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SINGULAR VALUE NOISE IN RSVD ALGORITHM FOR DAMAGE DETECTION

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Abstract. *The Robust Singular Value Decomposition algorithm (RSVD) was used to identify damaged structure using frequency response functions (FRFs) data from an aluminium plate. The routine uses robust statistics and the analysis was done in a data set from healthy and damaged structure. The effectiveness of the RSVD method was noticeable linked to optimum singular value basis reduction. The main problem experienced was to identify the correct noise level that has shown to be critical for data obtained in this work. A review for this kind of application has shown that there is no theoretical procedure to determine the optimal reduction for singular value basis. From experimental data this work will outline a practical procedure that presented good results.*

Keywords: *Singular value decomposition, Structural health monitoring, plate structure, damage detection, noise level, false positive*

INTRODUCTION

The use of modal analysis on structural health monitoring is a natural consequence of its use on structural dynamic. However, a major concern is that its validity is limited to linear structures Kerschen et al. (2005). Vanlanduit (Vanlanduit et al.) said that methods based on modal parameters are sensitive to changes in environment, operational conditions and structural uncertainties. The proper orthogonal decomposition (POD) method has been proposed aiming to remove the influence of these external factors (Vanlanduit et al., 2005; Shane and Jha, 2011).

The proper orthogonal decomposition (POD) is a powerful multi-variate statistical method for data analysis aimed to obtaining low-order approximate descriptions of a high-dimensional process, (Wu et al. (2003)). It is an essential technique for data reduction and feature extraction, and has been widely used in various areas including image processing, signal analysis, data compression, process identification, adaptive control, and many others Wu et al. (2003).

The POD is an orthogonal transformation that decorrelates the signal components and maximizes variance. By this way the POD method reduce a large number of interdependent variables to a much smaller number of uncorrelated variables while retaining as much as possible of the variation in the original variables (Kerschen et al. (2005)). The discrete implementation of the proper orthogonal decomposition is the popular singular value decomposition (Kerschen and Golinval, 2002; Wu et al., 2003; Shane and Jha, 2011).

There are different implementations of the proper orthogonal decomposition method to structural health monitoring (Ruotolo and Surace, 1999; Vanlanduit et al., 2005; Shane and Jha, 2011). Vanlanduit (2005), proposed the robust singular value decomposition method (RSVD) that uses the frequency response functions (FRFs) of displacement, force, acceleration or other output as a basic data for the method. Vanlanduit work presented experimental results using a statistical treatment to validate it. This work is closely related to implementation of the robust singular value decomposition method (RSVD) as published by Vanlanduit et al. (2005), which could find fault signals on a set with healthy and damaged data.

The SVD and RSVD complete procedure as presented by Vanlanduit et al. (2005) will be presented in the next Section. But in all failure detection using SVD methods the noise level must be evaluated. Usually the database is obtained from time or a frequency response function, FRF, a $M \times N$ matrix $H = [H_1, H_2, ...H_N]$ at N specified conditions, from specified positions. $N = N_u + N_d$ where N_u is the undamaged experimental FRFs from healthy structure and N_d is from damaged ones. The damaged database $N_D = \sum N_d$ ordinarily equal or lesser than $N_U = \sum N_u$. The first step is computing the SVD of the $M \times N$ matrix $H = USV^H$. Putting all singular values $s_{k+1}, ..s_N$, from $S = diag(s_1, s_2, .., s_N)$, bellow the noise level equals zero, $S1 = diag(s_1, s_2, .., s_k, 0, 0, .., 0)$. Then using singular vectors U and V from original matrix synthesize matrix $H1$, original size and rank k, $H1 = US1V^H$ and so computing residual matrix $E1$, $E1 = H - H1$.

Some techniques consider the failure as a outlier or an outstanding data, while there are specific statistical procedures that were developed for outlier measurements Vanlanduit et al. (2005). Of course if there are a great failure level this measurement will be a outlier, in this work the failure data come from added mass, 4.2×10^{-3} kg on a 2.7 kg plate, 0.16% of the total mass. Outlier will be considered here as a data biased by measurement like noise, non-linear behaviour, sensor or other measurement problem.

The main problem experienced was to identify the correct noise level that has showed to be critical for data obtained in this work. In previous works was retained only the first k singular values whose sum performed 99% of total sum, $\sum s_i$, $(\sum s_k < .99 \times \sum s_i)$. This criteria has worked fine for previous low noise measurement. For the FRF data presented in this work, a more noisy measurement, this criteria has not been so effective.

In Shane (Shane and Jha, 2011) work, different random force inputs are used to introduce variations in the loading conditions of a finite element model. The measurement matrix were assembled by results from 25 points. The first four singular value were chosen as having all the signal energy (or $\sum s^2_{k=4} \approx \sum s^2_{i=25}$). The remaining 21 singular values were considered under the noise level. Those data were taken from a graphic where the singular values were plotted in decreasing order so the highest values were selected. But there is no rule to stablish a threshold to indicate the reduction order. Although working with experimental results, in Valanduit work, (Vanlanduit et al., 2005), there is no discussion about this subject.

Ruotolo (Ruotolo and Surace, 1999) one first authors using SVD in failure detection, presents more detail on settlement of the dynamic reduction order avoiding noise in matrix reconstruction. This is graphical method, the singular value logarithmic are plotted in a decreasing order. The singular values related to measurement noise level will appear on a line, for a low noise data, almost parallel to horizontal axis. Any singular value greater than this reference line can be related to the state of the structure. The Shane energy method seems similar but there is not detail to settle a reference value.

The main objective of this work is to test different procedures to find out the best methods to establish a threshold reduction level to become more effective the identification failure procedure in a noise measurement. The experimental data was done with PZT sensors to fulfil measurement request at high frequency. This kind of sensor could be embedded or bonded to structure, a requisite to health structural monitoring procedure (Konstantinidis et al. (2006)), allowing good repeatability of measurements, essential in this frequency range.

The results presented in this work were made using FRF data from a Aluminum plate. The damaged structure was the same plate with an attached mass. The sensitivity analysis of the detection algorithm were performed using a set of FRFs from damaged and healthy structure. The measurements were taken at same PZT, for both healthy and damaged structure, the excitation PZT was also at same position. The detection algorithm was employed for a high frequency range, from 1.3 kHz to 2.3 kHz.

Although this frequency range can not considered high frequency in many Healthy Structure Monitoring applications indeed it is in modal analysis. The main idea is not work in high frequency but with wavelength dimensions close to failure dimension that could result in great FRF change in this range. To obtain those small wavelength, in this frequency range, it was used a slender plate.

DAMAGE DETECTION ALGORITHM

The classical Least-Squares SVD-based damage detection can be outlined as follow. The database is obtained from frequency response function, FRF, a $M \times N$ matrix $H = [H_1, H_2, ...H_N]$ at N specified conditions, each matrix H_n has FRF data from each P specified positions. $N = N_u + N_d$ where N_u is the undamaged experimental FRFs from healthy structure and N_d is from damaged ones. The damaged database $N_D = \sum N_d$ must be equal or greater than $N_U = \sum N_u$. The sequence steps of the damage detection algorithm are presented below.

1- Computing the SVD of the $M \times N$ matrix $H = USV^H$;
2- Putting all singular values $s_{k+1}, ..s_N$, from $S = diag(s_1, s_2, .., s_N)$, bellow the noise level equals zero, $S1 = diag(s_1, s_2, .., s_k, 0, 0, .., 0)$;
3- Using singular vectors U and V from original matrix synthesize matrix $H1$ of rank k $H1 = US1V^H$;
4- Computing residual matrix $E1$, $E1 = H - H1$;
5- Computing the variance s of the residuals:

$$s = \frac{1}{MN-1}\sum_{i=1}^{M}\sum_{j=1}^{N}(H_{i,j} - H1_{i,j})^2 \tag{1}$$

6- Computing the variance of each measurement j (each column j):

$$s_j = \frac{1}{M-1}\sum_{i=1}^{M}(H_{i,j} - H1_{i,j})^2 \tag{2}$$

7- Estimating the relative distance d_j^{SVD} of each measurement j to subspace computed by SVD:

$$d_j^{SVD} = \frac{s_j}{s} \tag{3}$$

As the relative distance d_j^{SVD} is defined as a variance ratio it obeys a χ^2 distribution. For a χ^2 distribution with a confidence level of $(100 - \alpha) = 95\%$ and for $(N_n - 1)$ degrees of freedom up to 30, $(N_n - 1) < 30$ (Vanlanduit et al., 2005), a threshold T can be defined as

$$T = \sqrt{\frac{\chi^2_{(100-\alpha)_{(N_n-1)}}}{2}} \tag{4}$$

If $d_j^{SVD} > T$ then there are a damaged sample. For N >30 a maximal value of T for N= 30 is taken in order to make sure that the threshold is not too tight, (Vanlanduit et al., 2005).

The Robust SVD

The problem with the LS-SVD based damage detection is that it is very sensitive to outliers in measurement, Vanlanduit (2005). To avoid this, the robust method was proposed, and it does not compute the whole database H, the SVD is computed only over $N/2$ observations, the original matrix is re-synthesized and the distances and the error are computed. The solution is the small cost function from all possible combinations of the database. The steps of the RSVD routine are:

1- Construct a matrix H_R from matrix H with $L = N/2$ combination columns from N columns of the original matrix.

a- computing the SVD of the matrix $H_R = U_R S_R V_R^H$;

b- computing the extended right singular vector $V_E = U_R^H S_R^{-1} H$;

c- computing re-synthesized matrix H_S using $H_S = U_R S_E V_E^H$, $S_E(i,i) = S_R(i,i)$ $i = 1, 2, ..., L$ and $S_E(i,i) = 0$ if $i > L$;

d- Computing the residuals $E_S = H - H_S$;

e- Computing the variance s of the residuals using the median absolute deviation MAD, modifying the step 4 of SVD-based damage detection, $s = MAD(E_S)$, where:

$$MAD(E_S) = 1.4826 * median(|E_S - median(E_S)|) \tag{5}$$

f- Computing the variance for each measurement j (each column j):

$$s_j = \frac{1}{M-1}\sum_{i=1}^{L}(H_{i,j} - H_{S(i,j)})^2 \tag{6}$$

g- Estimating the relative distance d_j^{SVD} for each measurement j to subspace computed by SVD:

$$d_j^{SVD} = \frac{s_j}{s} \tag{7}$$

h- computing the cost function $\kappa = \sum_j^L |d_j|^2$;

2- The smallest cost function κ is taken as the RSVD solution, $H_B = H_S$. The variance s and d_j^{SVD} are computed following the procedures from **d** to **g** in the previous steps. As the relative distance d_j^{SVD} is defined as a variance ratio it obeys a χ^2 distribution. For a χ^2 distribution, the threshold T is the same as defined on Eq. 4 and there was employed the same confidence interval.

EXPERIMENTAL SETUP

The experimental procedure was performed using dynamic response of a free Aluminium plate suspended by a thin nylon wire. The Aluminium plate is 1 m × 1 m, Fig.1. The measurement database was obtained from FRF response at measurement points A, B and D, Fig. 1, the reference was PZT 1 and excitation at PZT E, on plate opposite side just behind PZT 1 position, Fig. 1. The results from PZTs B and D was not used in this work.

Using the excitation signal, PZT E, as reference to obtain the FRFs at PZTs A, B and D has resulted in noisy FRFs at the intended frequency range. To improve the results, the reference signal was chosen to be from PZT 1, and not from the excitation signal PZT E. This procedure ensured a good signal to noise relation and improved the results.

Experimental data acquisition was performed by the PHOTON II from LDS. Measurements were done using two different excitation signals, white noise and sweep-sine signals both generated by the PHOTON II. From the generator the signal was send to amplifier and then to PZT E, Fig.2. All FRFs have units Volts/Volts (response/excitation) in spam

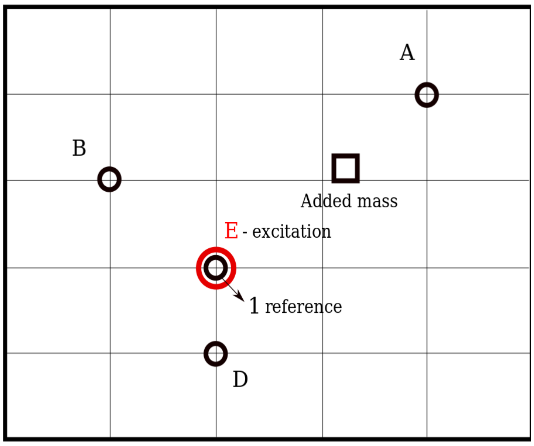


Figure 1: Aluminium plate and PZT sensor position.

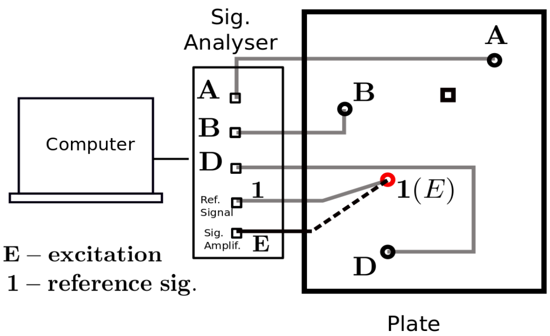


Figure 2: Measurement setup layout. PZT E (excitation), PZT 1 (signal reference), PZT A measurement.

of 2.4 kHz and 3200 line resolution. The employed frequency, in the range from 1.3 to 2.3 kHz, was obtained using a shaped excitation curve for random excitation signal, the same initial and final frequency defines the swept sine signal excitation in range employed.

The healthy structure has $1\,\text{m}\times1\,\text{m}\times1.0\times10^{-3}\,\text{m}$, 2.7 kg, data from vendor. The damaged structure is the same plate with 4.2×10^{-3} kg added mass which represents about 0.16% of the structure's mass.

The PZT effective diameter is $D_{PZT}\approx6.5\,\text{mm}$, with an area of $3.318\times10^{-5}\text{m}^2$. The passive or measurement PZT sensor, sensors 1, A, B and D are from Acellent Technologies. The position E excitation diaphragm PZT has an effective diameter $D^E_{PZT}\approx18\,\text{mm}$, placed behind reference sensor PZT 1. Its area is $2.5\times10^{-4}\text{m}^2$, approximately 0.03% of the plate area. The added mass was placed on squared mark, Fig. 1 and Fig. 2.

NOISE IDENTIFICATION

As stated before the core of failure identification by RSVD depends on correct reduction of the singular values avoiding measurement and numerical noise. The main methods employed was outlined on Introduction, the Ruotolo method (Ruotolo and Surace, 1999), the Energy method as in Shane (Shane and Jha, 2011) work.

Ruotolo Method

Ruotolo (Ruotolo and Surace, 1999) had developed a method for failure identification using the Damage Index as a criteria based on Singular Value Decomposition. To avoid non significant singular values related to measurement and numerical noise level there were employed a graphical procedure. A logarithm of singular values in decreasing order is plotted, Fig.3, best regression line matching the small singular value is chosen to be the threshold, the dashed regression line on Fig. 3. Singular values in this line were considered noise.

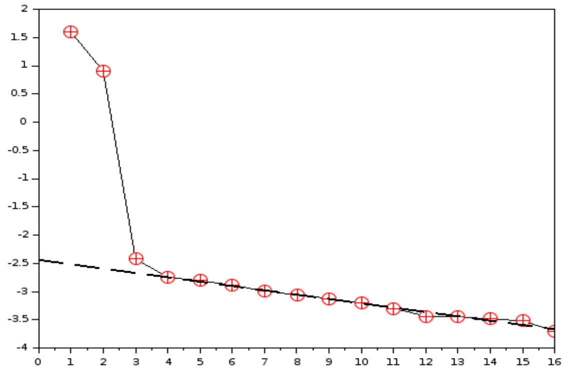


Figure 3: Ruotolo Method. Dashed threshold line.

In this work the threshold value was the point from regression line with the vertical coordinate axis, so all singular values below the threshold point value were discarded. The best regression line is those regression obtained by eliminating the first and highest values from singular value dataset until the inclination converge.

Energy Method

As in Shane work (Shane and Jha, 2011), the Energy Method is related to signal energy or sum of the N squared singular values, $E_s = \sum s^2_{i=N}$. The first L singular values ($L < N$) whose sum were close to total energy (or $\sum s^2_{k=L} \approx \sum s^2_{i=N}$) were chosen as having all the signal energy. The remaining singular values were considered under the noise level. This noise level was chosen from first representative singular values from a set assembled by damaged a healthy data, usually the first two singular values for this data.

RESULTS AND ANALYSIS

The analysis presented in following sections were made using FRF data from PZT at positions A using PZT 1 as reference, Fig. 1 and Fig. 2. At each position there are 16 FRFs for healthy structure, the reference set was compounded by a subset with 8 FRFs using Shaped Random signal and 8 measurements using Swept Sine signal. The set for $4.2{\times}10^{-3}$ kg added mass structure, the damaged set, were made up in similar way.

Ruotolo Method

On Fig. 4 there are the RSVD relative distance, d^{SVD}, graphic mark height, for each measurement, numbers on abscissa, threshold blue horizontal line. The first 8 measurements, on abscissa, are related to healthy reference set, and the following 8 marks to damaged set or 4.2×10^{-3} kg added mass structure. On Fig.4 there are the results using Shaped Random signal. On Fig.4-(a) there are no positive failure indications in the frequency range from 1.3 to 1.55 kHz. On Fig.4-(b), frequency range from 1.55 to 1.8 kHz, there is a positive failure on healthy measurement set. On Fig.4-(c) there are 3 and on Fig.4-(d) there are 2 failure indications for a frequency range from 1.8 to 2.05 kHz and for 2.05 to 2.3 kHz respectively.

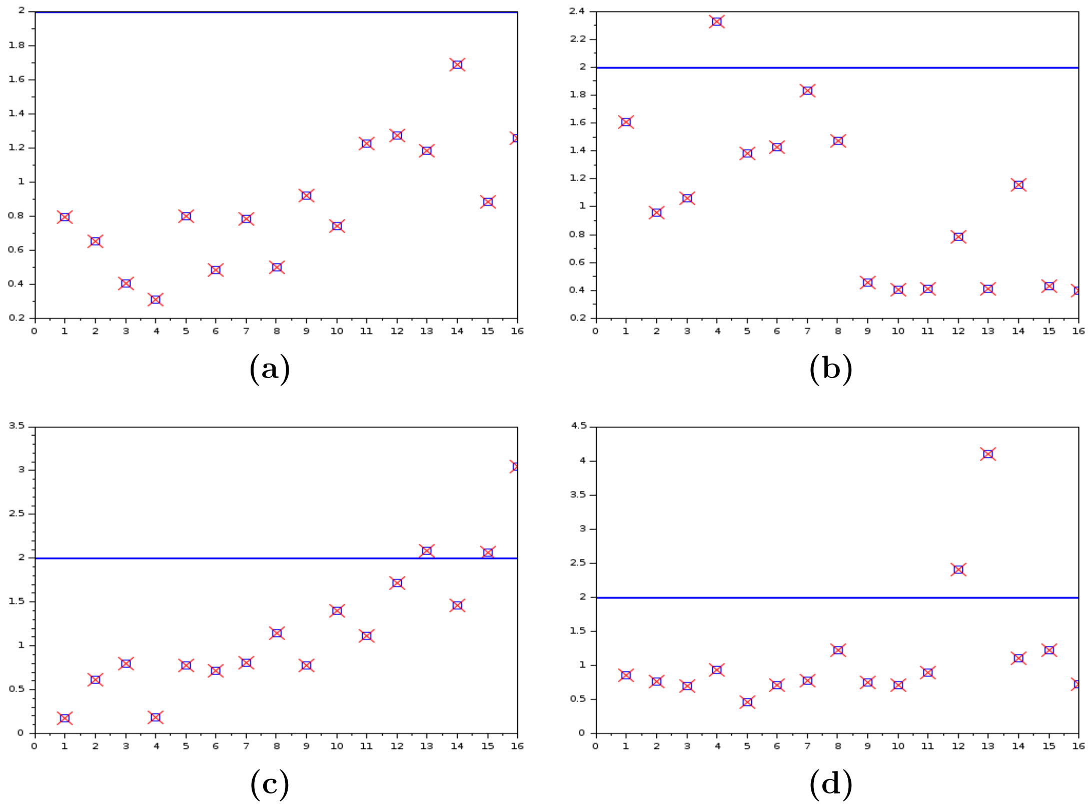


Figure 4: Ruotolo Method for healthy and 4.2×10^{-3} kg added mass structure. Shaped random signal, frequency range (a) 1.3 to 1.55 kHz, (b) 1.55 to 1.8 kHz, (c) 1.8 to 2.05 kHz, (d) 2.05 to 2.3 kHz.

On Fig.4-(b), the positive failure appear on healthy measurement set. The failure indication is expected to show up on damaged sets but knowing a healthy dataset in advance all distance above the acceptable level was considered a failure indication. The algorithm deals only with relative distances, d_j^{SVD}, between datasets and there is no mathematic basis to believe that the healthy set would be the reference.

Energy Method

As in the previous case on Fig.5 the first 8 measurements, on abscissa, are related to healthy reference set, and the following 8 marks to damaged set.

On Fig.5-(c), Fig.5-(d) there are respectively 3 and 2 marks showing relative distance above the threshold line or damage line. Those results are alike to previous one excepted by the positive failure indication for frequency range from 1.55 to 1.8 kHz on Fig.4-(b). The Ruotolo Method and Energy Method seem to show a similar performance.

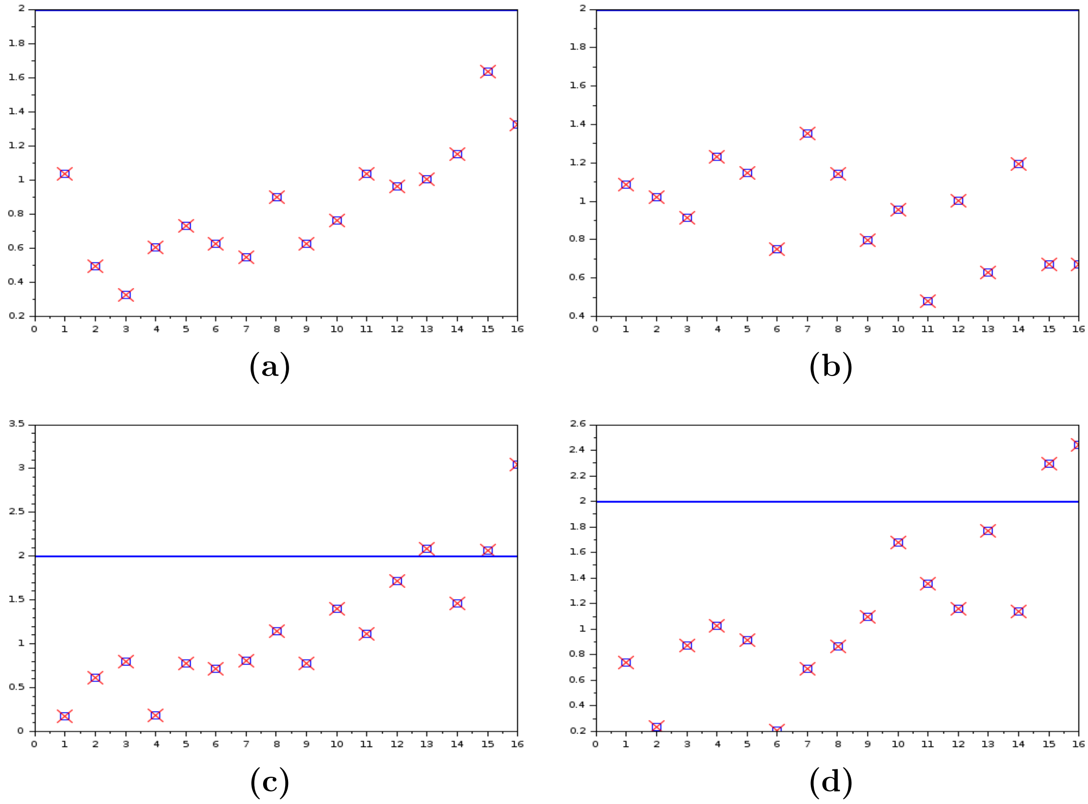


Figure 5: Energy Method for healthy and 4.2×10^{-3} kg added mass structure. Shaped random signal, frequency range (a) 1.3 to 1.55 kHz, (b) 1.55 to 1.8 kHz, (c) 1.8 to 2.05 kHz, (d) 2.05 to 2.3 kHz.

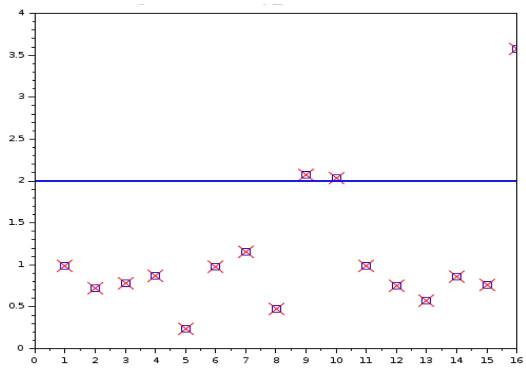
False Positive

The same procedure was run again but using only data from the healthy structure, the difference between them was the excitation signal. The Shaped Random signal set and Swept Sine signal set both from healthy structure were compared using the same Methods and same parameters as used in previous sections.

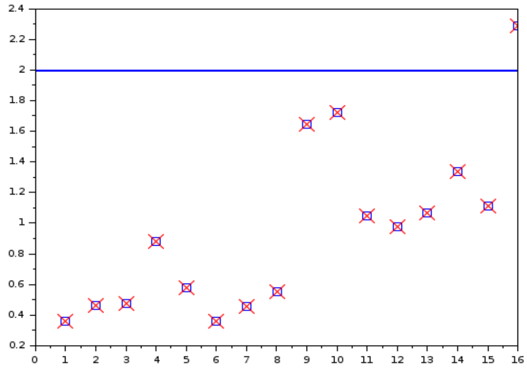
On Fig.6 there are the results for frequency range from 1.3 to 1.55 kHz, Fig.6-**(a)** and from 1.55 to 1.8 kHz, Fig.6-**(b)** using the Ruotolo Method. The Energy Method are presented on Fig.6-**(c)** from 1.3 to 1.55 kHz and Fig.6-**(d)** from 1.55 to 1.8 kHz.

For the same frequency range using the Ruotolo Method, there are remarkable differences from damaged structures, presented on Fig.4-**(a)**,**(b)** and those comparing different signal from healthy structure, Fig.6-**(a)**,**(b)**. The same behaviour can be seem for the Energy Method on Fig.5-**(a)**,**(b)** and Fig.6-**(c)**,**(c)** for the same frequency. No Method presented false positive indications in the range from 1.8 to 2.3 kHz. In this range the results were not showed.

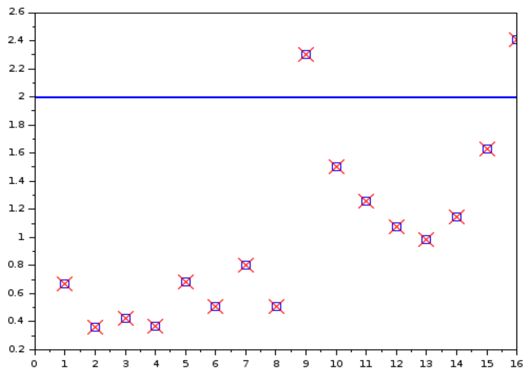
This behaviour became more curious when comparing the FRFs from healthy and 4.2×10^{-3} kg added mass structure, Fig.7, the damaged structure, with those FRFs from Shaped Random and Swept Sine signal, the healthy structure, Fig.8. On Fig.7 the differences between the two signals are more strong than those presented on Fig.8. Although the algorithm presented a strong failure indication for the healthy case it was not able to



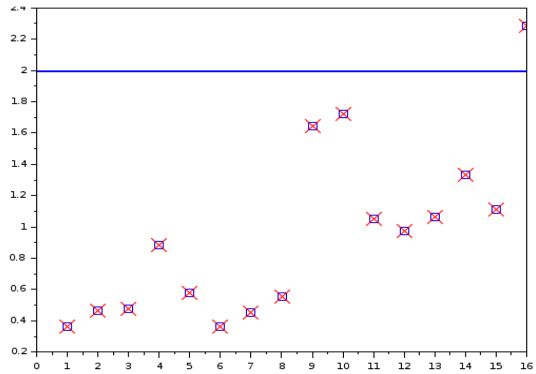
(a)



(b)



(c)



(d)

Figure 6: False Positive for Ruotolo Method, Shaped Random signal $\times$ Swept Sine signal both from healthy structure. Frequency range (a) 1.3 to 1.55 kHz, (b) 1.55 to 1.8 kHz. False Positive for Energy Method, frequency range (c) 1.3 to 1.55 kHz, (d) 1.55 to 1.8 kHz.

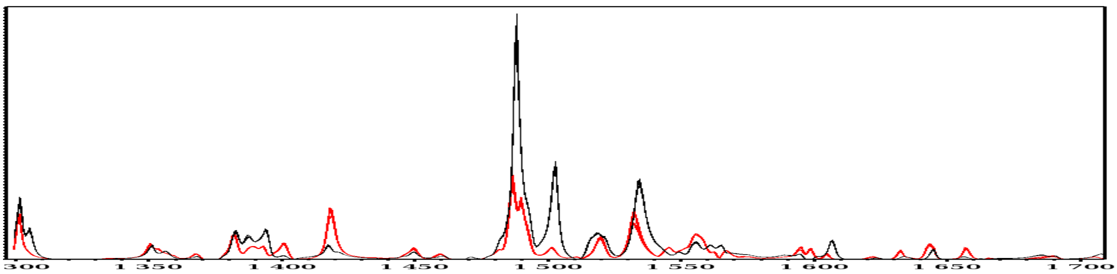


Figure 7: FRFs for healthy, black lines, and 4.2×10^{-3} kg added mass structure, red lines, from 1.3 to 1.7 kHz.

identify the real failure case.

To explain those behaviour the squared singular values was plotted on Fig.9 for both cases. The squared singular values were used to make clear the difference between the two cases. The singular value for healthy and 4.2×10^{-3} kg added mass structure problem are in black lines, the first two significant values received the "$\times$" mark. The singular values from Shaped Random and Swept Sine signal from healthy structure case are plotted using red lines, the first two significant values received the "$\circ$" mark.

Comparing the first two singular values the differences between the two cases are

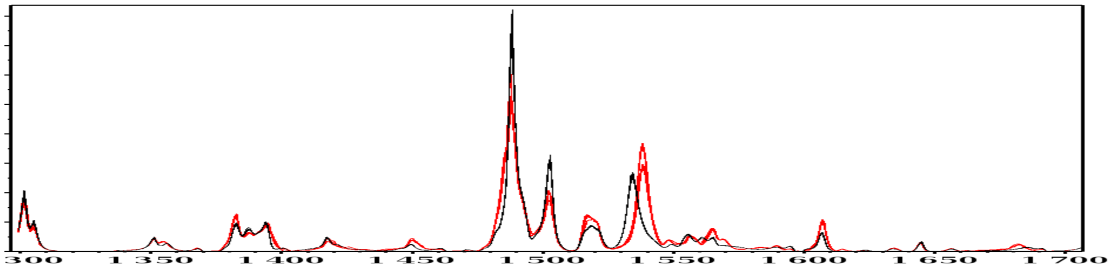


Figure 8: Shaped Random and Swept Sine signal from healthy structure. FRFs for shaped random, black lines, and swept sine red lines from 1.3 to 1.7 kHz.

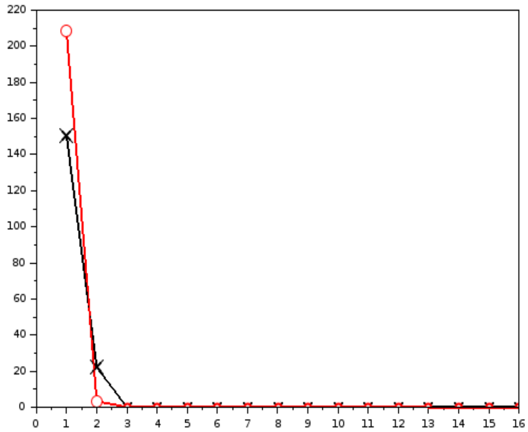


Figure 9: Squared Singular Values for healthy and 4.2$\times 10^{-3}$ **kg added mass structure result, black lines, and for Shaped Random and Swept Sine signal from healthy structure case.**

evident. For healthy and damaged structure the first two singular values are significant, the remaining are noise. For the healthy structure, or different signal case, only the first singular value is significant the second can be considered noise. So the false positive is related to the incapability of the Methods to eliminate this non significant singular value. To verify this statement the noise was adjusted to eliminate the second non significant singular value in the healthy case, on Fig.10 the results.

Adjusted the noise level, both methods presented the expected result for the health structure, Fig.10. To verify if this change in noise level parameter impair the results for damaged test case, the procedure was repeated for the healthy and 4.2×10^{-3} kg added mass. On Fig.11 there are the results for frequency range from 1.3 to 1.8 kHz. For the range from 1.8 to 2.3 kHz the results were equal for both methods and equal to those presented on Fig.4-**(c)(d)** and on Fig.5-**(c)(d)**.

On Fig.11 there are a great difference between the results for frequency range from 1.55 to 1.8 kHz. This could imply that the best noise level for this frequency range was not found.

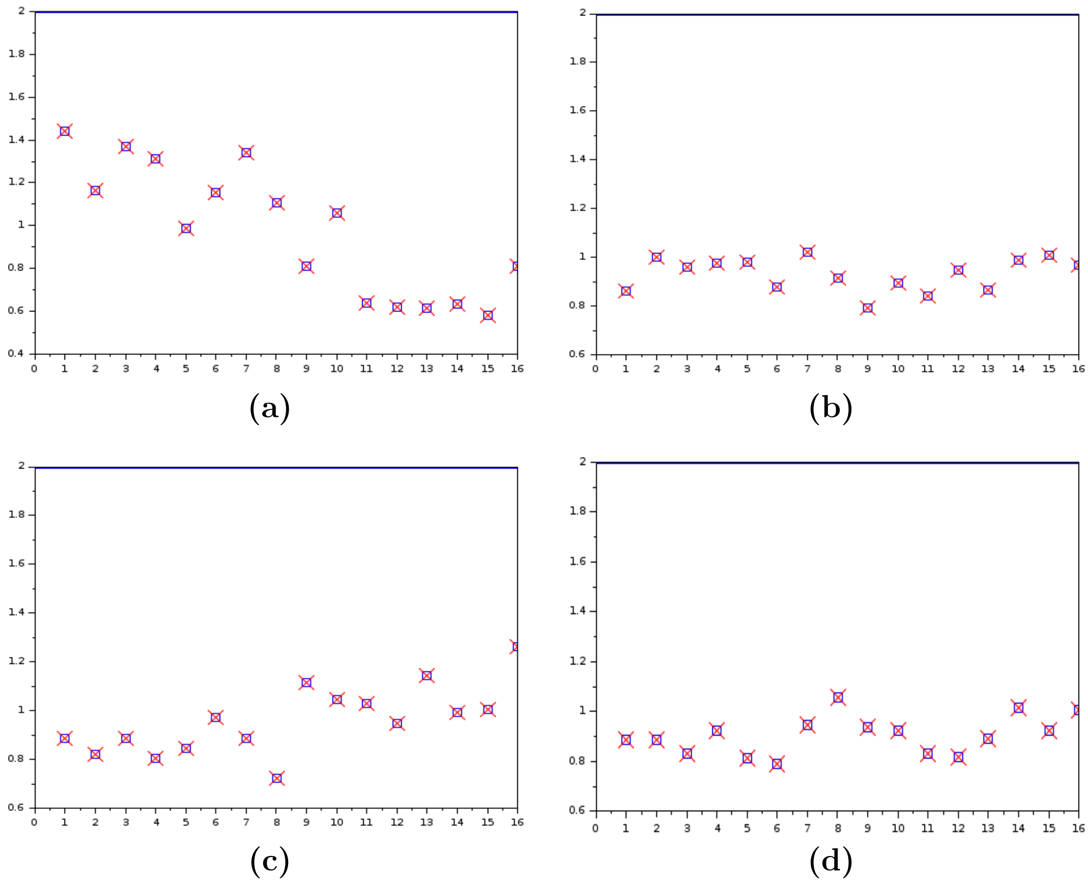


Figure 10: Adjusted noise parameters for Shaped Random signal × Swept Sine signal from healthy structure. Ruotolo Method, frequency range 1.3 to 1.55 kHz, (a), 1.55 to 1.8 kHz, (b). Energy Method, frequency range from 1.3 to 1.55 kHz, (c), from 1.55 to 1.8 kHz, (d).

CLOSING REMARKS

Both Methods were effective with similar results, although, the Energy Method is more fast. Using both methods is a procedure to assess the quality and to improve reliability of the results. The results showed sensibility to noise in both cases.

The results and following discussion about the procedures to find out the significance of singular values from the false positive case can be a practical method to adjusting the noise level. The Singular Value Decomposition Method identified the different signal FRFs from the healthy structure as similar data if there is only one significant singular value, a rank one matrix. A measurement matrix with rank one is a matrix with similar measurements and small deviations. So if there is only one kind of measurement the false positive between datasets were related to noise level.

So if there are changes in environment, operational conditions and structural uncertainties they should included in those reference measurement determining minimum noise level.

Comparing the FRFs from healthy structure and those from added mass damaged structure the singular decomposition method showed two singular value or two kind of measurement. Its a expected result for measurement at different positions, measurement

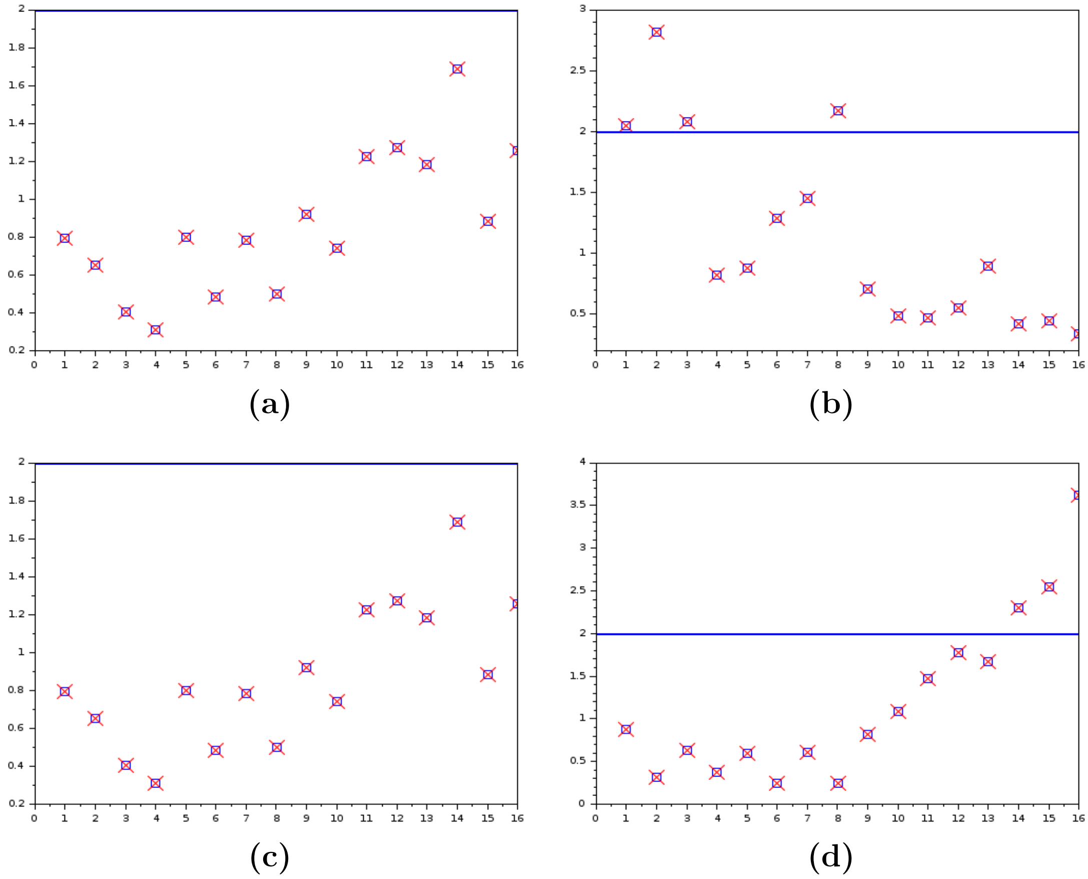


Figure 11: Adjusted noise parameters for damaged case. Ruotolo Method, frequency range from 1.3 to 1.55 kHz, (a), 1.55 to 1.8 kHz, (b). Energy Method, frequency range 1.3 to 1.55 kHz, (c), from 1.55 to 1.8 kHz, (d).

at same position is a damage indication if the significant singular values are above the noise level. Of course if the relative distance is greater than the statistical threshold, $d_j^{SVD} > T$.

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