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STRUCTURAL STABILITY OF LACED BUILT-UP COLD-FORMED STEEL COLUMNS

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Abstract. *Two main effects distinguish the behavior of laced built-up columns composed of cold-formed steel members from other types of columns: (i) the effect of shear deformations and (ii) the interaction between local and global buckling modes. The main design standards and specifications do not provide sufficient guidance for the design of this type of cold-formed structural system. From the literature, we verify that very few tests have been conducted on this type of built-up columns. The objective of this paper is to present the results of full-scale experimental tests carried out in spatial laced built-up columns, designed with stiffened channel cold-formed steel members. The five tested spatial laced columns have respectively 6, 12 and 16m long and 0.8 and 1.25mm plate thickness. Elastic buckling and nonlinear numerical analyses were performed, as well as four analytical methods applied to obtain global buckling including shear effect. The results indicate quite similar critical elastic buckling loads, and good agreement with those from the FEM buckling analyses. Experimental results of the critical loads, obtained by Southwell plot method, performed significantly lower than numerical and analytical results. The experimental collapse loads of the laced built-up columns showed good agreement with FEM nonlinear analysis.*

Keywords: *Laced built-up columns, Cold-formed steel structures, Buckling, Elastic critical load, Column strength*

1. INTRODUCTION

Cold-formed steel sections, CFS, are often used as structural members, providing economical solutions, due to its light weight and easy transportation and installation. On the other hand, thin-walled open section members are very much sensitive to local and distortional buckling modes. The great majority of researches dedicated to CFS have been dedicated to the study of buckling behavior of isolated members (local, global and distortional modes, L, D and G, respectively). In particular, the main design standards and specifications do not provide sufficient guidance for the design of cold-formed laced built-up columns.

Spatial laced columns, composed of steel CFS, are investigated in the present work, in which the four chords are connected with diagonal and bracing elements, conducting to trussed-type structural behavior. From the literature, we verify that very few tests have been conducted on this type of laced columns, especially in the case of spatial structures. Hashemi and Jafari (2009, 2012), Bonab et al (2013), Kalochairetis et al (2014), Hashemi and Bonab (2013), have investigated the elastic critical load and strength of laced (or batten) columns through laboratory tests, but in all cases there were only two hot rolled chord members, connected by lacing bars. Zhang and Young (2015) performed a numerical investigation and design of CFS built-up columns, although it was a numerical and experimental study with CFS members, it was not addressed to laced column.

The behavior of a laced built-up column depends on its bending and shear flexibility, as well as on the connections stiffness. The effect of shear deformations may cause distortion of laced panels. Engesser (1891) was the first to consider the shear deformation effect on the elastic critical load of columns. Bleich (1952), Timoshenko and Gere (1963), and several other researchers followed the original findings of Engesser. However, all the direct methods derived from Engesser original approach, do not consider the effect of different boundary conditions on built-up columns. Gjelsvik (1991) studied the buckling of laced built-up columns, considering shear stiffness for the most usual boundary conditions, and presented the analytical results in graphics. Gjelsvik modified Engesser's theory to consider the effect of end stay plates. A generalized form of the governing equations provided by Gjelsvik was presented by Paul (1995), applicable to any column composed by two or more chords. The columns buckling loads were obtained for different boundary conditions. Aslani and Goel (1991) presented a generalized version of the analytical equation proposed by Bleich, applicable to all built-up columns with general end conditions. Ziemian (2010) suggests the determination of the critical load of laced column, for three boundary conditions, through a graph based on μ parameter, which takes into account the additional distortion due to the axial force or bending in web elements. The shear flexibility was characterized by μ for the first time by Lin et al (1970). The elastic critical load for a laced column can also be determined using a method proposed by Razdolsky (2005), in which the buckling of laced columns is taken from statically indeterminate structure, ignoring higher order determinants. This method is applicable to any type of laced arrangement, but it is necessary to develop a computational solution. In addition, Razdolsky obtained buckling results of built-up columns with three types of laced arrangement: "V", "X" and "N" (2010, 2011, 2014a), and also for batten columns (2014b).

The critical load of a laced column is theoretically obtained considering the existing shear deformations, depending on the type of laced arrangement. There are analytical methods to obtain critical and collapse loads of laced built-up columns, but it is necessary to evaluate the precision of these methods when applied to thin-walled CFS. Therefore, experimental and numerical analyses are needed, especially in the present case of spatial built-up laced columns composed of CFS members.

Besides the effect of shear deformations, another issue that distinguishes the behavior of these columns from other structural systems is the interaction between local and global buckling modes.

The objective of this paper is to present the results of full-scale experimental tests carried out in spatial laced built-up columns, designed with stiffened channel cold-formed steel members. Five spatial laced columns were tested, with 6, 12 and 16m in length and 0.8 and 1.25mm plate thickness, and 400x400mm cross section. The laced column longitudinal members (chords), composed of two stiffened channel members connected with self-drilling screws, were previously analyzed by the finite strip method, in order to identify its buckling loads and modes. For the built-up column as a whole, elastic buckling and nonlinear FEM analyses were performed. Four analytical methods were applied to obtain the global buckling load including shear effect, for the pin ended condition: Engesser, Timoshenko, Bleich and Eurocode 3 (CEN, 2006b). In addition, experimental results of the critical loads were obtained by Southwell plot method and compared with numerical and analytical results.

2. EXPERIMENTAL INVESTIGATION

2.1 Test Samples

A total number of six spatial laced built-up columns, designed with stiffened channel CFS, have been tested at the Structural Laboratory (LABEST) of COPPE, at the Federal University of Rio de Janeiro. The laced column chords were composed of two stiffened channel members connected with self-drilling screws (Fig.2b). All tested columns have the same cross section (400x400mm, Fig.2a). Diagonals and bracings were composed of a single stiffened channel member (Fig.2c), connected with one self-drilling screws to the chords (Fig.1).

In order to evaluate the effect of global buckling, three lengths of columns were tested: 6200, 12200 and 16200mm. Two steel plate thicknesses were taken for each column length: 0.8 and 1.25mm (Table 1).The laced columns were tested in horizontal position, as illustrated in Fig.3. All diagonals and bracings of the built-up laced columns were connected to the chords with self-drilling screws, as shown in Fig.1. It can be observed that the chord's edge stiffeners have been cut off in the region of the arrival of diagonals and bracings, to enable the connection. The suppress of edge stiffeners to allow the direct connection is very common in laced structures with CFS. This type of connection, although practical, results in a discontinuity of the chord's cross-section, which may cause stress concentrations in this region. In addition, the loss of cross-section edge stiffeners, even in a limited region, causes a reduction of the cross section area, increasing the local buckling effect.

Figure 1 – Bracing and diagonal connected to the chord with self-drilling screws.

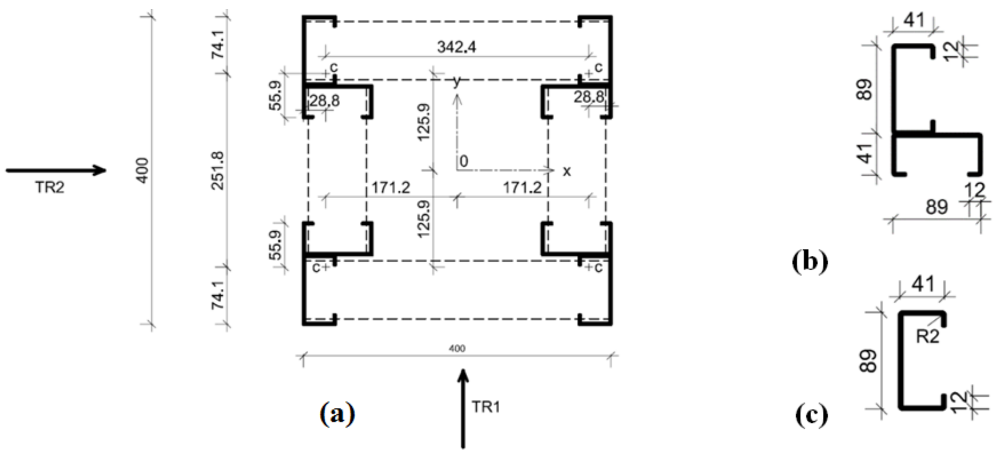

Figure 2 – (a) Laced built-up column cross section showing the four chords composed of two stiffened channel members Us89x41x12mm. The centroid of each chord is marked with "+C". (b) Chords: 2Us89x21x12mm, 0.8 or 1.25mm. (c) Diagonals and bracings: Us89x41x12mm, thickness 0.8 or 1.25mm.

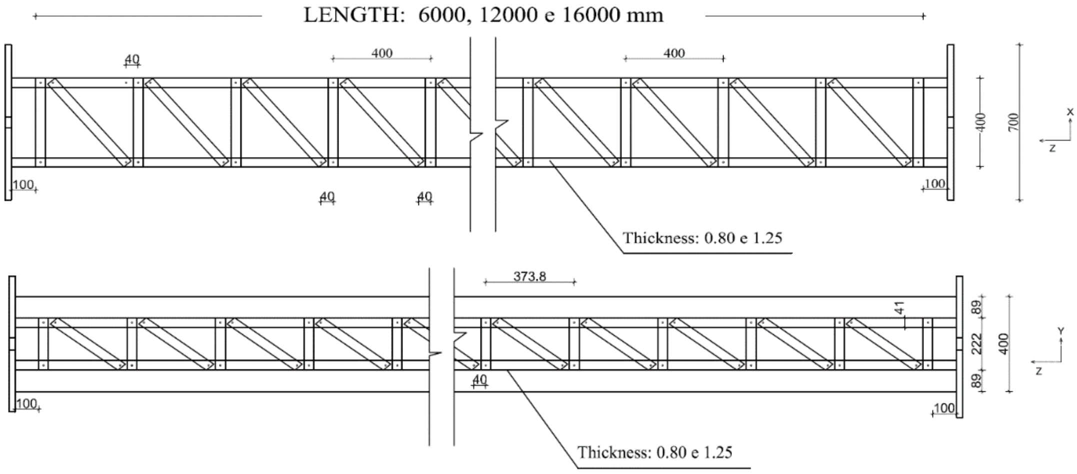

Figure 3 - Two faces of laced built-up column at the horizontal position for the test.

Table 1 – Columns IDs

Column ID	Built-up Laced Column
T6x0.8	Column 6200mm long, CFS members with 0.8mm
T12x0.8	Column 12200mm long, CFS members with 0.8mm
T12x1.25	Column 12200mm long, CFS members with 1.25mm
T16x0.8	Column 16200mm long, CFS members with 0.8mm
T16x1.25	Column 16200mm long, CFS members with 1.25mm

2.2 Testing procedure and measuring devices

The built-up laced columns were tested in the horizontal position, and the compression load was applied by a servo controlled hydraulic actuator and a steel rod. The actuator was placed at one end of the column, as shown in Fig.4a. The steel rod passes through the column length and is attached to a reaction stiff plate placed at the opposite end of the hydraulic actuator. The rate control of the applied displacement was equal to 0.02mm/s.

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P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

It was very difficult to create the end conditions for a double-hinged column condition. The end condition of the axial compression test was actually a semi-rigid type, through devices which allowed a certain degree of rotation, but were not perfect hinges. This condition was materialized by a rigid plate with a nut at the end of the actuator (Fig.4a), and a rigid plate with a pin end at the other extremity (Fig.4b). Neoprene plates with 10 mm thickness were placed at both ends to create smooth contact surface and obtain uniform load distribution between the four chords.

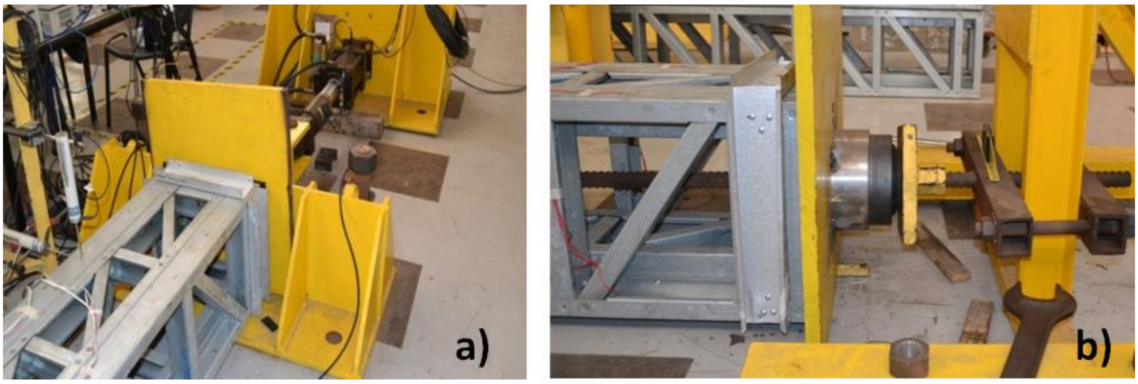

Figure 4 (a) Hydraulic actuator and end condition of the column; (b) compression loading transference between rod and stiff plate at the opposite end of the laced column.

Displacement Transducers (DT) were used for measuring deflections at mid-length, at quarters of the column length and near the actuator end. Their measurements were used for obtaining load-horizontal and vertical displacement behavior and identifying the global response of the columns. The DTs (FH and FV means horizontal and vertical displacements, respectively) were positioned according to figures 5 and 6: FH2 and FV1 near the actuator's end; FH4 and FV3 at the mid-length; FH9 and FH10 at the quarter lengths.

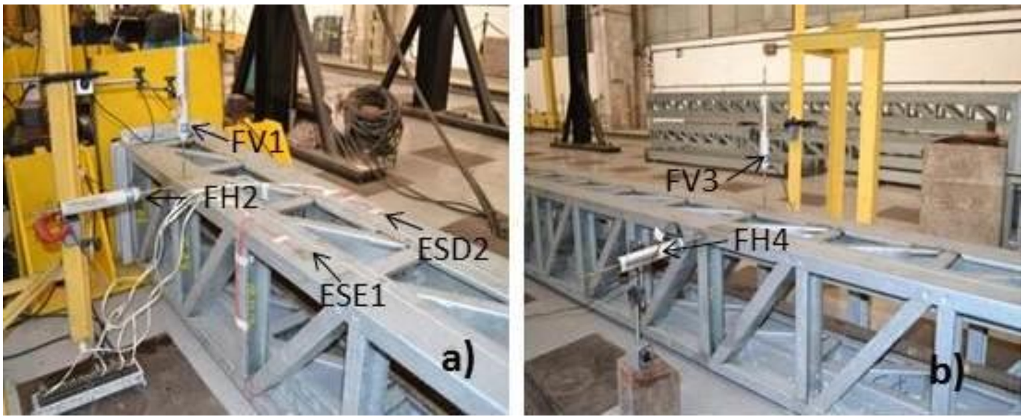

Figure 5 (a) DTs FV1 and FH2 positioned near the actuator's end. Strain gauges ESE1 and ESD2 placed on web chords. (b) DTs FV3 and FH4 positioned at the column's mid-length.

Strain gauges were placed externally in the web element of the chords. The strain gauge distribution included four sensors at each column end and four at the mid-length. For the column 16m long, four additional strain gauges were installed at the mid-length, in the internal side of the webs, thus performing four couples of strain gauges addressed to record the onset of local buckling deformation.

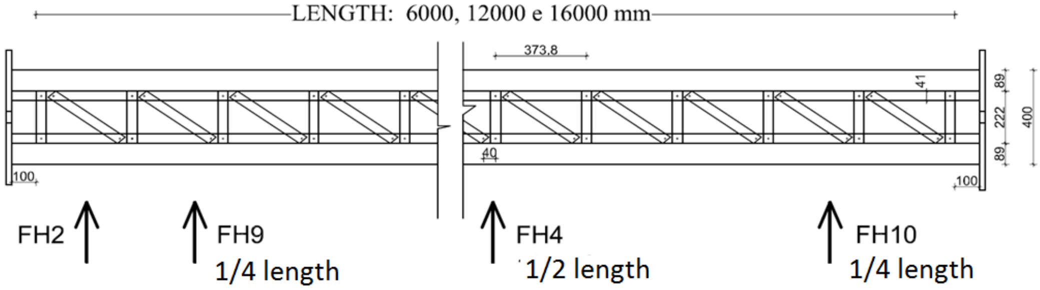

Figure 6- Locations where DTs were placed for horizontal displacement records.

In addition, two DTs were installed at each end of the laced column, with the help of wooden rods, allowing measurement of the flexural bending rotations, as shown in Fig.7.

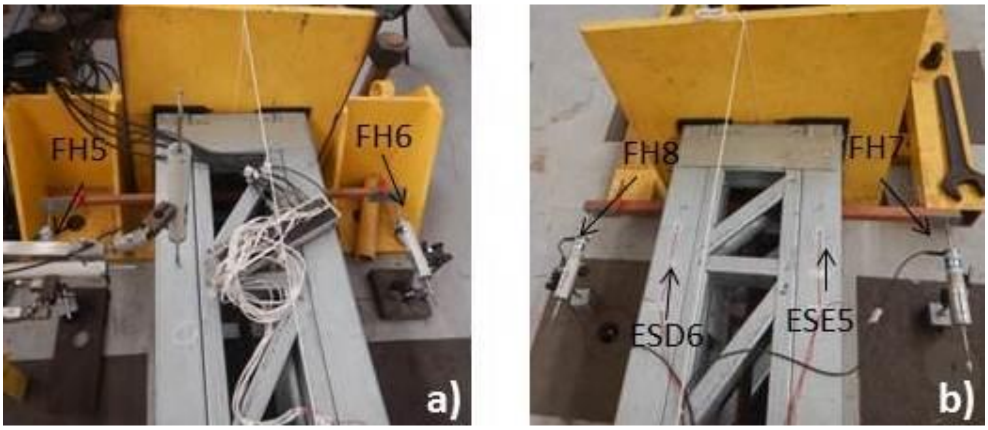

Figure 7 (a) DTs FH5 and FH6 positioned at actuator end (pointed to the wooden rod, to measure the rotation of the head). (b) DTs FH7 and FH8 positioned at hinged end. ESE5 and ESD6 strain gauges placed at chord´s web.

2.3 Material Properties

The laced columns were manufactured with steel (ZAR345) with nominal yield stress of 345MPa. The material properties of this steel were extracted through standard tensile tests of 14 coupons extracted from the chord members. The geometry of the coupons and the testing procedure were based on the guidance provided by ISO 6892 standard (ABNT, 2013). The average result of yield stress f_y, ultimate stress f_u, Young modulus E and residual strain after failure ε_r are given in Table 2.

Table 2 – Material properties

Thickness	Yield Stress	Ultimate Stress	Young Modulus	ε_r
t (mm)	f_y (MPa)	f_u (MPa)	E (GPa)	%
0.8	370	477	198	20
1.25	375	479	215	18

2.4 Tests Results

During the tests of the laced columns, it was possible to observe the development of typical local buckling deformation mode along the webs of the chords before collapse. Although this phenomenon could be observed in almost all the columns tested, as expected, it

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was more visible on the more slender 0.8mm thick built-up laced columns. Figure 8a shows the local buckling mode developing along the chords.

The shortest laced columns, 6m long, presented minor lateral displacements, according with the records of the DT's. In this case, global buckling was not visible. The collapse occurred due to a squash mechanism on the end section, close to the connection between the bracings and the chord, where the chord cross section reduction occurs, due to the suppression of edge stiffeners. Laced columns 12 and 16m long presented collapse mechanism at the column mid length, as shown in Fig.8b. The effect of global buckling could be observed in these columns, as shown in Fig.9. In this case, it was possible to observe the interaction between local and global buckling modes.

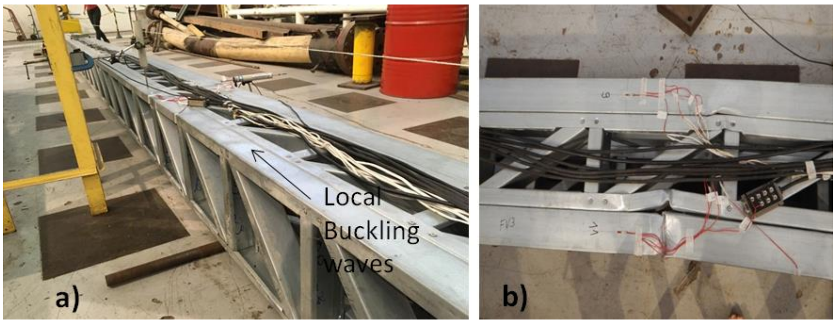

Figure 8 – (a) Local buckling deformation mode for 16m long and 0.8mm plate thickness column. (b) Collapse mechanism close to the column's mid length.

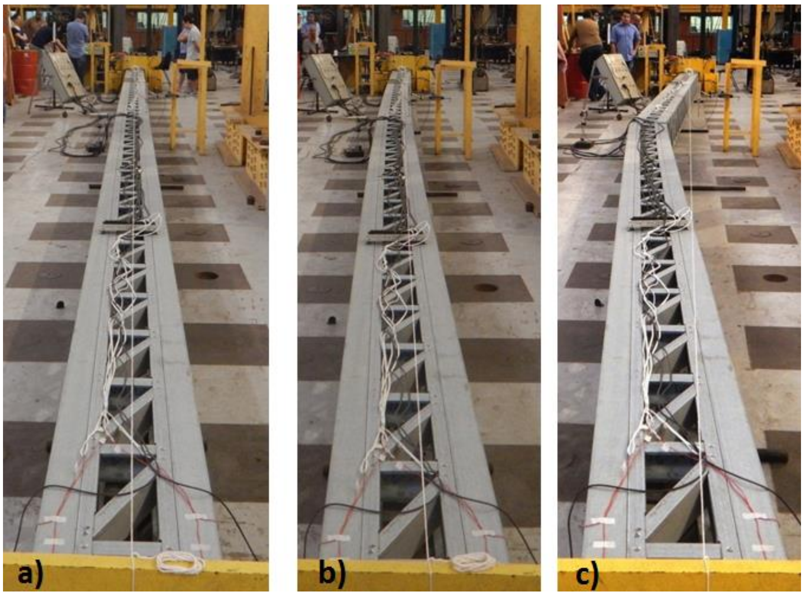

Figure 9 - Global buckling of laced built-up column 16m long. (a) Starting the test (low compression load). (b) Close to the collapse load. (c) After collapse.

The strain gauges registered a linear behavior until the onset of local buckling. Figure 10 shows the strain gauges results for the built-up laced column 12m long and 1.25mm thickness, where linear behavior can be observed until the applied load of 100kN. Local buckling effect was clearly recorded by strain gauge ESD10. The collapse load was 233kN.

Figure 11 shows the bending effect of column for 5 levels of applied load. The deformed shapes were obtained with the records of the displacement transducers FH4 (mid-length), FH9

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and FH10 (quarter length). The displacements were insignificants until applied load of 150kN. The effect of global buckling can be observed for applied loads of 200kN, 225kN and 233kN.

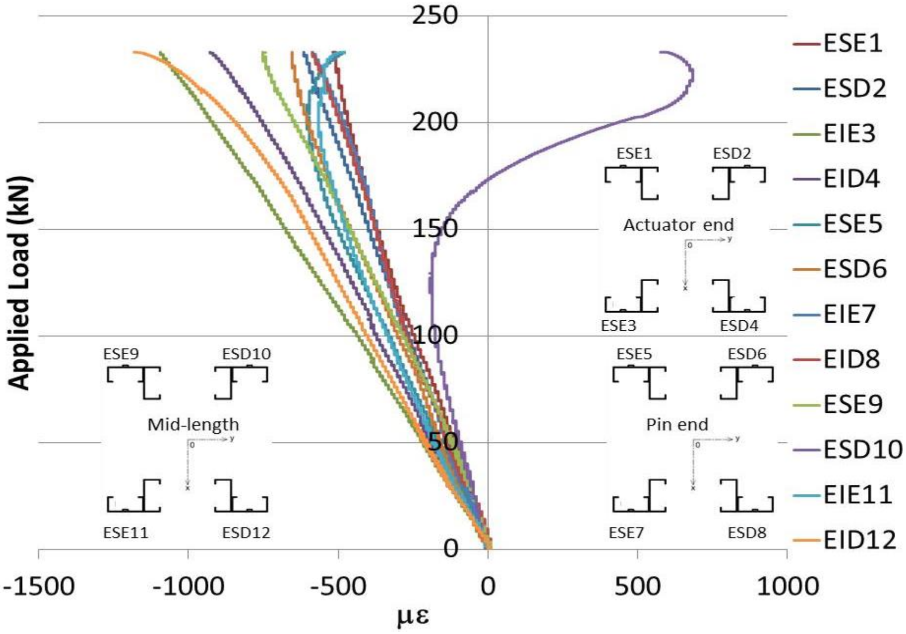

Figure 10 - Applied Force (kN) versus Strain (με) graph, for each strain gauge. Built-up laced column 12m # 1.25mm

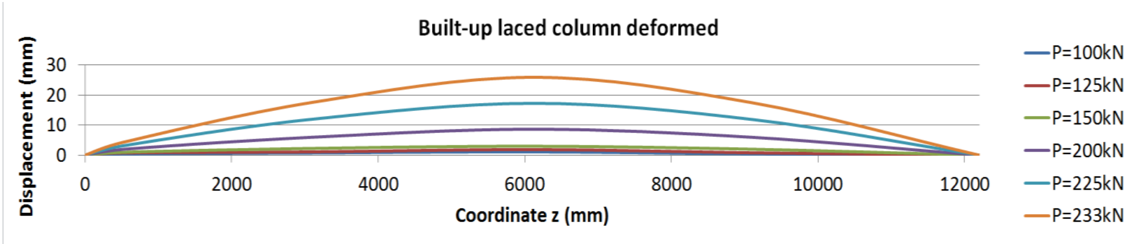

Figure 11 - Built-up laced column 12m # 1.25mm deformed, for 5 load levels

Table 3 – Built-up laced columns – summary results

Built-up Laced Column	Test date	Collapse load (kN)	FH4 displacement (mm)*	Shortening (mm)	Load (kN)**
6m #0.8mm	12/05/2016	110kN	<0.1	48.6	-
12m #0.8mm	22/06/2017	89kN	45	51.1	35
12m #1.25mm	01/06/2017	233kN	31	76.0	100
16m #0.8mm	11/04/2017	85kN	59	50.3	40
16m #1.25mm	17/03/2017	196kN	71	76.9	150

*Mid length horizontal displacement (mm)

** Applied load related to the local buckling onset. Until this load, displacement transducers registered linear behavior.

Experimental measurements from displacement transducers FH5 and FH6 (close to the hydraulic actuator) and FH7 and FH8 (opposite end of the column), indicate the rotation at the ends of the column are smaller than the expected rotation of a perfect pined-pined column. For example, the column with 12m # 0.8mm, presented 52% of expected rotation at actuator end, and 64% of expected rotation at opposite end. These results confirmed that the boundary condition of the axial compression test was actually a semi-rigid type. The end section devices allowed rotations but were not perfects hinges. Table 3 shows a summary of the results for 5 built-up laced columns tested.

3. EXPERIMENTAL ELASTIC CRITICAL LOAD

The Southwell plot method (Southwell, 1932) is a graphical method that calculates the column's critical elastic load using experimental data. Although the technique may be used for any structural component affected by buckling deformation, the conventional Southwell plot method was developed for solid columns, so it does not take into account the effect of shear deformation. In this article, the modified Southwell Plot for built-up columns proposed by Bonab et al (2013) was adopted to obtain the elastic critical load using the tests results. Figure 12a illustrates the method, where deformations (Δ_1) are plotted against the values of:

$$\left(\frac{k_s - P}{k_s P}\right)\Delta_1 \tag{1}$$

In this modified Southwell plot, P is the applied load and the slope is P_{e1} (first mode buckling load). The experimental critical load for the first mode is given by the expression below:

$$P_{cr} = \frac{P_{e1}}{1+\frac{P_{e1}}{k_s}} \tag{2}$$

where: k_s is the column's shear stiffness, Δ_1 is the displacement at the mid-length of the column and Δ_{01} is the initial imperfection.

When k_s approaches infinity, the result is the conventional Southwell´s equation:

$$\Delta_1 = P_{cr1}\frac{\Delta_1}{P} - \Delta_{01} \tag{3}$$

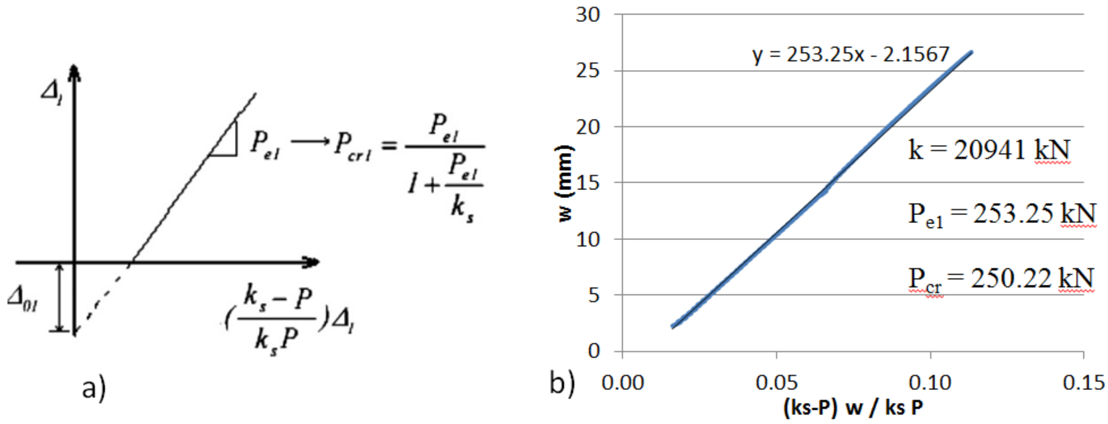

Figure 12 – (a) Modified Southwell Plot for built-up columns (Bonab et al, 2013). (b) Modified Southwell Plot for built-up laced column 12m long #1.25mm

The shear stiffness (k_s or S_v as Eurocode notation) for built-up columns was calculated using Eurocode 3 method (CEN, 2006b) as included in session 5.4 of the present article.

Experimental displacements recorded at the mid-length section of the columns (DT FH4) were used to obtain the experimental elastic critical load of columns 12 and 16m long, using the modified Southwell Plot. As FH4 registered very small values for column 6m long, it was not possible to obtain the experimental elastic critical load for this length (short column with negligible global buckling effect). As an example, the modified Southwell Plot for a column 12m long, 1.25mm thickness is shown in Fig.12b. The experimental critical load obtained through Modified Southwell Plot was similar to those obtained through Conventional Southwell Plot, as shown in Table 4.

Table 4 - Experimental critical load

Built-up Laced Column		Experimental Critical Load P_{cr} (kN)	
Length (mm)	Thickness (mm)	Southwell	Modified Southwell
12200	0.8	103.1	102.7
12200	1.25	250.0	250.2
16200	0.8	82.0	81.8
16200	1.25	204.1	204.6

4. STIFFENED CHANNEL MEMBERS (CRITICAL AND ULTIMATE LOAD)

The direct strength method (DSM), proposed by Schafer and Peköz (1998) has been adopted by ABNT NBR14762 (2010) as an alternative design method for cold-formed steel structural members (instead of the effective width method). However, the DSM does not cover the design of built-up members. The chords of the laced column of this study are built-up members, as they are composed of two stiffened channel members connected with self-drilling screws (Fig.2b). Zhang and Young (2012, 2015) studied the appropriateness of the DSM for built-up open section compression members, by a FEM parametric study, and compared with the results of compression tests of cold-formed steel I-shaped open section with edge and web stiffeners. Although the authors have proposed a modified direct strength method for this section, it was concluded that the current DSM, based on rational buckling analysis, can be used for the design of cold-formed steel built-up open section columns.

The present study adopted the DSM to obtain the strength of the laced column longitudinal members (chords), composed of two stiffened channel members as shown in Fig.2b.

Elastic Buckling stresses for the chord member

The analysis of the buckling modes and the corresponding elastic buckling stresses is the first step for the design calculation using the DSM. They can be obtained by using a rational elastic buckling analysis, i.e., the finite strip method proposed by Papangelis and Hancock (1995).

Software CUFSM (Schafer, Ádány, 2006), based on the finite strip method, was applied to obtain the buckling modes and corresponding elastic buckling stresses of the chord member.

The laced column longitudinal member (chord), composed of two stiffened channel members 2U89x41x12 (Fig.2b), was modeled using the software CUFSM to obtain the local (N_L) and distortional (N_{dist}) critical buckling forces. As the length of one laced panel (the chord length between two bracings) was 400mm, the study was until this length. The boundary conditions were considered hinged and free warping. The contact part between the two channels was modeled with double thickness (2t). Thus, the following members were modeled in CUFSM, as illustrated in Fig.13: (i) 2Us 89x41x12mm stiffened channel member with thicknesses 0.8 and 1.25mm; (ii) 2U 89x41x12mm channel member thicknesses 0.8 and 1.25mm.

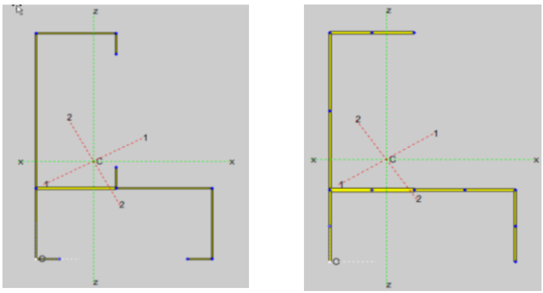

Figure 13 Cross sections modeled in CUFSM: 2Us stiffened channel 89x41x12mm and 2U unstiffened channel 89x41x12mm.

The buckling curve is the main result of a Finite Strip Method (FSM) analysis, and it is also called the "cross section signature curve". The minimum value of this curve is of special interest because indicates the critical loads associated with each buckling mode (local and distortional). The result of the analysis for one cross section is shown in Fig.14. In this figure the buckling curve is presented and the buckling modes identification. Below the minimum value in the curve, two values are presented: the first corresponds to the length in which that minimum value occurs, and the second one corresponds to the value of the critical buckling stress (local or distortional). In order to obtain the value of the local (N_L) or distortional (N_{dist}) critical load, was applied unitary uniform stress. Table 5 shows the results of the local (N_L) and distortional critical load (N_{dist}).

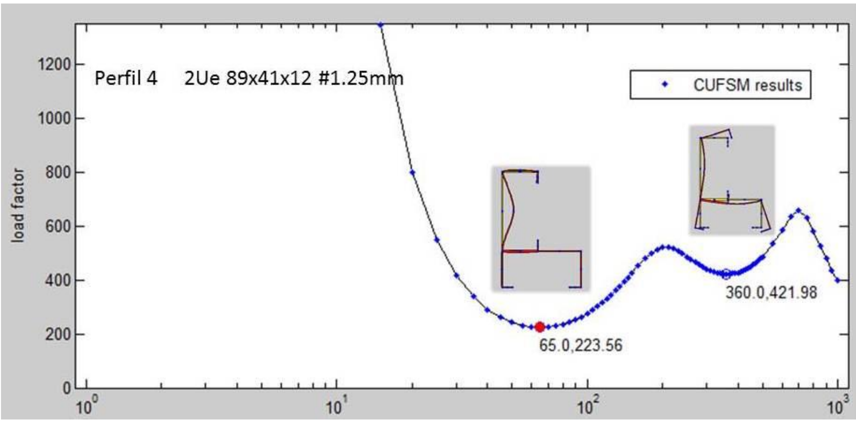

Figure 14 - CUFSM Results: buckling curve for cross section 2Us 89x41x12#1.25mm

Table 5 - Local and distortional critical stress (MPa) and load (kN)

		Critical Stress (Mpa)		Critical Load (kN)	
Description	**Area (mm^2)**	$\sigma_{critl\ (Local)}$	$\sigma_{critd\ (Distorcional)}$	$N_{L\ (Local)}$	$N_{dist\ (Distorcional)}$
2Ue 89x41x12 #0.8mm	309	91	252	28	78
2U 89x41 #0.8mm	271	58	-	16	-
2Ue 89x41x12 #1.25mm	480	224	422	107	203
2U 89x41 #1.25mm	420	143	-	60	-

Compression strength according with the Direct Strength Method

The design rules of the DSM according to the Brazilian standard, NBR14762, Annex C (ABNT,2010) for columns are described below.

The value of the strength ($N_{c,R}$) will be the lowest value calculated for global ($N_{c,Re}$), local ($N_{c,Rl}$) and distortional ($N_{c,Rdist}$) buckling.

The strength due to global buckling ($N_{c,\,Re}$) is:

$$N_{c,Re} = \left(0.658^{\lambda_0^2}\right) A f_y \qquad \text{for } \lambda_0 \leq 1.5 \tag{4}$$

$$N_{c,Re} = \left(\frac{0.877}{\lambda_0^2}\right) A f_y \qquad \text{for } \lambda_0 > 1.5 \quad \text{where} \quad \lambda_0 = \left(\frac{A f_y}{N_e}\right)^{0.5} \tag{5}$$

N_e is the elastic flexural buckling in accordance with NBR14762 item 9.7.2.

The strength due local buckling ($N_{c,\,Rl}$) is:

$$N_{c,Rl} = N_{c,Re} \qquad \text{for } \lambda_l \leq 0.776 \tag{6}$$

$$N_{c,Rl} = \left(1 - \frac{0.15}{\lambda_l^{0.8}}\right) \frac{N_{c,Re}}{\lambda_l^{0.8}} \qquad \text{for } \lambda_l > 0.776 \quad \text{where} \quad \lambda_l = \left(\frac{N_{c,Re}}{N_l}\right)^{0.5} \tag{7}$$

N_L is the elastic local critical load, obtained from CUFSM (Table 5).

The strength due distortional buckling ($N_{c,Rdist}$) is:

$$N_{c,Rdist} = A f_y \qquad \text{for } \lambda_{dist} \leq 0.561 \tag{8}$$

$$N_{c,Rdist} = \left(1 - \frac{0.25}{\lambda_{dist}^{1.2}}\right) \frac{A f_y}{\lambda_{dist}^{1.2}} \qquad \text{for } \lambda_{dist} > 0.561 \text{ where} \quad \lambda_{dist} = \left(\frac{A f_y}{N_{dist}}\right)^{0.5} \tag{9}$$

N_{dist} is the elastic distortional critical load, obtained from CUFSM (Table 5).

Table 6 shows the compression strength $N_{c,R}$ (without strength safety factor) for the members with 0.8 and 1.25mm thickness (single chord and four chords contribution).

Table 6 - Ultimate strength for chord´s member ($N_{c,R}$)

Description	$N_{c,R}$ (kN)	4 Chords (kN)
2Us 89x41x12 #0.8mm	55.1	220
2U 89x41 #0.8mm	40.1	160
2Us 89x41x12 #1.25mm	117.5	470
2U 89x41 #1.25mm	86.4	346

5. ANALYTICAL METHODS FOR GLOBAL BUCKLING LOAD OF BUILT-UP LACED COLUMNS

In this section, the analytical values of the elastic critical loads for the built-up laced columns are calculated according to four methods: Engesser (ENGESSER, 1891); Bleich (BLEICH, 1952); Timoshenko (TIMOSHENKO, GERE, 1963); Eurocode (CEN, 2006b).

The built-up laced columns were considered as pined-pined end condition and the calculations have been done considering the distance between rotations centers measured at the experimental tests, as follows: 6448, 12448 and 16448mm for built-up laced column 6, 12 and 16m long, respectively. For all theoretical calculations, the Young Modulus (E) has been adopted as 200GPa.

5.1 Engesser Method

Engesser (1891) was the first engineer who studied the influence of shear deformation in built-up laced columns. Based on the Engesser method, the critical load value for the built-up laced column, considering the shear effect, is calculated using the equivalent slenderness, as follows:

$$P_{cr} = \frac{\pi^2 EA}{(\alpha\lambda)^2} \tag{10}$$

where A is the cross-sectional area and λ is the slenderness of the column.

The value of α is given by the formula:

$$\alpha = \sqrt{1 + \frac{\pi^2 EA}{k_s \lambda^2}} \tag{11}$$

where k_s is the shear stiffness of built-up laced column (which is calculated at item 5.4 as S_v).

5.2 Bleich Method

Bleich method uses the same Engesser's formula for the critical load of a built-up laced column (P_{cr}), but with a simplified value for α. In view of the usual small effect of shear in laced columns, Bleich (1952) has suggested that a conservative estimate of the influence of 60º or 45º lacing, can be made by modifying the effective-length factor, so α is given by:

$$\begin{cases} \lambda \leq 40 \rightarrow \alpha = 1.1 \\ \lambda > 40 \rightarrow \alpha = \sqrt{1 + \dfrac{300}{\lambda^2}} \end{cases} \tag{12}$$

5.3 Timoshenko Method

According to the Timoshenko method (Timoshenko and Gere, 1963), the critical load for laced column, with "N" lacing, hinged ends and considering shear effect, is given by:

$$P_{cr} = \frac{\pi^2 EI}{l^2} \frac{1}{1 + \frac{\pi^2 EI}{l^2}\left(\frac{1}{A_d\, E\, sen\phi\, cos^2\phi} + \frac{b}{aA_b E}\right)} \tag{13}$$

where:

ϕ is the angle between the diagonals and the horizontal bracings;

A_d is the cross-sectional area of 2 diagonals (one at each face of laced column);

A_b is the cross-sectional area of 2 horizontal bracings (one at each face of laced column);

b is the length of horizontal bracings ;

I is the moment of inertia of the cross section of the structure (considering the 4 chords).

When this equation is used, the actual built-up column is replaced by an equivalent column of a modified length given by:

$$L_{eq} = l\sqrt{1 + \frac{\pi^2 EI}{l^2}\left(\frac{1}{A_d E\, sen\phi\, cos^2\phi} + \frac{b}{A_b Ea}\right)} \tag{14}$$

The column slenderness is given by:

$$\lambda = \frac{L_{eq}}{r} \tag{15}$$

where r is the radius of gyration of the cross section of the laced column (including four chords).

5.4 Eurocode Method

Although Eurocode part 1-3 (CEN, 2006a) is dedicated to cold-formed steel structures, it does not present any guidance to design built-up laced columns. Therefore, Eurocode 3 part 1-1 (CEN, 2006b) will be adopted, as it presents a design methodology for this type of structural system. According to Eurocode 3 part 1-1 item 6.4, the critical load for laced columns is:

$$N_{cr} = \frac{\pi^2 E I_{eff}}{L^2} \tag{16}$$

I_{eff} is the effective second order moment of area of laced built-up members and is given by:

$$I_{eff} = 0.5\, A_{ch}\, h_0^2 \tag{17}$$

Where h_0 is the distance between the centroids of the chords and A_{ch} is the cross-sectional area of one chord.

The effective critical load of built-up column considering shear effect is given by:

$$N_{cr,v} = \frac{1}{\frac{1}{N_{cr}} + \frac{1}{S_v}} \tag{18}$$

S_v is the shear stiffness of built-up columns. In item 6.4.2.1, figure 6.9 of Eurocode 3 (CEN, 2006b), formulas for shear stiffness calculation for different types of lacing arrangements are presented. In the case of laced built-up column with “N” lacing arrangement, the shear stiffness is given by:

$$S_v = \frac{n E A_d a {h_0}^2}{d^3\left[1 + \frac{A_d h_0^3}{A_v d^3}\right]} \tag{19}$$

where “n” is the number of lacing plans; A_d is the diagonal cross-sectional area and A_v is the horizontal bracings cross-sectional area, of a single plan; “a” is the length of one panel; “d” is the diagonal length.

Table 7 presents the computed results of shear stiffness and global buckling critical loads for modes 1 (Fig.1 axis X-X) and 2 (Fig.1 axis Y-Y).

Table 7 – Eurocode shear stiffness and global buckling critical loads

Laced Colum		Shear Stiffness		Critical Buckling Loads	
Length (mm)	Thickness (mm)	S_{v2} (kN)	S_{v1} (kN)	N_{cr2} (kN)	N_{cr1} (kN)
6448	1.25	23937	20941	2406	1353
6448	0.8	15465	13529	1548	871
12448	1.25	23937	20941	697	381
12448	0.8	15465	13529	448	245
16448	1.25	23937	20941	404	220
16448	0.8	15465	13529	260	142

5.5 Analytical Elastic Critical Buckling Loads Summary

The analytical results presented in Tables 8 and 9 indicate quite similar critical elastic buckling loads. Eurocode method presented critical loads slightly smaller.

Table 8 - Global buckling critical loads - axis X-X (mode 1)

Laced Column		Global Buckling Critical Loads Axis X-X (kN)			
Length (mm)	Thickness (mm)	EUROCODE	TIMOSHENKO	BLEICH	ENGESSER
6448	1.25	1353	1473	1414	1493
6448	0.8	871	974	910	961
12448	1.25	381	418	413	420
12448	0.8	245	271	266	270
16448	1.25	220	242	240	242
16448	0.8	142	156	155	156

Table 9 - Global buckling critical loads - axis Y-Y (mode 2)

Laced Column		Global Buckling Critical Loads Axis Y-Y (kN)			
Length (mm)	Thickness (mm)	EUROCODE	TIMOSHENKO	BLEICH	ENGESSER
6448	1.25	2406	2356	2286	2443
6448	0.8	1548	1600	1471	1573
12448	1.25	697	709	701	717
12448	0.8	448	464	451	461
16448	1.25	404	414	411	417
16448	0.8	260	269	265	268

6. FEM ELASTIC BUCKLING ANALYSIS

In order to evaluate the global buckling of laced columns and obtain the respective modes and critical loads, numerical models were developed with the help of the finite element method (FEM), for comparison with the analytical and experimental results. For this, SAP2000 computational program was achieved. Previous studies from Kalochareits and Gantes (2011) concluded that the use of more sophisticated shell elements is not expected to give significantly different results for global modes of laced built-up columns, from the ones obtained with beam element numerical analyses and analytical procedures. Thus, the model was developed adopting beam elements for the laced column members.

The boundary conditions in the FEM models were provided by rigid plates at the ends and rigid bars connecting the center of the rigid plates to the center of rotation, in order to have the same buckling lengths adopted in experimental tests and analytical calculations (6448, 12448 and 16448mm). So, in all models, the columns were hinged at both ends.

The laced columns were modeled by the centroid of the chord section as shown in Fig.1. For all FEM models, the Young Modulus (E) has been adopted as 200GPa.

Figure 15 illustrate the first four buckling modes for built-up laced column 12m long. The columns 6m and 16m long presented similar buckling modes. It is observed that in general the first two buckling modes were global buckling around X and Y axes respectively. Table 10 presents a comparison, for the first buckling mode, of the laced columns critical loads: analytical, experimental and numerical values. As all analytical methods resulted in similar values, this table presents the critical loads obtained by the Bleich method.

Proceedings of the XXXVIII Iberian Latin-American Congress on Computational Methods in Engineering
P.O. Faria, R.H. Lopez, L.F.F. Miguel, W.J.S. Gomes, M. Noronha (Editores), ABMEC, Florianópolis, SC, Brazil, November 5-8, 2017.

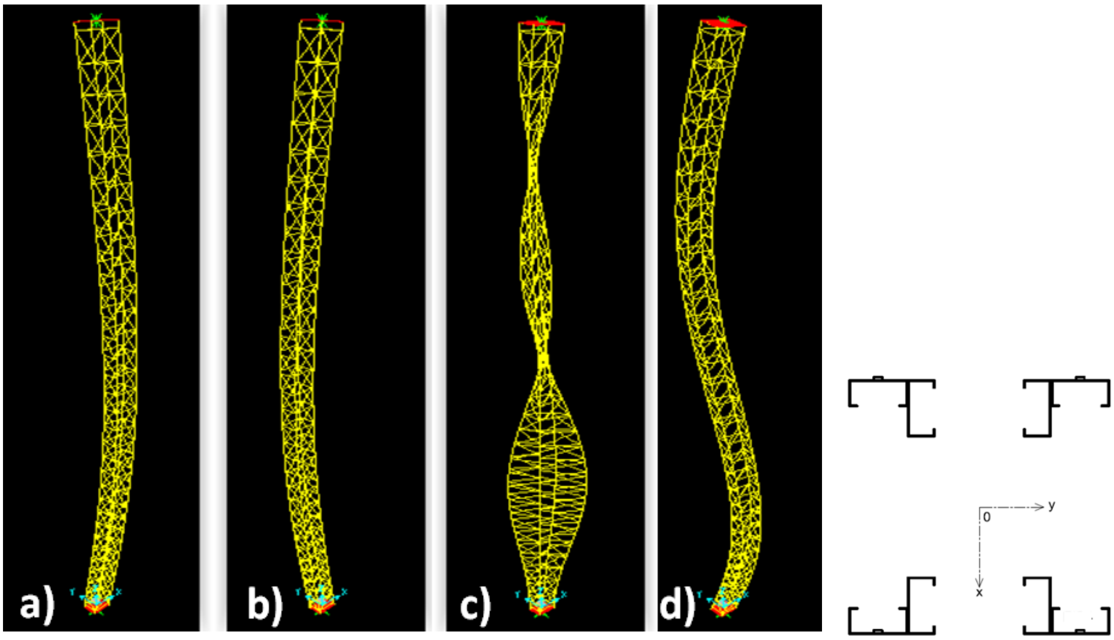

Figure 15 – SAP2000 elastic buckling modes for built-up laced column 12200mm long, 0.8mm thickness: (a) Mode 1: global flexural axis X-X (b) Mode 2: global flexural axis Y-Y (c) Mode 3: torsional (d) Mode 4: flexural- torsional axis X-X, with 2 waves.

Comparison of theoretical critical loads with the tests results

Table 10 - Analytical, experimental and numerical laced columns critical loads (mode 1, kN)

Laced Column		Analytical	Experimental	Numerical
Length (mm)	Thickness (mm)	BLEICH	Southwell Modif.	SAP2000
6448	1.25	1414	-	1505
6448	0.8	910	-	969
12448	1.25	413	250	413
12448	0.8	266	103	266
16448	1.25	240	205	236
16448	0.8	155	82	152

It is observed that the analytical and numerical critical loads are very close, practically coincident for laced columns 12m and 16m long. However, experimental results of critical loads, obtained by Southwell plot method and modified Southwell plot method, performed significantly lower than the numerical and analytical results.

The analytical and numerical analyses are simplified, as both consider the chord's member to be uniform along the longitudinal direction. Obviously, the built-up channel members connected by screws cannot meet this requirement. In addition, the chord cross section is not continuous, as there is the suppression of edge stiffeners in order to allow connection between chords, diagonals and bracings. The analytical methods and FEM buckling analysis with beam elements were used to study the global behavior of the laced column. This type of model does not take into account the suppress of edge stiffeners at chord´s member, what may affect the critical load. There are no explicit and direct design rules for cold-formed steel built-up laced columns. So it is necessary to develop FEM models with shell elements to simulate the connections and the variation of the channel cross section along the chord members, in order to evaluate the changes in critical loads.

7. FEM NON-LINEAR ANALYSIS

The first laced column deformed shape mode, resulted from the elastic buckling analysis, was used as the initial geometry for the FEM elastic geometrical nonlinear analysis. As recommended in Eurocode 3 part 1-1 (CEN, 2006b), the initial imperfection should be adopted as L/500. So, the maximum displacement at the column's mid length was adopted as L/500 for initial geometry.

According to Eurocode 3 part 1-1 (CEN, 2006b), the collapse of built-up laced columns occurs when one of the members reaches its strength. So, the maximum compression force at the chord members, resulted from the non-linear analysis, was compared with the strength of the chords ($N_{c,R}$ included in Table 6). Table 11 contains the collapse loads for built-up laced columns, considering the chord cross sections with and without stiffeners. Considering the collapse mechanism occurred in the section where there was suppression of edge stiffeners during experimental tests, the strength of the cross section with unstiffened channel members was compared with the experimental collapse loads (bold values in Table 11).

The last column in Table 11 shows the strength taking four chords with unstiffened channel members, from Table 6. It can be observed that the experimental collapse load is much less than compression strength of four chords. The difference increases with the column length, because of the influence of global buckling. So, the experimental results indicate that shear deformations and the interaction between global and local buckling modes have a significant effect on reducing the strength of the tested laced columns.

Table 11 - Experimental and numerical collapse loads for built-up laced columns

Laced Column		**Experimental**	**SAP2000 Non-linear analysis (kN)**		P_u (kN)
Length (mm)	Thickness (mm)	Southwell Modif.	edge stiffener	no edge stiffener	4 chords (*)
6448	0.8	**110**	198	**143**	160
12448	1.25	**233**	289	**240**	346
12448	0.8	**89**	154	**120**	160
16448	1.25	**196**	192	**176**	346
16448	0.8	**85**	115	**97**	160

*4 chords strength without edge stiffeners (see Table 6)

Although the model was developed with beam elements, the experimental collapse loads of the laced built-up columns showed good agreement with FEM nonlinear analysis for laced column (considering the strength of the chord member without edge stiffeners). As the chord member strength was used, resulted from the finite strip analysis and the DSM (Table 6), the local buckling has been considered.

Figure 16a shows the mid-length displacements, resulted from SAP2000 FEM nonlinear analysis, for each built-up laced column. From this graph it can be observed the global buckling influence and the nonlinear behavior of columns 12m and 16m long, until the columns reach the collapse load (P_u). As the column 6m long did not suffer global buckling, the response is linear. Figure 16b shows the comparison between numerical and experimental results, for displacement at mid-length for built-up laced column 12m long and 1.25mm thickness (displacement transducer FH4). It can be observed that the initial imperfection of

L/500 adopted at the numerical nonlinear analysis probably is exaggerated, as in the experimental tests the displacements in the beginning are much lower. As the boundary conditions were not perfect hinges, it could be observed that in the beginning of the tests, the columns had insignificant displacements. The displacements started to increase after the applied load reaches a certain level (110kN for the column with 12m#1.25mm, Fig.16b).

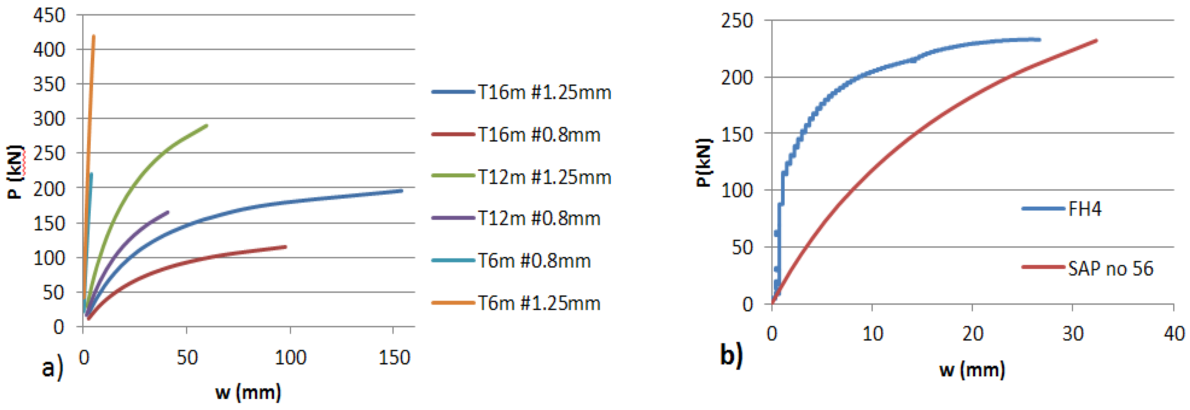

Figure 16 – (a) Mid-length displacements from SAP2000 FEM nonlinear analysis for each built-up laced column. (b) Comparison between numerical and experimental displacements at mid-length for built-up laced column 12m long and 1.25mm thickness (DT FH4).

8. CONCLUSIONS

Experimental results are presented, for full-scale experimental tests carried out in spatial laced built-up columns, designed with stiffened channel cold-formed steel members. The 5 tested spatial laced columns had respectively 6, 12 and 16m long and 0.8 and 1.25mm plate thickness, all with 400x400mm cross section. The laced column longitudinal members (chords), composed of two stiffened channel members connected with self-drilling screws, were previously analyzed by the finite strip method, in order to identify its buckling loads and modes. For the built-up column as a whole, elastic buckling and nonlinear analyses were performed. Four analytical methods were applied to obtain global buckling including shear effect: Engesser, Timoshenko, Bleich and Eurocode 3.

The analytical results indicated quite similar critical elastic buckling loads, and good agreement with those from the FEM buckling analyses. Experimental results of the critical loads, obtained by Southwell plot method, performed significantly lower than numerical and analytical results. Shell FEM model should be developed in order to simulate the connections and variation of the channel cross section along the chord members.

The collapse mechanism for the 6m columns was typical squash-load. For the columns 12m and 16m long, global buckling occurred in both cases. Although the numerical model was developed with beam elements, the experimental collapse loads of the laced built-up columns showed good agreement with FEM nonlinear analysis for laced columns (considering the strength of the chord's section without edge stiffeners).The experimental results also indicated that shear deformations and the interaction between global and local buckling modes have a significant effect on reducing the strength of the tested laced columns.

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