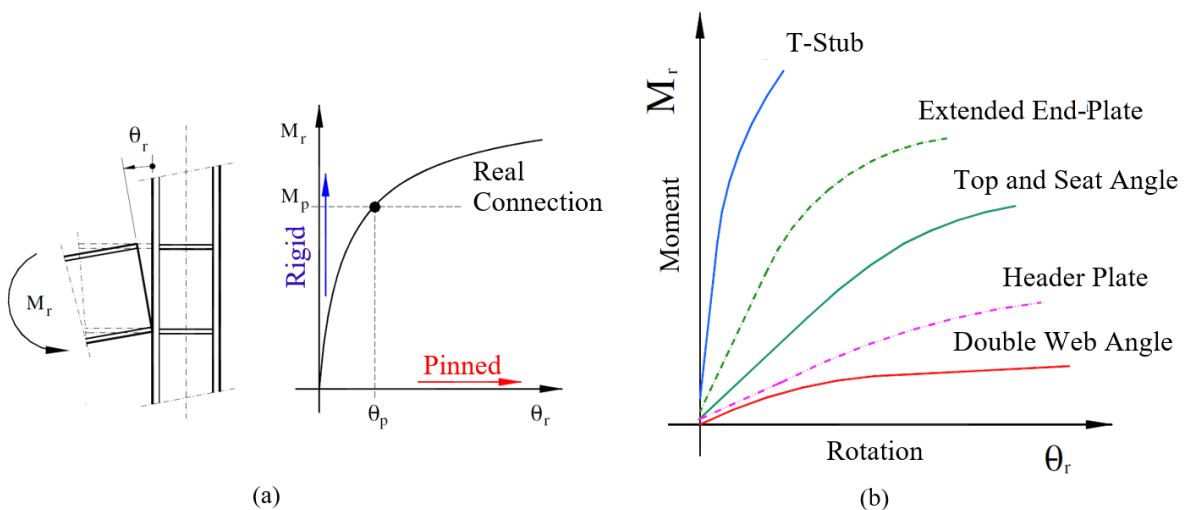


Figure 3. Behavior of the connections: (a) relative beam-column rotation and moment-rotation diagram; and (b) moment-rotation curves for several connections (Alvarenga, 2010).

The beam-column connections are generally subjected to axial forces, shear forces, bending moments and torsion. Usually, torsion is disregarded and the effects caused by axial and shear forces are very small compared to those caused by bending moments (Chen et al., 1996). Consequently, only the effect caused by the bending moment will be considered in this work.

3.1 Moment-Rotation curve models

The researches have sought to develop simple models that represent the moment-rotation curves, which describe the behavior of the joints and are used in the structural analyzes (Higaki, 2014). Considering the scope of this work, only a few mathematical methods will be addressed, which are presented in the next subsections. In this study, the moment-rotation curves will be introduced in the structural analysis by means of mathematical models with parameters obtained from experimental studies or from literature.

3.1.1 Linear models

Linear models are the simplest and most usual models, which depend on only one parameter to determine the joint stiffness. Several works (Batho, 1931; Baker, 1934; Rathbun, 1936; Monforton & Wu, 1963; Lightfoot and Lemesurier, 1974) present this consideration of initial constant (elastic) stiffness and are even applied for vibration analysis (Chan, 1994). The linear hypothesis is convenient for small load values, however, in analyzes with large deflections, the degradation of the joint stiffness should be considered (Alvarenga, 2010; Chen et al., 2000, Chen et al., 1996). The expression for a linear model is given by:

$$M_r = R_k \theta_r \quad (1)$$

where R_k is the initial tangent stiffness constant (R_{ki}) of the connection or the secant stiffness (R_{ks}).

The multilinear model allows describing a stiffness trajectory with as many line segments as possible, such that the difference between the arc and the linear segments is indistinct (Alvarenga, 2010). Thus, each linear segment is represented by the expressions:

$$M_r = R_{kab} (\theta_r - \theta_a) + M_a, \text{ with} \quad (2)$$

$$R_{kab} = \frac{(M_b - M_a)}{(\theta_b - \theta_a)} \quad (3)$$

where R_{kab} is the relative stiffness to the points “a” and “b”; θ_r the relative rotation between the beam and the column; θ_a the rotation relative to point “a”; θ_b the rotation relative to point “b”; M_a the moment relative to point “a”; M_b the moment relative to point “b” and M_r the relative moment between the beam and the column, as shown in Fig. 4.

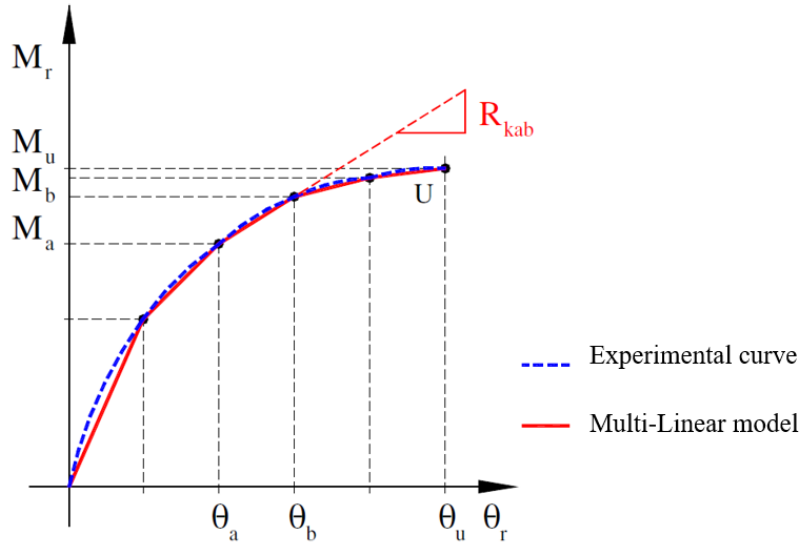


Figure 4. Moment-rotation curve with multilinear model (Alvarenga, 2010)

The experimental tests provide joint behavior similar to the multilinear model, which justifies the possibility of using this model in any computational program (Alvarenga, 2010). However, the multilinear models have the deficiency of discontinuity in the slopes (Pinheiro, 2003).

3.1.2 Kishi & Chen potential model (1987)

The potential model of Kishi & Chen (1987) is an adaptation of the potential model of Richard & Abbott (1975) when considering the joint plastic stiffness equal to zero. Thus, the formulation of the potential three-parameter model is given by:

$$M_r = \frac{R_{ki} \theta_r}{\left[1 + (\theta_r / \theta_0)^{C_1}\right]^{\frac{1}{C_1}}} \quad (4)$$

where C_1 is the curve fit parameter; R_{ki} the initial stiffness; θ_r the relative rotation between the beam and the column; and the reference rotation is defined by $\theta_0 = M_u / R_{ki}$.

Alvarenga (2010) presents an alternative form (Fig. 5) to represent the potential model of three parameters, where the dimensionless rotation $r = \theta_r / \theta_0$ is defined, the dimensionless moment $m = M_r / M_u$, and taking $\theta_0 = M_u / R_{ki}$. This alternative form is given by:

$$m = \frac{r}{\left[1 + |r|^{C_1}\right]^{\frac{1}{C_1}}} \quad (5)$$

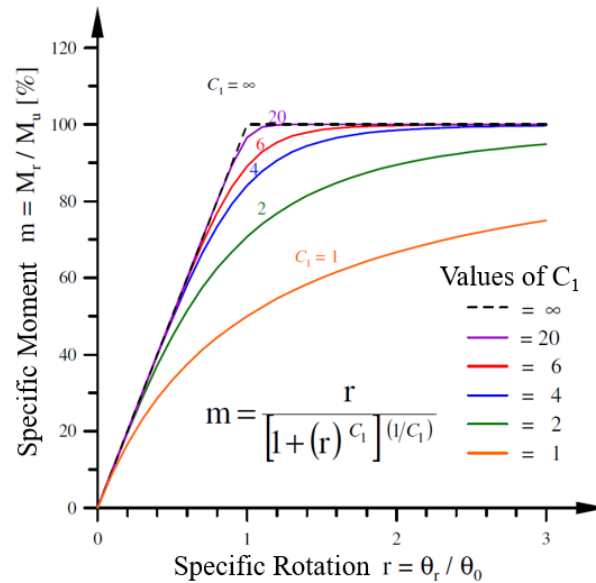


Figure 5. Typical form of the power model of Kishi & Chen (1987) (Alvarenga, 2010)

3.2 Column–base connections

Early studies of the column-base connections appeared in Salmon et al. (1955). Since then the studies began to deepen in the behaviors and effects of the base-column connections.

At the construction or design levels, there are traditionally two types of base, as discussed by Alvarenga (2010): labeled (Fig. 6(a)) or crimped (Fig. 6(b)).

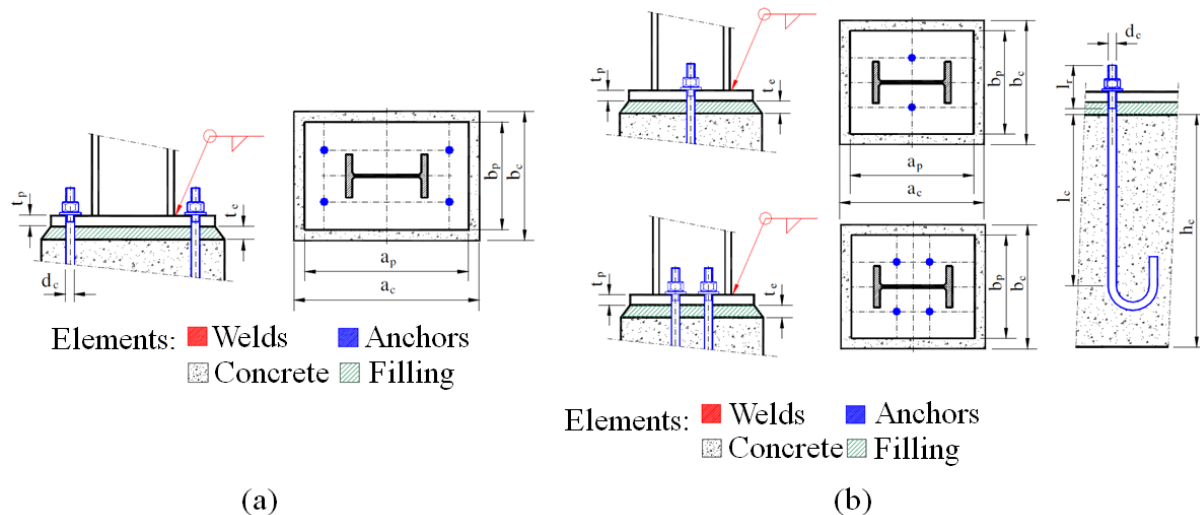


Figure 6. (a) Column base labeled: a) with two anchors; B) with 4 anchors; C) detail of the anchor; and (b) Base of crimped columns (Alvarenga, 2010).

As shown in the introduction of this work, its influence on structural behavior was confirmed by considering the column-base connections as semi-rigid, both in isolated columns, as well as in frames. This fact justifies the inclusion of the analysis of these connections in this study. The partial stiffness of the column-base joint will be determined

from the formulation presented by Chan & Chui (2000) and its behavior will be considered linear.

4 STIFFNESS COUPLING

The literature presents several models for stiffness coupling in a semi-rigid element (Chan & Chui, 2000; Torkamani et al., 1997; Yang & Kuo, 1994; Sekulovic & Salatic, 2001). In this work the semi-rigid element proposed by Chan & Chui (2000) and implemented in NIDA software is considered.

A semi-rigid connection can be modeled as a spring element inserted at the point of intersection between the beam and the column, as shown in Fig. 7. Such spring element has zero length and does not change the characteristics of the connection.

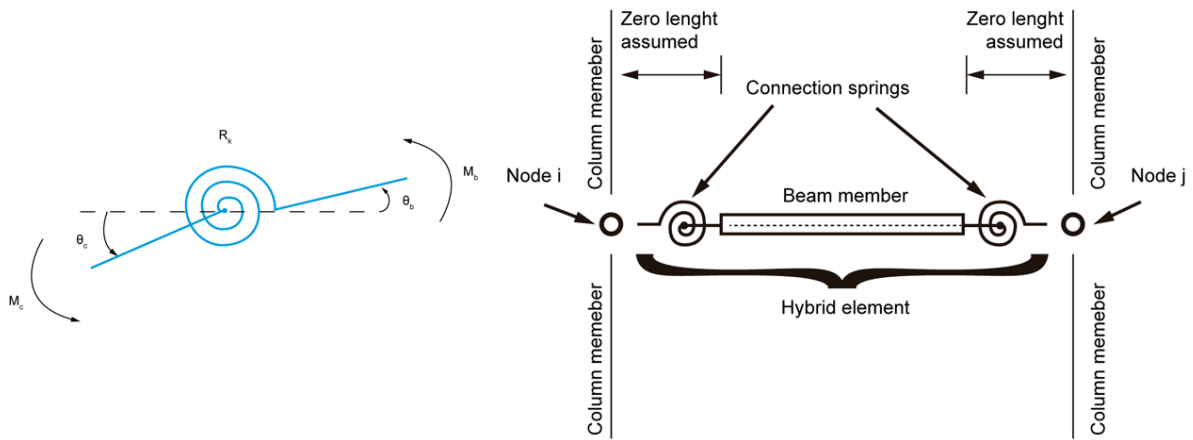


Figure 7. (a) Spring element simulating a connection; (b) Idealized element model (Chan & Chui (2000) adapted by the author)

Due to the flexibility of a semi-rigid connection, the rotations of the ends of the spring connected to the columns and the ends connected to the beams are generally different. In this work the rotation of the connection and the rotation of the beam will be called θ_c and θ_b , respectively.

The link spring and beam-column element are combined to form the hybrid element shown in Fig. 8. One side of the spring member is connected to the beam member while the other side is connected to the overall node (or column). With the connecting springs, added to the ends of the beam, the conventional stiffness element matrix should be modified in order to take into account the effect of the semi-rigid links. The resulting stiffness relation is:

$$\begin{Bmatrix} \Delta M_{ci} \\ \Delta M_{bi} \\ \Delta M_{bj} \\ \Delta M_{cj} \end{Bmatrix} = \begin{bmatrix} R_{ki} & -R_{ki} & 0 & 0 \\ -R_{ki} & R_{ki} + K_{ii} & K_{ij} & 0 \\ 0 & K_{ji} & R_{kj} + K_{ji} & -R_{kj} \\ 0 & 0 & -R_{kj} & R_{kj} \end{bmatrix} \begin{Bmatrix} \Delta \theta_{ci} \\ \Delta \theta_{bi} \\ \Delta \theta_{bj} \\ \Delta \theta_{cj} \end{Bmatrix} \quad (6)$$

where the subscripts ‘i’ and ‘j’ refer to the nodes i and j of the beam element (Fig. 7 (b)); the terms K_{ij} are the flexural stiffness components of the beam; R_{ki} e R_{kj} are the tangent stiffness components of the coupling springs and $\Delta\theta_i$ and $\Delta\theta_j$ are the incremental rotations of the two ends of the element on the basis of the last equilibrium configuration.

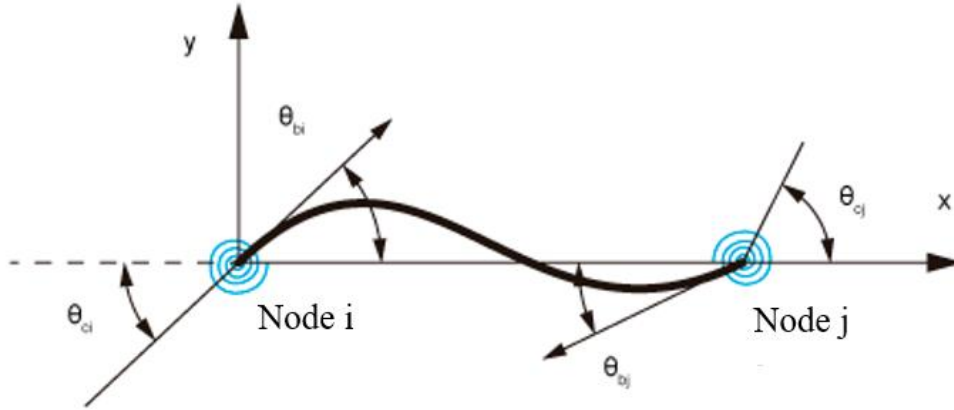


Figure 8. Hybrid element (beam) with added connection springs (Chan & Chui (2000), adapted by the authors)

5 NUMERICAL APPLICATION

Figure 8 shows the structural model of a frame presented in the work of Chan & Chui (2000). They performed only first order elastic linear analysis and compared the bending moment forces obtained by varying the beam-column connections between rigid, flexible and semi-rigid (linear), and considering the column-base connection as rigid and semi-rigid (linear). However, the work proposed by Chan & Chui (2000) only indicates the area and inertia values for each structural element, without the use of usual metallic profiles. In this work, real steel sections with geometric properties equivalent to those indicated in Chan & Chui (2000) were chosen to represent structural elements used in the analyzes.

In this numerical application beam-column connections were considered: rigid, semi-rigid with linear behavior, semi-rigid with multilinear behavior and semi-rigid with Kishi & Chen (1987) potential behavior. The column-base connections were considered rigid and semi-rigid with linear behavior. The initial beam-column connection stiffness considered as $R_{kini} = 11070,00$ kNm/rad and the column-base connection stiffness as $S_{cini} = 3228,75$ kNm/rad were calculated by (Chan & Chui, 2000):

$$R_{kini} = \frac{4E_{beam}I_{beam}}{L_{beam}} \quad (7)$$

$$S_{cini} = \frac{E_{column}I_{column}}{L_{column}} \quad (8)$$

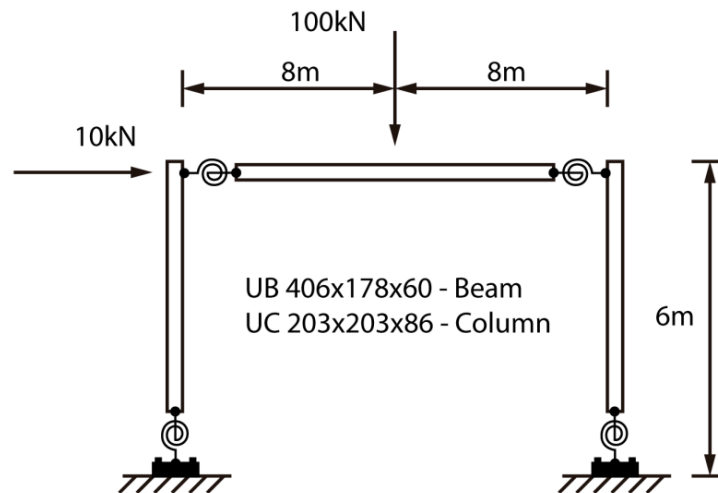


Figure 9. Frame: Geometry and loading (Chan & Chui (2000), adapted by the authors)

The parameters adopted to describe the nonlinear curve of the potential model of Kishi & Chen (1987) were the initial stiffness presented previously, the curve parameter (C_1) equal to 1, and the resistant moment of the beam adopted as a value higher than the maximum moment obtained in the second order analysis of the frame with flexible connections (case where the greatest moment occurs in the center of the beam span), in this case 550 kN.m.

Table 1 shows the models analyzed in this work, classified according to the analyzes performed (first order linear analysis and second order analysis) and the stiffness considerations for each joint and varying the beam-column connection models, as well as of the column-base connections.

Table 1. Types of models and respective performed analyzes

Beam-Column Connection	Column-Base Connection	Type of analysis / Model number		
		Linear 1 st Order	2 nd Order	Modal
Rigid	Rigid	1	9	17
	Linear semi-rigid	2	10	18
Linear Semi-rigid	Rigid	3	11	19
	Linear semi-rigid	4	12	20
Multi-linear Semi-rigid	Rigid	5	13	21
	Linear semi-rigid	6	14	22
Non-linear Semi-rigid Kishi-Chen	Rigid	7	15	23
	Linear semi-rigid	8	16	24

In the second-order analyzes the Newton-Raphson method was used (with constant loading), with 120 load cycles and the value of each load increase equal to 0.01 of the total, thus applying 120% of the load in the structure. The mass matrix was considered to be consistent, and the effects of physical and geometric non-linearities were considered.

The NIDA – *Non-linear Integrated Design and Analysis* – software was developed in 1996 by Siu Lai Chan of the Polytechnic University of Hong Kong - China. The software allows all the considerations presented previously in this work. In this work will be analyzed the influence of different semi-rigid connection models in the following structural parameters: 1) internal efforts; 2) displacements; 3) load factor $\times$ displacement curve; and 4) vibration frequencies.

Figure 10 shows the redistribution of efforts caused by the type of analysis employed to obtain the efforts of the structure. The analyses are: (a) all rigid connections with first-order linear analysis; (b) all rigid connections with second order analysis; (c) rigid beam-column and semi-rigid column-base connections with first-order linear analysis; and (d) rigid beam-column and semi-rigid column-base connections with second-order analysis. We can easily observe the redistribution of the efforts in the second-order analysis comparing with the first-order analysis, thus obtaining larger bending moments of up to 21.87% in the middle of the beam span and up to 22.45% in the base of the columns, due to the consideration of physical and geometric non-linearities.

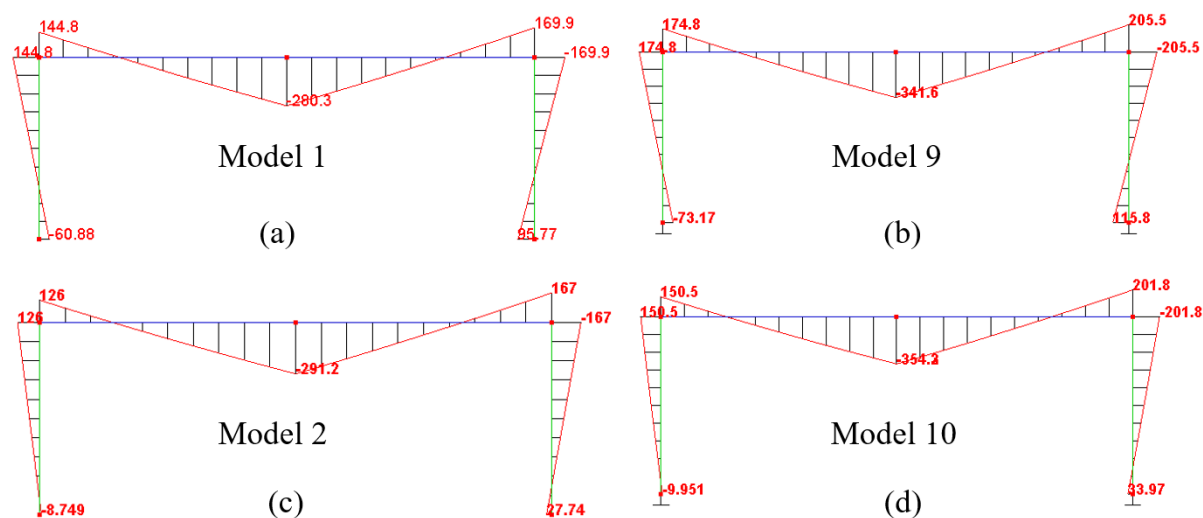


Figure 10. Distribution of bending moments by varying the analysis used and the rigidity of the column-base connections (description of the models in Table 1)

Figure 11 shows the bending moment redistribution by changing the beam-column connection among rigid, linear semi-rigid, multilinear semi-rigid and semi-rigid with Kishi & Chen (1987) model, considering rigid base-column connections and performing second-order analysis. The greatest influence on the bending moment redistribution occurs when we compare the multilinear model with the rigid model, with an increase of 18.20% in the middle of beam span and a decrease of 50.79% in the base of the column. The Figure 12 presents the same considerations described in Fig. 11, however considering the base-column connection as linear semi-rigid. Again, the greatest influence on bending moment redistribution occurs when we compare the multilinear model with the rigid model, with an increase of 15.42% in the center of the span of the beam and a decrease of 82.52% at the base of the pillar.

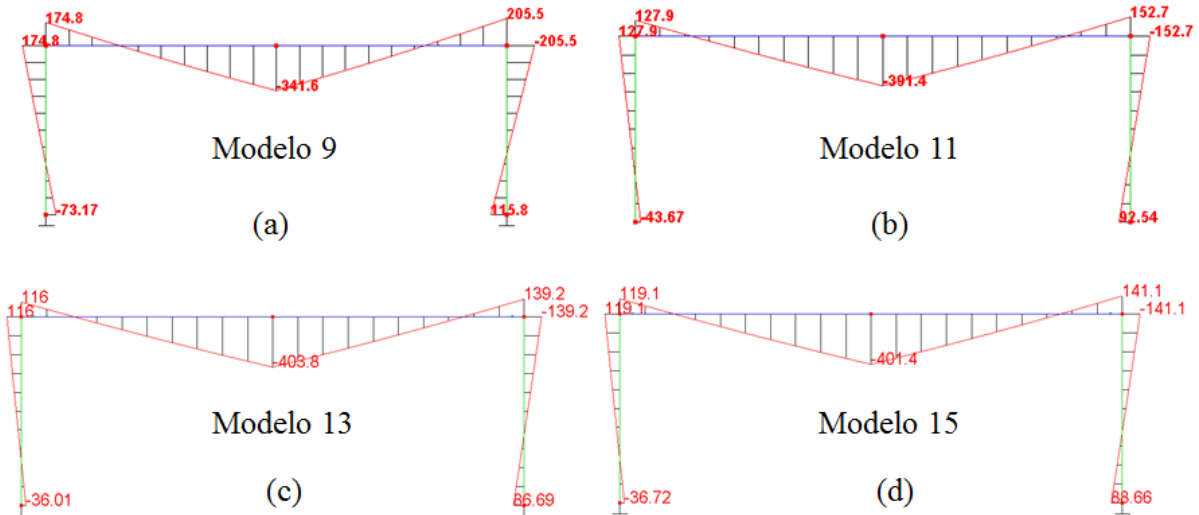


Figure 11. Influence of the beam-column semi-rigidity model on the distribution of bending moment considering rigid column-base connections and second-order analysis (description of models in Table 1)

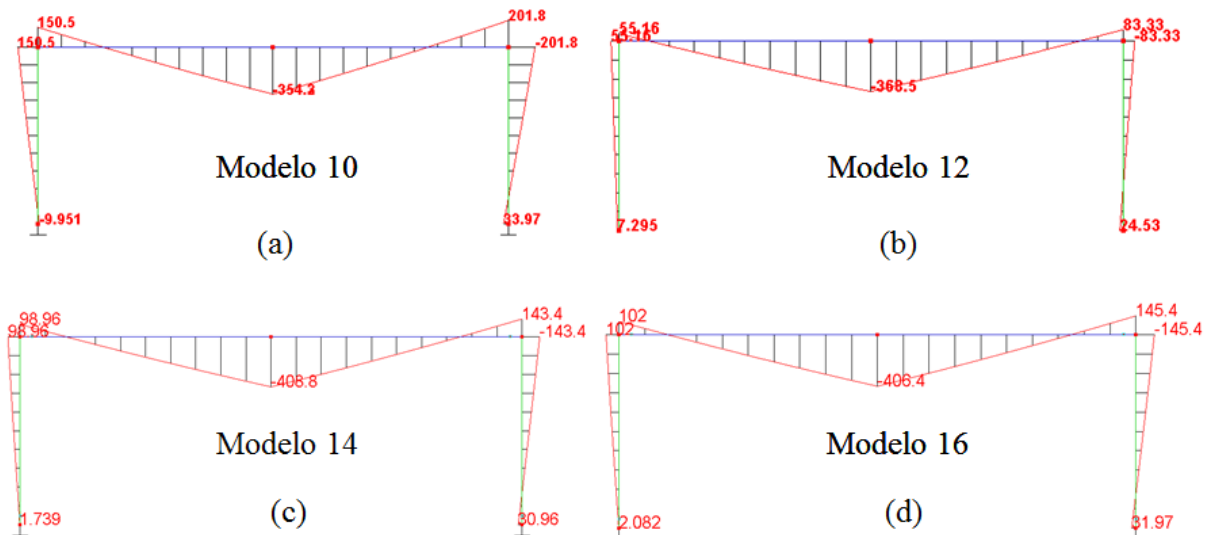


Figure 12. Influence of the beam-column semi-rigidity model on the distribution of bending moments considering linear semi-rigid column-base connection and second-order analysis (description of models in Table 1).

. Table 2 shows the values of the displacements of the node 3 (Fig. 13) for each one of the 16 models. We can easily observe that the consideration of the semi-rigidity in the column-base connection favored the redistribution of the internal forces, however a significant increase occurs in the displacements in first and second order analyzes. The displacements shown in Table 2 correspond to the UX direction (horizontal displacement in plane).

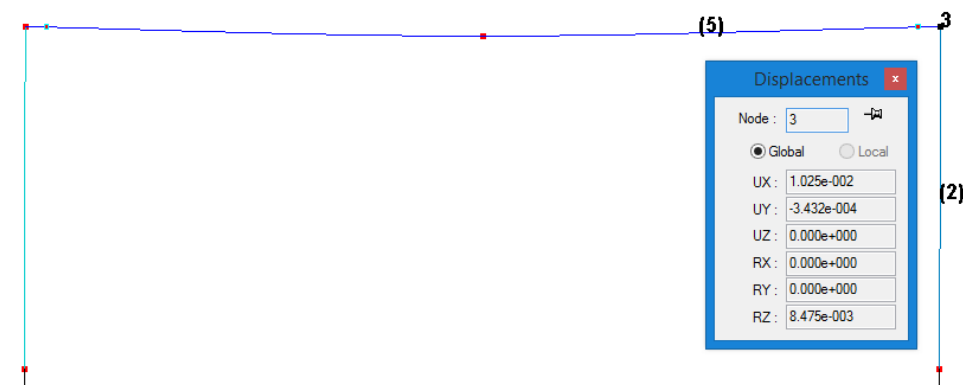


Figure 13. Node 3 of the frame

Table 2. Horizontal displacement (UX) of node 3 for the 16 models (indicated in Table 1)

Beam-Column Connection	Column-Base Connection	Horizontal displacement [Node 3] (cm)	
		Linear 1 st Order	2 nd Order
Rigid	Rigid	0,669	0,7842
	Linear semi-rigid	1,701	2,001
Linear Semi-rigid	Rigid	0,9032	0,9019
	Linear semi-rigid	3,498	2,131
Multi-linear Semi-rigid	Rigid	0,9032	0,962
	Linear semi-rigid	1,504	3,093
Non-linear Semi-rigid Kishi-Chen	Rigid	0,9032	1,025
	Linear semi-rigid	2,375	3,283

In the linear first-order analysis with rigid column-base connection, the model used to describe the semi-rigid beam-column connection did not influence the displacement values. However, in the other analysis and column-base connection considerations, the refinement of the model describing semi-rigid beam-column connection causes an increase in displacements of up to 64.07% when comparing Model 10 with Model 16.

As described previously, the second order analysis allows to describe the load cycles and the value of each increment of load, being possible to analyze the curve "load factor x displacement" of the structure. Again, node 3 is used as the reference point for the analyzes. Figure 14 shows the displacement load factor curves for the second-order analyzes of the models with rigid column-base connection, while Fig. 15 shows the displacement load factor curves for the second-order analyzes of the models with the linear semi-rigid column-base connection.

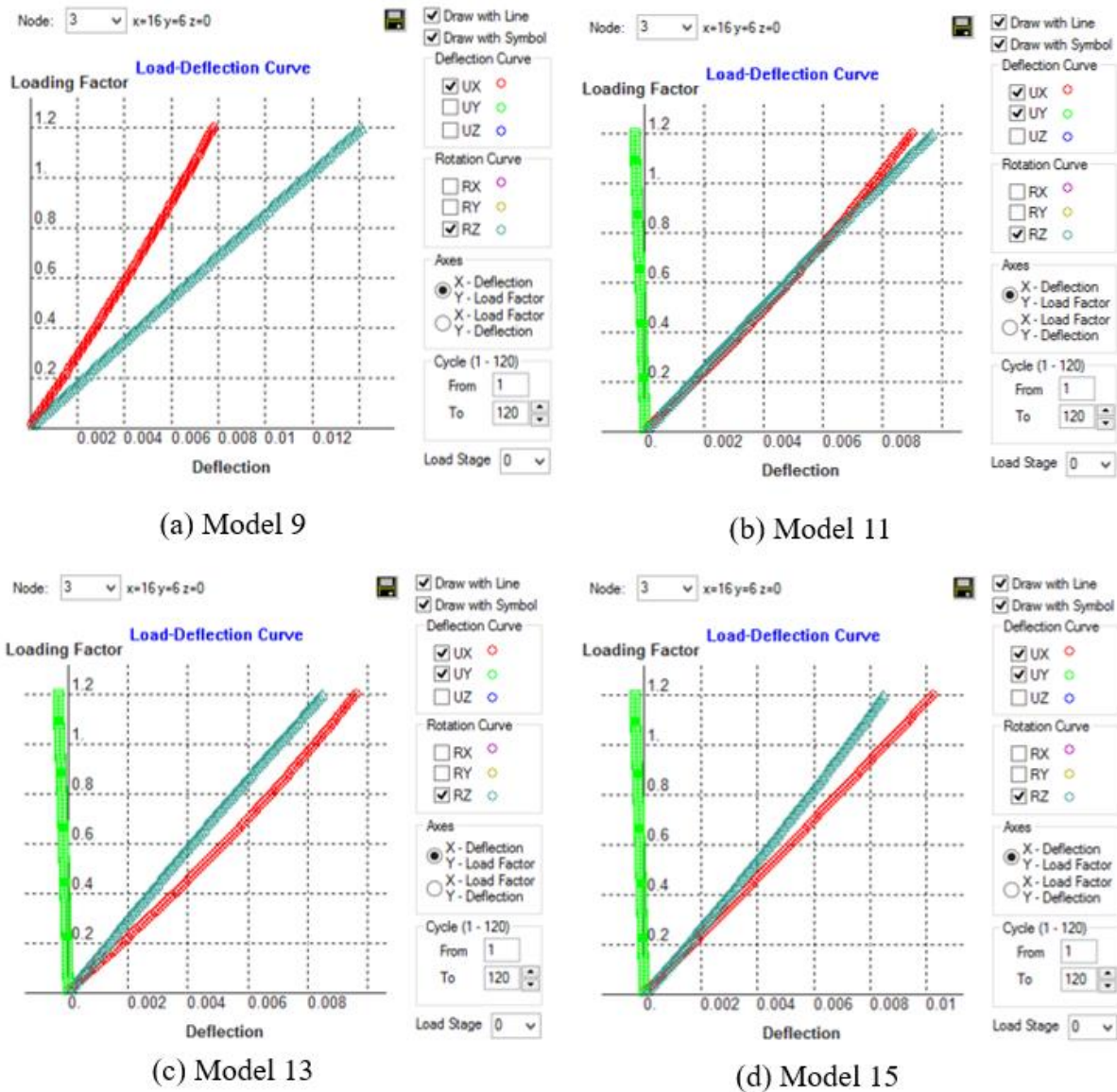


Figure 14. Load factor x displacement curves for the second-order analyzes of the models with the rigid column-base connection (models described in Table 1)

Finally, it was performed a modal dynamic analysis of the frame in order to analyze the influence of the semi-rigid considerations on beam-column and base-column connections in the natural frequencies of vibration. The first three frequencies of the frame are shown in Table 3. It can be seen that the frequencies are strongly dependent on the connection stiffness. According to Wyatt (1989) the vibration frequency of a building should be maintained above 5 Hz. Looking at Table 3, we observe that the fundamental frequency of this structure is in the range of 2,7 – 5,2 Hz, which is below the minimum recommended value for the maintenance requirement except for model 17.

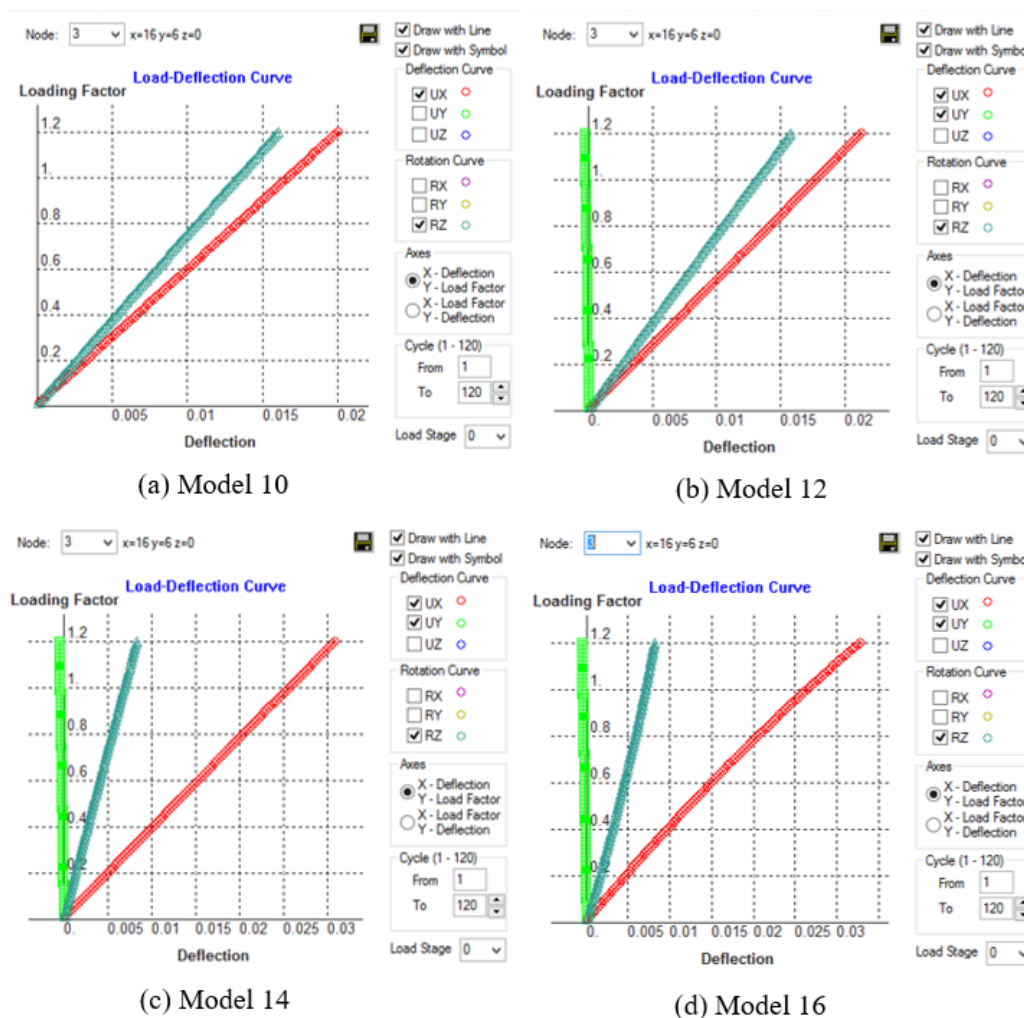


Figure 15. Load-factor x displacement curves for the second-order analyzes of the models with the rigid column-base connection (models described in Table 1).

Table 3. Vibration frequency of the frame for different beam-column and base-column connection considerations.

Beam-Column Connection	Column-Base Connection	Modal Analysis (Hz) - Present Work		
		1 st Mode	2 nd Mode	3 rd Mode
Rigid	Rigid	5,2561	7,9165	26,8856
	Linear semi-rigid	3,3498	7,5912	25,9299
Linear Semi-rigid	Rigid	4,5339	7,2561	25,7840
	Linear semi-rigid	2,8336	7,0981	24,3599
Multi-linear Semi-rigid	Rigid	4,3752	7,1130	25,5645
	Linear semi-rigid	2,7110	6,9835	24,0381
Non-linear Semi-rigid Kishi-Chen	Rigid	4,4427	7,1740	25,6562
	Linear semi-rigid	2,7672	7,0362	24,1820

6 CONCLUSIONS

The present work presented a comparative numerical study of the influence of different semi-rigid connection models in the structural behavior. Four beam-column connection models were used: rigid, linear semi-rigid, multi-linear semi-rigid and potencial semi-rigid (Kishi & Chen, 1987). Other two column-base models were: rigid and linear semi-rigid. It was also verified the influence of the first and second order analysis in the results.

It was analyzed the bending moment redistribution due to different considerations of stiffness and analysis, as well as the variation of displacements and frequencies of vibration. Clearly it is possible to observe a significant variation of these parameters due to the change of the models that describe the moment-rotation curve of the connections. The results obtained lead to the conclusion that more complete studies about the influence of semi-rigid connections in the structural projects are necessary in order to better describe the real structural behavior.

It is intended to extend this study to an evaluation of structures more consistent with reality, as well as to describe the type of connection used and its parameters for both beam-column and base-column connection and the application of other moment-rotation curve models that better describe the behavior of the connection.

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