# COMPUTATIONAL MODELING OF DYNAMIC CONTROL MECHANISMS WITH VISCOELASTIC MATERIALS

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***Abstract****. In this paper numerical modeling of composite structures and damping devices conceived with one or more layers of viscoelastic materials (VEM) is applied towards practical solutions of vibration problems in structural systems subjected to dynamic loadings.*

*Simple examples of composite structural systems are used to explore the potential of practical application of sandwich and multilayered VEM mechanisms to damp out vibrations induced by dynamic actions. The performance of these VEM damping devices are demonstrated by comparing response amplitudes of the uncontrolled and controlled structural systems.*

*The validation of the developed computational tool and the dynamic numerical modeling is done by means of correlation between obtained numerical results and their experimental counterpart for selected case examples.*

***Keywords:*** *Structural dynamics, Viscoelastic materials, Dynamic control, Damping, Numerical modeling*

# 1 INTRODUCTION

Mechanisms of dynamic structural control have been developed to reduce vibrations induced in civil structures by environmental actions like wind and earthquakes and also by human activities and machines. Among these mechanisms, dynamic control devices which make use of viscoelastic materials (VEM), have proved successful thanks to its great damping capacity in reducing substantially the amplitudes of the displacements and accelerations (Vasconcelos and Battista, 2004). The analytical modeling of viscoelastic materials in time-domain is complex because its behavior is clearly dependent on frequency and temperature, and governed by non-linear constitutive laws.

A software based on the Finite Element Method (FEM) was developed in recent past (Barbosa, 2000; Vasconcelos, 2003) for the dynamic analysis of composite structures with layers of viscoelastic materials taking advantage of the Golla-Hughes Method (GHM) (Golla et al., 1985) for the inclusion of VEM elements. The performed analyses included as examples different composite structures like sandwich-slabs and beams and also viscoelastic mechanisms typical of dampers. This computational tool has been recently extended to model and analyze the behavior and performance of multilayered viscoelastic mechanism intended to be applied as seismic base-isolation systems. Correlations between the dynamic responses of a multilayer damper element obtained with the updated numerical tool with those measured by other authors in experimental tests were made in order to validate the use of the computer software. The dynamic responses obtained from composite structures with layers of VEM corroborate previous assessments of the damping performance which is much influenced by the thickness of the viscoelastic layer and also by the exciting load frequencies. Based on the numerical results from the analyzed examples one can state that the design of a dynamically controlled structure by means of layers of VEM must consider in a first stage, the dynamic analysis of the original non-controlled structural system to determine its modal properties and to estimate the maximum displacements and accelerations amplitudes for the design dynamic loading, to then define the type and the location of the multilayered mechanisms of VEM controllers. The mechanisms of dynamic control that are object of the present study correspond mainly to viscoelastic devices which are applied to reduce vibrations in civil structures.

# 2 VISCOELASTIC MATERIALS: AN OVERVIEW

The constitutive law of a viscoelastic material depends not only of the dynamic load magnitude, but also of the exciting frequency range and temperature. Altogether these characteristics have influence on the capacity of a viscoelastic material to dissipate energy.

Viscoelastic properties of polymers and damping properties of rubberlike materials may be found in the classical technical literature (Nolle, 1950; Ferry, 1970; Nashif et al., 1975; Jones, 2001).

Viscoelastic materials share elastic properties of solids with viscous properties of fluids. In high frequencies, the response is analogue with elastic solids. In that state, there is a linear relationship between stress and strains. For lower range of frequencies, the behavior is similar to fluids, for which there is a linear relationship between the shear stress and the strain rate.

Figure 1 illustrates the response of different materials to an oscillatory deformation. In case of elastic solids, the phase angle is zero, while in viscous fluids the phase angle is 90°. Viscoelastic materials are in a middle state, with a phase angle between 0° and 90°.

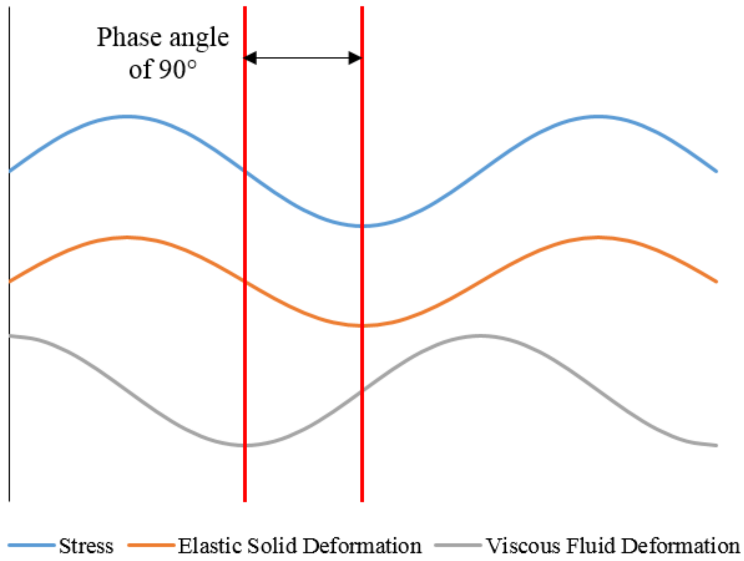


**Figure 1. Response of elastic solids and viscous fluids to an oscillatory deformation**

The Eq. (1) represents the oscillatory response in terms of shear strain (ϒ) for an applied harmonic dynamic load.

$$\gamma = \gamma o \times sen(\omega t) \tag{1}$$

where "ϒo" is the maximum amplitude of shear strain, and "ω" is the circular frequency. If the response corresponds with an elastic body, the relation between stresses and strains is given by Eq. (2).

$$\tau = G \times \gamma o \times sen(\omega t) \tag{2}$$

where "G" is the shear modulus, and "τ" is the shear stress. For a viscous body, the phase angle between stress and strains is 90°, and the relation between them may be written as follows:

$$\tau = \mu \times \omega \times \gamma o \times cos(\omega t) \tag{3}$$

where "μ" is the coefficient of viscosity. The shear stress response of a viscoelastic body to a cyclic force is a combination of that obtained for an elastic body, with another similar to the one obtained for a viscous body. Hence, stress versus strain relationship in viscoelastic bodies is given by Eq. (4).

$$\tau = \gamma o \times \left[ G' \times sen(\omega t) + G'' \times cos(\omega t) \right] \tag{4}$$

where G' is known as the storage modulus, and represents the capacity of the material to restore deformation energy in every loading cycle. G" is the loss modulus, that is related to the dissipated energy in the form of heat. An alternative form to represent the relation between the storage and loss modulus is by means of the complex modulus (G*), as given in the Eq. (5) and Eq. (6).

$$\tau = \gamma o \times G^{*} \tag{5}$$

$$\tau = \gamma o \times \left[G' + iG''\right] \tag{6}$$

The longitudinal modulus may be written in terms of the transverse modulus with the aid of Poisson's coefficient. Eq. (7) and Eq. (8) show this relation:

$$E' = G' \times \left(1 + 2\upsilon\right) \tag{7}$$

$$E'' = G'' \times \left(1 + 2\upsilon\right) \tag{8}$$

The loss factor (η) in Eq. (9), by the ratio between the loss modulus and the storage modulus, expresses the capacity of the material to dissipate energy.

$$\eta = \frac{E''}{E'} = \frac{G''}{G'} \tag{9}$$

Figure 2 adapted from Nolle (1950) shows the dependence of the storage modulus and the loss factor with the temperature and frequency. The first zone indicated on the left of the figure, is where the values of the parameters are the lowest, and correspond to a state of high temperatures or low frequencies. This is known as relaxation state, without great variations of stresses and zero phase angle between the excitation and the response.

The second zone on the middle of the figure is reached when the temperature decreases or when the frequency is higher. In this zone there is an increase in the values of the storage modulus and of the loss factor. In this transient zone there is a phase angle between the excitation and the response, and the loss factor reaches its peak value, helping to dissipate energy with more efficiency.

The last zone of the right of the figure corresponds to a state of the highest frequencies or the lowest temperatures. The storage modulus reaches a maximum value and keeps approximately constant. The loss factor decreases and the potential of dissipation of energy is lower in relation to the transient state. The behavior of the material is somewhat vitreous, similar to a hard elastic body.

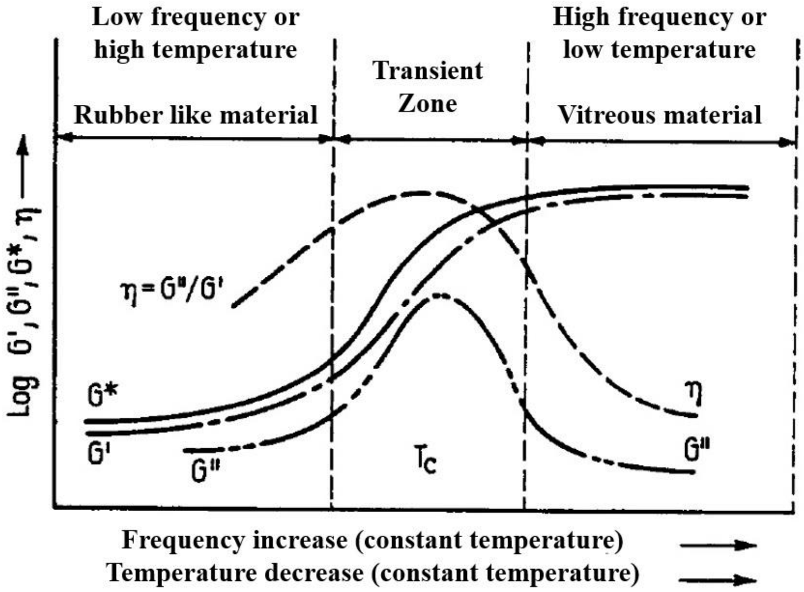


**Figure 2. Influence of frequency and temperature on the dynamic mechanical properties of VE materials (Adapted from Nolle, 1950)**

In the theory of linear elasticity the relationship between stress and strain is set by the elastic modulus. Taking advantage of this it is possible to adapt the constitutive law to viscoelastic materials (Eq. (10)), by considering that in a constant temperature state the complex modulus is not constant, with variation of frequency.

$$\sigma(\omega) = E^{*}(\omega) \times \varepsilon \tag{10}$$

The complex modulus can be written as a combination of the storage and loss modulus, and the Eq. (11) expresses the constitutive law in frequency domain. Equation (11) may be transformed to Laplace domain, as given by Eq. (12):

$$\sigma(\omega) = \left[E'(\omega) + iE''(\omega)\right] \times \varepsilon \tag{11}$$

$$\sigma(s) = E(s) \times \varepsilon \tag{12}$$

where E(s) is the complex modulus in Laplace domain, which is represented by the sum of an elastic component ε and a dissipation function h(s) as follows:

$$E(s) = \in + h(s) \tag{13}$$

# 3 DYNAMIC EQUATIONS OF A COMPOSITE STRUCTURE WITH LAYERS OF VEM

Some dissipation functions have been proposed to characterize the complex modulus in Laplace domain. In this work, the Golla Hughes method (Golla et al., 1985) is applied to formulate the dynamic equations of a composite structure with layers of VEM. The dynamic response is obtained in time domain by solving the equation of motion expressed in complex form, written in the Laplace domain. In this work, it has been considered a constant temperature and the influence of frequency on the dynamic mechanical properties of VEM.

In the present work it is adopted the dissipation function proposed by Biot (1955), expressed by a summation showed in Eq. (14). According to some researchers (Golla et al., 1985) just two terms of the series as written in Eq. (15) are necessary to get a good approximation of the VEM properties.

$$h(s)=\sum_{i=1}^{n}\frac{a_i s}{s+b_i} \tag{14}$$

$$h(s)=\frac{a_1 s}{s+b_1}+\frac{a_2 s}{s+b_2} \tag{15}$$

It is possible to make some variables changes, expressing the dissipation function in terms of “α”, “β”, “ϒ” and “δ”, as follows:

$$\alpha=a_1+a_2>0 \tag{16}$$

$$\beta=b_1+b_2>0 \tag{17}$$

$$\gamma=a_1b_2+a_2b_1>0 \tag{18}$$

$$\delta=b_1b_2>0 \tag{19}$$

The Eq. (20) shows the constitutive laws in dependence of the new variables. In this expression, it is possible to identify the storage modulus (Eq. (21)) and the loss factor (Eq. (22)).

$$\sigma(s)=\left[\in+\frac{\alpha s^2+\gamma s}{s^2+bs+\delta}\right]\times\varepsilon \tag{20}$$

$$E'(\omega)=\in+\frac{\alpha\omega^2(\omega^2-\delta+\beta^2)}{(\delta-\omega^2)^2+\beta^2\omega^2} \tag{21}$$

$$\eta(\omega)=\frac{\alpha\beta\omega\delta}{(\delta-\omega^2)^2+\beta^2\omega^2}\times\frac{1}{E'} \tag{22}$$

Figure 3 adapted from Barbosa (2000) shows the storage modulus and loss factor for a wide range of frequencies ($\epsilon$=1MPa, $\alpha$=5MPa, $\beta$=6.000s$^{-1}$ and $\delta$=1.200.000s$^{-2}$).

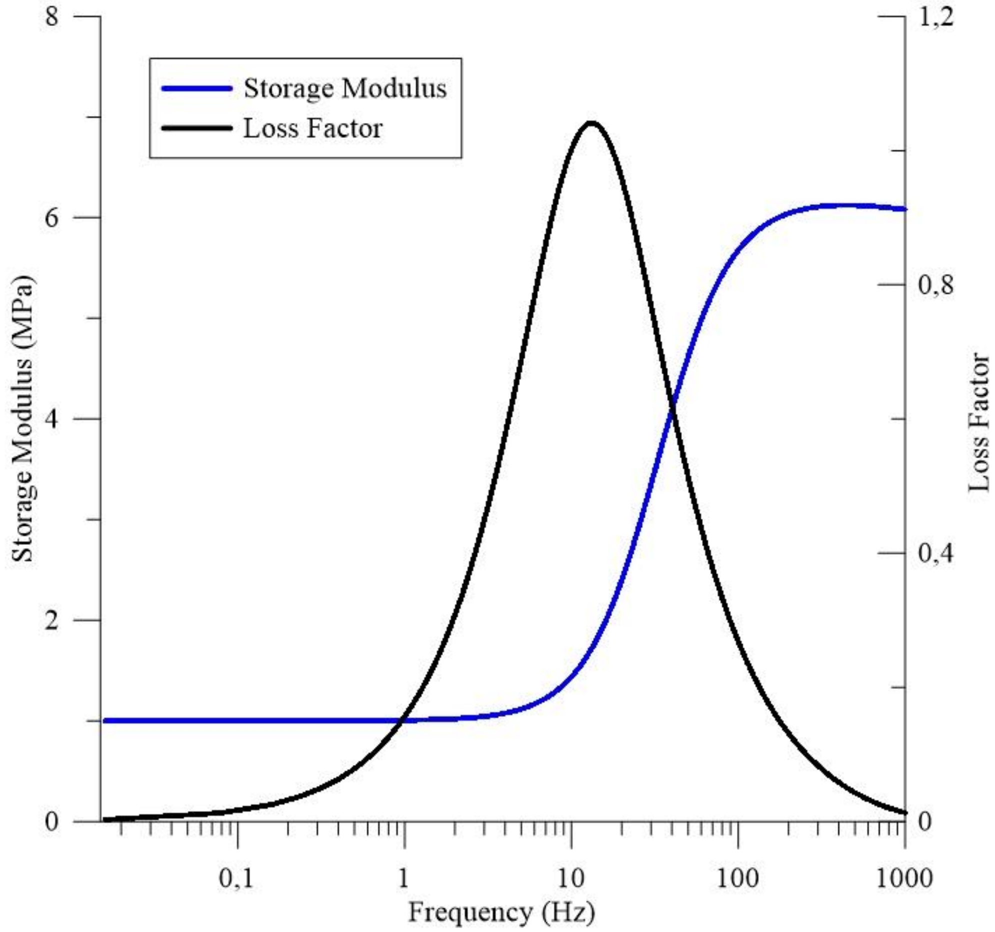


**Figure 3. Variation of the Storage Modulus and Loss Factor**

# 4 NUMERICAL MODELING OF VISCOELASTIC DAMPED MECHANISMS

The numerical modelling is made with a program based in the finite element method (FEM). The software was first developed by Barbosa (2000), and later augmented and optimized for research purposes (Vasconcelos, 2003). In the last version of the program three types of elements are available: frame element, plane shell element for elastic materials and hexahedral elements for viscoelastic materials. The last two elements were used in the modeling of the composite structures presented in this work.

The plane shell element is based in Reissner-Mindlin theory for plates in bending which considers shear deformations. A process of reduced selective integration is used for calculate the bending deformation energy with just some points in the dominium. This shell element compared by membrane and plate elements has four nodes, with three degrees of freedom (DOF) in every node (two rotations about the main axis that defining the plane element plus the transversal displacement), resulting in a total of 24 DOF.

Mass and stiffness matrices are formed for elements of elastic materials and the damping matrix is built considering Rayleigh's damping, leading to a damping matrix proportional to the mass and the stiffness matrices, according to Eq. (23).

$$[C] = a_0[M] + a_1[K] \tag{23}$$

The element that characterizes viscoelastic materials is the 8-node hexahedron with three translational DOF in every node. The GHM method establishes that for every elastic DOF, it is necessary to add a dissipator DOF. In this case, the total number of DOF is 48, but after eliminating the six spurious modes of rigid body, the final number of DOF is 42.

Equations (24), Eq. (25) and Eq. (26) show, respectively, the mass, damping and stiffness matrices for considering an 8-node hexahedron element of viscoelastic material.

$$[M^V] = Diagonal\left[M^E, \frac{\alpha_1}{\delta_1}, \ldots, \frac{\alpha_m}{\delta_m}\right] \tag{24}$$

$[M^E]$ represents the elastic mass matrix of the hexahedral element.

$$[C^V] = Diagonal\left[0, \frac{\alpha_1\beta_1}{\delta_1}, \ldots, \frac{\alpha_m\beta_m}{\delta_m}\right] \tag{25}$$

$$[K^V] = \begin{bmatrix} (\in + \sum_{i=1}^{m} \alpha_i)K^E & \alpha_i R_i & . & . & . & \alpha_m R_m \\ \alpha_i R_i{}^T & \alpha_i & 0 & & & 0 \\ . & 0 & . & & & . \\ . & & & . & & . \\ . & & & & . & 0 \\ \alpha_m R_m{}^T & 0 & . & . & 0 & \alpha_m \end{bmatrix} \tag{26}$$

where $[K^E]$ represents the elastic stiffness matrix of the hexahedral element and [R] is defined as follows:

$$[R] = [R_f]\Lambda_f^{1/2} \tag{27}$$

being $[R_f]$ a matrix formed with the eigenvectors of flexible body modes of the hexahedral element, and $[\Lambda_f]$ a diagonal matrix with the eigenvalues related to these modes.

The global matrices of mass, stiffness and damping matrix for elastic and viscoelastic materials, resulting from the numerical modeling of a composite structure are then taken into the modal equations of motion which may be solved by Newmark's numerical integration method (Newmark, 1959) to obtain time-history responses in terms of displacements, velocities and accelerations.

# 5 NUMERICAL EXAMPLES

## 5.1 Numerical validation of viscoelastic coupling dampers (VCD)

Validation of the numerical modeling briefly described here in has been made through correlation with the results of a group of experimental tests of viscoelastic coupling dampers, carried out by Montgomery and Christopoulos (2015). These damping devices were devised to enhance the performance of buildings subjected to seismic and wind loads.

Figure 4 represents a general scheme of the viscoelastic dampers, characterized by four layers of 150x150mm viscoelastic ISD111 material bonded in steel plates, manufactured by Sumitomo 3M. The thickness of the VE layers is 5mm. Table 1 shows the experimental values of the storage modulus and of the loss factor for a frequency range between 0.1Hz and 0.3Hz. These values were used to construct the curves showed in Figure 5 that were used in the modeling by GHM method.

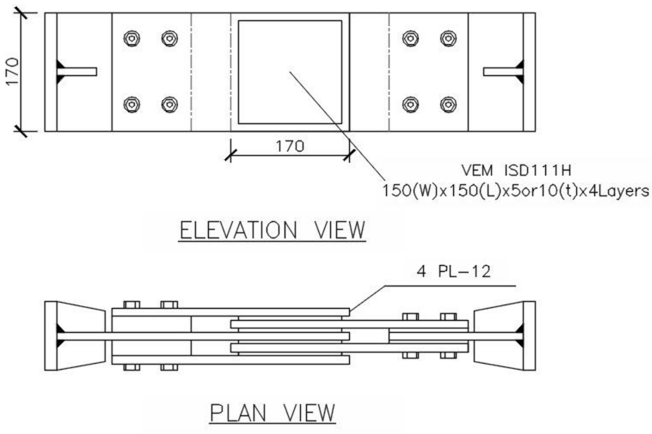


**Figure 4. Viscoelastic dampers details. Dimensions in millimeters (adapted from Montgomery and Christopoulos (2015))**

Table 1. Experimental values of storage modulus and loss factor. Adapted from Montgomery (2015)

| f (Hz) | E' (MPa) | η |
|---|---|---|
| 0.10 | 0.110 | 0.79 |
| 0.15 | 0.137 | 0.88 |
| 0.20 | 0.134 | 0.91 |
| 0.25 | 0.170 | 0.99 |
| 0.30 | 0.162 | 0.96 |

For the experimental evaluation of the viscoelastic dampers, it was applied a displacement controlled axial loading along the longitudinal axis of the dampers. The axial harmonic loading was applied within a frequency range 0.1Hz to 0.3Hz, in order to achieve different ranges of shear strain. Montgomery and Christopoulos (2015) also showed hysteresis cycles for those tests which considered a shear strain of 50%, equivalent to a relative displacement of 2.5mm between the surfaces of a 5mm thick viscoelastic layer.

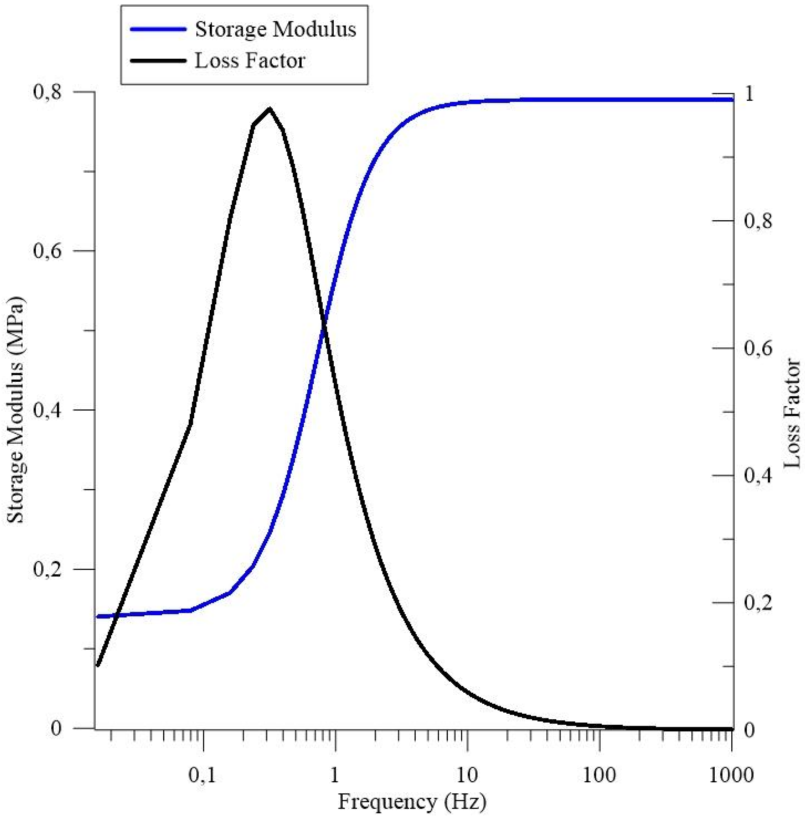


**Figure 5. Storage modulus and loss factor used for the VE material**

Figure 6 to Figure 8 show the hysteresis cycles for three frequency values of the harmonic loading applied in the experimental tests. Those figures also show comparisons between experimental results and the theoretical results obtained with the developed finite element program.

What can be readily observed in Figure 6 to Figure 8 is the good correlation the experimental and numerical results. In general, these hysteretic cycles show for each load frequency a fair closeness of theoretical and experimental results in terms of the amplitudes of displacements and force magnitude. Moreover, the cycling responses are according to the VEM behavior described in the Figure 2, and there are two aspects that demonstrate the influence of the frequency value and load magnitude.

The first aspect to note is when the frequency is higher the loss factor increases yielding a great capacity for dissipation of energy, that can be measured by the area of the hysteresis cycle. In Figure 8, related to f=0.3Hz, this area is greater than those related to f=0.2Hz and f=0.1Hz. The hysteresis cycle of Figure 6 has the smaller area among the three hysteresis cycles. Consequent, in this lower frequency the lowest capacity of energy dissipation.

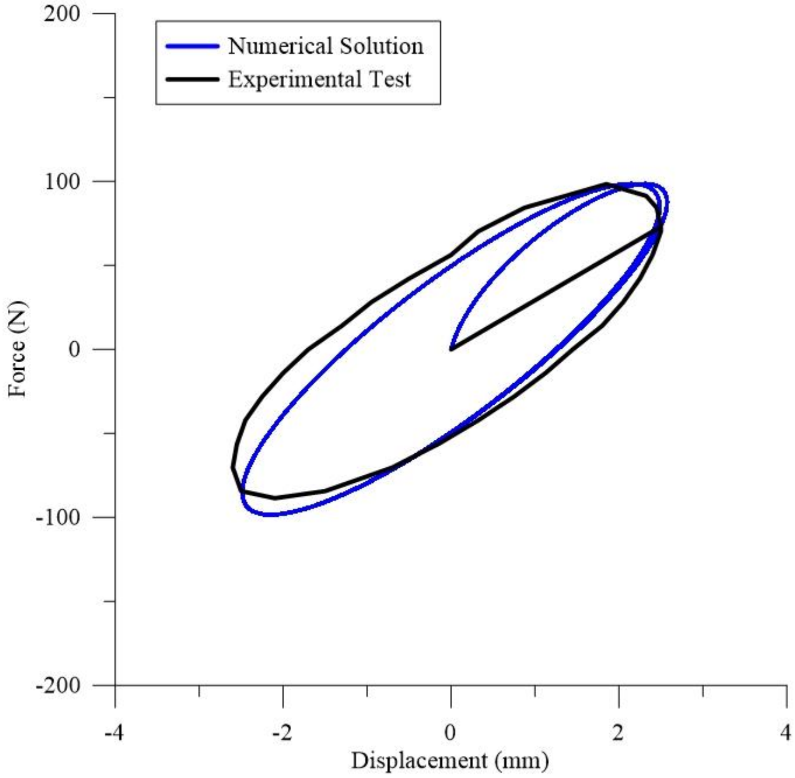


**Figure 6. Hysteresis cycle for the VCD. f=0,1Hz**

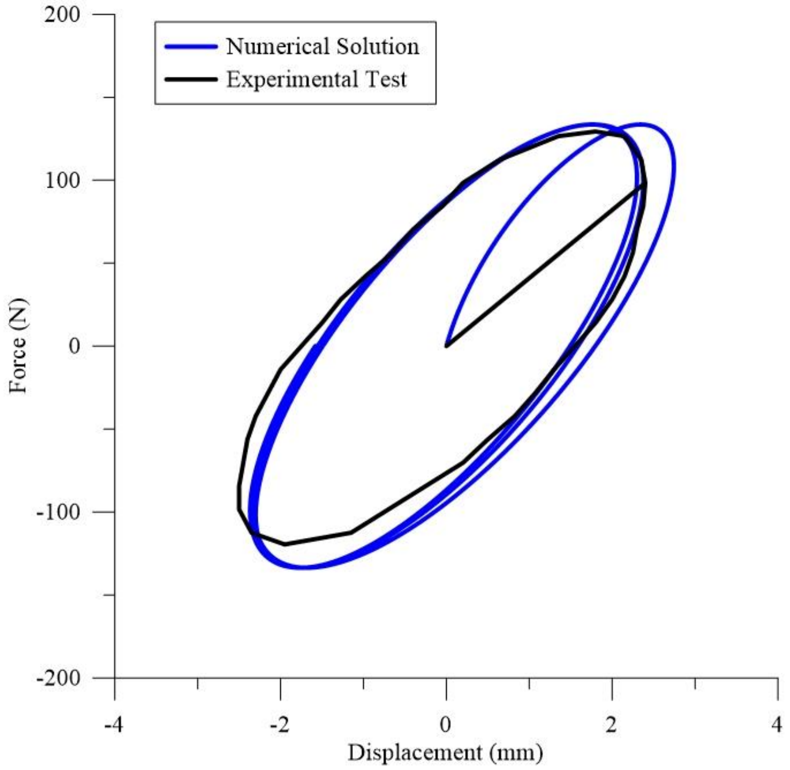


**Figure 7. Hysteresis cycle for the VCD. f=0,2Hz**

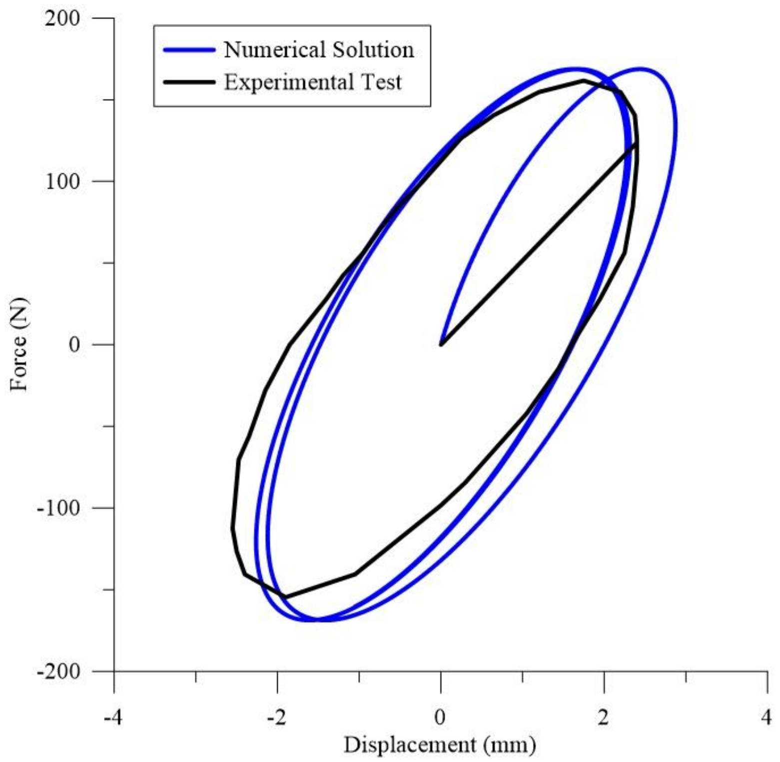


**Figure 8. Hysteresis cycle for the VCD. f=0,3Hz**

The second aspect is associated to the force magnitude to produce the same level of displacement. It can be showed that an increasing of frequency demands a higher force to produce the same controlled relative displacement, and this is related to the increase of the loss factor when the frequencies are higher. If the loss factor increases, the dissipation tends to be greater, demanding a major level of force to produce the same relative displacement.

It is worthwhile observing that the inclination of the major axis of the elliptical hysteresis cycle is related to the elastic modulus of the VEM at a given temperature and load frequency.

## 5.2 Dynamic Response of a sandwich slab

The dynamic response of a sandwich slab has been evaluated. The geometry dimension of the rectangular slab is 12x5m, with the four edges simply supported. Figure 9 shows a unit length of the slab cross section composed by constrained layers of concrete (100mm) and steel (10mm) and the viscoelastic core layer of a certain thickness. Mechanical properties of these materials are showed in Table 2.

For this numerical evaluation it has not been considered the weight of the materials neither the damping of elastic materials in order to appreciate in an easier way the damping capacity of the VEM.

The dynamic loading is defined by a triangular impulse (Figure 10), with a magnitude of 10 kN applied at the point “F” within the first 0.05s. “O” is the observation of the time-history displacement, as indicated in Figure 11.

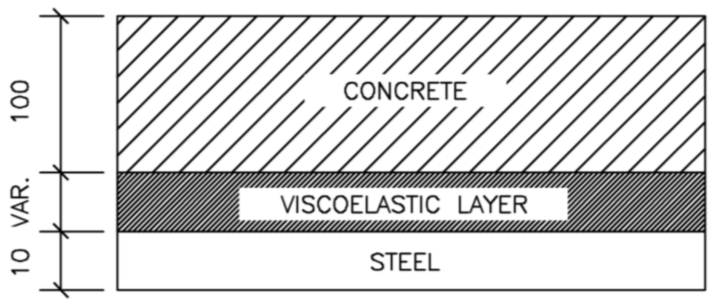


**Figure 9. Cross section of the sandwich slab. Dimensions in millimeters**

Table 2. Mechanical Properties of the materials

| Mechanical Properties | Steel | Concrete | Viscoelastic Material |
|---|---|---|---|
| Modulus of Elasticity (GPa) | 200 | 25 | * |
| Specific Mass (kg/m$^3$) | 7827 | 2400 | 1100 |
| Poisson's Coefficient | 0.3 | 0.2 | 0.25 |

(*) Property that shows dependence of frequency, according with GHM

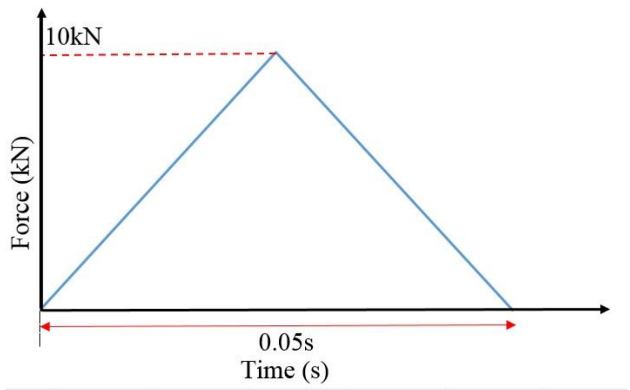


**Figure 10. Triangular impulse loading**

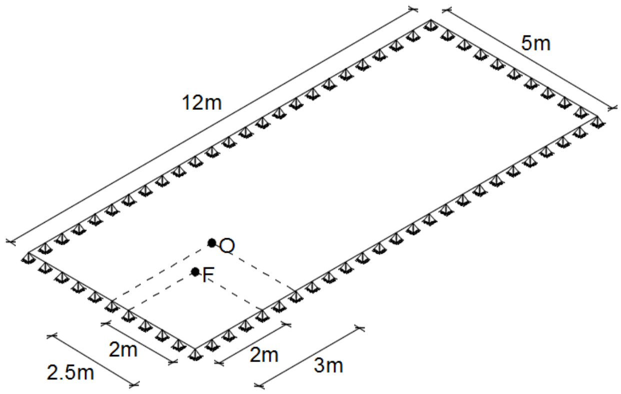


**Figure 11. Geometry of the sandwich slab**

Figure 12 shows the frequency-domain response of the composite concrete-steel slab, i.e. before the inclusion of the viscoelastic layer. According to this representation, the range of frequencies that is mainly excited lies between 5 and 15 Hz. Based on this, it must be chosen a viscoelastic material that offers an acceptable dissipation capacity within this range of frequencies.

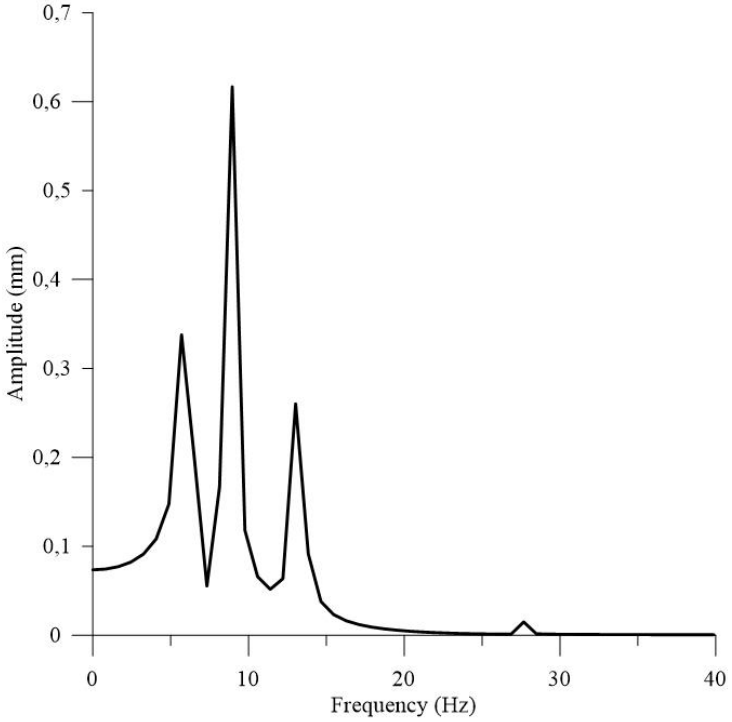


**Figure 12. Frequency domain response**

The material whose characteristics are given in Figure 3 was chosen for the viscoelastic layer. It is noticeable that this VEM has peak values of the loss factor close to 10 Hz. Three numerical tests have been carried out for three different thickness of the viscoelastic layer: 1mm, 5mm and 10mm. Figure 13 shows the time history responses in terms of the displacement at point “O” for three numerical tests.

It can be observed in Figure 13 at the beginning of the graph evidences that the dynamic responses of the concrete-steel composite slab and the other three composite slabs which have the viscoelastic layer are all them very similar at the beginning during the transient load time interval. This behavior is typical of structures provided with passive control devices when subjected to impulsive loads.

Thereon energy start to be dissipated being more effective in those slabs that have thicker viscoelastic layers between 5 and 10mm thickness thickness. However, it should be observed that does not exist a great difference in damping efficiency between the composite slabs with VEM layers of 5mm and 10mm of thickness; a VEM layer of 5mm results in an acceptable damping level.

The impulsive loading applied at point “F” yields at point “O” a dynamic response for which contribute various vibration modes. The multimode dynamic response represented by the black line does not impose a limit for the damping effectiveness of the VEM layer, as it is adaptable to random dynamic loadings being therefore suitable in applications that involve environmental actions.

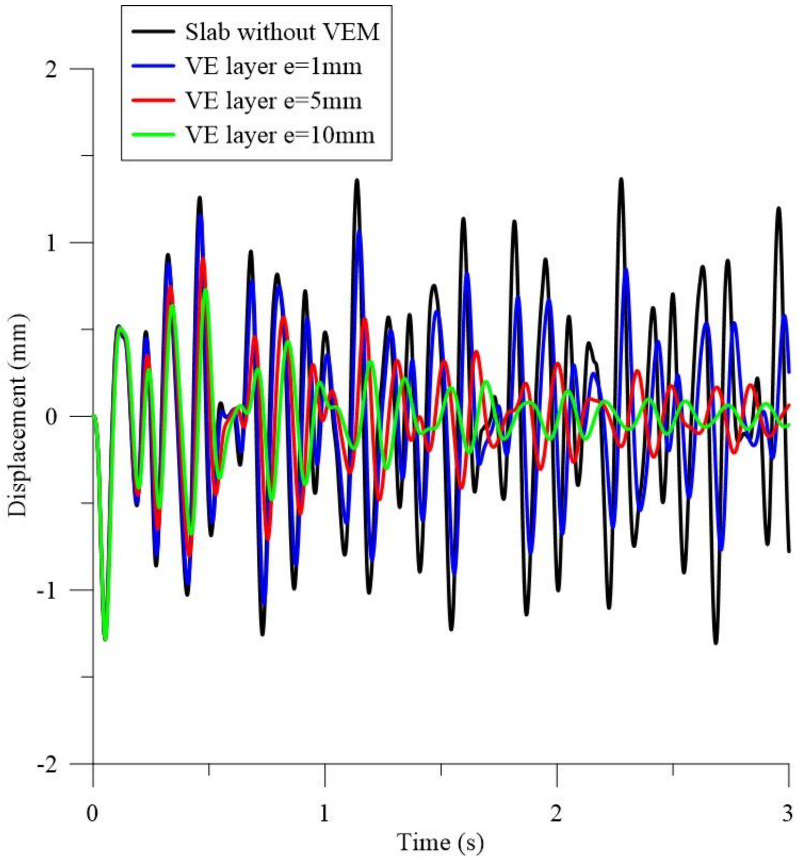


**Figure 13. Time History response for the sandwich slabs**

# 6 CONCLUSIONS

Based on the numerical results from the case examples presented in this paper, one can draw some conclusions related to the modeling of VEM and the behavior and effectiveness of VEM dampers mechanisms applied to civil engineering structures. Firstly, it must be pointed out the strong sensitivity that the damping capacity of a VEM layer has from the parameters that characterize the dynamic mechanical properties of this material. For this reason, when it is desired to validate the numerical results through correlation with the results obtained from experimental tests, it is needed a fine experimental characterization of the storage modulus and loss factor of the VEM and their functional dependence on the frequency range and temperature.

With regard to the multilayered viscoelastic dampers effectiveness, it can be concluded that these mechanisms, if properly design and fabricated, may have the necessary damping capacity to reduce the displacements, velocities and acceleration of a structure subjected to distinct dynamic loading. It was demonstrated in one case example of sandwich slab that increasing thickness of the VE layer helped to reduce the displacement magnitude, but this is not a rule that may be establish to be applied to every case of study. The selection of the layer material and its thickness is strongly dependent on the dynamic properties of the original non-controlled structure and the characteristics of the dynamic load.

## REFERENCES

Barbosa, F. S., 2000. *"Modelagem computacional de estruturas com camadas viscoelásticas amortecedoras"*. DSc. Thesis. Universidade Federal do Rio de Janeiro, Brasil.

Barbosa, F.S., Battista, R.C.. 2003. "A computational modeling of sandwich viscoelastic dampers", CILAMCE 2003 – XXIV Iberian Latin-American Congresso n Computaionla Methods in Engeineering, Ouro Preto, Brazil.

Biot, M.A.,1955. "Variational Principles in Irreversible Thermodynamics With Applications to Viscoelasticity". *Physical Review*, vol. 97, n. 6, pp. 1463-1469.

Ferry, J. D., Viscoelastic Properties of Polymers. 2 ed. New York, John Wiley & Sons, Inc.,1970.

Golla, D.F. and Hughes, P.C., 1985."Dynamics of viscoelastic structures: A time-domain finite element formulation". *Journal of Applied Mechanics*, vol.52, pp. 897-906.

Jones, D.I.G 2001. *Handbook of viscoelastic vibration damping*, John Wiley & Sons, USA.

Montgomery, M. and Christopoulos C., 2015. "Experimental Validation of Viscoelastic Coupling Dampers for Enhanced Dynamic Performance of High-Rise Buildings". *Journal of Structural Engineering*, vol.141, Issue 5.

Nashif, A.D, Jones, D.I.G, Henderson, J.P. 1985. *Vibration damping*, John Wiley & Sons, USA.

Newmark, N. M., 1959. "A method of Computation for Structural Dynamics". *Journal of the Engineering Mechanics Division*, ASCE, vol. 85, n.3, pp. 67-94, 1959.

Nolle, A. W., 1950. "Dynamic Mechanical Properties of Rubberlike Materials", Journal of Polymer Science, Vol. V, nº. 1, pp. 1-54.

Vasconcelos, R. P., 2003. "Controle Dinâmico Estrutural via Mecanismos Viscoelásticos". DSc. Thesis, Universidade Federal do Rio de Janeiro, Brazil.

Vasconcelos, R. P., and Battista, R.C., 2004. "Análise Numérica de um Modelo Físico do Tabuleiro Ortotrópico de uma Ponte com Material Viscoelástico". *XXXI Jornadas Sulamericanas de Engenharia Estrutural*. Mendoza, Argentina.